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BY FRANK EGLESTON ROBBINS

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generally been accepted as final.¹ More may be drawn from the study of Sextus, however, than Schmekel took account of.

In the first place, Schmekel used the passage designated almost to the exclusion of another in the same work, iv. 2 ff., which, since it agrees practically word for word with vii. 91 ff. throughout much of its length, is obviously ultimately derived from the same source. A proper diagnosis of the situation surely demands, first of all, that the two passages be examined side by side and their meaning analyzed;² hence one may infer whence each came and for what purpose it was written. If we do this, and take iv. 2 ff. first, we find that its outline is as follows: At the very beginning the general theme of the passage is stated, which is simply that the Pythagoreans ascribed great power to numbers in the government of the universe; this caused them to use the expression "all things are like to number," and in their well-known oath by Pythagoras to speak of the tetraktys as the "source that holds the roots of everlasting nature." The following sections develop the implications of this epithet and are therefore thoroughly consistent with the central theme of the passage; the numbers of the tetraktys, 1, 2, 3, 4, in their capacities as point, line, surface, and solid, respectively, represent all bodily natures, and, furthermore, soul; for since 1, 2, 3, and 4 contain the fundamental concords they control the harmony whereby the world is ordered and the soul constituted. This passage, then, is thoroughly consistent within itself, its main thesis is kept in view throughout and supported by argument, and its logical structure is admirable.

The other, vii. 91 ff., treats an entirely different theme, the criterion. It may be summarized thus: Anaxagoras declared that the criterion was λόγος, using the term in a general way; but the Pythagoreans restrict it to the mathematical λόγος, because, as Philolaus said, if it takes cognizance of the universe it must be akin

¹ Cf. G. Borghorst, *De Anatolii fontibus*, Berlin, 1905; G. Altmann, *De Posidonio Timaei Platonis commentatore*, Berlin, 1906; Gronau, *Poseidonios und die jüdisch-christliche Genesisexegese*, Berlin, 1914, p. 197, n. 1.

² Zeller, *Philosophie der Griechen*, III, 1, p. 514, n. 5, concludes from such a comparison that the Pythagorean element in *Adv. math.* vii, 92 ff. is not from Posidonius. Against this view Schmekel argues, *op. cit.*, p. 407, n. 1. A proper comparative analysis of the two passages has not, however, been made.

to the universe, on the principle of the perception of like by like; Empedocles, who is quoted,¹ clearly states this principle. And as the light is perceived by the kindred sight, so Posidonius says in his *Commentary on the Timaeus*, and sound by the hearing, which is akin to air, so the universe ought to be apprehended by a λόγος of the same nature. But number is the source of the universe, wherefore the λόγος that is the criterion of all, not without its share in this, would be called number. It is to declare this that the Pythagoreans often say "all things are like to number" and use their accustomed oath. Its author was Pythagoras; the tetraktys is 10, or 1+2+3+4, which is the "source of everlasting nature," first, because it contains the basic concords; second, because it contains point, line, surface, and solid; furthermore, because, all things being numerable, number is fundamental. This is the outline of the passage; I have more carefully reproduced the first portion in order to make clear the logical connection; but the last part, from the mention of the Pythagorean sayings, is that which is closest to iv. 2 ff., and no argument is needed to show that ultimately the two must have been derived from the same source.

Now it must be equally clear, it seems to the writer, that there are signs of disjointedness and of compilation in the second, but not in the first, passage. For in the former there is one theme only, and that sustained consistently throughout; in the latter we begin with the criterion and then abandon it for a detailed discussion of certain Pythagorean theories about number, the transition being of the most formal character. The last portion of vii. 91 ff. leaves the impression of being an argument, already existent and ready to hand in the work of some earlier author, awkwardly forced into a new context by the writer of the part about the criterion, and betraying its alien origin by the noticeable lapse in logical connection. This conclusion commends itself the more strongly because we know iv. 2 ff. also; having the latter, and seeing in it none of the marks of patching together, but on the contrary an admirable unity of construction, we must conclude that in iv. 2 ff. we have Sextus' quotation of certain material in its original form and used as it was

¹ Fragment 109, Diels; cited also by Aristotle *De anima* A 2 404 b 8; *Metaphysica* B 4 1000 b 5; and by Chalcidius in *Plat. Tim.* c. 50.

originally intended to be used, whereas in vii. 92 ff. he has employed a passage of another author who had himself cited this same material.

There is still further proof that the Pythagorean part of vii. 92 ff. existed in another form (i.e., that of iv. 2 ff.) before it was taken into its present context; not only that, but we may infer the original purpose of its composition. This is afforded by the discovery of a probably identical origin for *Adv. math.* iv. 2 ff. and two passages of Theon of Smyrna and Anatolius respectively.

