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PYTHAGOREAN NUMBER THEORY AND ITS IMPLICATIONS FOR PSYCHOLOGY¹

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WHEN Lois and I had the delightful course by Ruth Benedict at Columbia entitled "The Religions of Preliterate Peoples," we learned the following from the philosophy of the Dakotahs:

As the units of time are four, the day, the night, the month and the year, and as the seasons of man's life are four, infancy, childhood, adulthood and old age, and as the fingers are four, and the toes are four, and the sum of the thumbs and big toes is four (1) . . .

We decided we were encountering a number-obsessed people. Perhaps we ourselves are another number-obsessed people. I thought I would explore this question. Numbers are vital to our life. It may be worth while to understand their role in our thinking.

It is not my intention to regale you, like Major General Stanley, with "many cheerful facts about the square on the hypotenuse." Rather, I hope to direct your attention to the "golden numbers" of arithmetic and geometry, inclining to the view that "God made the whole numbers, man everything else." I shall ask you to accept from me a few little signposts planted by Egyptians in the earth to mark distances; a few little numbers that make for Mendeleev the all-or-none difference between one traditional atom and another; a few little odd facts about black-body radiations that give us Planck's world of the quantum and discontinuity; a few little reminders that we must, in two profound meanings of the term, find "safety in numbers"; and a few unanswerable questions, I believe, as to where number cathexis carries us from the soundest reality testing to the wildest extravaganza of irrationality.

Not much is known about Pythagoras, nor about any of his immediate followers. They shared the fascination and the excitement of those who made discoveries that transcend the world of ordinary communication, and maintained the secret fraternity of their silence through the whole great period of

their discoveries. They taught a wide variety of things that puzzle you and me. The basic thing about them for the history of science is that they discovered the role of number in that vast enterprise which we call science; together with the atomists on the one hand and the Platonists on the other, they defined the basis for an ordered cosmic structure, an ordered structure of human individuality, and a social order.

We may make many mistakes as to what they taught, but this is of secondary importance. The main thrust of what they taught is very plain, and the danger is not that we falsely attribute this or that to them, but that we go wrong in our own contemporary thinking about what numbers are; how they can be used; where they hold a priceless key to reality; and where they lead us off into a solipsistic world in which the manipulation of numbers bends back upon itself in a self-contained world of fantasy, remote from that numbered world which it is our scientific task to perceive and to use.

Absolutely nothing that I shall say will be new to the mathematical specialist. But there is enough, I believe, of direct importance for psychologists in the rich Pythagorean tradition to be worth our spending an hour with it, and it is only in these terms that I ask for your attention.

HERACLITUS, DEMOCRITUS, AND PYTHAGORAS

There are three great personalities from pre-Socratic philosophy to whom I would invite your attention: (a) There is Heraclitus, the man who said that we never step twice into the same river, and the man who, in the expanding trade between Greeks and Mesopotamians, reminded us that all things can be changed into gold, or into fire, and back again. He taught that strife is the father of all things, and above all, that all things flow. (b) There was the atomist, Democritus, who conceived the world to be made of minute moving particles, some larger, some smaller, some constituting what we call our bodies, some what we call our minds. He laid the foundation for all modern material atomism. (c) There was Pythagoras, of

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the Island of Samos, who fled a local despot and set up his school in Croton in the south of Italy about 530 B.C. His school was a fraternity, or secret society, lost in the immensities of number and of measurement, and like many Greek schools, concerned both with cosmic and with human problems; both with epistemology and with aesthetics and ethics. He taught that quantity, and specifically, number, was the key to all reality, and it is for this that we revere him.

MATHEMATICS AND BEAUTY

Two great discoveries heralded the arrival of Pythagorean number theory: (a) in tuning the lyre it was discovered that there is a certain unity or blending of tone when one string is twice as long as another, the octave; that there is a good, acceptable, sweet chord when the ratio of length is three to two, and again that good chords are produced by relations such as four to three. Using four strings, it was found that the combinations one to two, three to two, and four to three were especially delicious to the lover of tone. (b) There was something about simple numbers that gave beauty. More complex numerical relationships appeared when the lyre was not quite properly tuned, and the relation was something like fifteen to eight. These, when simultaneously struck, gave a discord. There was thus a direct relationship between arithmetical simplicity on the one hand, and beauty on the other hand. It was not far from this to a doctrine of harmony in nature, for example, in the courses of the stars or the rhythms and ordered time relations of pageantry and the dance, and the basic physiological rhythms of life.

