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A. W. VERBALL.

WHAT LED PYTHAGORAS TO THE DOCTRINE THAT THE WORLD WAS BUILT OF NUMBERS?

ARISTOTLE, when comparing Plato's doctrine of causation with that of the Pythagoreans, states in the familiar passage of the *Metaphysics* (A. 6) that Plato took the Pythagorean doctrine, merely changing the terminology: *τὴν δὲ μέθεξιν τοῦ νομα μόνον μετέβαλεν οἱ μὲν γὰρ Πυθαγορεῖοι μῆσαι τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν, Πλάτων δὲ μετέχει, τοῦ νομα μεταβαλὼν.*

What did Pythagoras mean by the imitation of numbers? First let us ask what kind of numbers does he mean? Did he mean nothing more or less than the modern scientific doctrine that all natural phenomena may be expressed in mathematical formulae? This seems to be reading into Pythagoreanism, the first faltering step towards a scientific theory of the universe, the most advanced doctrines of our own age. Mankind always advances to the abstract from the concrete, and this principle must have prevailed in the first gropings of the early philosophers, as it did and still does in all else. As every one knows, *Arithmos* with the Greeks was far wider in use than our word Number. *Arithmos* included the whole field of mathematics. When Aeschylus represents Prometheus as the discoverer of *Arithmos* for mankind—*ἀριθμὸν, ἔξοχον σοφισμάτων ἑξέσπον*—meaning thereby that he was the founder of all which we call mathematics, he is using the term in its ordinary use among the Greeks of the fifth century. With

Plato geometry and number still run together. The very terminology, as seen in the expressions *ἐπιτεδοὶ ἀριθμοί, στερεοὶ ἀριθμοί*, 'superficial' and 'solid numbers,' is sufficient to prove how indissoluble was the bond between number and geometry proper. When Socrates gives his demonstration of the doctrine of Anamnesis on the slave in the *Meno*, he treats the construction of a square twice the size of a given one in a thoroughly concrete manner. The size of the square and the length of its side are expressed in feet. If Plato finds it so hard to deal with simply abstract or mere numerical numbers, how much more difficult was it for his forerunner, Pythagoras! It is therefore more probable that Pythagoras held that the world was made up of geometrical solids than that he held the modern doctrine. This too is the view held by the chief modern writers who have dealt with Pythagoreanism. Mr. Grote says (*Plato* I. p. 10), 'Numbers were not separate from things (like the Platonic ideas) but mere *fundamenta* of things, their essence or determining principles; they were moreover conceived as having magnitude and active force.'

But there is a passage in the *Timaeus* of Plato which almost puts beyond doubt that Pythagoras held the doctrine that the universe (*τὰ ὄντα*) exists by the imitation of *solid numbers*.¹

¹ Plato, *Tim.* 58-61 C.

Comment Pythagore a-t-il conçu sa doctrine du monde formé avec des nombres [W. Ridgeway]. Pythagore s'est élevé à cette conception de l'univers par la considération des nombres solides, comme il est dit dans le *Timée*, 53-54. γ

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1896

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14 Pythagorica. — Bibliographie crit., par P. Lortzino (1876-1897): JAW EXII 187-223.

*Jahresbericht über die Fortschritte der klassischen
Alterthums Wissenschaft*

VV48

212. W. R. Ridgeway, What led Pythagoras to the doctrine, that the world was built of numbers? Class. Review X (1896) S. 92-95.