Sextus Empiricus *Adv. math.* iv. 2 ff.:

καθόλου μὲν οὖν οἱ ἀπὸ τῶν μαθημάτων Πυθαγορικοὶ μετὰ τὴν ἀποτέμνουσιν δυνάμιν τοῖς ἀριθμοῖς ὡς τῆς τῶν δυνάμιν φύσεως κατ' αὐτοὺς διοικουμένης. ὅθεν καὶ αἰεὶ ποτε ἐπεφώνουν τὸ "ἀριθμῶ δέ τε πάντ' ἐπέοικεν," ὁμνύοντες οὐ μόνον τὸν ἀριθμὸν ἀλλὰ καὶ τὸν ὑποδείξαντα αὐτοῖς τοῦτον Πυθαγόραν ὡς θεὸν διὰ τὴν ἐν ἀριθμητικῇ δυνάμιν, λέγοντες "Οὐ μὰ τὸν ἀμετέρῳ ψυχῇ παραδόντα τετρακτὸν, Πηγὴν ἀενάου φύσεως ριζώματ' ἔχουσιν." τετρακτὸς δὲ προσήγορέετο παρ' αὐτοῖς ὁ ἐκ τῶν πρώτων δ' ἀριθμῶν

Theon, p. 99, ll. 8-23:¹

ὁ δὲ καὶ ἐκ τῶν τετρακτῶν τούτων συστάς κόσμος ἔσται <τέλειος> ἡρμοσμένος κατὰ γεωμετρίαν καὶ ἁρμονίαν καὶ ἀριθμὸν, δυνάμει περιεληφώς πᾶσαν ἀριθμοῦ φύσιν πᾶν τε μέγεθος καὶ πᾶν σῶμα ἀπλοῦν τε καὶ σύνθετον, τέλειός τε, ἐπειδὴ τὰ πάντα μὲν τούτου μέρη, αὐτὸς δὲ οὐδενός. διὸ πρότερον τῷ ἐξημένῳ ὄρκῳ οἱ Πυθαγορικοὶ ἐλέγοντο³ . . . καὶ ἀριθμῶ δέ τε πάντ' ἐπέοικε. καὶ τοῦτο εἶναι τὸ σοφώτατον· πάντα μὲν γὰρ τὸν ἀριθμὸν εἰς δεκάδα ἡγαγον, ἐπειδὴ ὑπὲρ δεκάδα οὐδεὶς ἔστιν ἀριθμός, ἐν τῇ αὐξήσει πάλιν ἡμῶν

Anatolius, p. 29, 1-10:²

ἡ δεκάδος καὶ τῶν ἐν τῇ αὐτῇ ἀριθμῶν φύσει μυρία μὲν περιέχεται καὶ ἐπιδείκνυσιν κάλλη τοῖς ὀξυδερκῶς τῷ νῶ καθορᾶν τὰ τοιαῦτα δυνάμενοις. ὅσα μὲν οὖν ἔαν οἶδον τε ἢ λέξομεν ἐφ' ἐκάστου τῶν ἀριθμῶν, τοσοῦτον δὲ προλέγομεν νῦν,

ὅτι οἱ Πυθαγορεῖοι τὸν πάντα ἀριθμὸν εἰς δέκα ἀνήγον, ὑπὲρ δέκα δὲ οὐδεὶς ἔτι ἀριθμός, ἐν πάσῃ αὐξήσει πάλιν ἡμῶν ἐπι-

¹ *Expositio rerum mathematicarum ad legendum Platonem utilium*, ed. Hiller, Leipzig, 1878. The work is generally acknowledged to be a compilation. Schmekel, *Op. cit.*, p. 410, believes this passage to come from Thrasyllus; so also Switalski, *Des Chalcidius Kommentar zu Platos Timaeus*, Beiträge z. Gesch. d. Phil. d. Mittelalters, III, 6, Münster, 1902, pp. 85-86; Borghorst, *op. cit.*, pp. 17 ff., assigns it to Moderatus, and Altmann, *op. cit.*, pp. 19 ff., to Adrastus.

² Ed. J. L. Heiberg, *Annales internationales d'histoire, Congrès de Paris, 1900, 5^e section, histoire des sciences*, Paris, 1901. The title is *περὶ δεκάδος καὶ τῶν ἐν τῇ αὐτῇ ἀριθμῶν*. Many excerpts from it appear in the anonymous *Theologumena Arithmeticae*, ed. Ast, 1817.

³ Theon has already quoted the oath given by Sextus in p. 94, ll. 6-7, a passage which contains probably more of the material found in *Adv. math.*, IV, 2 ff. For other citations of this oath cf. Hiller, *ad loc.* Delatte, *op. cit.*, p. 250, note. On the evident textual difficulties at this point see Hiller, *ad loc.*

Sextus:

συγκείμενος ἰ' ἀριθμός·
 α' γὰρ καὶ β' καὶ γ' καὶ δ'
 ἰ' γίνεται· ὅς ἐστι τελει-
 ότατος ἀριθμός, ἐπείπερ ἐπ'
 αὐτὸν φθάσαντες πάλιν
 ἀναλύομεν ἐπὶ τὴν μονάδα
 καὶ ἐξ ὑπαρχῆς ποιούμεθα
 τὰς ἀριθμήσεις. πηγὴν
 κτλ. . . . εἰρήκασιν αὐτὸν
 διὰ τὸ κατ' αὐτοὺς ἐν αὐτῷ
 τὸν λόγον τῶν ἀπάντων
 κείσθαι συστάσεως, ὅσον εὐ-
 θέως τοῦ τε σώματος καὶ
 τῆς ψυχῆς

Theon:

ὑποστρεφόντων ἐπὶ μονά-
 δα καὶ δυνάδα καὶ τοὺς ἐξῆς·
 τὴν δὲ δεκάδα ἐπὶ τετράδα
 συνίστασθαι· ἐν γὰρ καὶ
 β' καὶ γ' καὶ δ' ἐστι ἰ',
 ὥστε τοὺς δυνατωτάτους
 ἀριθμοὺς ἐντὸς τῆς τετράδος
 θεωρεῖσθαι.

