

THE "PYTHAGOREAN" THEORY OF THE DERIVATION OF MAGNITUDES

J. A. PHILIP

MANY OF THE GREAT DEBATES in the history of thought have had their origin in, or turned on, what to later times seem minor problems—whether the earth moves, whether transubstantiation in fact occurs, whether we are descended from apes. Early in the fourth century B.C. we begin to hear echoes of what appears to have been such a debate. References then first occur to point, line, plane surface, solid, as the constituent elements in ideal and physical magnitudes. These elements play a major role in the mathematical and metamathematical theories of Plato and the Early Academy, and against such theories Aristotle directs much of the polemic of the *Metaphysics*. Plato, Speusippus, and Xenocrates, each in differing ways, derive magnitudes from the sequence point, line, surface, solid, and their speculations gave rise to a lively controversy concerning indivisible lines—a controversy that influenced Aristotle's notions of space, indivisibility, continuum. At the end of the fourth century we find Euclid defining the four terms as basic concepts in his *Elements*.

Our earlier tradition is silent on the origin of these theories of derivation. I propose to discuss here the time when we may reasonably suppose that they arose, and the milieu in which their origin is most probable. The nature of the several theories, and their mutual relations, will be discussed only incidentally, in so far as is relevant to the theme.¹ It will be suggested that:

(1) We must distinguish between cosmogonical number theories, such as that attributed by Aristotle to the Pythagoreans, and theories of the derivation of solids.

(2) A theory of the derivation of solids—point, line, plane surface, solid, or some modification of this sequence—was expounded

¹References are made as follows:

- Metaph.* W. D. Ross, *Aristotle's Metaphysics* (Oxford 1924) quoted by page numbers only.
- Phys.* W. D. Ross, *Aristotle's Physics* (Oxford 1936).
- De. An.* W. D. Ross, *Aristotle: De Anima* (Oxford 1961).
- Theory* W. D. Ross, *Plato's Theory of Ideas* (Oxford 1951).
- Robin L. Robin, *La Théorie Platonicienne des Idées et des Nombres d'après Aristote* (Paris 1908).
- Eudemus F. Wehrli, *Die Schule des Aristoteles VIII: Eudemos von Rhodos* (Basle 1955).
- Cherniss H. Cherniss, *Aristotle's Criticism of Plato and the Academy* (Baltimore 1944).
- Heath T. L. Heath, *The Thirteen Books of Euclid's Elements*² (Cambridge 1926) 3 vols.
- Vors.* H. Diels-W. Kranz, *Die Fragmente der Vorsokratiker*³ (Berlin 1951–1952) 3 vols.

138
PHILIP, J. A.
1966

by Plato, Speusippus, Xenocrates. Against them, and against such theories as a prevailing trend of contemporary philosophy, Aristotle argues.

(3) In the Early Academy a Pythagorean origin was attributed to such theories, not on historical grounds but because there was in the Academy a vogue for attributing the beginnings of such speculation to Pythagoras. In the first century B.C. and in succeeding centuries modified forms of the theory reappear as Neopythagorean.

(4) Any such theories must be post-Zenonian, and are unlikely to have been evolved before the beginning of the fourth century.

In the first place a distinction must be made between cosmogonical number theories and theories of the derivation of physical magnitudes: that is, between a primary derivation of the universe *qua* One from numerically determined first principles and a secondary derivation of magnitudes or physical bodies from numbers, whether by the agglomeration of atomic magnitudes—which we might call arithmetical derivation—or by some variety of geometrical derivation based on point, line, plane, surface, solid. The primary or cosmogonical theories are said by Aristotle to have early precursors. For the secondary or physical theories we have no authoritative statement of the time or place of origin beyond the fact that such theories were propounded in the Early Academy. And whereas the secondary theories clearly have a background of mathematical inquiry we need assume no such background for the cosmogonical theories.

Any inquiry into these problems must begin from the *locus classicus* of the *Metaphysics* (985b 23–988a 17). This passage, and the elaborations on it of the commentators, bristles with difficulties; but for our present purposes we need consider only such parts as are immediately relevant to our theme. "Our purpose" Aristotle remarks (986a 13) "is" (not to outline Pythagorean theories in general but) "to find out in their case too what first principles they posit and how these coincide with the causes we have mentioned. Now they notoriously consider number to be a first principle, both as material cause of existents and as their modifications and states."² The elements of Pythagorean number are the odd and the even, the former being limited, the latter unlimited. The

²It is easy to see how, for the Pythagoreans, numbers as physical existents could be the material cause. By the following phrase Aristotle should allude to some cause *other than the material*. Ross *ad loc.* suggests that he is alluding here, as he alludes elsewhere, to a Pythagorean formal cause. It is equally probable that he is alluding to an efficient cause, or cause of movement, numbers somehow also accounting for qualitative change. Some colour is lent to this suggestion by 987a 5–9 where Aristotle says that the Presocratics anticipated the doctrine of causes in respect of only two of them, the material and the efficient. It seems more likely however that he is simply saying that number is the material substrate *and also* accounts for the characteristics of existents.

PHILIP J. A., *The "Pythagorean" theory of the derivation of magnitudes: Phoenix XX 1966 32–50.* | The theory of the derivation of solids—points, line, plane surface, solid, or some modification of this sequence—expounded by Plato, Speusippus, and Xenocrates and opposed by Aristotle, though attributed in the Early Academy to the Pythagoreans, must be post-Zenonian. Distinction must be made between theories of the derivation of solids and cosmogonical number theories, such as that attributed by Aristotle to the Pythagoreans.

One proceeds from both these elements, being both odd and even. Number proceeds from the One. "And numbers, as I explained earlier, are the whole universe." Here we have a generation of the universe and then a brief allusion to a post-cosmogonical episode in which number-particulars proceed from the One. Aristotle goes on to complain (989b 29-990a 8) that, though their first principles are abstract and would lend themselves to investigations of another character, nevertheless the Pythagoreans are to be ranged with the physical philosophers in that their whole approach is physical. For they generate the universe, discuss its constitution and phenomena, and to these ends expend their first principles and causes, "as agreeing with the rest of the physical philosophers that being is just the perceivable comprised within what we call the universe." On a later occasion he insists again on this interpretation of Pythagorean theory (1091a 12-20). "It is absurd, and indeed impossible, to have eternal existents generated. Yet there can be no doubt that the Pythagoreans do indeed generate them. For they state explicitly that when the one has been constituted (whether from physical surface or geometrical plane surface or some yet more mysterious constituent) immediately the nearest part of the unlimited is drawn in and delimited by limit."

