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The argument from coin-types is perhaps not Euclidean in its compactness, but neither is it a mere parcel of sticks.

4. Professor Halliday lays stress on the fact that Greek writers in general, and Thucydides in particular, only mention the Corinthian, but not the Euboean foundation on Corcyra.

Here is a real crux of historical method: how far is it legitimate to use the *argumentum ex silentio*? On this point it may suffice to make two remarks:

(i.) The entire history of the Greek

colonial movement is largely built up out of stray allusions and ἀπαξ λεγόμενα: discard these, and our chapters on colonisation will shrink visibly. Under these conditions it is surely risky to draw conclusions from the lack of references to our Euboean colony, which bore an early date, had a short life, and was obliterated by a famous and lasting settlement on or near the same site.

(ii.) In the case of so economical a writer as Thucydides it is not enough to say that he *could* have worked in an allusion *somewhere*. We must be prepared to show at what point such an insertion was *requisite*. Where could Thucydides have usefully mentioned a Euboean settlement on Corcyra? In the *Κερκυραϊκά* of Book I., in his reflections on Corcyraean στάσις, in his Sicilian 'Αρχαιολογία? In none of these passages would such a reference have been relevant. In any case, it is as unsafe to argue from the reticences of Thucydides as to presume on the silences of Colonel Bramble.

Conclusion.—The evidence in favour of a Euboean settlement on Corcyra is, of course, not conclusive; but it is as strong as that on which many generally accepted statements about early Greek history are founded, and the proof based upon it cannot fairly be quoted as a classical example of making sunshine out of cucumber. M. CARY.

TWO PYTHAGOREAN PHILOSOPHEMES.

1. *The connexion between τὸ ἄπριον and bisection ad indefinitum.*

It is well known that the later writers of antiquity explain the connexion asserted by the Pythagoreans between τὸ ἄπριον and the ἀπειρον by saying that 'the even' can be bisected εἰς ἀπειρον (see on this point Burnet *EGPh*³ 288-9, with the references given there). This looks like nonsense, since after ($n-1$) divisions of 2^n by 2, you get 2 as a quotient, and 2 can no longer be divided into 'even' parts. The n^{th} quotient will be 1, and thus the 'halving' will come to a stop. The real meaning becomes clear if we understand that we are dealing with a theorem of Pythagorean geometry. We have to think of a

terminated segment *AB* of a straight line as made up of a finite number of 'points,' or 'units with position' (μονάδες θέσιν ἔχουσαι); between any two adjacent 'points' there is an empty interval, what Aristotle calls a κενὸν ὃ τὰς φύσεις διορίζει (*Phys.* 213b 24); if there were not this 'gap' the two 'points' would be identical. In other words, on the view which is in question, it is *not* true that 'between any two points on a straight line there is always a third point.' It seems also to be assumed, as is only natural, that the intervals between adjacent 'points' are all equal.

Now apply this to the bisection of a terminated segment of the straight line.

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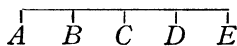
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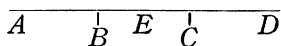
Now apply this to the bisection of a terminated segment of the straight line.

First, let the number of 'points' in the segment be odd, thus:



If you could bisect AE at all, the bisection would fall on C . You would have to 'split' the 'unit' C , and the 'splitting of the unit' is logically impossible (Plato, *Rep.* 525d 8, οἶσθα γάρ που τοὺς περὶ ταῦτα δεινοὺς αὖ ὥς, εἴν τις αὐτὸ τὸ ἐν ἐπιχειρῇ τῷ λόγῳ τέμνειν, καταγελῶσί τε καὶ οὐκ ἀποδέχονται). Such a line cannot be bisected at all. This is why τὸ περιττόν and πέρασ are associated.