Anatolius:

στρεφόντων ἐπὶ μονάδα
 μετὰ τὸ συμπληροῦσθαι
 πᾶσαν δεκάδα· ἀλλὰ καὶ
 ὅτι ἐκ τετράδος συνίστα-
 ται ἡ δεκάς εἰς τὰ μάλιστα
 τὴν τετρακτὺν ἐτίμων.

That all three of these passages occur in different contexts is the reason why there is divergency among them. No two contain exactly the same material, yet by pairs they show close agreement, and the same thought is implicit in all three. The close similarity of the latter parts of Anatolius and Theon leaves no doubt that both go back to the same ultimate source,¹ and their differences are to be ascribed to the position of the passage in Theon, as a link between the discussion of the tetraktys and that of the first decade, and to Anatolius' tendency to condense, noticeable throughout his whole treatise. He has here left out all mention of the Pythagorean oath and the other saying,² but his own last words, "they honored the tetraktys especially,"³ its inclusion in both Sextus and Theon, and the fact that his statements about the Decad have point only as an explanation of the oath by the tetraktys, make it hardly doubtful that this is his own arbitrary omission. As for Sextus, his passage contains all the thought of the other two—the two Pythagorean sayings: the notion that 10 is made up of 1, 2, 3, and 4, the tetraktys;

¹ If there were any doubt, the verbal correspondence of Theon and Anatolius in the other passages dealing with numbers would dispel it.

² This, however, is due only to the desire to condense, for the saying is found in a collection of excerpts from Anatolius, in this form: *ὅτι τὴν ἀριθμητικὴν οὐ μόνος ἐτίμα Πυθαγόρας, ἀλλὰ καὶ οἱ τοῦτον γνώριμοι ἐπιλέγοντες 'Ἀριθμῷ δὲ τε πάντ' ἐπέκειν* (see F. Hultsch, *Heronis Alexandrini Geometricorum et stereometricorum reliquiae*, Berlin, 1864, p. 279).

³ This phrase, and the concluding phrase of Theon, should be compared with the end of the passage of Sextus, which is quoted below, p. 318.

that 10 includes all the numbers (he says that it is "most perfect"), because we find no new ones in counting above it. The order of the last two is reversed in Anatolius and Theon as compared with this, and in what follows the part quoted. Sextus elaborates further the powers of the numbers of the tetraktys. In general, we may conclude that we have here two versions of the same thing, a longer one represented by Sextus, and a condensed one in Theon and Anatolius, the latter having still further condensed on his own account.¹ Possibly more material that originally belonged in this context and is still seen there in Sextus is to be found in Theon's foregoing discussion of the tetraktyes.² To make the relationship even more probable, we shall see later that in another section Anatolius agrees word for word with Sextus in the latter's continuation of the citation above.

As to the original nature of the passage, the clue, I believe, is given by Theon and Anatolius. In each it appears as the introduction to a discussion of the numbers of the first decade; in Theon this is not so apparent, since it also happens to conclude the section about the tetraktyes, but Anatolius' employment of it should be taken as the proper one. This conclusion will recommend itself if one simply reads over the Theonian context and notes the plain evidence of its disjointedness, and furthermore that in citing these words Theon is led to make an awkward repetition of statements already introduced relative to the first tetraktys.³ We have, then, in this passage the introduction to a Pythagorean treatise on the first ten numbers, and surely nothing could be more appropriate for this purpose, from the Pythagorean standpoint, than a forceful exposition of the general powers of number in the cosmos, as epitomized in 10 and the tetraktys.

¹ There are other versions of the material presented in the three passages quoted. The original arithmological document, of which I assume these to cite the preface, apparently repeated most of the statements later, in the chapters devoted to 4 and 10, and from these probably are derived, as far as any connection exists, the reports of Philo *De mundi opificio* cc. 15-16, Lydus *De mensibus* iii. 4, iv, 64, Wünsch, Chalcidius in *Tim.* c. 35, Hierocles in *carm. aur.* c. 20, Martianus Capella, Favonius, etc.

² See p. 312, n. 3, above.

³ Cf. Theon, p. 99, ll. 21-23, with p. 93, ll. 19 ff., Hiller.