Eine originelle, aber sehr gewagte Vermutung über den Ursprung der Zahlenlehre des P. spricht Ridgeway aus. Indem er es, ohne jedoch zwingende Gründe dafür beizubringen, als wahrscheinlich hin- stellt, daß die kosmischen Theorien in den beiden Platostellen Tim. 58-61C und Phädon 108D-111C auf pythagoreischer Grundlage be- ruhen, schließt er aus der ersten, daß nach P. die Welt aus geometri- schen festen Gebilden (geometrical solids) besteht (vgl. Platons $\epsilon\pi\iota\kappa\tau\epsilon\iota\sigma\iota$ und $\sigma\tau\epsilon\phi\alpha\iota\ \delta\iota\sigma\tau\upsilon\lambda\omicron\varsigma$), und aus der zweiten, daß P. durch die Beobachtung der mathematischen Gestalt natürlicher Krystalle, insbesondere der Edelsteine wie Jaspis und Smaragd, die in Platons mythischer Dar- stellung als Überreste der glänzenden und reinen Steine einer herr- lichen, von uns nicht gesehenen Erde geschildert werden, zu der Meinung geführt wurde, die Welt sei aus Zahlen aufgebaut. Damit sei zugleich der Schlüssel dafür gegeben, was P. unter „Nachahmung der Zahlen“ (Aristot. Metaph. 987b 11) verstanden habe. Bestätigt werde diese Annahme durch die Notiz bei Laert. VIII 1, daß der Vater des P. ein Steinschneider gewesen sei (?). Vielleicht habe P. sogar ur- sprünglich dasselbe Geschäft betrieben. In Ägypten habe er dann seine krystallographischen Kenntnisse mit denen der ägyptischen Geometrie kombiniert und so die Welt erkannt als aufgebaut aus einer Reihe materieller Körper, die geometrische Körper nachahmen; er habe also einen materiellen Ursprung der Welt mit dem formalen geometrischen Element verbunden. Daher (?) komme der Zweifel des Aristot., ob die pythagoreische $\delta\epsilon\upsilon\chi\eta$ materiell oder formal sei. Schließlich zeigt R., daß P. Krystalle in Pyramiden-, Kubus- und Dodekaederform gekannt haben kann, während das Eikosiäedron (s. Platons Tim.) nicht in der Natur vorkommt. Auch für die Zahl 24, auf die die Pythagoreer großen Wert legten, gab es ein Prototyp in der Natur. — Diese Hypothese ist ein geistreicher Einfall, veranlaßt durch eine gelegentliche Bemerkung in einer phantastischen kosmologischen Konstruktion Platons, von der aber durchaus nicht feststeht, daß sie auf die Pythagoreer oder gar auf P. selbst zurückgeht.

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¹ Plato, *Tim.* 53-61 C.

Plato there enumerates the several varieties of each element, fire, water, earth: he then proceeds to mention the attributes. The Demiurgus brought the four elements out of confusion into definite bodies and regular movements. He gave to each a body constructed upon the most beautiful proportions of arithmetic and geometry as far as this was possible.¹ Respecting such proportions the theory which Plato here lays out is admitted by himself to be a novel one, but it is most probably borrowed with more or less modification from the Pythagoreans. Every solid body is circumscribed by plane surfaces; every plane surface is composed of triangles: all triangles are generated out of two—the right-angled isosceles triangle, and the right-angled scalene or oblong triangle. Of this oblong there are infinite varieties, but the most beautiful is a right-angled triangle having the hypotenuse twice as long as the lesser of the two other sides (*Tim.* 53–54).

From this sort of oblong triangle are generated the tetrahedron or pyramid, the octahedron, and the eikosihedron; from the equilateral triangle is generated the cube. The cube, as the most stable and solid, was assigned by the Demiurgus for the fundamental structure of earth; the pyramid for that of fire; the octahedron for that of air; the eikosihedron for that of water. Lastly the dodekahedron was assigned as the basis of structure for the spherical Kosmos itself, or Universe. Upon this arrangement, each of the three elements—fire, water, air—passes into the other; being generated from the same radical triangle. But earth does not pass into either of the three, nor either of these into earth, being generated from a different radical triangle. The pyramid, as sharp and cutting, was assigned to fire as the quickest and most piercing of the four elements; the cube, as the most solid and difficult to move, was allotted to earth, the stationary element. Fire was composed of pyramids of different size, yet each too small to be visible by itself, and becoming only visible when grouped together in masses; the earth was composed of cubes of different size, each invisible from smallness; the other elements in like manner each from its respective solid in exact proportion and harmony, as far as necessity could be persuaded to tolerate. All the five regular solids were thus employed in the configuration of the new

structure of the Kosmos. I have given Mr. Grote's summary of chapters xix.–xxi. of the *Timæus*: as he has no thesis to prove such as I have in view, his statements will be free from all suspicion of being *ex parte*.

The notion that the Kosmos itself is a spherical dodekahedron naturally suggests another passage of Plato still more familiar than that of the *Timæus*.

In the *Phædo* (chapp. lviii. lix. § 109 *seq.*) Plato gives us a set of kosmical views, which are again based on Pythagorean doctrines.

If one could look down on the earth from space, it would appear just like a ball made up of twelve pieces of leather (*ὥσπερ αἱ δωδεκάσκντοι σφαῖραι*), variegated, picked out with colours, of which the colours known here are samples.