You will notice that in this conception of order there is a preoccupation and fascination with whole numbers. Of course, the mystic three, the mystic four, the mystic seven, are virtually universal human concerns. There is luck in odd numbers, specifically three and seven; and bad luck in thirteen, etc., according to the culture. This feeling that numbers are almost persons, almost benign and malignant entities in nature adds to the emotional investment or cathexis in the number system as a whole. A child is early aware that he has two hands, two eyes, etc. He must "count out"; he must reestablish rhythm after falling or sobbing or getting lost in his own little diadic patty-cake of life.

This idea of the sacredness of number was, of course, enormously accentuated with the discovery of the amazing fact that the right triangle, familiar from Egyptian measurements of the land, was so invested with nature's universal harmonies that the square of the hypotenuse was exactly equal to the sum of the squares on the other two sides; not somewhere near it, not by an approximation to a 1% error, not probabilistically, or in terms of averages, but literally, absolutely, eternally, and indisputably, as a necessary numerical relationship.

Many a Greek temple showed likewise a subtle and marvelous concern with the manipulation of arithmetical relations between columns, capitals, the interspaces between columns, and the relations of lengths and widths appreciated by the beauty-loving eye long before they are discovered by the individual to be based upon numerical realities. In other words, the world of geometry speaks to us in the same language as the world of the musician's lyre, and both speak in terms of the basic numerical relationships of the body. Music and the time arts, geometry and the space arts, gave the Pythagoreans boldness to speculate that order, rhythm, balance, numerical simplicity are likewise the clue to the ordering of human affairs through systems of morals, politics, and law.

Another celebrated discovery of the Pythagoreans related to the interdependence of the integers which add up to ten. Imagine the front elevation of a traditional pile of cannon balls with 1 at the top, 2 in the line below, 3 below that, and 4 at the base. The total is 10; 10 contains in rational order the first four cardinal numbers. But the form of the two-dimensional figure, if properly ordered, is an equilateral triangle, so that the 3 enters into the company of the 10 in an especially intimate way.

If you think that "this way madness lies," I will not dispute you. I learned once from a numerologist in New Haven why the prevailing color on the face of the earth is green. Count out from the sun: the sun is one; Mercury, two; Venus, three; the earth, four. Now count the colors of the rainbow: red is one; orange, two; yellow, three; and green, four. Does it not follow that in the kingdom of the number four the earth must be green? Or does it? If it is not self-evident to you, you are lacking in that fine flavor of number mysticism which stretches all the way from the greatest mathematical genius to the schizoid confusion of symbols with the things symbolized.

But this is not the only kind of passion with which the lovers of number are tortured. Deep in our culture is the belief that numbers have distinctive qualities, like people: the clean, square four, and the ragged, cruel, unfortunate thirteen. They have family resemblances and tribal affinities. Indeed, who can quite say that they have not? In anthropology we encounter the moieties, and the people who live on one or the other side of the river; and in the number world we have the odds and the evens, the prime numbers and those that can be factored, and countless other categories. These qualities often lead to their being loved and hated, sought and avoided.

THE "PERSONALITIES" OF NUMBERS

Let me see if I can persuade you that there are, in our tradition, easily recognizable and very intense feelings about the qualities and the importance of the individual integers. Take them, then, from 1 to 10. One is, of course, the center and root of all numerical thinking. There is no number at all if the one is itself uncertain. "One is one and all alone and evermore shall be so." Two represents the fundamental duality—the male and the female, the Yang and the Yin, the odd and the even—as bases of the eternal polarities based on opposition or directionality. Three is the great tripod, the universal system in which simple opposition is transcended—the father, the mother, and the child; the trinity of body, mind, and spirit; Osiris, Isis, and Horus; of course, three is lucky; "third time never fails."

The four is a pair of pairs; the consummation of the polarities, north, south, east, west. Five is a hand, a fist full of fingers, the combination of the polarity and the mystic triad so structured in its "fearful symmetry" that it appears in evolution as the five-toed form from which endless diversification has appeared in feet, paws, and hands. There are countless creatures to whom this supple and majestic symmetry is essential for balance, for locomotion, and for grasping. How much is biology, how much culture; how much the inscrutable "fiveness" of elementary symmetry? Six is another kind of two and three—not a simple summation, but a multiplication which appears in the hexagonal cell of the insects' constructions, and appears in the perfect cube, because there are really three dimensions, and there really are six basic faces of the simplest

reality. Mathematicians, with their theories of the group, have done many wonderful things with boxes of sixes. Now as we go on, we realize another enormously mysterious and mystifying number—the seven, which is a six plus a one, a four plus a three, and you know intuitively why there had to be seven heavenly bodies to constitute the harmony of the spheres, seven wonders of the ancient world, seven candles in the mystic candelabra. Seven is a lucky number. More than that, it is a number fundamental in that kind of human constructiveness which gives symmetry and asymmetry their equal power. Another challenging relation of two and three appears in the eight, which is a two to the third power; and the sublime rhythm which represents the number of the muses, the three times three, the nine.

