

¶ 3^e livr. Πέρας et
 "Ἀρχή" dans la philosophie pythagoricienne [W. A. Heidel]. Discute les
 opinions de Preller plus connues que celles des autres historiens de la phi-
 losophie sur la base ou la racine (root) de la doctrine pythagoricienne. ¶

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XII.

Πέρας and Ἀρχή in the Pythagorean Philosophy.

Von

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The question as to the root of Pythagoreanism has occupied scholars for a century, but does not yet seem to have reached a definite conclusion. The divergencies of opinion are not such, however, as to make an understanding appear hopeless. Among those who have addressed themselves to this problem we may especially mention Schleiermacher, Ritter, Brandis, Heyder, and Zeller; and if in the following discussion account is taken chiefly of the views of the last mentioned, it is both because they are more generally known, and because they deserve more consideration.

We may begin by granting two propositions rightly insisted upon by Zeller: first, Pythagoreanism owed its origin to an ethico-religious interest; second, the Pythagorean philosophy was, in its first intention, addressed primarily to the explanation of physical phenomena, and only secondarily concerned itself with problems of theoretical ethics. We have in these propositions a statement of what appears to me to be the actual truth, but not in sufficient detail to afford a satisfactory account of the facts.

It is not a new observation in the history of philosophy that ethical ideas determine physical and metaphysical thought. There is much truth contained in this view, but to be rendered available

for purposes of intelligent discussion, it requires to be more carefully defined. Without detriment to this contention, it may be granted that the ethical ideas which a philosopher accepts and announces may or may not be in accord with his metaphysics: for, like any other two sets of opinion entertained by one and the same man, these also may be flagrantly inconsistent. But it is not of such opinions that we speak when we say that ethical ideas determine metaphysical thought. There are other ideas that are much more deeply rooted. While inferences and derivative notions are often diametrically opposed one to another, there is a certain core of life and interest too central to be thus rent by contradictions; for round it revolves the entire circle of immediate feeling and untutored emotion. How famous the Πυθαγόρειος νόμος τοῦ βίου was even in Plato's day is evident from Rep. 600A foll.

It is within this inner circle of feeling and emotion that we must institute our quest for the ethical ideas which effectively influence speculation. And we shall look for them not so much in the views expressly stated as in the latent presuppositions and preconceptions. "All things in the universe arrange themselves to each person anew, according to his ruling love. Man is such as his affection and thought are. . . . As he is, so he sees." In these words of Emerson we have the suggestion that there are levels of being too deep to be influenced by considerations of self-interest or by superficial currents of thought. On these levels are built the foundations of the character that distinguishes an individual or an age. Here, oftentimes, is to be found the common ground of systems of thought which in their superstructure stand very far apart. And it is precisely this common ground that must constitute the scope of our especial study if we would learn to seize the trend of the history of thought in its entirety. We need not now pause to enquire why this should be: suffice it for the present to say that the direction of life defines itself in a given set of interests whose organizing influence upon thought may be roughly compared to the lines of force in a magnetic field along which the stray iron-filings leap into place.

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the root of Pythagoreanism. It is in fact none other than „the habitual Greek preference for limitation“ which was shared and first formulated by the Pythagoreans in abstract terms¹⁾. It found expression in the characteristic Πυθαγόρειος τρόπος τοῦ βίου, in the pronounced love of order and the declaration of war on lawlessness²⁾, and in the moral demand for ἀρμονία and a κόσμος³⁾. This Zeller also acknowledges, but he seems to strain his point⁴⁾; for to admit that ethical ideas determine the form and substance of physical and metaphysical speculations, is not necessarily to regard the body of doctrine as a system of ethics. It is quite true, as Zeller contends⁵⁾, that the Pythagoreans themselves thought of the order, which they demanded in the world, as natural rather than as moral; but the demand for that natural order is itself at bottom ethical. Nothing could be more enlightening on this point than the Stoic prayers for an overruling fate and the Epicurean requirement of a causal nexus in the world to dispel the

somnia quae uitae rationes uertere possint

fortunasque tuas omnis turbare timore.

For this is but one phase of our demand that the world be rational⁶⁾, which is the ultimate ethical postulate.

It is not at all surprising, therefore, that such ethical ideas should become embodied in the Pythagorean philosophy. Associated

¹⁾ Benn, The Philosophy of Greece, p. 85.

²⁾ Laert. Diog. III. 23, νόμος βοηθεῖν, ἀνομία πολεμεῖν.

³⁾ For the ethical notions associated with this term in the Pythagorean mind, see Plato, Gorg. 507 E: φασὶ δ' οἱ σοφοί, ὧς Καλλικλεις, καὶ οὐρανὸν καὶ γῆν καὶ θεοὺς καὶ ἀνθρώπους τὴν κοινωνίαν συνέχειν καὶ φιλίαν καὶ κοσμιότητα καὶ σωφροσύνην καὶ δικαιοσύνην, καὶ τὸ ὅλον τοῦτο διὰ ταῦτα κόσμον καλοῦσιν, ὧς ἑταῖρε, οὐκ ἀκοσίαν οὐδὲ ἀκολασίαν.

⁴⁾ Zeller, Phil. der Griechen I. a. p. 469. „So gerne ich daher zugestehe, dass die Pythagoreer vielleicht nicht auf diese Bestimmungen gekommen wären, wenn ihnen die ethische Richtung des pythagoreischen Bundes den Sinn für Mass und Harmonie nicht geschärft hätte, so kann ich doch ihre Wissenschaft selbst deshalb nicht für Ethik, sondern ihrem wesentlichen Inhalt nach nur für Physik halten.“

⁵⁾ Zeller, ibid., p. 468.