He then describes at length the glories of that unseen region, enumerating the various hues, such as gold and purple and blue, which it presents; he proceeds to describe the perfection of things, then their perfect purity and freedom from all corruption, and finally the structure of the earth itself is described—‘the mountains in like fashion and the stones in similar proportion possess both a smoothness and a transparency and colours more beautiful than those here; and of these the little stones in this world, the precious stones, are parts, such as sards and jaspers and smaragdi’ :—

τὰ ὄρη ὡσαύτως καὶ τοὺς λίθους ἔχειν ἀνὰ τὸν αὐτὸν λόγον τὴν τε λειότητα καὶ τὴν διαφάνειαν καὶ τὰ χρώματα καλλίω· ὃν καὶ τὰ ἐνθάδε λιθίδια τὰ ἀγαπώμενα μόρια· οἷον σάρδια τε καὶ ἰάσπιδας καὶ σμαράγδους.

Plato argues thus from the most beautiful, most pure, and most imperishable of all things in this world to substantiate his doctrine of the unseen world. The natural crystals are indeed the most perfect and most enduring of all things that we know.

In later times the writer of the Apocalypse forms his conception of the Holy City, the New Jerusalem, on the same analogy. The foundations of the city were garnished with all manner of precious stones, the first a jasper, the second sapphire, the third a chalcedony, the fourth an emerald, the fifth sardonyx, the sixth sardius, etc.

As Plato follows Pythagoras in the *Timæus*, so also he seems to be following him in the *Phædo*. The doctrine of the Transmigration of Souls embedded in this same description is beyond doubt Pythagorean. Moreover it is generally agreed

¹ *Timæus* 53.

that Pythagoras was the founder of the doctrine that the earth is a sphere, and to the Pythagoreans must be ascribed the first use of the word Kosmos in the sense of an ordered universe.

The key to what Pythagoras meant by saying that τὰ ὄντα had their existence by the imitation of numbers seems to be given us here. The great mass of the earth's crust which we see around us is corrupt, and formed of amorphous matter, the rocks and stones are eaten away by the impure atmosphere and the brine of the sea. Were it not for these agencies we might see them in glorious intact forms and colours. There are certain objects however which lead us to this conclusion, the little stones called precious stones which are fragments of those diaphanous stones of perfect purity of which the unseen region is wholly compact. Is it overbold to suggest that Pythagoras from observing the perfect mathematical shapes of natural crystals was led to the conception that the world was built of numbers? If the objection is raised that it is a groundless assumption to suppose that Pythagoras ever had his attention called to any such objects as natural crystals, my answer is not far to seek. Diogenes Laertius says (viii. 1), Pythagoras was the son of the Samian Mnesarchus, a *signet-engraver* (δακτυλιόγλυφον). Thus above all men Pythagoras had the shapes of precious stones forced upon his attention from his earliest days. We are not told anywhere that he was himself brought up to the same trade as his father, but from our knowledge of the way in which arts and trades were hereditary in Greece, as they are at this day in Oriental countries, we may not unreasonably conjecture that he was brought up to his father's trade, though he may have abandoned it when he came to manhood.

That he would have approached the treatment of philosophy under the influence of his boyish training is rendered highly probable by the analogous case of Socrates. The latter introduces references and analogies borrowed not only from the trade of his father, Sophroniscus the statuary,¹ but also from the calling of his mother Phaenarete the midwife.²

If any fact in the life of Pythagoras is well attested, it is that he went to Egypt, and there studied mathematics. Geometry was the branch of that subject which was the creation of the Egyptians. Combining

then his knowledge of crystallography gained from his father's trade with that of Egyptian geometry, Pythagoras conceived the world built up of a series of material bodies imitating geometrical solids.

Aristotle is in doubt as to whether the Pythagorean cause is material or formal.³

The view that I have put forward explains this doubt; for the Pythagorean cause is material, combined with the formal element of geometry.

Plato mentions the pyramid, the octahedron or double pyramid, the eikosihedron, the cube, and the dodekahedron. Let us see what crystals suggesting such forms Pythagoras could have seen. An ordinary form of quartz crystal would give him a perfect pyramid and a double pyramid. The quartz crystal has been in use among primitive men everywhere as an amulet and ornament from the earliest times. There are many Assyrian cylinders made of it and, what is still more to our purpose, it was regularly used by the Greeks who engraved that class of signet known as the Island gems.⁴

Iron pyrites is widely diffused and was certainly known to the Greeks. It is found in cubes massed together.

Theophrastus (*Lap.* § 14) most probably alludes to it. Galena ore has been found in great quantities in the ancient mines of Laurium. This substance crystallizes in cubes.

Fluor spar exhibits the same form of crystallization, though I am not aware that any archaic Greek gems made of it have been brought to light. Assyrian cylinders made of this substance are known.