In these passages we observe Aristotle's insistence on the fact that Pythagorean theories are cosmogonical. He does however make it clear that Pythagorean cosmogony went beyond the creation of a Parmenidean One to describe, however briefly and unsatisfactorily, the generation of discrete quantity within the One. It apparently "breathes in" a part of the surrounding void (*Phys.* 213b 22-27; *Frs.* [Ross] p. 137), this void separating things, and apparently also the number constituents of things, from one another. It is significant that, whereas Aristotle does tell us what are the elements constituting the Pythagorean One or universe, he is at a loss to explain the origin of particular substances in their theory. The One breathes in the surrounding void, and somehow out of a One substance the many particulars of the physical world come to be. He does however tell us in general terms how the Pythagoreans conceived these number-particulars. For them "things were numbers" (*Metaph.* 987b 28) and they thought they saw in number, rather than in elements such as fire, water, air, earth, many similarities or resemblances to existents and things in the process of becoming (985b 27-29). That is, they believed in a numerical structure of the physical universe, and fancied they could observe this structure in physical existents.

Aristotle's insistence that Pythagorean theory had as its primary object the explanation of the origin of the physical world has its counterpart in the distinction he draws between Pythagorean and Platonic theories (987b 22-32). By and large both held that the One was a substance, and that underlying all existents was number. But for the Pythagoreans things were numbers, and there were no mathematical to mediate

their existence. Plato on the other hand believed that numbers existed independently of sensible things, and this *χωρισμός* marks the difference between Platonism and Pythagoreanism.³

It is obvious that if Plato's theory of ideas has the characteristics Aristotle describes, and if Plato distinguishes between intelligibles and sensibles, then sensibles cannot proceed directly from number first principles, as they do in Pythagorean theory. For Plato sensibles have the geometrical structure of their intelligible counterparts, and these are the products of his first principles. Sensibles have such being as they have by their participation in intelligible mathematica, and their elements or constituents are the elements of the mathematica, these imposing order on the "receptacle." For our purposes we need not consider how these elements are derived from higher principles, nor how Plato, Speusippus, and Xenocrates differed regarding the stages of their derivation. They were all agreed as to the number and identity of the elements except in one particular. Speusippus held that the elements were point, line, plane, and solid. Plato and Xenocrates held that the first element was a minimum indivisible line rather than a point. "Plato," says Aristotle (992a 20-22), "used to oppose the notion that the point is more than a convention of geometry. He used to assert that the indivisible line was the *ἀρχή* or first principle of the line, and frequently maintained this position."⁴

That a theory of the derivation of solids such as Aristotle describes

³Aristotle may seem to deny the validity of the distinguishing mark he has recognized, *χωρισμός*, when he says (987b 11-13) that Plato's *μέθεξις* (participation) and Pythagorean *mimesis* are equivalent and interchangeable terms. This statement is surprising and isolated. Nowhere else do we hear of Pythagorean *mimesis*; and if this *mimesis* implies *χωρισμός*, as we would assume, the statement is a contradiction of what Aristotle always elsewhere assures us is the Pythagorean position, that "things are numbers." The simplest solution is to regard this as a piece of careless terminology on the part of Aristotle. He tells us (985b 27) that the Pythagoreans thought they saw *δμοιώματα* (likenesses) of numbers in existing things, i.e., that they thought they could observe the underlying number structure which things might (loosely) be said to imitate or better represent. Plato likewise says that their real nature is to be explained by their participation in the ideas. As Aristotle is drawing a parallel between Plato and the Pythagoreans, he may have considered that there was a real similarity in their approach to physical existents. For this similarity see also 1090a 4-7. For discussions of the passage see Ross *ad loc.* and *Theory* 217; Robin 74; Cherniss 180.

⁴See Ross *ad loc.* and his further discussion in *Theory* 206-212. See also Robin 286-293, 472-474 and his *Rapports de l'Etre* (Paris 1957) 71-73. We are not here concerned with the ramifications of Plato's Theory of Ideas, nor with the modifications of that theory effected by Speusippus and Xenocrates. Nor are we concerned with the *Timaeus* theory of how ideal magnitudes materialize. Our interest is confined to the general features of the doctrines to which Aristotle testifies. The existence of these doctrines in the Early Academy is questioned only by those who completely reject the testimony of Aristotle on *agrapha dogmata*. When Aristotle is not referring to the doctrines of a particular thinker—Plato, Speusippus, or Xenocrates—he refers to the derivation theory as point-line-plane, as, e.g., *Metaph.* 1090b 5, *An. Post.* 73d 35-38.

was propounded within the Early Academy we cannot reasonably doubt. We may, however, ask ourselves *why* they constructed ideal magnitudes, as "things after the numbers," to mediate the coming-to-be of physical body. There was no reason why the ideal numbers should not produce physical body without the intervention of intermediates. A theory of number-atomism would have resulted. Though geometrical derivation may better account for order and structure, that is not in itself sufficient reason for so surprising an innovation. We can only suppose that in the Academy thinking in geometrical terms either had the charm of novelty or was induced by the mathematical discipline that prevailed. In any case it is a striking fact that Plato and his immediate successors in the headship of the Academy, whatever their other differences, were with one slight exception in agreement as to the derivation of solids.

It was to such theories, and to the mathematical tendencies of the Academy, that Aristotle took exception when he wrote (992a 32; cf. Robin 318): "Mathematics has become for our contemporaries the whole of philosophy, though they allege that it is to be pursued (not for its own sake but) for other ends." That the derivation theory was for him an important aspect of these tendencies is obvious from the arguments he directs against it. In Books M and N of the *Metaphysics* we have two versions of his polemic against the theory of ideas in its various forms. As Jaeger (*Aristotle*² [Oxford 1948] 176–193) has shown, Book M to 1086a 12 is the later of the two versions that together constitute these two books; and it is significant that in the later version Aristotle's arguments against the derivation theory assume a larger place. In Book N, apart from occasional allusions, there is only one major attack (1090a 2 ff.) and that attack is directed against the doctrine of point, line, plane as separate substances.

At the time of the writing of Book M it may be that the emphasis on mathematical aspects of the theory of ideas had increased, or it may be that Aristotle had more clearly appreciated their importance. In any case the theory of ideas in its earlier, non-mathematical form, though it is introduced to make the treatment complete, is touched on only cursorily. Elsewhere in the book the theory is discussed as one having mathematical aspects, and as presenting two major problems; (1) Can mathematical objects be said to exist? and (2) Are numbers separate substances and first principles? The mathematical objects Aristotle envisages are either mathematical, i.e., numbers as separate substances, or geometrical, i.e., line, plane, and solid as separate substances. Both are derived, by differing methods, from the same first principles. Aristotle is concerned to show that they are to be considered not as separate substances but as mathematical abstractions from sensibles.