⁶⁾ Compare Münsterberg, on Psychological Atomism, Psychological Review, vol. VII. p. 6.

with them are other conceptions, like that of ἁρμονία, which are quite as much esthetic as ethical. A particular phase of this general condition has long engaged the attention of historians of philosophy. Aristotle twice refers to a single instance, that of the counter-earth. Once⁷⁾ he complains of the Pythagoreans as οὐ πρὸς τὰ φαινόμενα τοὺς λόγους καὶ τὰς αἰτίας ζητοῦντες, ἀλλὰ πρὸς τινὰς λόγους καὶ δόξας αὐτῶν τὰ φαινόμενα προσέλκοντες καὶ πειρώμενοι συγκοσμεῖν. Again he blames them for allowing esthetic prepossessions to bias their conclusions⁸⁾. Diels likewise remarks upon the effect of similar considerations among the Eleatics in shaping their doctrine of the sphericity of the world⁹⁾. The fact that a conclusion so reached maintained itself for so long a time sufficiently proves that the preconceptions on which it rested were basic in Greek life and civilization.

It may be well at this point to guard against a misconception¹⁰⁾. It is quite in the nature of things that the minds of the Pythagoreans, however predisposed to apply ethical standards in passing judgment on the world, should turn first to that which we regard as the objective, — the problems of cosmology. Only at a later time would they naturally return, as it were, from the outer world to matters of ethics and politics, making it evident that their speculations in these fields were wholly secondary and derivative.

⁷⁾ Arist. de Caelo II. 13. 293a 25.

⁸⁾ Arist. Met. I. 5. 986a 3: καὶ ὅσα εἶχον ὁμολογούμενα δεικνύναι ἐν τε τοῖς ἀριθμοῖς καὶ ταῖς ἁρμονίαις πρὸς τὰ τοῦ οὐρανοῦ πάθη καὶ μέρη καὶ πρὸς τὴν ὅλην διακόσμησιν, ταῦτα συνάγοντες ἐφήρμοττον. Κἀν εἰ τί που διέλειπε, προσεγγίχοντο τοῦ συνειρομένην πᾶσαν αὐτοῖς εἶναι τὴν πραγματείαν. λέγω δ' ὅλον, ἐπειδὴ τέλειον ἢ δεκάς εἶναι δοκεῖ καὶ πᾶσαν περιελιφέναι τὴν τῶν ἀριθμῶν φύσιν, καὶ τὰ φερόμενα κατὰ τὸν οὐρανὸν δέκα μὲν εἶναι φασιν, ὄντων δὲ ἐννέα μόνον τῶν φανερῶν διὰ τοῦτο δεκάτην τὴν ἀντίχθονα ποιοῦσιν.

⁹⁾ Diels, Parmenides, p. 56: „Zu diesen Resten der überwundenen Weltanschauung des Eleaten gehört nun auch die wunderliche Vorstellung, sich das gesamte Sein unter dem Bilde einer allseitig wohlgerundeten Kugel vorzustellen. Die Vollkommenheit seines Ἐόν sollte sich in dem vollkommensten Körper abspiegeln, ein Gedanke, der pythagoreischer Grübeleien entsprungen, wunderbar lange nachgewirkt hat. Die Kosmologie des Platon wie des Aristoteles ruht darauf.“

¹⁰⁾ This in view of the polemic of Zeller, *ibid.*, 468, foll.

Just so, in regard to musical harmony, they noted the purely physical phenomena, failing even to perceive the psychological problems raised by their discovery.

We may now proceed to consider the case of the Pythagorean *πέρας* and *ἄπειρον*. Zeller contends that the distinction of these contraries is primarily metaphysical¹¹⁾, and that the distinction itself is not a part of the original Pythagoreanism, but acquired its place at the head of the list of the ten contraries only with Philolaus¹²⁾. Again he asserts that the distinction of the odd and the even was earlier than that of the limit and the unlimited¹³⁾. All of these statements seem to me to be very closely interconnected, and they must stand or fall together.

It may be well first of all to remark how closely associated the notion of *πέρας* is with all the ethico-esthetic ideas whose importance for the Pythagorean mind we have already noted¹⁴⁾. Aristotle, in a very instructive passage¹⁵⁾, says: τῷ γὰρ τιμιωτάτῳ οἶονται προσήκειν τὴν τιμιωτάτην ὑπάρχειν χώραν, εἶναι δὲ πῦρ μὲν γῆς τιμιώτερον, τὸ δὲ πέρας τῶν μεταξὺ, τὸ δὲ ἔσχατον καὶ τὸ μέσον πέρας. In this connection we may compare also the use made by Simplicius of the adjectives *ἀστεῖος* and *φαῦλος*¹⁶⁾. Theophrastus¹⁷⁾

¹¹⁾ Zeller, *ibid.*, 388, 467 foll. So too Windelband, *Gesch. der alten Phil.*, p. 173. Of course there is no doubt of its application in metaphysics; the question here is as to its origin.

¹²⁾ Zeller, *ibid.*, p. 389: „Erst einer weiter entwickelten Reflexion kann der abstraktere Ausdruck des Begrenzten und Unbegrenzten angehören, wenn er gleich später, bei Philolaus und in der zehngliedrigen Kategorientafel, an die Spitze des Systems gestellt wird.“

¹³⁾ Zeller, *ibid.*, p. 388. He gives for proof his inability to account for the derivation of the odd and even from the limit and the unlimited. More of this anon.