But now consider the segment AD —



which contains an even number of 'units.' Here bisection will divide AD at E , in the middle of the 'empty' interval BC , and is *therefore* possible. It is then easy to show that ED can in turn be bisected, because the division will not fall on the 'unit C ,' the only 'unit' in the interval ED , but somewhere between C and D , and that, in like manner, the same process can be repeated endlessly. Every 'bisection' after the first will fall within the 'empty' interval between C and D , and D itself will never be actually reached. Hence the 'even' can, as the commentators say, always be bisected *ad indefinitum*.

This is strictly equivalent to the arithmetical proposition that, if n be any natural integer, the sum of n terms of the series $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}$ steadily approximates to the value 1 as n is taken greater and greater, but never reaches it. Thus the proposition that the 'even' can always be bisected *in indefinitum* is strictly true, when we understand what it means. But since, on the given assumptions, not all segments, but only those with an even number of 'points' can be bisected, we see at once that the Pythagorean conception of the point as μονὰς θέσιν ἔχουσα is actually incompatible with the most elementary constructions of the geometry which the Pythagoreans themselves had created.

2. *The One and the 'Gnomons.'* Aristot. *Phys.* 203a 13 περιτιθεμένων γὰρ τῶν γνωμόνων περὶ τὸ ἐν καὶ χωρὶς, ὅτε μὲν ἄλλο ἀεὶ γίγνεσθαι τὸ εἶδος, ὅτε δὲ ἔν (see *EGPh*³ 103 and *ib. n. 2*).

Milhaud and Burnet seem to have wholly misunderstood this passage, though the meaning is rightly indicated by Themistius in his paraphrase. As to the words, (a) the 'gnomons' meant, are clearly, as both Milhaud and Burnet say, the successive series of odd numbers which have to be 'put round' 1 to produce the series of squares. [$1+3=2^2$, $1+3+5=3^2$, and generally $1+3+5+\dots+(2n-1)=n^2$]; (b) καὶ χωρὶς means 'and in the other case,' *e contrario*, much as at Aeschylus, *Agam.* 637 χωρὶς ἡ τιμὴ θεῶν, where χωρὶς is apparently adverbial, and the sense is that telling bad news and rejoicing over good fortune are 'clear contrary' duties, which should not be blended; (c) as the emphasis given to περιτιθεμένων τῶν γνωμόνων by its position shows, the two contrasted procedures are not 'putting the gnomons round the one' and 'dispensing with the one'—this is the mistake committed by Milhaud and Burnet—but 'putting the gnomons round the one' and 'putting something else round the one'; (d) εἶδος has its standing meaning when it occurs in connexion with geometry, 'regular polygon.' (The rectangles Milhaud and Burnet produce by putting successive even numbers round one another are not 'regular polygons,' and are never spoken of as εἶδη.)

Aristotle means, then, that if you take the series of sums, 1, 1+3, 1+3+5 . . ., you get 1, 4, 9 . . ., the second powers of the successive integers. The 'pattern' (εἶδος) remains the same throughout the series; it is a square. But if you add to 1 successive sums of *even* numbers you get the series 1, 1+2, 1+2+4, 1+2+4+6 . . ., that is 1, 3, 7, 13 . . . Now here the 'pattern' changes at every step; 3 is a 'triangle,' 7 a 'regular heptagon,' and so on. Burnet has misunderstood the words he quotes from [Plutarch] *Stromat.* at *EGPh*³ n. 2. In them τῶν δὲ ἀρτιῶν ὁμοίως περιτιθεμένων can only mean, as the antithesis with the preceding clause shows, περιτιθεμένων τῇ