Much of Aristotle's criticism, however, is applicable both to ideal

magnitudes and physical magnitudes. First he shows that mathematical objects cannot be *in* sensibles because we then have a meaningless duplication (1076a 38; cf. 998a 7–19), and because, if mathematical and physical magnitudes are derived from point, line, plane, then they must be divisible into these elements, and if the solid is to be divisible the point must be divisible. Next he points out (1076b 11) that if mathematical magnitudes are separate substances there results a preposterous pullulation of points, lines, and planes. For there must be points prior to the points of the line, lines prior to the lines of the plane, and so forth. And surely points, lines, and planes cannot be prior in substance to body, which possesses unity and completeness and can be ensouled. We need not here discuss the merits of Aristotle's arguments against the theory of derivation (Robin 427–435), to which we have reason to believe that he once subscribed (Walzer, *Frs.* [Florence 1934] 28 and n. 3).⁵ It is sufficient for us to note the importance of the role it plays.

When, after a rather perfunctory discussion of the theory of ideas in its earlier, non-mathematical form, Aristotle reverts to ideal numbers and idea numbers, he turns to the second question he propounded: Are numbers separate substances and first principles? In this discussion he is concerned with the comparability of the units constituting idea numbers and with the stratification of substances from the one down to sensibles. Lines, planes, and solids are referred to only incidentally, to ask whether we must distinguish between mathematical lines and those that come after the ideas, i.e., between an idea of line and mathematical line (1080b 23–30). (That such a distinction was recognized is suggested also by 1084a 37–b 2.) It is only towards the end of his survey that he turns to the special problems of the *generation* of point, line, plane as the constituents of magnitudes (1085a 7–b 33; Robin 370–371). He argues that Xenocrates, for whom the ideas were numbers, must generate numbers and geometrical objects from identical first principles, and that this involves a *μετάβασις εἰς ἄλλο γένος*; that Speusippus does not explain how the many points of geometrical objects are to be generated from the one unique point that must be assumed; and finally (this applies also to Plato) that, if there exist indivisible lines, then no line and no continuum can be infinitely divisible. With these and other arguments Aristotle shows the difficulties that arise in generating the points, lines, and planes that are said to constitute magnitudes.

⁵That Aristotle in an early period accepted some geometrical theory of derivation is also suggested by his remark in the *Protrepticus* (I. Düring, *Aristotle's Protrepticus* [Göteborg 1961] fr. 33, p. 61 = Iambli. *Protrep.* [Festa] 38.14–22P.) that "things that are prior are causes . . . , for if the former are removed (*ἀναίρεται*) the things that have their being from them are removed; lines if numbers are removed, planes if lines, solids if planes, the so-called syllables if letters are removed."

In the *Metaphysics* Aristotle shows little or no sympathy for the efforts of Platonists to derive physical magnitudes from ideal magnitudes having the three dimensions of the solids of geometry. He regards such attempts as a deplorable aberration of contemporary philosophy. For him sensible particulars are substances, and it is absurd to attempt to account for them by suggesting that they owe what reality they possess to their participation in a non-sensible geometrical paradigm, itself derived from higher principles. That he was not, however, unaware of the problems that were being discussed in the Academy while he was a member is shown in his other writings, in particular in the *Physics*. In Book 5 of that treatise he argues for the infinite divisibility of all continua, "the infinite divisibility of movement forming the middle term by which that of extension and that of time are seen to imply one another" (*Physics* 70, n. 1). As a consequence, when we speak of a point in space, a point in a path of movement, or an instant in time, we are not speaking of parts of, or entities constituting, space, movement, or time (*Physics* 212b 24; 220a 4; 222a 15). In the course of his discussion Aristotle alludes casually and with some contempt to the notion of indivisible lines (206a 16–18) and he touches briefly on the problems of three-dimensional entities (209a 2–13; 227a 27–32; 231a 20–231b 1). It is obvious from these allusions, and also from his whole reformulation of physical thought in terms of matter, form, and privation that he has faced the problems that were agitating the Academy and is offering solutions for them in his own terms.

Aristotle's account in the *Metaphysics*, implying as it does lively discussion and important differences within the Academy, and also his own physical theory, both suggest that the problem of the derivation of ideal and physical magnitudes was a live contemporary problem. It arose, or at least its solutions found their application, within the theory of ideas. Point (or indivisible line), line, plane, solid—the three spatial dimensions of geometry—were borrowed from that discipline to serve in explaining how physical magnitudes were derived from first principles; that is, how the intelligibles are to be related to sensibles. The fact that this derivation theory was a theme of lively discussion, and a theme on which Plato, Speusippus, Xenocrates differed radically, suggests that it had been evolved within the Academy. It is usually claimed, however, that it is a borrowing from Pythagoreanism and represents the pythagorizing tendencies of the Early Academy. Aristotle tells us that the Platonic philosophy was based on Pythagorean theories (987a 29–31). The ideas as "concepts" were a Socratic legacy, but their mathematization was Pythagorean. Plato, however, is said to have recognized the existence of "numbers apart from sensibles, whereas the Pythagoreans assert that things themselves are numbers" (987b 27–29). Now if "things

are numbers"—a tenet Aristotle repeatedly imputes to the Pythagoreans—then there is no need of a derivation theory, whether arithmetical or geometrical, to account for the derivation of number-things. We must suppose that someone within the Academy, and probably Plato himself, was inspired by the cosmogonical number theories of the Pythagoreans, but at the same time was impressed (like Aristotle) by the fact that they did not account for the procession of number-things from their cosmogonical first principles. So he devised a theory of derivation, but not on Pythagorean lines. For a Platonist sensibles, as being in continual flux, could not be derived from intelligibles. The derivation theory propounded was not an arithmetical one, such as Pythagorean first principles would have suggested, but a geometrical one in which the three spatial dimensions were as it were hypostasized to mediate the derivation of physical objects.

It may, however, be argued that we cannot exclude the possibility of a Pythagorean derivation theory simply because the testimony of Aristotle appears to exclude it. The Pythagoreans may have developed a derivation theory known to Plato but not to Aristotle, a theory not involving the separation of intelligibles and sensibles. That the Pythagoreans had evolved any such theory does not seem probable, but we can reject the possibility only if we can determine when the geometrical notions and procedures which the theory assumes were developed. But before we attempt to consider this difficult problem let us first turn briefly to the post-Aristotelian history of the derivation theory, and ask ourselves why it then appears as Pythagorean rather than Platonic.