¹⁴⁾ Cp. Windelband, *Gesch. der Phil.* (Tufts' transl. p. 46, n. 3): „Nor must we here overlook the factor which had already asserted itself with Xenophanes and Parmenides, viz. that to the Greek the conception of measure was one that had a high ethical worth; so that the infinite, which derides all measure, must to him appear imperfect, while the definite or limited (*πεπερασμένον*) was necessarily regarded as more valuable.“

¹⁵⁾ Arist. *de Caelo* 293a 30.

¹⁶⁾ Simplicius, *Phys.* 181, 7 D.

¹⁷⁾ Theophrastus, *Met.* p. 12 § 33 W. Compare also Plut. *adv. Colot.* 1114 B who speaks of *ἀπειρία* as *ἀτακτος*, *ἄλογος*, *ἀπερληπτος*.

speaks of the ἄπειρον in a way to disclose its emotional connotations: ἐν ᾗ καὶ τὸ ἄπειρον καὶ τὸ ἄτακτον καὶ πᾶσα ὡς εἰπεῖν ἀμορφία καθ' αὐτήν. But we need not content ourselves with matters so inconclusive. The fundamental conception of the list of contraries was evidently ethical, intended to explain the physical world. Thus Simplicius¹⁸⁾ says: τὸ οὖν δεξιὸν καὶ ἄνω καὶ ἔμπροσθεν ἀγαθὸν ἐκάλουν, τὸ δὲ ἀριστερὸν καὶ κάτω καὶ ὀπίσθεν κακὸν ἔλεγον, ὡς αὐτὸς Ἀριστοτέλης ἰστόρησεν ἐν τῇ τῶν Πυθαγορείois ἀρεσκόντων συναγωγῇ. Aristotle himself¹⁹⁾ declares the same even more unequivocally: πιθανώτερον δὲ εἰκάσιν οἱ Πυθαγόρειοι λέγειν περὶ αὐτοῦ, τιθέντες ἐν τῇ τῶν ἀγαθῶν συστοιχίᾳ τὸ ἐν. It was to have been expected that πέρασ, which to the Pythagorean mind denotes so essential a characteristic of the good, should be employed typically by Aristotle to represent the entire σύστοιχία, just as ἀγαθόν was used above²⁰⁾. Aristotle, in fact, readily perceived the affinity between the Pythagorean πεπερασμένον and his μέσον (for, as we have just seen, τὸ μέσον πέρασ), and acknowledged it²¹⁾: ἔτι τὸ μὲν ἀμαρτάνειν πολλαχῶς ἔστιν (τὸ γὰρ κακὸν τοῦ ἀπείρου, ὡς οἱ Πυθαγόρειοι εἶχαζον, τὸ δὲ ἀγαθὸν τοῦ πεπερασμένου), τὸ δὲ κατορθοῦν μοναχῶς. Had he desired to display his wit instead of giving his sober interpretation of their meaning, he would have adjusted his statement to their contrast ἐν × πλῆθος rather than to πέρασ × ἄπειρον.

From the account of Aristotle²²⁾ it appears to be unquestionable that there was some difference between the Pythagoreans in regard to the list of contraries, some setting up only the two pairs, odd and even, and limit and unlimited, while others constituted the table of ten pairs. I agree with Zeller²³⁾ in regarding the former

¹⁸⁾ de Caelo, 173a 11.

¹⁹⁾ Arist. Eth. Nich. A. 4. 1096b 5 foll. By Plutarch's day the ethical interpretation was fully established, cf. de Iside 48: οἱ μὲν Πυθαγορικοὶ διὰ πλειόνων ὀνομάτων κατηγοροῦσι τοῦ μὲν ἀγαθοῦ τὸ ἐν τὸ πεπερασμένον, κ. τ. λ.

²⁰⁾ Arist. Met. 987a 13 foll. Aristotle here clearly employs πεπερασμένον and ἄπειρον instead of their respective series.

²¹⁾ Arist. Eth. Nic. B. 5. 1106b 28 foll.

²²⁾ Arist. Met. A. 5. 986a 15 foll.

²³⁾ Zeller, *ibid.*, 355, notes 1 and 2.

as presenting the more original views of the Pythagoreans. It will be noted that in the briefer table the distinction of good and evil is not specifically mentioned; but probably nobody would maintain that it was not present to the thought. We may therefore assume that it is implicit in one or both of the pairs enumerated, and few, I fancy, would hesitate to affirm that, of the two, limit and the unlimited most adequately represent it. In the longer table, which Zeller, perhaps rightly, refers to Philolaus, $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ heads the list, while $\acute{\alpha}\gamma\alpha\theta\acute{\omicron}\nu \times \chi\alpha\chi\acute{\omicron}\nu$ stands near the end. It is as manifestly unfair to conclude from the latter fact that ethical ideas were introduced as an afterthought, as to infer from the former, without further evidence, that $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ contained the groundwork of the entire system. No conclusion whatever can be properly drawn from the order in which the contraries are enumerated.