In order to clarify the situation still further, the words of Sextus immediately after what has been quoted may be compared with the following:

Sextus *Adv. math.* iv. 4
(p. 722, 15 Bekker.):

ἡ μὲν οὖν μονὰς ἀρχὴ τις ὑποκεῖται τῆς τῶν ἄλλων ἀριθμῶν ἀπεργαστικῆ συνστάσεως, ἡ δὲ δυὰς μήκους ἐστὶ ἀπεργαστικῆ. καθάπερ γὰρ ἐπὶ τῶν γεωμετρικῶν ἀρχῶν ὑπεδείξαμεν¹ πρῶτον τίς ἐστὶν ἡ στιγμὴ, εἴτα μετ' αὐτὴν ἡ γραμμὴ μήκος ἀπλατὲς τυγχάνουσα, τὸν αὐτὸν τρόπον καὶ ἐπὶ τοῦ παρόντος ἡ μὲν μονὰς τὸν τῆς στιγμῆς ἐπέχει λόγον, ἡ δὲ δυὰς τὸν τῆς γραμμῆς καὶ τοῦ μήκους· ποθὲν γὰρ ποῦ ἐφέρετο ὁ νοῦς, καὶ προστιθεμένης τῇ κατὰ μήκος διαστάσει τῆς κατὰ πλάτος διαστάσεως ἐπιφάνεια νοεῖται, ἀλλὰ κἂν ἐπιθεωρήσῃ τις τῇ τριάδι τέταρτην μονάδα, τουτέστιν τέταρτον σημείον, γίνεται πυραμὶς, στερεὸν σῶμα καὶ σχῆμα· καὶ γὰρ μήκος ἔχει καὶ πλάτος καὶ βάθος· ὥστε ἐν τῷ τέσσαρα ἀριθμῷ τὸν τοῦ σώματος περιέχεσθαι λογον.

Philo *De mundi opificio* 16:

πρῶτῃ γὰρ αὕτῃ τὴν τοῦ στερεοῦ φύσιν ἔδειξε, τῶν πρὸ αὐτῆς ἀριθμῶν τοῖς ἀσώματοις ἀνακειμένων. κατὰ μὲν γὰρ τὸ ἐν τάττεται τὸ λεγόμενον ἐν γεωμετρικῇ εἶναι σημείον, κατὰ δὲ τὰ δύο γραμμῇ, διότι φύσει ἐκ σημείου συνίσταται. ἡ γραμμὴ δὲ ἐστὶ μήκος ἀπλατὲς· πλάτους δὲ προσγενομένου γίνεται ἐπιφάνεια, ἣ τέτακται κατὰ τριάδα· ἐπιφάνεια δὲ πρὸς τὴν στερεοῦ φύσιν ἐνὸς δεῖται τοῦ βάθους, ὃ προστεθὲν τῇ τριάδι γίνεται τετράς. . . . ὃ δὲ μὴ συνιεῖς τὸ λεγόμενον ἐκ τινος παιδιᾶς εἴσεται πάνυ συνήθους. οἱ καρνατίζοντες εἰώθασι τρία ἐν ἐπιπέδῳ προτιθέντες κάρνα ἐπιφέρειν ἐν, σχῆμα πυραμοειδὲς ἀπογεννῶντες. τὸ μὲν οὖν ἐν ἐπιπέδῳ τρίγωνον ἵσταται μέχρι τριάδος, τὸ δὲ ἐπιτεθὲν τετράδα μὲν ἐν ἀριθμοῖς, ἐν δὲ σχήμασι πυραμίδα γεννᾷ, στερεὸν ἥδη σῶμα.

Anatolius, p. 32, 3 Heiberg:

πρῶτος ἔδειξε τὴν στερεοῦ φύσιν·

σημείον γάρ,
εἴτα γραμμῇ,

εἴτα ἐπιφάνεια,

εἴτα στερεόν, ὃ ἐστὶ σῶμα.

τοῦτο τῶν καρνατίζοντων παιδιὰ, ποιούσα σχῆμα πυραμίδος.

The passage of Anatolius comes, not from the introduction, but from the chapter on the tetrad, and there is a close parallel to it in Johannes Laurentius Lydus *De mensibus* iv. 64 (Wünsch).² There can be no doubt that Philo, Anatolius, and Lydus ultimately go

¹ This is probably a reference to *Adv. math.* iii. 19 ff.

² πρῶτος οὖν τετράγωνος ἀριθμὸς οὗτος καὶ τετρακτὺς, ἀλλὰ μὴν καὶ πρῶτος ἔδειξε τὴν στερεὰν φύσιν· σημείον γάρ, εἴτα γραμμῇ, εἴτα ἐπιφάνεια, εἴτα στερεόν, ὃ ἐστὶ σῶμα. Theon, p. 101, 11 Hiller has so abbreviated his notice of the number 4 that little can be made of the parallel passage: ἡ δὲ τετράς στερεοῦ ἐστὶν εἰκὼν, πρῶτός τε ἀριθμὸς [καὶ] τετράγωνός ἐστιν ἐν ἀρτίοις, κτλ.

back to an identical source; the former, however, apparently used it in an unabridged form, while Anatolius and Lydus both drew from an epitomized version. Without delaying to speculate upon these matters, in support of which further arguments could be stated, let us note that, although Sextus treats of similar topics, the form of his statement is very different from that of the other three. This is probably because he drew from the introduction, the others from the body, of the common source.