The pseudo-Aristotelian treatise *De lineis insecabilibus* is generally thought to have been written by a member of the Peripatos of the first generation, probably under the headship of Theophrastus. It begins by recapitulating the case for indivisible lines, in part deriving its arguments from the Aristotelian treatises, but in part adducing new arguments of its own.⁶ Once the case has been presented it is answered without preamble, point by point. The answer has the character and develops its arguments after the manner of a formal *précis* used in instruction. Some

⁶"Parts are by nature prior to their wholes" (986a 10). S. Pines in "A New Fragment of Xenocrates and its Implications" (*TAPHilosA* 51.2 [Philadelphia April 1961]) discusses this doctrine as one originating with Xenocrates, to be connected with his theories of genus and species (Cherniss 13–17) and indivisible lines and solids. Aristotle (*Metaph.* 1017b 19) discusses the priority of genus and species without alluding to Xenocrates. The argument from commensurate lines (*De lin. insec.* 986b 4) is not an argument deriving from Aristotle, because of the obvious fallacy. Throughout the *De lineis insecabilibus* the geometrical references are more immediate and technical than in Aristotle; see the references to Euclid in the translation of H. H. Joachim, *The Works of Aristotle* (Oxford 1908). See also G. Vlastos, "Minimal Parts in Epicurean Atomism," *Isis* 56/2 (1965) 125–129.

of its arguments are as valid against the points of Speusippus as they are against the indivisible lines of Xenocrates (971a 5-7). Reference is made throughout to geometry as an organized discipline, and the meaning can frequently be elucidated by a reference to Euclid, whose *Elements* must have been published at about this time. No allusion is made to a Pythagorean origin for the derivation theory, one basic aspect of which is being discussed, nor to any Pythagoreans holding it. There is indeed no mention of persons, but the immediate reference to the Aristotelian treatises makes it certain that the indivisible lines being discussed are the lines of Xenocrates (and Plato).

The *De lineis insecabilibus*, probably written in the third century, is the last work making reference to the derivation theory as Platonic or Academic. When first we encounter it again it reappears as Pythagorean. Alexander Polyhistor (first century B.C.) claims to have discovered Pythagorean writings—many such writings appear from the third century on⁷—from which he quotes as follows (D.L. 8. 25): "The first principle of all things is the monad. From the monad proceeds the indefinite dyad, constituting the material substrate for the monad, the monad being cause. From the monad and the indefinite dyad proceed the numbers, from numbers points, from points plane figures, from planes three-dimensional figures, from these sensibles. The elements of sensibles are four: fire, water, air, earth."

This is obviously a pastiche of doctrines of the Early Academy. It grossly over-simplifies, and it deforms the theory of ideas from which it springs. In the place of the One we have the monad. The indefinite dyad has become material substrate (ὕλη). Point, line, plane, and solid now proceed from one another.⁸ Sensibles, once constituted, are further

⁷See H. Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period* (Abo 1961).

⁸The procession of line from point, plane from line, solid from plane implies a kinetic nature or principle that is absent from Aristotle's account of derivation in the *Metaphysics* and *Physics*. He himself defines the unit or monad as indivisible quantity not having position, the point as indivisible quantity having position. Line, plane, and solid are the divisible in one, two, and three dimensions. These definitions are coloured by geometrical thinking and were probably developed within the Academy, Book D of the *Metaphysics* where they occur being an early book. Aristotle's account of Platonic derivation theory is conceived in the same terms and is essentially static. So are the definitions of Euclid. There the unit or monad is defined arithmetically. (*Elements* 7.1; cf. Heath 1.279), and we find the same static notion of plane and solid numbers (*Elements* 7, def. 16, 17; cf. Heath 287-291).

In the *De Anima*, where the theme of discussion is the soul, we discover a radically different point of view or, perhaps better, a new aspect of the derivation theory. There, at 409a 4, we read: "It is said that a line being set in motion creates a plane, a point a line; but then the movements of monads are lines." This passage occurs in the context of a discussion of Xenocrates' definition of the soul as a self-moving number. The

analysable into elements. In this caricature of Platonic doctrines the most significant feature is that while the doctrines remain recognizably Platonic they are now reported as Pythagorean. Why have they been rebaptized?

Reinhardt (*RE*, s.v. "Poseidonios" 647) has pointed out that Stoic pantheism has important similarities with Pythagorean cosmology, the structure of the physical world being for both schools mathematical and so intelligible. But the doctrine that the physical world is understandable *qua* mathematical is Platonic, and no earlier source suggests that it was also Pythagorean. It seems, however, probable that they were treated as Pythagorean in the school of Posidonius, and we may find some confirmation of this in the Stoic terminology that colours the *Hypomnemata* quoted by Alexander Polyhistor. Why were doctrines that were

phrase κινηθείσαν γραμμήν, which I have translated by "a line being set in motion," is usually rendered by "the movement of a line." But the whole purport of the passage is not that sensibles are being created by a method of fluxion (so Cornford, *Plato and Parmenides* [London 1939] 12). Here they are being "ensouled"—that is, an aspect of the relation between soul and mathematical is being explained, a relation to which Aristotle alludes again in arguing against mathematics as separate substances (*Metaph.* 1077a 20-36). Here he argues that both souls (as being in physical bodies) and monads (as points) have position. If both have position how can we have soul as a self-moving number?

Cherniss (396-397) argues that this is a fluxion theory held by Speusippus. For such a theory we have no other evidence, and in view of the "episodic" character of Speusippus' theories it seems unlikely. It seems more probable that it is a loose way of speaking of extension, that you advanced a line as you advanced a military formation in line; and that in this rather captious argument Aristotle is hinting that a point when it moves along a line to another position produces a line, not a soul. Similarly, in a much disputed passage (*De An.* 404b 19-24), we have the form of the one, and primary length, breadth, and depth referred to the autozoon; the one, the two, and the numbers of surface and solid referred to cognitive and perceptive faculties. Though the themes may be germane, they are not directly relevant to the derivation of solids from mathematical. For souls and mathematical see P. Merlan, *From Platonism to Neoplatonism*² (The Hague 1960) 222 and *passim*; Robin 487-491.