Without resorting to means so untrustworthy, we may be able to lend a high degree of probability to the thesis that in a certain sense the contrast $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ is the basis of all the contraries. We have seen that the earlier Pythagoreans probably contented themselves with setting up two pairs of contraries, $\pi\epsilon\pi\epsilon\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\iota\tau\omicron\nu$ and $\pi\epsilon\pi\epsilon\rho\alpha\sigma\mu\acute{\epsilon}\nu\omicron\nu \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$. But what is the relation of these two pairs? The key to this question seems to be given by Aristotle²⁴) when he says: τοῦ δὲ ἀριθμοῦ στοιχεῖα τὸ τὲ ἄρτιον καὶ τὸ περὶττόν. For nothing is more evident to even the casual reader of the documents of Pythagoreanism, than the fact that there are here two streams of interest, the ethico-religious and the mathematico-scientific, which never entirely blend, though they tend ever more and more to merge into one another. These interests are represented in the original table, consisting of two pairs of contraries, respectively by $\pi\epsilon\pi\epsilon\rho\alpha\sigma\mu\acute{\epsilon}\nu\omicron\nu \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ and $\pi\epsilon\pi\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\iota\tau\omicron\nu$. Now since the ethico-religious interest is manifestly the root of Pythagoreanism we might readily be led to affirm that $\pi\epsilon\pi\epsilon\rho\alpha\sigma\mu\acute{\epsilon}\nu\omicron\nu \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ constitutes the basis of the system and that $\pi\epsilon\pi\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\iota\tau\omicron\nu$ must somehow be deduced directly from it. But I fear we

²⁴) Arist. Met. A. 5 986a 17.

should merely be yielding to the insidious temptation, always strong upon us, to simplify matters overmuch.

Philosophies do not grow in that abstract manner. Man lives and gathers experience and acquires convictions and preconceptions before he proceeds to synthesize his thought. Hence it is likely that neither of these pairs of contraries was derived from the other, but that each represented, as we have said, one of the two paramount and parallel interests of the brotherhood. For all that, though recognized as parallel, the two pairs were not regarded as equally basic, as may be seen from the questionable fragments of Philolaus, where $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ has quite crowded out its rival.

This parallelism finds expression in the identification²⁵⁾ of the pairs $\pi\epsilon\rho\iota\tau\tau\acute{o}\nu \times \acute{\alpha}\rho\tau\iota\omicron\nu$ and $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$. Just here we meet one of the most interesting problems connected with the Pythagorean doctrine of $\pi\acute{\epsilon}\rho\alpha\varsigma$ and $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$. Aristotle²⁶⁾, in speaking of the relation of the terms $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ and $\acute{\alpha}\rho\tau\iota\omicron\nu$, endeavors to illustrate it by the case of the gnomon. Unfortunately the illustration throws very little light on the problem, being itself scarcely intelligible²⁷⁾. But even if Aristotle's account is rightly interpreted and accepted, it must at once be evident, considering the complicated nature of the mathematical relations assumed, that the explanation is at best a late refinement rather than the original conception²⁸⁾. Now the commentaries here offer a different explanation which has been set aside by modern scholars as of a value

²⁵⁾ Arist. Phys. Γ 4. 203a 10: καὶ οἱ μὲν τὸ ἄπειρον εἶναι τὸ ἄρτιον. Elsewhere, as, e. g. Met. A. 5. 986a 18, Aristotle subsumes ἄρτιον under ἄπειρον, which is logically the more natural as well as the more in keeping with the historical order pointed out above.

²⁶⁾ Arist. Phys. Γ 4. 203a 10 foll.

²⁷⁾ Cf. Zeller, *ibid.*, p. 351, n. 2, and Prantl, *Arist. Physik*, p. 489; see also Burnet, *Early Greek Philosophy*, p. 310 foll.

²⁸⁾ Cf. Burnet, *ibid.*, p. 311, foll.: „The artificial character of all this shows that it does not belong to the groundwork of the system“. Compare Windelband, *Gesch. der alten Phil.*, p. 173, n. 1: „Die Begründung dieser Identifikation ist so künstlich, dass man deutlich sieht, sie ist ad hoc gemacht, kein natürliches Produkt der Zahlentheorie.“ It does not follow, however, as Burnet seems to think, that the identification has no significance for Pythagoreanism.

inferior to that of Aristotle's. Simplicius, Phys. 455, 20 D, who gives the fullest account, in commenting on the above mentioned passage, says: οὗτοι δὲ τὸ ἄπειρον τὸν ἄρτιον ἀριθμὸν ἔλεγον διὰ τὸ πᾶν μὲν ἄρτιον, ὥς φασιν οἱ ἐξηγηταί, εἰς ἴσα διαιρεῖσθαι, τὸ δὲ εἰς ἴσα διαιρούμενον ἄπειρον κατὰ τὴν διχοτομίαν· ἡ γὰρ εἰς ἴσα καὶ ἡμίση διαίρεσις ἐπ' ἄπειρον²⁹). τὸ δὲ περιττὸν προστεθὲν περαίνει αὐτό· κωλύει γὰρ αὐτοῦ τὴν εἰς τὰ ἴσα διαίρεσιν. οὕτως μὲν οὖν οἱ ἐξηγηταὶ τῷ ἀρτίῳ τὸ ἄπειρον ἀνατιθέασιν κατὰ τὴν εἰς τὰ ἴσα διαίρεσιν, καὶ δηλονότι οὐκ ἐπ' ἀριθμῶν ἀλλ' ἐπὶ μεγεθῶν λαμβάνουσι τὴν ἐπ' ἄπειρον τομήν . . . ὁλως δὲ οὐδὲ ὁ Ἀριστοτέλης φαίνεται τὴν εἰς ἴσα διαίρεσιν αἰτιασάμενος τοῦ ἀπείρου. μήποτε οὖν ἐν πάσῃ τομῇ τὸ ἄρτιον αἰτιὸν ἐστὶ πάσης διαιρέσεως.