Comparing the immediately succeeding words of Sextus, again, with others of the arithmological writers, a somewhat different situation appears:

Sextus <i>Adv. math.</i> iv. 6 ff. (Bek- ker, p. 723):	Anatolius, p. 32, 15 Heiberg:	Philo, <i>De mundi</i> <i>opificio</i> 15:	Chalcidius, <i>Comm.</i> <i>in Tim.</i> c. 35:
καὶ μὴν καὶ τὸν τῆς ψυχῆς· ὥς γὰρ τὸν δλον κόσμον κατὰ ἀρμονίαν λέγουσι δι- οικεῖσθαι, οὕτω καὶ τὸ ζῶον ψυχοῦσθαι. δοκεῖ δὲ ἡ τέλειος ἀρμονία ἐν τρισὶ συμφωνίαις λαμβά- νει τὴν ὑπόστασιν, τῇ τε διὰ τεττάρων καὶ τῇ διὰ πέντε καὶ τῇ διὰ πασῶν. ἡ μὲν οὖν διὰ τεσσά- ρων ἐν ἐπιτρίτῳ κεί- ται λόγῳ, ἡ δὲ διὰ πέντε ἐν ἡμιολίῳ, ἡ δὲ διὰ πασῶν ἐν δι- πλασίονι. ἐπιτρι- τος ² δὲ λέγεται ἀριθμὸς ὁ ἐξ δλου	οὐ μόνον δὲ τὸν τοῦ σώματος ἐπέχει λό- γον ἐν ἀριθμοῖς τε- τράς, ¹ ἀλλὰ καὶ τὸν τῆς ψυχῆς· ὥς γὰρ τὸν δλον κόσμον φασὶ κατὰ ἀρμονίαν διοικεῖσθαι, οὕτως καὶ τὸ ζῶον ψυχου- σθαι. δοκεῖ δὲ τέλε- ος ἀρμονία ἐν τρισὶ συμφωνίαις ὑφιστά- ναι, τῇ διὰ δ', ἥτις ἐν ἐπιτρίτῳ κείται λόγῳ, τῇ διὰ ε' ἐν ἡμιολίῳ, τῇ διὰ πα- σῶν ἐν διπλασίονι.	περιέχει δὲ ἡ τετράς καὶ τοὺς λόγους τῶν κατὰ μουσικὴν συμ- φωνιών, τῆς τε διὰ τεττάρων καὶ διὰ πέντε καὶ διὰ πασῶν καὶ προσέτι δις διὰ πασῶν, ἐξ ὧν σύστη- μα τὸ τελειότατον ἀπογεννᾶται· τῆς μὲν γὰρ διὰ τεττά- ρων ὁ λόγος ἐπιτρι- τος, τῆς δὲ διὰ πέντε ἡμιόλιος, διπλάσιος δὲ τῆς διὰ πασῶν, τετραπλάσιος δὲ τῆς δις διὰ πασῶν. οὗς ἅπαντας ἡ τετράς ἔχει περιλαβοῦσα· τὸν μὲν γοῦν ἐπιτρι- τον ἐν τῷ τέσσαρα	symphoniae quo- que ratio ex eorum- dem numerorum qui decimanum numerus con- pleant quasi quo- dam fonte de- manat: siquidem ex his epitriti et sescuplares et du- plices et triplices et quadruplices numeri sonique nascuntur. epi- tritii quidem ut quattuor adver- sum tres. habent enim totum nu- merum trientem et eius tertiam partem, id est

¹ Note that this parallels the final words of Sextus in the last passage quoted above. It is a very significant parallel.

² The following section, containing matter explaining certain arithmetical terms, since it is not paralleled by Anatolius, might be regarded as an addition by Sextus; on the other hand it could as easily be an instance of an omission by Anatolius for the sake of brevity. Chalcidius presents similar details, without, however, the distinctive marks of close relationship present in Sextus and Anatolius.

Sextus:

τινὸς ἀριθμοῦ συνεσ-
τηκὼς καὶ ἐκ τοῦ
τρίτου μέρους ἐκεί-
νου, ὡς ἔχει ὁ ὀκτώ
πρὸς τὸν ἕξ· καὶ γὰρ
αὐτὸν τὸν ἕξ περιέ-
σχηκε καὶ τὸ τρίτον
αὐτοῦ, τουτέστι τὴν
δυάδα. ἡμιόλιος δὲ
καλεῖται, ὅταν πε-
ριέχῃ ἀριθμὸς ἀριθ-
μὸν καὶ τὸ ἥμισυ
ἐκείνου, ὡς ἔχει ὁ
ἐννέα πρὸς τὸν ἕξ·
συνέστηκε γὰρ ἐκ τοῦ
ἕξ καὶ ἐκ τοῦ ἡμίσεος
αὐτοῦ, τουτέστι τῶν
τριῶν. διπλασίων
δὲ προσαγορεύεται ὁ
δυσὶν ἀριθμοῖς ἴσος,
ὡς ὁ τέσσαρα πρὸς
τὸν δύο· δις γὰρ τὸν
αὐτὸν περιέσχηκεν.
ἀλλὰ γὰρ τούτων
οὕτως ἐχόντων, καὶ
κατὰ τὴν ἀρχῇθεν
ὑπὸθεσιν τεσσάρων
ὄντων ἀριθμῶν, τοῦ
τε ἑνὸς καὶ δύο καὶ
τρία καὶ τέσσαρα,
ἐν οἷς ἐλέγομεν καὶ
τῆς ψυχῆς ἰδέαν
περιέχεσθαι κατὰ
τὸν ἑναρμόνιον λό-
γον, ὁ μὲν τέσσαρα
τοῦ δύο καὶ ὁ δύο τῆς
μονάδος ἐστὶ διπλα-
σίων, ἐν ᾧ ἐκείτο ἡ
διὰ πασῶν συμφω-
νία, ὁ δὲ τρία τοῦ
δύο ἡμιόλιος (καὶ
γὰρ αὐτὸν τὸν δύο