μονάδι, 'if we put the even numbers round 1.' The writer understands Aristotle, not as Burnet does, but exactly in the way I have just explained. His statement that the numbers which result from the proceeding are *ἐτερομήκεις καὶ ἄνισοι πάντες* must not be pressed to mean that they are one and all 'oblongs' (products of two unequal factors), still less must 'oblong' be taken in the strict sense in which it means a number of the form $n(n+1)$. Such numbers are necessarily even, and none of them could be produced by the method Aristotle and [Plutarch] are describing. Milhaud and Burnet want to find the 'oblongs' in the texts, but they have to mistranslate in order to do so. If we write down the first few terms of the series really meant, 1, 1+2, 1+2+4 . . ., we get 1, 3, 7, 13, 21, 31, 43, 57 . . . Out of these eight terms all are prime except 21 and 57, which are products of two unequal factors, 3×7 and 3×19 . It is these products of two unequal factors, both odd, which [Plutarch] means by *ἐτερομήκεις*. At any rate this is how

Themistius also understood the passage, and I feel sure he is right (*In Phys. Γ.* Spengel, p. 222, οἱ δὲ ἄρτιοι προστιθέμενοι τῇ μονάδι κατὰ τοὺς ἐφεξῆς αἰεὶ τι καινὸν εἶδος ποιοῦσι, καὶ ἡ διαφορὰ πρόεισιν εἰς ἄπειρον, τρίγωνον, εἴτα [καὶ] ἐπτάγωνον εἶθ' ὅτι καὶ τύχοι). It is interesting that St. Thomas understood the passage correctly. Cf. *Comment. in de Anima* I. 7, c, *si enim unitati addatur binarius qui est primus par, consurgit ternarius, qui est numerus triangularis; quibus si rursus addatur quaternarius, qui est secundus par, consurgit septennarius, qui est septangulae figurae, et sic in infinitum.* The true sense must have been lost in quite modern times, since Pacius, in his commentary *ad loc.*, only goes wrong on one point; he supposes that τὸ ἐν means 4, because 4 is the 'first square'! But he correctly points out that if you add 6 to 4, and get 10, the 'pattern changes,' since 10 is a 'triangle.' Thus he seems to have understood that an εἶδος means a 'regular polygon,' not the sort of rectangle some modern interpreters bring into the discussion.

A. E. TAYLOR.

SOME NOTES ON AESCHYLUS, *EUMENIDES*.

10. κέλσας ἐπ' ἀκτὰς ναυπόρους τῆς Παλλάδος.

This description of Apollo's journey from Delos to Delphi is connected in the Scholia and by modern editors with the route of the Sacred Way through Attica to Delphi. The particular landing-place, however, is in dispute. Is the allusion, perhaps, to Prasiae? Mr. Seltman (*Athens, its History and Coinage*, pp. 12, 30) argues that Prasiae, on the east coast of Attica, was the original harbour of Athens, whence the Theoria of the Theseus-ship sailed to Delos. The evidence for the view that it sailed from Prasiae in the fifth century is not conclusive, but the traditional connexion between Delos and Prasiae would support Aeschylus' assumption of a landing by Apollo 'on the shores of Pallas,' and Prasiae lies in the direct line between Delos and Delphi.

285. χρόνος καθαιρεῖ πάντα γηράσκων ὁμοῦ.

An interpolation, as most editors agree, yet surely not 'spurious,' but an Aeschylean line quoted by some commentator which has strayed into the text. Cp. *Prom. Vinct.* 981: ἀλλ' ἐκδιδάσκει πάνθ' ὁ γηράσκων χρόνος.

328-9.) ἐπὶ δὲ τῷ τεθυμένῳ

341-2.) τόδε μέλος . . .

This is generally translated (*sc. ἐστὶ*) 'Over the victim this is the song.' For ἐπὶ I suggest ἐπι—'This is the song appointed for the victim.' Cp. 543: ποινα γὰρ ἐπέσται, and 393-4 (*codd.*): ἐπι δέ μοι γέρας παλαιόν; also *Agam.* 547.

632. Is not Verrall's translation of εὐφροσι, 'for loyal hearts,' supported not merely by the general sense of the passage (788-798) in the *Agamemnon* to which he refers, but also by *Choeph.* 109: φθέγγον χέουσα κεδνὰ τοῖσιν εὐφροσιν? Cp. also *Eumen.* 992.

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