The attentive reader will perceive at a glance that we have here presented by the ἐξηγηταί a point of view not quite comprehended by Simplicius. He thinks the reasoning erroneous, and assures us that he finds nothing in Aristotle to countenance it. Finally, he clearly associates it with the conception of the divisibility of space or matter ad infinitum popularly current since the days of Zeno³⁰). Philoponus (Phys. 391, 25) goes farther, quite unconsciously confounding the two notions: τὸ μὲν γὰρ περιττὸν περατοῖ καὶ ὁρίζει, τὸ δὲ ἄρτιον τῆς ἐπ' ἄπειρον τομῆς αἰτιὸν ἐστίν, αἰεὶ τὴν διχοτομίαν δεχομένην. It is quite clear that he owes his explanation to the same ἐξηγηταί, although he reproduces their statements more freely, according to his own mistaken understanding of them.

I hope to make it appear very probable that this explanation, rejected or disregarded by modern scholars, is the reason originally assigned for the identification of the ἄρτιον and the ἄπειρον, and that it can be traced back in substance to the time of Aristotle. If it should prove to rest upon a conception of number which Aristotle has elsewhere shown his inability to understand, this

²⁹) I regard this clause (ἡ γὰρ . . . ἐπ' ἄπειρον) as an attempted elucidation added by Simplicius. The reasons for this belief will shortly appear.

³⁰) Windelband (Tufts) p. 46, n. 2. betrays a lurking doubt as to the correctness of the interpretation given by Simplicius, while it likewise becomes evident that he does not understand the ἐξηγηταί: „The reason presented for this, viz. that even numbers permit of bisection to infinity(?), is indeed very questionable and artificial.“

would sufficiently account for his adopting the later and more refined explanation.

It would seem plain enough from Aristotele's own account what the Pythagorean theory of number really was. Let us then glance at a few of the passages³¹⁾. Met. A, 5. 985b 25: τὰς τούτων (sc. τῶν μαθημάτων) ἀρχὰς τῶν ὄντων ἀρχὰς ψήθησαν εἶναι πάντων . . . 986a 1: τὰ τῶν ἀριθμῶν στοιχεῖα τῶν ὄντων στοιχεῖα πάντων ὑπέλαβον εἶναι, καὶ τὸν ὅλον οὐρανὸν ἀρμονίαν εἶναι καὶ ἀριθμὸν. *ibid.* M. 6, 1080b 16: καὶ οἱ Πυθαγόρειοι δ' ἓνα τὸν μαθηματικὸν (sc. ἀριθμὸν εἶναί φασιν) πλὴν οὐ κεχωρισμένον (Aristotle had just defined ἀριθμὸν μαθηματικόν as τῶν ὄντων κεχωρισμένον τῶν αἰσθητῶν). M. 8. 1083b 12: τὸν ἀριθμὸν τοῦτον εἶναι μαθηματικόν, ἀδύνατον εἶναι . . . N. 3. 1090a 32: κατὰ μέντοι τὸ ποιεῖν ἐξ ἀριθμῶν τὰ φυσικὰ σώματα, ἐκ μὴ ἐχόντων βάρος μηδὲ κουφότητα ἔχοντα κουφότητα καὶ βάρος, εἰκόασι περὶ ἄλλου οὐρανοῦ λέγειν καὶ σωμάτων ἀλλ' οὐ τῶν αἰσθητῶν. A. 8. 990a 21: ἀριθμὸν δ' ἄλλον μηδένα εἶναι παρὰ τὸν ἀριθμὸν τοῦτον ἐξ οὗ συνέστηκεν ὁ κόσμος. Two things become very evident as one reads these quotations: first, Aristotle is carrying his own conceptions into his discussion of the Pythagorean doctrine, as he is wont to do³²⁾; second, he is very much perplexed by the theory he is considering and is endeavoring to reproduce it in part in his own terminology. The Pythagoreans assume to be explaining physical phenomena by physical entities, known as numbers; in his desperation, Aristotle pronounces these numbers mathematical but not abstracted from things of sense, though just what that could mean from his point of view is past comprehension. He then says that it is impossible for these numbers to be mathematical, but, assuming that they must be, he berates these philosophers for attempting to explain ponderable objects by elements not pon-

³¹⁾ These passages have been very diligently collected by Zeller, *ibid.*, p. 343, foll., although it must be said, in justice to him, that he does not draw the same inferences from them. I may add that I arrived at my interpretation independently of Burnet, *ibid.*, p. 306, foll. Our general agreement I regard as a strong corroboration of the view here set forth.

³²⁾ Compare Zeller, *ibid.*, p. 381, n. 3.

derable³³). For there is only one kind of number with which the Pythagoreans have to do.

It would seem that from all this only one conclusion can be drawn. The numbers of the Pythagoreans, as Aristotle himself tells us, are not abstracted from objects of sense. They are not the „mathematical“ numbers with which all men of intelligence in his day were perfectly familiar, although Aristotle finds it well-nigh impossible to conceive of any other. He does, indeed, speak of ἀριθμοὶ ἐνοπάρχοντες³⁴), but he must have been as keenly conscious as the modern reader that this conception is itself a striking novelty, born of his criticism of the Platonic Ideas. Hence it was altogether right that he did not attribute it to the Pythagoreans, although it was the nearest approach to the truth of which his mind was capable. We may say that the Pythagorean numbers were concrete, that is to say, things were naively regarded as numbers because the abstraction of the mathematical from the other qualities that go to constitute things had not yet become a stereotyped fact. To speak of concrete numbers is to employ unfamiliar language, although the conception is not quite lost to us. Within a very limited range we are well enough acquainted with them and speak of them as denominate numbers.