The first explicit mention of a line being described by the motion of a point is by Proclus (185.6-25) ascribed to Geminus (Heath 1.40). But Geminus may have referred simply to the describing of a line by a geometer in the drawing of diagrams and not to a principle of fluxion in a point by reason of which a point in a spatial continuum tends to flow into a line. In a passage in the *Laws* (894a) where he classifies motions Plato says: "In the case of all things *genesis* occurs when an ἀρχή increases and makes the transition to a second dimension, and from that to the next; and having achieved three dimensions acquires sensible properties for percipients." A. E. Taylor (*The Laws* [Everyman 1960] 285) interprets this as implying the fluxion theory. But Plato has been talking of collisions of bodies, moving with moving and moving with stationary. As a consequence of these collisions bodies disintegrate and reintegrate in new masses which *come to be*. This *genesis* occurs when we have "something to start with" and it acquires three dimensions in the process of integration, thus becoming sensible. Though elliptical this account of *genesis* is such as to exclude fluxion of point, line, plane.

clearly Platonic in the fourth century said to be Pythagorean in the second?⁹

Burkert (*Weisheit und Wissenschaft* [Nuremberg 1962] 82–85) has suggested that when with Arcesilaus the Middle Academy turned to scepticism, alleging Socratic origins for the tendency, they could no longer interpret as Platonic the mathematical metaphysics of Plato's latest period. They therefore took literally the pythagorizing of the Early Academy, and considered as Pythagorean the doctrines that were no longer comparable with what they chose to teach as Platonism. The Early Academy undoubtedly was influenced by, and to a greater or lesser extent borrowed from, a preceding Pythagoreanism. But what the post-Aristotelian tradition presents as Pythagoreanism is largely the pythagorizing Platonism of the Early Academy. This reappears, greatly simplified and sometimes contaminated, in the doxographical account as Pythagoreanism. This thesis of Burkert's is the only plausible explanation of a very curious change.

The derivation theory reappears in Neopythagorean writings as characteristically Pythagorean, and as such is implied in the *Introduction to Arithmetic* 2.6–7 (tr. M. L. D'Ooge [New York 1926] 236–241) of Nicomachus of Gerasa. It is no part of our purpose to trace the post-Aristotelian tradition of the theory. A brief outline of the manner in which it is presented by Sextus Empiricus, our most important source for its later form, will suffice to show how the features of the theory were changed in transmission, and will suggest why his account cannot be used for the reconstruction of early doctrines, as was suggested by Cornford.

Sextus, as a Sceptic, aims at showing the shortcomings and fallacies of all non-Sceptical doctrines, but his discussion shows an interest in the history of thought and considerable philosophical insight. He is not, however, a careful reporter. Of Pythagoras, Empedocles, and "the rest of the Italiots" he tells us (*Adv. Phys.* 1. 127) that they believe there exists a community between men and animals because there is "one *pneuma* pervading the whole universe, like a soul," and that this (Stoic) persuasion led them to abstain from meat. Again he tells us that Pytha-

⁹G. Borghorst, *De Anatolii Fontibus* (diss. Berlin 1904) 65 and *passim*, argues *totam de Pythagorico criterio disputationem depromptam esse a Posidonio in Timaeum commentario*, and that as a consequence our accounts of the derivation of solids in the mathematical tradition and in commentators on Plato like Chalcidius and Macrobius derive from Posidonius. This thesis perhaps oversimplifies (Pohlenz, *Die Stoa*² [Göttingen 1959] 386–388). Pohlenz points out (255–256) that the tendency to present Posidonius as the first Neoplatonist "verkennt freilich die tiefe Kluft die diesen von Transzendentalismus trennt." Neopythagorean derivation theories, however, usually deform or neglect the transcendental implications of their Platonic original, or bypass them by reference to the world soul.

goras was the leader of those who conceive physical body as "what is capable of affecting anything else or being affected" (*Pyrr. Hyp.* 3.38; *Adv. Phys.* 1. 366). Plato in the *Sophistes* (247E) in almost the same words offers this as the mark of the moderate materialist. Again Sextus tells us (*Adv. Phys.* 2. 261) that for Pythagoras the One "when compounded with itself in respect of difference creates what is called the indefinite dyad"—a curious garbling of Platonic doctrine. What is approximately Aristotle's definition of time is attributed to Aristotle "or, as some say, to Plato" (*Pyrr. Hyp.* 3. 136) and a distinction in kinds is made among opposites that derives from the Aristotelian *Categories* (6b 15) rather than from any Pythagorean source. In all these cases we must suspect a Neopythagorean source that sweeps everything even distantly congruous into the Pythagorean net, and we must expect the same tendency to be at work in the account of Pythagorean doctrines.

In Sextus' discussions of Pythagorean theory in general we are given a sketchy account of its metaphysical presuppositions and a not altogether consistent account of the theory itself. The dyad, of Platonic origin, is generated from the monad directly (*Pyrr. Hyp.* 3. 153), and monad and dyad are the first principles from which numbers are generated. Numbers are invisible and incorporeal. Physical bodies are a composite of corporeal and incorporeal. They are numerable, the point having the *logos* of the monad, the line of the dyad, plane surface of the triad, solid of the tetrad. (The priority of the numbers to the corresponding geometrical dimensions in 154 is Platonic.) The universe is made up of these number bodies, and is ordered after the ratios of the musical chord, these being expressible in the first four numbers.

These numbers (those of the *tetractys*) not only constitute the cosmos, they also render it intelligible. For "it is by the ratios of these four numbers that we conceive both corporeals and incorporeals" (*Adv. Log.* 1. 99). The Pythagorean *kriterion* of truth is the *logos* deriving from mathematics (*Adv. Log.* 1. 92). It is number because everything is measured by number (*Adv. Log.* 1. 105) and numbers are the first principles and elements of all things (*Adv. Phys.* 2. 248).

Aristotle in the *Metaphysics* complained that the Pythagoreans generate their universe of number-things from the One, but do not explain how it is generated. In Sextus we have an account of its generation in which elements of Platonic, Stoic, and Epicurean theory (*Pyrr. Hyp.* 3. 152) are borrowed. The most characteristic feature of this account is a derivation theory (which for Aristotle is Academic rather than Pythagorean) and that theory is given a peculiar twist. Earlier Pythagoreans, Sextus tells us (*Adv. Phys.* 2. 270–284), generated the numbers from the One and the indefinite dyad. They then generated from numbers point, line,

plane, solid (282). But *later* Pythagoreans assert (281) that physical body is generated from the point which "flows" to produce the line, line *flowing* to produce plane, and plane *flowing* to produce solid.