Anatolius:

ὄντων δὲ ἀριθμῶν
τεττάρων τῶν πρῶ-
των α' β' γ' δ', ἐν
τούτοις καὶ ἡ τῆς
ψυχῆς ἰδέα περιέχε-
ται κατὰ τὸν ἑναρ-
μόνιον λόγον, ὁ μὲν
δ' τοῦ β' καὶ ὁ β'
τοῦ α' διπλασίος, ἐν
ᾧ κείται ἡ διὰ πα-
σῶν συμφωνία, ὁ δὲ
γ' τοῦ β' ἡμιόλιος,
περιέχων αὐτὸν καὶ
τὸ ἥμισυ, τὴν διὰ
πέντε συμφωνίαν
ὑποβάλλει, ὁ δὲ δ'

Philo:

πρὸς τρία, τὸν δὲ
ἡμιόλιον ἐν τῷ τρία
πρὸς δύο, τὸν δὲ δι-
πλάσιον ἐν τῷ δύο
πρὸς ἓν ἢ τέσσαρα
πρὸς δύο, τὸν δὲ τε-
τραπλάσιον ἐν τῷ
τέσσαρα πρὸς ἓν.

Chalcidius:

unum. sescu-
plares uero, ut
tres aduersum
duo; habent enim
tres totum nu-
merum dualem et
eius partem dimi-
diam, id est unum.
duplices uero ut
sunt duo aduersus
unum. triplices
porro ut sunt tres
aduersus unum.
et quadruplices ut
sunt quattuor ad-
uersus unum. epi-
tritus autem in
calculando idem
est qui diates-
saron dicitur in
canendo. sescu-
plaris uero idem
est qui diapente
dicitur in can-
endo. duplex uero
qui diapason dici-
tur in canendo,¹
quadruplex qui di-
diapason dicitur
in canendo.

¹ At this point Wrobel would add "triplex qui diapason et diapente dicitur in canendo," following Fabricius and thinking that Chalcidius omitted the words, which appear neither in the MSS nor the first edition, through error or forgetfulness. Note, however, that the passage agrees with Theon, p. 58, 13 ff., as it stands.

Sextus:

περιέσχηκε καὶ τὸ
ἡμῖσι τούτου, ὅθεν
καὶ τὴν διὰ πέντε
συμφωνίαν ὑπέβαλ-
λεν), ὁ δὲ τέσσαρα
τοῦ τρία ἐπίτριτος,
ὑπέκειτο δὲ καὶ ἐν
τούτῳ ἡ διὰ τεσ-
σάρων συμφωνία.
ὥστε εἰκότως τὸν
τέσσαρα ἀριθμὸν
παρὰ τοῖς Πυθαγο-
ρικοῖς εἰρησθαι πη-
γὴν ἀενάου φύσεως
ρίζωματ' ἔχουσιν.

Anatolius:

τοῦ γ' ἐπίτριτος, ἐν
ᾧ ἡ διὰ δ' συμφωνία.
εἰ δὲ ἐν τῷ δ' ἀριθμῷ
τὸ πᾶν κείται ἐκ
ψυχῆς καὶ σώματος,
ἀληθὲς ἄρα καὶ ὅτι
αἱ συμφωνίαι πᾶσαι
κατ' αὐτὸν τελούν-
ται.¹

Philo:

Chalcidius:

In Theon's arithmology, p. 101, 12 Hiller, he merely says καὶ αἱ συμφωνίαι δὲ πᾶσαι κατ' αὐτὸν συμπληροῦνται, ὡς ἐδείχθη. This reference leads us back to p. 93, 17: ἐπεὶ δὲ πάντες οἱ τῶν συμφωνιῶν εὐρέθησαν λόγοι, καθὰ δέδεικται, ἐν τῇ τῆς δεκάδος τετρακτύν, καὶ περὶ τούτων πρότερον λεκτέον. τὴν μὲν γὰρ τετρακτὺν συνέστησεν ἡ δεκάς. ἐν μὲν γὰρ καὶ β' καὶ γ' καὶ δ' ἰ'· α' β' γ' δ'. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἐστὶν ἡ τε διὰ τεσσάρων συμφωνία ἐν ἐπιτρίτῳ λόγῳ καὶ ἡ διὰ πέντε ἐν ἡμιολίῳ καὶ ἡ διὰ πασῶν ἐν διπλασίῳ καὶ ἡ δις διὰ πασῶν ἐν τετραπλασίῳ· ἐξ ὧν συμπληροῦται τὸ ἀμετάβολον διάγραμμα. And in turn the reference here is to p. 58, 13: πάσας δὲ τὰς συμφωνίας περιέχει ἡ τετρακτὺς. συνέστησε μὲν γὰρ αὐτὴν α' καὶ β' καὶ γ' καὶ δ'. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἐστὶν ἡ τε διὰ τεσσάρων συμφωνία καὶ ἡ διὰ πέντε καὶ ἡ διὰ πασῶν, καὶ ὁ ἐπίτριτος λόγος καὶ ἡμιόλιος καὶ διπλάσιος καὶ τριπλάσιος καὶ τετραπλάσιος.