Such then, it would appear, are the Pythagorean numbers by

³³) Yet Aristotle himself says, Met. 1080b 18: τὸν γὰρ ὅλον οὐρανὸν κατασκευάζουσιν ἐξ ἀριθμῶν, πλὴν οὐ μοναδικῶν; where Ps.-Alex. remarks: μοναδικὸν γὰρ τὸ ἀμερὲς καὶ ἀσώματον ἐνταῦθα δηλοῖ. How solid and averse to division the unit was to the Greek mind might be illustrated by a thousand passages. Cp. Plat. Repub. 525D, foll.: οἶσθα γὰρ πού τοὺς περὶ ταῦτα δεινοὺς ὡς, ἐάν τις αὐτὸ τὸ ἐν ἐπιχειρῇ τῇ λόγῳ τέμνειν, καταγελῶσί τε καὶ οὐκ ἀποδέχονται, ἀλλ' ἐάν σὺ κερματίζῃς αὐτό, ἐκείνοι πολλαπλασιοῦσιν, εὐλαβούμενοι μὴ ποτε φανῇ τὸ ἐν μὴ ἐν ἀλλὰ πολλὰ μέρη. Lest anyone should suppose that this is wholly due to Plato's realism, compare the naiveté of Arist. Met. 987a 25: πρῶτον ὑπάρχει τοῖς δυοῖ τὸ διπλάσιον. Greek mathematicians did not deal as familiarly with fractions as we moderns do. The solid, indivisible, concrete unit was what underlay the Pythagorean conception. It was not a violent revolt, but rather a natural development of the original view, when Ecphantus assimilated his doctrine to the atomism of Leucippus and Democritus. For his teachings, cf. Diels, Dox. p. 566, l. 11, foll.

³⁴) Arist. Met. M. 6. 1080b 3.

which Aristotle is as much perplexed as is Simplicius by the explanation which the ἐξηγηταί offered for the identification of the ἄρτιον and the ἄπειρον. It is of such numbers as these that the περιττόν and the ἄρτιον are the elements. And Simplicius tells us that the ἄπειρον was identified with the ἄρτιον „because every thing that is even is divided into equal parts, and that which is divided into equal parts is unlimited in respect of bipartition; whereas the odd, when added, limits it, since it prevents its division into equal parts.” Now, bearing in mind the Pythagorean conception of number, we may illustrate the subject of the odd and the even with the following figures.

$$(A) \quad \begin{array}{c} \cdot \cdot \cdot \cdot \\ \hline \cdot \cdot \cdot \cdot \end{array} \rightarrow$$

$$(B) \quad \begin{array}{c} \cdot \cdot \cdot \cdot \\ \hline \cdot \cdot \cdot \cdot \end{array} \rightarrow \cdot$$

Let us first take ten, an even number (A). The process of halving, represented by the arrow, goes on without let or hindrance, there being no limit set to it by a solid unit³⁵). But if we take eleven, an odd number, we find that the unit added sets a limit, preventing the indefinite continuance of the process (B). That this illustration is really more than a plausible guess at the meaning of the ἐξηγηταί, which Simplicius did not quite understand, is rendered exceedingly probable by a number of passages to which we may next address ourselves. Plutarch³⁶) in an excerpt preserved in Stobaeus, after reciting the facts relative to the gnomon, adduced by Aristotle in his Physics, proceeds as follows: καὶ μὴν εἰς δύο διαιρουμένων ἴσα τοῦ μὲν περισσοῦ μονὰς ἐν μέσῳ περίεστι, τοῦ δὲ ἀρτίου κενὴ λείπεται χώρα καὶ ἀδέσποτος καὶ ἀνάριθμος, ὡς ἂν ἐνὸς οὗ καὶ ἀτελοῦς ὄντος. Again Plutarch says³⁷): πᾶν γὰρ κακὸν ἐκ διαστάσεως καὶ διαφορᾶς γίνεται· οὕθεν καὶ τῶν ἐφεξῆς ἀριθμῶν τὸν μὲν ἄρτιον ἐνὸς καὶ ἀτελοῦς, τὸν δὲ περισσοῦν πλήρη τε καὶ

³⁵) Compare for the solid unit note 33 above and Arist. Phys. I. 7. 207 b 5: αἴτιον δ' ὅτι τὸ ἐν ἔστιν ἀδιαίρετον, ὅτι περ ἂν ἐν ᾧ, ὅσον ἄνθρωπος εἰς ἄνθρωπος καὶ οὐ πολλοί· ὁ δ' ἀριθμὸς ἔστιν ἓνα πλείω καὶ πᾶς ἅττα· ὥστε ἀνάγκη στήναι ἐπὶ τὸ ἀδιαίρετον.

³⁶) Cp. Stobaeus, vol. I (Wachsmuth) p. 21 foll.; Diels, Dox. p. 96, foll.

³⁷) Plutarch, de Vita et Poesi Homeri, 145.

τέλειον ἀπέφηνεν. The same author elsewhere writes as follows³⁸): ταῖς γὰρ εἰς ἴσα τομαῖς τῶν ἀριθμῶν, ὁ μὲν ἄρτιος πάντη διισταμένος ὑπολείπει τινὰ δεκτικὴν ἀρχὴν οἷον ἐν ἑαυτῷ καὶ χώραν, ἐν δὲ τῷ περιττῷ ταὐτὸ παθόντι μέσον αἰεὶ περίεστι τῆς νεμήσεως γόνιμον.