This fluxion theory is novel, but there can be no doubt that it was current. Sextus alludes to it several times (*Pyrr. Hyp.* 3. 154; *Adv. Log.* 1. 99; *Adv. Phys.* 1. 375; *Adv. Math.* 3. 20). Once, not in the course of discussing Pythagoreanism (*Adv. Math.* 3. 28) he reports a doctrine of the school of Eratosthenes. They hold that the point does not occupy place nor does it measure the breadth of a line: by *flowing* it creates the line. The line is "a flux of the point." This flux variation of the derivation theory may be an attempt to escape the consequences of atomic points and indivisible lines. It may however derive, as Sextus suggests, from the theories of later Pythagoreans. "The Pythagoreans specializing in mathematics" (*Adv. Math.* 4.2)¹⁰ held that "the monad as first principle underlies and produces all other numbers in their constituted form. The dyad produces length. The monad imposes the logos of the point, the dyad that of the line. . . . For in conceiving the line the mind moved from one point to another, and this is length" (*Adv. Math.* 4. 4). The other dimensions have a similar origin. Like the logos of the soul, that of the body is comprised in the number four; and living creatures are, like the cosmos, ensouled in accordance with musical ratios. Here it is the movement of the mind, rather than the flowing of the point, that determines the four dimensions. We are reminded of Aristotle's account of the *autozoon* (*De An.* 404b 16-27) where it is said that knowledge is the two because it moves in one direction towards a single object, and of Xenocrates' definition of the soul as a self-moving number (*De An.* 408b 30-409a 10). At a time when the derivation theory and the definition of the four dimensions was a critical problem (*Adv. Math.* 3. 19) the flux theory may have been suggested by earlier Platonic theories. It appears to have arisen in connection with the diagrams of geometers. We have no grounds for believing that the flux theory, as it is reported by Sextus, had much earlier origins.

Neopythagorean doctrine, as it is reflected in Sextus' account, is a conflation of Stoic, Platonic, and Aristotelian theories. The extent of its earlier Pythagorean content seems questionable. In particular the derivation theory has been modified by Neopythagorean mathematical speculation. The distinction between intelligibles and sensibles has been blurred. The dyad no longer has its Platonic role. The function of the theory is to explain how physical body is generated from first principles.

¹⁰Here we at once think of the *mathematici/acusmatici* division of Pythagoreans of which Iamblichus speaks. But it seems more probable here that the reference is to a contemporary difference. Mathematicians like Moderatus of Gades or Nicomachus must have been separated by a great gulf from Apollonius of Tyana and his followers.

It may, however, be objected that the arguments we have adduced so far are not decisive. We have argued (1) that the problem of the derivation of magnitudes was a critical one for the Early Academy, (2) that the theories evolved in the Academy, and by the Academy loosely called "Pythagorean" were by succeeding generations considered to be of Pythagorean and not of Academic origin, and (3) that the Neopythagoreans ascribed the origins of the theory to their Founder. Even if all this is conceded it may still be argued that the theory, or some form of the theory, originated long before Plato. And this may be argued even if we are unwilling (as indeed criticism is increasingly unwilling) to believe in a Pythagoras who is the source from which flows, in the secrecy of a verbal tradition through a century and a half, most mathematical knowledge. Let us consider how probable it is that such a theory should have evolved in the fifth century and have been merely adopted by Plato.

If it did so evolve it is unlikely that the development occurred in Athens and among Athenians. In the fifth century Athens was a centre of the political and artistic but not of the scientific life of Greece. Though all the great sophists visited Athens and some few of them spent a longer time there, none was Athenian. A Callias or a Pythodorus could—and at great expense to themselves did—entertain eminent visitors such as Parmenides and Protagoras. Anaxagoras was an intimate of Pericles, and each contributed to the other's unpopularity. But the status of these visitors was something like that of the Greek philosophers who later visited or settled in Rome. They were courted, flattered, well-paid. But in one essential respect they were *inferiors*. From politics, the only life in which *arete* could really be displayed, the sophist was excluded. He was a foreigner, and a teacher for pay. The contempt for their profession that Plato expresses must have been shared by others of his class. It was a paradoxical development that Plato himself first created a milieu in which native Athenians could pursue scientific studies; and the first Athenian mathematician of any stature was Theaetetus, himself a pupil of Theodorus of Cyrene but later a member of the Academy.

It does not therefore seem probable that a theory of derivation should have developed in the Athens of the fifth century and simply have been adopted by Plato. There are two other possibilities; the first, that it evolved in Ionia, then the centre of mathematical studies (W. A. Heidel, *AJP* 71 [1940] 1-33); the second, that it was a product of Western Greek speculation. We know that Oinipodes and Hippocrates, both of Chios, visited Athens and that the numerous mathematicians subsequently attracted to the Platonic Academy were predominantly Ionians. It is only Archytas, and a Pythagorean tradition that we have seen reason to regard as suspect, that induce us to think of the West. Archytas, despite his own achievements as a geometer, regarded arithmetic as a more

exact science. None of his writings suggest an interest in physical theory such as might inspire a doctrine of the derivation of magnitudes. However, no investigation of the complex and controversial history of Greek mathematics is likely to provide us with a satisfactory answer to our problem. We must therefore attempt a secondary line of inquiry—a δεύτερος πλοῦς.

In a discussion of the principles of the sciences Aristotle writes (*Anal. Post.* 76a 37–b 5): “The premisses used in demonstrative sciences are some of them common, some of them peculiar to the particular science. Common premisses are common analogically, since their use is restricted to the field of the particular science. An instance of the premisses of a particular science is that the line and the straight line have such and such characteristics. An instance of common premisses is that if equals are subtracted from equals the remainders are equals. It is enough that each of these premisses shall be valid within the field of the particular science, even if it is not assumed to be true of all sciences, but only in geometry of magnitudes, in arithmetic of numbers. Peculiar to a science, and of necessity assumed to exist, are those entities whose essential attributes the science investigates—in the case of arithmetic these are the units, in the case of geometry point and line.”

The first two definitions of the first book of Euclid's *Elements* define respectively the point and the line, and it is obvious that even if in theory the assumption of their existence was not made, without point and line there could be no geometry. If magnitudes were said to derive from point and line (or *a fortiori* from indivisible lines) such a theory could have arisen only in connection with geometrical thinking. At what period is it credible that geometry ceased to be empirical—as when Thales calculated the height of a pyramid—and became a body of knowledge with its characteristic assumptions, methods, proofs? Obviously it did not spring fully armed from the head of any one geometer.

Oinipodes of Chios, a younger contemporary of Anaxagoras (*Vors.* 41.1) appears to have been interested primarily in astronomy. He is accused of having stolen from Pythagoras the notion of the ecliptic (*Vors.* 41.7), the typical device by which the Neopythagoreans annexed for the Master a scientific discovery. As Diels suggests (*Vors.* 41.2 note), it is probably to the ecliptic that allusion is made in the *Erastes* (132A), and it may be that Oinipodes first introduced that notion to Greece. (But cf. O. Neugebauer, *The Exact Sciences in Antiquity*² [Brown UP 1957] 102.) At all events his other geometrical achievements are said by Proclus (*Vors.* 41.13) to have had their application in astronomy.