The dodekahedron is found in nature in the common garnet. This was a stone well known to the Greeks and held in high favour both in the noble kind, which came from Carthage and Massilia, and also in the common coarse varieties which were found in Greece itself, both at Orchomenus and in the island of Chios (Theophrastus, *Lap.* §§ 18 and 33). It was so highly esteemed that Theophrastus devotes a special section to it, just as he does to the smaragdos. Both of these are placed at the head of his list of stones used by the engravers for signets.

That the engravers of Samos were well

³ *Metaph.* A. 6. This I owe to my friend Dr. Jackson.

⁴ *British Museum Cat. of Gems*, Nos. 33, 57, 72. There is an early scaraboid gem in rock crystal in the Fitzwilliam Museum (No. 5).

¹ Plato, *Euthyphro* 11 C.

² *Ib.*, *Theaetetus* 161 E.

acquainted with the *smaragdos*, a term which included down to the time of Theophrastus (315 B.C.) all the three kinds of the same beautiful crystal, the beryl, the emerald, and *aqua marine*—is put beyond doubt by the fact that the renowned signet of Polycrates, the tyrant of Samos (560–522 B.C.), which he cast into the sea to avert Nemesis, was a *smaragdos* engraved for him by the famous sculptor and engraver, Theodorus of Samos (Herod. iii. 41). The beryl was found in Cyprus, as we learn from Theophrastus (*op. cit.* 26), who alludes to the beautiful cylindrical hexagons in which it is found as rods (*ῥάβδοι*). The Greeks used these elegant natural crystals as earrings. Such have been found in Cypriote graves. Long cylindrical beads of emeralds and beryls have been found in the archaic tombs of Rhodes.

As Theophrastus certainly knew the difference between crystalline and amorphous substances, there can be no reasonable ground for doubting that the engravers of archaic gems must have learned very early this difference. In fact it is absolutely certain that the observation of such a difference must have been first made by those whose profession it was to seek after crystals.

I have purposely left to the last the eikosihedron of the *Timaeus*. No such crystalline form is known in nature. It is strange that Plato should have taken a number which gives no relation to the octahedron. The Pythagoreans held the number 24 of great value. It was the product of $1 \times 2 \times 3 \times 4$, just as the sum of these first four digits was 10. If Plato had taken a 24-sided figure, it would have been in relation to 4 and 8 (the pyramid and double pyramid), and it would have had a prototype in nature. But for our purpose it is unnecessary to discuss what Plato meant. With him the mathematical side was completely detached from the natural phenomenon, the observation of which had probably led Pythagoras to conceive that the world existed by the imitation of natural crystals.

Imitation was an excellent term to employ. Every one conversant with crystallography knows how frequently crystals are mis-shapen, the facets irregular. Pythagoras as a practical engraver could not help observing this and feeling that they frequently were not perfect mathematical solids, but attempted imitations of such, more or less imperfect.

WILLIAM RIDGEWAY.

THE BATTLE OF MARATHON.

THE second volume of the excellent English translation of Holm's *History of Greece*¹ contains some of the best work of the historian. When we come into the clear field of historical fact, Holm's narrative and exposition are masterly. It is in the dimmer regions where we find anecdote, legend, and history mixed that he is less satisfactory; and his first volume is the weakest of the four. The weakness consists in a certain credulous caution, if I may use the expression, in dealing with such a source, for example, as Herodotus. His excessive distrust of scepticism leads him into distrust of criticism. This defect is illustrated in vol. ii. in the account of the Persian war. The narrative of the campaign of Marathon given by Herodotus is simply reproduced by Holm, without any adequate recognition of the difficulties besetting that narrative,

in which the Persians are represented as acting like children. Any one who reads critically the Herodotean account must see that Herodotus had not the smallest idea why the battle was fought, and had a very inadequate notion of how it was fought. He has collected a number of details, some true, others absurd; which, as he relates them, are without any inner connexion.

In his extremely interesting and important historical studies on Herodotus (vol. ii. of his recent edition of Books iv., v., vi.) Mr. Reginald Macan has devoted a hundred pages to an elaborate examination of the problems connected with Marathon. He has not only done good service by his minute criticism of all the extant evidence, but he has made a distinct contribution to the reconstruction of the battle.

The first important step was taken by Leake who saw that the Athenian camp was near Vrana, at the mouth of the valley of Avlona; and this discovery was reinforced

¹ *History of Greece*, by Adolf Holm. Translated from the German. Vol. ii. The Fifth Century B.C. London and New York: Macmillan. 1896. Price 6s.