I trust that I have made it evident that the explanation of the identification of the pairs $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ and $\pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\tau\iota\omicron\nu$ offered by the $\acute{\epsilon}\xi\eta\gamma\eta\tau\alpha\acute{\iota}$ had nothing to do with infinite divisibility of space, as seems to have been supposed by Simplicius and as Philoponus clearly believed³⁹). We do not know certainly who these $\acute{\epsilon}\xi\eta\gamma\eta\tau\alpha\acute{\iota}$ were, but we may readily conjecture that Alexander was one of them. Although his sources were of the best, this fact, even if guaranteed, would not enable us to determine the age of the conception we have been trying to establish. Fortunately we possess a bit of evidence, which, though standing alone, yet suffices to date it back to the time of Aristotle. Aristoxenus⁴⁰), in his work on the theory of number, writes as follows: τῶν δὲ ἀριθμῶν ἄρτιοι μὲν εἰσιν οἱ εἰς ἴσα διαιρούμενοι, περισσοὶ δὲ οἱ εἰς ἄνισα καὶ μέσον ἔχοντες . . . ὁ περισσὸς καὶ ἀρχὴν καὶ τελευτὴν καὶ μέσον ἔχει⁴¹). The mention of odd numbers as μέσον ἔχοντες

³⁸) Plutarch, de E apud Delphos, c. 8, 388A. He is speaking of the identification of the $\theta\eta\lambda\upsilon$ with the $\acute{\alpha}\rho\tau\iota\omicron\nu$ and the $\acute{\alpha}\rho\acute{\rho}\epsilon\nu$ with the $\pi\epsilon\rho\iota\sigma\sigma\acute{\omicron}\nu$. Compare also Plut., Aetia Rom. 102: καὶ διαιρουμένων εἰς τὰς μονάδας, ὁ μὲν ἄρτιος, καθάπερ τὸ $\theta\eta\lambda\upsilon$, χώραν μεταξὺ κενὴν ἐνδίδωσι, τοῦ δὲ περισσοῦ μόριον αἰεὶ τι πλεονεξ ὑπολείπεται. If we bear in mind that $\acute{\alpha}\rho\acute{\rho}\epsilon\nu \times \theta\eta\lambda\upsilon$ is one of the ten pairs of contraries enumerated by Aristotle and assigned by Zeller to Philolaus, it seems not improbable that the general conception underlying these illustrations was a part of the groundwork of the theory. Certain it is that $\acute{\alpha}\rho\acute{\rho}\epsilon\nu \times \theta\eta\lambda\upsilon$ had to be associated in some way with $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu = \pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\tau\iota\omicron\nu$, and this conception is undoubtedly more primitive than the physiological and astrological theories current among the Neo-Pythagoreans. For the latter cp. Sext. Empir. adv. Math. V. 6, foll.

³⁹) Compare note 30 above.

⁴⁰) Aristoxenus, apud Stobaeus (Wachsmuth) vol. I. p. 20.

⁴¹) This clause, ὁ περισσὸς . . . ἔχει, may be merely a paraphrase of the adjective, τέλειος, repeatedly used above; cp. Arist. Poetica, 6, 1450b 24, foll. On the other hand it may be a development of the afore-mentioned view reflected perchance in the enigmatic words of Aristotle, de Caelo I. 1, 268a 10, foll.: τελευτὴ γὰρ καὶ μέσον καὶ ἀρχὴ τὸν ἀριθμὸν ἔχει τὸν τοῦ παντός.

shows clearly that the view above set forth goes back to Aristoxenus, from whom Aristotle himself may have had much of his own knowledge of Pythagoreanism.

I believe that I have now shown that Aristotle did not give the only explanation for the identification of $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ with $\pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\tau\iota\omicron\nu$ that could have been known to him. Anybody who carefully considers the matter will agree, I fancy, that he chose the more artificial account instead of a more primitive one, which to him was not readily intelligible. The reasons that would determine his choice were doubtless such as vexed the mind of Simplicius as he grappled with the statements of the ἐξηγηταί⁴²). If the above argument holds, then it cannot be strictly true of the oldest Pythagorean doctrine that $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu = \kappa\epsilon\nu\acute{\omicron}\nu$ ⁴³), although the $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ was later made to account for the $\kappa\epsilon\nu\acute{\omicron}\nu$, since $\acute{\omicron}\ \mu\acute{\epsilon}\nu\ \acute{\alpha}\rho\tau\iota\omicron\varsigma\ [\acute{\alpha}\rho\iota\theta\mu\acute{\omicron}\varsigma]\ \dots\ \chi\acute{\omega}\rho\alpha\nu\ \mu\epsilon\tau\alpha\acute{\xi}\iota\ \kappa\epsilon\nu\eta\nu\ \acute{\epsilon}\nu\delta\acute{\iota}\delta\omega\sigma\iota\nu$.

The foregoing account of the original conception of the relation between $\pi\acute{\epsilon}\rho\alpha\varsigma \times \acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ and $\pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu \times \acute{\alpha}\rho\tau\iota\omicron\nu$ is readily seen to be in harmony with the view to which we were led by the consideration of the historical root of Pythagoreanism. The odd and the even are characterized with reference to their perfection or imperfection as determined by the inherence or the absence of the limit. Thus the supreme worth of the limit is vindicated, and the ethico-religious interest comes to dominate the mathematico-scientific.