Hippocrates of Chios is a more serious contender for the title of “Father of Geometry.” He belonged to the generation after Anaxagoras and Oinipodes (*Vors.* 42.1) and it seems probable that Athens was the

scene of his principal scientific activity. He had enunciated a theory of the comets before, but probably not long before, 427 B.C. (W. Burkert, *Weisheit und Wissenschaft* [Nuremberg 1962] 291, n. 76). But his importance for our present purposes lies in the fact that he is said to have written the first *Elements*. “After these (Anaxagoras and Oinipodes), Hippocrates of Chios, the discoverer of the quadrature of the lune, and Theodorus of Cyrene distinguished themselves in geometry . . . Hippocrates wrote a book of *Elements*” (*Vors.* 42.1; Wehrli, fr. 133). This information comes to us through Proclus (*In Eucl. El.* 66.4) from Eudemos' *History of Geometry*, and so from the first generation of the Peripatos. Eudemos of Rhodes, himself a scientist and an historian of the sciences, was a candidate for the succession to Aristotle in the headship of the Peripatos when Aristotle died in 322/1 B.C. The choice fell on Theophrastus, probably a contemporary of Eudemos, who lived until 288/7 B.C. The date of Eudemos' death is not known, but he may well have lived to see the publication of Euclid's *Elements* at about the turn of the century. If so, his mention of Hippocrates' *Elements* acquires added significance, as does the mention of another of Euclid's predecessors, Theudius of Magnesia, who is said to have written “a good book on the *Elements*” and to have “given general application to particular inquiries” (Eudemos fr. 133).¹¹ But even if Eudemos did not see Euclid's

¹¹The generally accepted floruit of Euclid is 300 B.C. (T. L. Heath, *The Thirteen Books of Euclid's Elements*² [Cambridge 1926] 1.2). It is assumed that Eudemos cannot have known his work because the Proclus lemma, deriving however indirectly from Eudemos, is made to end at the words translated by Heath (1.37) as follows: “Those who compiled histories bring the development of this science up to this point.” This is understood to refer to Eudemos. Proclus then continues: “Not much younger than these is Euclid.” On the basis of these two sentences it is argued that Eudemos is chief among those who compiled histories, and that what follows derives from some source other than Eudemos, though Heath (37) concedes that “the style of the summary after this point does not show any such change from that of the former passage as to suggest different authorship.”

The sentence translated by Heath reads in the Greek: οἱ μὲν οὖν τὰς ἱστορίας ἀναγράφαντες μέχρι τούτου προάγουσι τὴν τῆς ἐπιστήμης ταύτης τελείωσιν. It can also be translated: “Those who made written record of mathematical investigations up to this time advance the perfecting of their science.” (And then Euclid. . .) ἱστορία can mean, and in this context is more likely to mean (*LSJ*), some kind of scientific inquiry. Proclus has been listing geometers and their achievements, with special emphasis on publication. Our sentence does not render it impossible that Eudemos should have said: “Thus far . . . and then Euclid.”

It is, however, generally conceded that Proclus has rearranged the material he found in Eudemos (and has probably added the incongruous tribute to Pythagoras). His lemma does not permit us to maintain that Eudemos did in fact know Euclid and looked to Hippocrates as his earliest predecessor, but it does not exclude the possibility that he did. In any case the explicit mention of Hippocrates' *Elements* suggests that they were a landmark in the development of geometry, and probably its first general formulation.

predecessors in the perspective of Euclid the fact that the name *Stoicheia* or *Elements*, with its implications of system, was used to refer to the treatises of Hippocrates of Chios, Leon, Theudius, and Euclid suggests that they were roughly comparable in theme and intention, if not in scope.

Hippocrates, in the Athens of about 430/410 B.C., is the first mathematician known to us who attempted a systematic exposition of the elements of geometry. But even if we had no tradition of Hippocrates' treatise we would not expect that before about the last quarter of the fifth century an attempt could be made to present geometry as a science capable of formulating its first principles and presenting its deductions systematically. Any such development must have been preceded by a period in which geometrical problems, both practical and theoretical, were not necessarily related to one another nor to first principles. It must also have been preceded, or at least accompanied, by an evolution of logical methods such as we find in the fifth century particularly in the Eleatic school. Geometry as a science having *some* organization must have preceded the derivation theory. For we can perhaps imagine the crude number-thing theory of the Pythagoreans of which Aristotle speaks (*Metaph.* 985b 23–986a 21) evolving into a number atomism earlier than the time of Hippocrates, even though our first account of such a theory having scientific pretences ascribes it to Leucippus and Democritus, who were his contemporaries. But a *geometrical* theory of derivation assumes a pre-existing geometry.

If Hippocrates gives us a *terminus post quem* for a geometrical theory of derivation, at what time and in what place is the theory most likely to have arisen? Those who favour a relatively late date often suggest that the theory is a product of the Pythagorean tradition and elaborated in the Tarentum of Archytas. But even if we assume that our accounts of a Pythagorean tradition, though largely deriving from Neoplatonist sources, are in substance true, we must ask ourselves what is the character of early Pythagorean mathematics. Aristotle tells us (*Metaph.* 987b 28) that for the Pythagoreans things were *numbers*, these number-things proceeding in some unexplained fashion from the One. This number was not *χωριστός*, but the constituent of sensibles (1086b 18). "For they construct the whole universe out of numbers." The constructs Aristotle describes (985b 26–986a 13) are altogether unscientific, but they are such as could evolve into an arithmetical theory of derivation, the discrete quantities being atomic magnitudes. If there was in fact some such tradition it enables us better to comprehend the saying of Archytas (*Vors.* 47 B4) that arithmetic is the queen of the sciences. His own principal achievements were apparently in the fields of geometry and of harmonics, and yet he maintains that arithmetic rather than geometry

investigates first principles. He is unlikely to have accepted or to have devised a theory of the derivation of magnitudes having geometrical characteristics.

If there are no good grounds for looking to Magna Graecia as the milieu in which such a theory could develop, we must then consider whether it could have arisen with Plato and the Early Academy. We know:

- (1) that the geometrical presuppositions for a geometrical theory of derivation were present only at about the time of Plato's youth;
- (2) that geometrical speculations, and in general an intense mathematical activity, characterized the Early Academy;
- (3) that geometrical theories of derivation were in fact advanced in the Early Academy;
- (4) that these theories took various forms and occasioned lively debate, suggesting that the theme was a new one;
- (5) that the One and the indefinite dyad were first principles such as might be expected to give rise to a geometrical theory of derivation.