Among these passages, there certainly exists an agreement between Sextus and Anatolius which is indicative of a common source; the others do not share this verbal similarity, and, besides, they include among the harmonic ratios the diapason, making four, while Sextus and Anatolius report only three. Chalcidius and Theon, here as elsewhere,² show marks of close relationship; they

¹ The concluding sentence of Anatolius is obviously modeled after what is seen in Sextus, but modified to suit its new context.

² This alliance is manifested by several significant minor identities of phrase; for instance, of the triad, Theon says *πρώτη λέγεται πάντα εἶναι* and Chalcidius

both mention the triple ratio, and each regards the number four as the tetraktys. The conditions may be explained by concluding that Sextus and Anatolius for this passage depend on the introduction of the arithmological treatise, Philo, Theon, and Chalcidius upon its chapter on 4. The latter two, however, are probably derived from some common ancestor between the ultimate common source and themselves; this will explain their slight disagreement with Philo. The likeness between Sextus and Anatolius, however, is a most important confirmation of the conclusion reached above with regard to the first set of passages cited.

Thus the passage of Sextus is seen to be connected most intimately with a lost book, the existence of which, however, is as certain as that of the *Iliad*—namely, the common source of Anatolius, Theon, Philo, and Lydus, which determines the form of the first two throughout the whole length of their arithmologies,¹ and from which Philo and Lydus excerpted lengthy fragments identifiable by their close likeness to Theon and Anatolius.² Besides these, Chalcidius³ was certainly affected by it, probably also the *Theologumena Arithmeticae*,⁴ Varro,⁵ Hermippus of Berytus,⁶ and indirectly Macrobius,

tria . . . dicta sunt omnia; but Anatolius and Lydus (ii. 8) both have *ὅτι πρῶτος τὰ πάντα σημαίνει*. Otherwise the passages are much alike. Again, in the chapter on 7, Chalcidius inserts a block of material much resembling a similar block inserted by Theon, which disagrees with the corresponding parts of Anatolius, Philo, and Lydus.

¹ It is to be identified with the "grand traité d'époque alexandrine" mentioned by Delatte, *op. cit.*, pp. 140, 207.

² See Philo *De mundi opificio* cc. 3, 15-16, 20, 30-42; Lydus *De mensibus* ii. 7, 9, 11, 12; iii. 4; iv. 64 (all in part). The connection of other passages of both authors with this source is debatable.

³ In his commentary on the *Timaeus* cc. 35-37.

⁴ An anonymous treatise first edited by Ast in 1817, thought by most to have been compiled by Iamblichus; its chief sources are Anatolius and the *Theologumena Arithmeticae* of Nicomachus of Gerasa (this latter is otherwise known only through Photius, *Bibliotheca*, cod. 187). Probably Nicomachus' *Theologumena* also was influenced by the anonymous arithmologist.

⁵ In the first book of the *Hebdomades* or *Imagines*, ap. Aulus Gellius *Noct. Att.* iii. 10, and in *Tubero* or *De origine humana*, ap. Censorinus *De die natali* 9, 1 ff.

⁶ Clement of Alexandria, *Strom.* VI. xvi (see especially sec. 145, 2) quotes his *περί ἐβδομάδος*.

Martianus Capella, Favonius, and Isidore of Seville.¹ By comparisons among these and other writers it would still be possible to reproduce a large part of this source in its original form. It consisted of an introduction and ten chapters dealing with the ten numbers.² The study of its transmission, which would be too lengthy to attempt here, would show that it circulated in several versions and epitomes,³ and that its influence was spread still farther by those who derived their arithmology from it. But although it was the most generally quoted of all ancient arithmologies, its very name and that of its author are lost and probably will never be known. It was probably written by at least 100 B.C., for Varro seems to have been influenced by it; Philo,⁴ in the early first century A.D., certainly was.

With regard to the question of the share of Posidonius in promulgating Pythagorean lore, the conclusion must be that even if the whole of Sextus *Adv. math.* vii. 91-109 is his, the Pythagorean part of it must nevertheless be a citation on his part from the introduction of an already existent work, which was known to Sextus also from another source, that used in iv. 2 ff. This treatise, and not Posidonius, was the ultimate source of information used by the long list of writers just named.