Having thus brought to bear upon my main thesis all the evidence and the considerations that support it, I shall now briefly

⁴²) Even Burnet, *ibid.*, p. 310, n. 37, seems to share the same views: „The commentators usually say that even numbers were called unlimited because they could be halved indefinitely, which, as Simplicius points out in *Phys.* p. 455, 20D., is not the case.“

⁴³) For this view, see Ritter, *History of Ancient Philosophy*, I. p. 374, and Burnet, *ibid.*, pp. 108, 201, 310. Ritter's use of the above mentioned passage of Philoponus is peculiarly unfortunate, as I have shown that it is based upon a thorough misunderstanding of his authorities. Zeller, *ibid.*, p. 386, foll., criticizes this view somewhat, at length. Compare also Shorey's review of Patin's *Parmenides im Kampfe gegen Heraklit*, in the *Amer. Journ. of Philol.* vol. XXI. 2, p. 208.

touch upon a few, more or less related, problems having to do with the beginnings of Pythagoreanism. It is now admitted on all hands, I believe, that the *δοξὰς ἀόριστος* is a later addition to the theory, and indeed it seems very likely that the introduction of the *μονάς*, instead of the *ἐν*, belongs to the same stratum. In view of what has been said of the primitive Pythagorean conception of number, one is tempted to conjecture that neither of these terms, with the accompanying refinements of theory, was added until, as the science of mathematics advanced, a knowledge of abstract numbers was diffused throughout Greece. It is clear that this stage was reached at latest in Plato's day⁴⁴), and in Aristotle, as we have seen, the ancient conceptions appear only to work confusion. When one has fully realized the hopelessness of attempting to follow Aristotle in his tortuous efforts⁴⁵) to state the matter, one will gladly dispense with Zeller's learned discussion⁴⁶) of the question whether the Pythagorean numbers were spatial or arithmetical. The simplest and truest answer would seem to be that the earliest members of the school had not raised the question, but that, once asked, it could be answered only in favor of the latter view, as being more in keeping with an advanced mathematical science. Hence, also, we may readily infer how much credence should be accorded to the statement of Stobaeus⁴⁷) concerning Ecphantus: τὰς γὰρ Πυθαγορικὰς μονάδας οὕτως πρῶτος ἀπεφάνητο σωματικὰς⁴⁸).

There is perhaps another point at which we may distinguish the early doctrine from the later. There can be no doubt that in the days of Plato⁴⁹) and Aristotle the Pythagoreans constructed surfaces out of lines, and bodies out of surfaces⁵⁰). But when we

⁴⁴) Here Plato's distinction between ἀριθμητικὴ and λογιστικὴ is of interest, although it is not exactly the distinction between concrete and abstract number.

⁴⁵) See especially, Arist. Met. M. 6. 1030b 16, foll.

⁴⁶) Zeller, *ibid.*, p. 378—389.

⁴⁷) Stobaeus (Wachsmuth) vol. I. p. 127, l. 17.

⁴⁸) Compare note 33 above.

⁴⁹) For Plato, see Arist. Met. N. 3. 1090b 20, foll., and Ps.-Alex. p. 756, 14, Bonitz.

⁵⁰) This statement is not intended to be exhaustive. Zeller, *ibid.*, p. 405, foll., discusses the matter at length and supplies the references.

remember what Aristotle himself reports of Eurytus⁵¹), one of the earlier Pythagoreans, it would seem reasonable to suppose that their method was far simpler. Aristotle says⁵²): οὐδὲν δὲ διώριστα: οὐδὲ ὑποτέρως οἱ ἀριθμοὶ αἵτιοι τῶν οὐσιῶν καὶ τοῦ εἶναι, πότερον ὡς ὅροι, οἷον αἱ στιγμαὶ τῶν μεγεθῶν, καὶ ὡς Εὐρύτος ἔταττε τίς ἀριθμὸς τίνος, οἷον ὁδὸν μὲν ἀνθρώπου, ὁδὸν δὲ ἵππου, ὥσπερ οἱ τοὺς ἀριθμοὺς ἄνουντες εἰς τὰ σχήματα τρίγωνον καὶ τετράγωνον, οὕτως ἀφομοιωὺν ταῖς ψήφοις τὰς μορφὰς τῶν φυσικῶν. Aristotle here speaks of a method of 'limits' represented by points. Eurytus used pebbles in lieu of points to mark the outlines of things, then, forming a rude likeness, took the number of pebbles required as the representative or typical number of the thing. We are told⁵³) that Eurytus lived in Tarentum and was a contemporary of Philolaus. Surely, it is altogether more probable that his method was a survival from the earlier procedure of the Pythagoreans than an invention of a contemporary of Plato. We should then have to think of the early Pythagoreans as employing this method of 'limits'⁵⁴) in accordance with which they represented the line by 2, the number of points or limits necessary to define it, the surface by 3, the body by 4. Therefore they could also use 3 to denote a triangle, and 4 to represent a quadrangle, though each had but two dimensions. One can readily see how, with the advance of mathematics, the procedure in vogue in Plato's day could come to supplant the earlier and cruder method.

⁵¹) On Eurytus see Zeller, *ibid.*, 338, n. 5.

⁵²) Arist. Met. N. 5. 1092b 7, foll.

⁵³) Laert. Diog. VIII. 46. Zeller admits that we cannot rely on later accounts regarding the date of Pythagoreans. Eurytus may have been considerably older than Philolaus, however, and yet have been the instructor of the Pythagoreans whom Aristoxenus knew. Burnet, *ibid.*, p. 314, says of the method of Eurytus: „This was simply a graphic way of showing how many dimensions a thing had, taking a simple pebble as one dimension.“ I confess that I am utterly unable to understand his meaning. How many dimensions should we have to conceive of Eurytus' man as having, if he used pebbles to delineate his form and took „a single pebble as one dimension?“

⁵⁴) These 'limits' are the ὅροι of Aristotle's account, not the 'limits' technically known as such in modern mathematics.