The early history of Greek mathematics is a tissue of fable and conjecture. Facts are few and far between. An inquiry into origins must remain at best a plausible account. It is, I would suggest, a reasonable belief and one that casts light on the evolution of mathematical thinking within the Early Academy, that the point-line-plane-solid theory was devised in that milieu as an essential part of their mathematical and physical doctrines.

But what of Zeno and his paradoxes? Must we not see implied in them a theory of the derivation, or at least of the constitution, of solids such as the theory we are discussing? To this objection we must answer that Zeno is indeed a forerunner, but not therefore a progenitor. He was the first to explore logically problems of space, time, and movement. As such he was "the father of dialectic." His method naturally led to geometrical inquiry, the precondition of any derivation theory. But though for us his problems are mathematical problems, Zeno's approach is logical. In none of the fragments need we assume that he possessed mathematical knowledge, or that he is using a mathematical method. When he uses the word *μέγεθος* (*Vors.* 29B 1, 2), it cannot be given its later and technical sense "magnitude" as it cannot when used by Anaxagoras (*Vors.* 59B 1), or by Gorgias (*Vors.* 82B 3, 73). H. Fraenkel (*Weg und Formen* [Munich 1955] 220–221) treats *μέγεθος*, *πᾶχος*, *δύκος* (B2) as alluding to the three dimensions. There is no justification for this. The words, if they are cited from Zeno, are practically synonymous. But in our fragment they are used by Simplicius, in the passage introducing the lemma, and not by Zeno. If in the "race-course" (Aristotle *Phys.* 239b 34) Zeno used

the word *δῶκος*, he can only have used it as a synonym for *μέγεθος* in the sense of *σῶμα*. But this is Aristotle's own usage (Bonitz, *Index s.v. σῶμα*). And so he is probably paraphrasing. In all the other paradoxes Zeno uses visual objects to illustrate.

However the paradoxes may be interpreted, it is clear that Zeno employed, and employed effectively, the notion of infinite divisibility. But he is not concerned with surfaces and solids, nor with geometrical space. He applies divisibility to lines, and envisages points. His arrow is at a point and a "now." Achilles and the tortoise progress from point to point. In the "dichotomy" lines are traversed at points. But there is no suggestion that the points are atomic points having magnitude; and the implication of infinite divisibility is that the line divided is a continuum. Let us, however, concede that in the fourth paradox or "race-course" he may have envisaged indivisible lines and times (*Phys.* 81-82). If in fact he did so, there is no suggestion that he regarded them as the ultimate constituents of magnitude, though the paradox may have suggested this development to later mathematicians.

The notion of indivisible lines, however, implies that all lines are commensurable in terms of these lines. If Zeno had propounded any such revolutionary theory, we should expect our tradition to allude to it; or at least we should expect to hear some echo of it, perhaps in connection with Theaetetus' surds, in discussions of irrationals, or in the tenth book of Euclid where problems of incommensurability are treated. But there are no such echoes. When first we hear of indivisible lines we hear of them in connection with the derivation theory of the Early Academy, as an answer to difficulties arising within that theory when the problems of points having magnitude were realized.

It seems therefore reasonable to assume that the derivation theory was posterior to the geometrical developments it implies; and that it arose in the milieu in which for some half century it caused lively debate—in the Early Academy.¹²

¹²If, however, we accept that the derivation theory was developed within the Academy (or in allied mathematical circles of the same time) we are not thereby committing ourselves to an esoteric metaphysic such as is constructed by H. J. Krämer in his article "Die Platonische Akademie und das Problem einer systematischen Interpretation der Philosophie Platons" (*Kant-Studien* 55/1 [1964] 69-101) and in his preceding book *Arete bei Platon und Aristoteles* (Heidelberg 1959). For a criticism of such tendencies, and especially of metaphysical structures erected on point-line-surface-solid see K.-H. Ilting's review of K. Gaiser's *Platons Ungeschriebene Lehre in Gnomon* 37/2 (April 1965). The derivation theory is certainly not a foreign body in the thought of the Early Academy. It is not a borrowing from elsewhere that remains unrelated to principal themes. But neither is it a corner-stone having an obvious and necessary place in a metaphysical structure of which we have ground-plan and elevation. Because Plato's later metaphysic is so problematical it has seemed opportune to confine ourselves here to vindicating the derivation theory for the Early Academy.

SOPHIA AND SOPHROSYNE IN EURIPIDES' *ANDROMACHE*

PATRICIA NEILS BOULTER

LIKE SO MANY Euripidean plays, the *Andromache* lacks a central unifying character, and for this reason seems to fall into two parts. Criticism has centred upon this disunity, either attempting to explain it away¹ or simply condemning it as a major flaw in construction.² A frequent explanation of the two-part structure is that it reflects the joining of two separate legends, one concerning Andromache, the other, Neoptolemos and Hermione.³ But in fact, the legendary material concerning Andromache before Euripides' time seems to have been very slight. In the *Iliou Persis* Andromache was given to Neoptolemos after the fall of Troy. Euripides undoubtedly used local material for Thetis' prophecy that Andromache would marry Helenos and that her son Molossos would rule over Epirus.⁴ Between these two given points Euripides freely invented the action of the drama. Similarly with Hermione; earlier writers noted that she had been promised by Tyndareus at the same time to both Orestes and Neoptolemos. Euripides altered this detail for his characterization of Menelaos; in the *Andromache*, Menelaos had himself made the promise to Orestes and then reneged when it seemed expedient (966-970). According to earlier legend Neoptolemos was killed at Delphi, and after his death Hermione was returned to Orestes. Here again Euripides made a slight change, for Orestes enters the story before the death of Neoptolemos and in fact contrives it. Thus the idea that two separate legends were patched together and resulted in a two-part plot structure does not really explain much. Euripides was clearly free to invent his story in whatever way he chose, and even to alter certain details of the legend between the two given points, beginning with the known fact that Neoptolemos took Andromache after the fall of Troy, and ending with his death at Delphi and Hermione's subsequent marriage to Orestes. Therefore we must assume that Euripides deliberately constructed the plot as we have it, and that he did so with a purpose.

Many attempts have been made to find a unifying element that binds the two parts of the play together, but they have met with little success.

¹H. D. F. Kitto, *Greek Tragedy* (London 1939) 230; G. M. A. Grube, *The Drama of Euripides* (New York 1961) 81-82.

²A. R. F. Hyslop, *The Andromache of Euripides* (London 1900) xiii.

³L. Méridier, *Euripide* 2 (Paris 1927) 90-98.

⁴The *Andromache* may well have been written for the young king of Molossia, Tharpylos, and produced at his court. Cf. D. S. Robertson, *CR* 37 (1923) 58-60; Schmid-Stählin, *Geschichte der griechischen Literatur* 3 (Munich 1940) 405.