In conclusion, let us consider briefly the question whether or not the citation of vii. 91 ff. belongs to Posidonius, reviewing the arguments that connect him with the arithmological traditions. First, it has been claimed⁵ that the mention of Posidonius' name at

¹ The sources of these writers are not so easily determined. Macrobius in his commentary on the *Somnium Scipionis* (i. 6) parallels extensive passages of the *Theologumena Arithmeticae*.

² There is a possibility that it formed a part of a large Pythagorean work dealing with the mathematical sciences generally.

³ Anatolius, Theon, and others seem to have used epitomes; Philo, discussing the number 7 in *De mundi opificio* 30-42, and Sextus, quoting the Introduction, apparently had access to the unabridged work. As has been seen above, Anatolius also quoted a section of this Introduction.

⁴ Philo was the author of a book, now lost, called *περὶ ἀριθμῶν*, mentioned by title in *Vit. Mos.* iii. 11, *Quaestiones et solutiones in Genesim* iv. 110, 151, and less definitely in *De mundi opificio* cc. 16 and 43, *Quaest. et sol. in Gen.* ii. 14 and iii. 49. Apparently he collected here all that is said of the numbers in his extant works, and the treatise was doubtless affected, as they are, by the anonymous arithmologist.

⁵ By Schmekel, *loc cit.*

the beginning marks him as the author. This is a double-edged argument and by no means conclusive; for ancient writers quite as often concealed the name of their primary source as they revealed it, and frequently they veiled their action by mentioning freely the authorities named in their real source.¹ Furthermore, we have now seen that in any case Posidonius himself must have been quoting; and this will also apply to the purely subjective argument that attempts to decide, from the nature of the passage, that the whole belongs to him.²

More satisfactory in some respects is an argument based on the commentary of Chalcidius, c. 50, where the following occurs:

uult igitur animam sensibilis mundi tamquam permissa usurpandi
licentia nasci, cognitricem tamen rerum omnium, quae sunt tam intellegibiles
quam sensibiles. est porro Pythagoricum dogma similia nonnisi a similibus
suis comprehendendi. quod etiam Empedocles sequens ait in suis uersibus:

terram terreno comprehendimus, aethera flammis,
humorem liquido, nostro spirabile flatu,
pacem tranquillo, litem quoque litigioso.

haec quippe constituebat elementa et initia uniuersitatis, ex quibus animae
quoque censebat constare substantiam.

The thought of this passage is parallel to Sextus *Adv. math.* vii. 91 ff., and the citation of the same verses of Empedocles, and the possible reference to Philolaus, are remarkable. The reappearance of all these matters in conjunction could be offered as evidence that the passage in Sextus is a unity, and from the pen of Posidonius; it has, in fact, been argued that Chalcidius (probably through Adrastus) and Sextus both depend upon him.³ Even if this is so, however, it does not prove that the Posidonian quotation in Sextus extends over the arithmological sections as well, of which in any case he could not have been the author.

¹ Cf. Eyssenhartd's introduction to his edition of Martianus Capella, p. xxxii, and Hiller in *Rheinisches Museum*, XXVI (1871), pp. 582 ff., cited by Switalski, *op. cit.*, p. 62.

² For the controversy over the exact extent of the Posidonian quotation, cf. Zeller and Schmekel, cited above, p. 310, n. 2, and R. M. Jones, *The Platonism of Plutarch*, Chicago dissertation, Menasha, 1916, p. 77, n. 21. The latter agrees with Zeller that the Pythagorean material does not belong to Posidonius. It may be noted that at the conclusion of the debated passage Sextus (vii. 110) appends the words ταῦτα μὲν οἱ Πυθαγόρειοι.

³ Borghorst, *op. cit.*, p. 60.

Finally, there is some interest in the citation of Posidonius by Theon in his arithmology, p. 103, 16 ff.: *ἐπόμενος δὲ τῇ φύσει καὶ ὁ Πλάτων ἐξ ἑπτὰ ἀριθμῶν συνέστησε τὴν ψυχὴν ἐν τῷ Τιμαίῳ. ἡμέρα γὰρ καὶ νύξ, ὥς φησι ὁ Ποσειδώνιος, ἀρτίου καὶ περιττοῦ φύσιν ἔχουσι. μὴν δὲ καθ' ἑβδομάδας τέσσαρας συμπληροῦται, τῇ μὲν πρώτῃ ἑβδομάδι διχοτόμου τῆς σελήνης ὁρωμένης, κτλ.* At this point, it may be noted, begins a block of material¹ which, unlike most of the chapter on the number 7, seems to be taken from a source different from that of Anatolius—that is, probably not from the anonymous arithmology of which we have spoken.² Opinions as to the extent of the Posidonius citation may, of course, differ; there seems to be no reason, however, to think that it did not at least include the sentence describing the influence of the number 7 on the moon and the lunar month. Nevertheless, all that can be held proven is that Posidonius was interested in Pythagorean arithmology, and that he may have influenced some lines of its tradition, not that he was the author of any part of the general source of Theon, Anatolius, Philo, and the rest. At most, he may have been responsible for introducing alien elements into the descendants of this arithmology.

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¹ Especially p. 104, ll. 1–5, but in all the rest of the section about the hebdomad there are striking differences of detail between Theon and Anatolius.

² This special source, however, was probably itself influenced by the Anonymous.