You have only now to put your two hands together and find suddenly the root of all Western numerical construction, the decimal or digital system. As Frank Lorimer says, a decade is a "two hands of years, and a century is a two hands of two hands of years." Alone in its oddity stands 11, claiming its arch intimacy with the 10 on the one side and the tremendous 12 on the other; the 12, with its facets of 2 and 3, its divisibility into all the fundamental forces we have tried to describe. When this magnificence has been exhausted, is there anything strange about the threatening quality of the lonely, residual 13? It negates the supreme 12, and in utter Satanism, rejects the sublime unity of the number ritual which all the foregoing have laid down.

I will not hold you longer on this point, but ask you to play this game in your own way, remembering the sacredness of the "score," the diabolism of "40 stripes save 1," the 70 years which define the span of man's life, the 144,000 of the majestic vision of the Revelation, and so on, to your heart's content. I read from the Family Section of *Grit* for July 3, 1966:

No. 40 Plays Unusual Role

The number 40 has played a role of great importance in world history.

Jesus fasted 40 days. Moses spent 40 days and 40 nights on Mount Sinai at the time he got the Ten Commandments . . .

A quarantine extends to 40 days. In old English law, the privilege of sanctuary was for 40 days. . . ."

From the sublime to the ridiculous: I myself discovered, at the age of 5, a basic numerical clue to

human personality: I discovered that there were odd-numbered people—generous, friendly, free-roving, simple, direct, earthy, lovable people; and that there were even-numbered people—restrained, orderly, correct, demanding, and ultimately inscrutable people. The odd-ball, minority-group kinds of people with Irish names like myself were comfortable in their oddness. Most of the Anglo-Saxons who lived in the little Massachusetts town which was my home were even-numbered in every respect—in their politics, their religion, their philosophy of life, the way they said “good morning,” and the way they called in the dog. Think back. Did you not also make a great discovery about numbers and people? And when you took statistics, did you really find all numbers equal, or some “more equal than others?”

NUMBERS IN WESTERN PHILOSOPHY

Aristotle, of course, had the theory of the golden mean. He was speaking a language already somewhat familiar to his fourth-century audience who flourished long after the time of the Pythagoreans. But Aristotle had virtually no interest in mathematics; virtually no understanding of the Pythagorean vision. Despite his brilliant derivation of an ethical system from the theory of the golden mean, he left the main course of Greek thought to develop number theory in other directions. The Platonists, however, through the extraordinary mathematical cosmogony of the *Timaeus* saw the implications for man's moral and ideal fulfillment. Plato indeed made a journey to the Pythagorean establishment in southern Italy, and came back a man broadened in terms of his mathematics. Over the gate of the Academy the inscription was written: “Only Mathematicians Enter Here,” and in one of his wisest intellectual bouts with an especially alert questioner, Socrates himself is reported to have said: “They err not only with respect to lack of knowledge in general, but in particular with respect to that kind of knowledge which is called measuring.”

If we think of Socrates and Plato as drawing away from crass mathematical atomism and the early forms of a hard-boiled physicalist scientific naturalism we wholly miss the boat, and flounder helplessly in a sea of confusion. For the whole point, from the Greek outlook, was to find the value of numbers as guides to order, and not only

to physical but to aesthetic and ideal order. Naturally, this led to mystical applications which neither the Pythagoreans nor the Platonists shunned. It is not, however, with these mystical applications that we are here concerned, except insofar as we must note that preoccupations with numbers and attribution of vast importance to them in the basic cosmic and human meanings was inevitably associated with the conception of cognitive drives as superior to the simpler physical or physiological drives, and to the idea, especially associated with Plato, that man is most completely man when he thinks in terms of generalizations and abstractions representing a pure, cognitive attachment to the known or knowable world, man being driven on by an orderly nature of the knowing process syntonic with—we might almost say isomorphic with—the ordered nature of the world which he hopes to know.

GALILEO

Pythagorean number theory was kept alive in the West largely through Plato's *Timaeus*. The historian and the philosopher are concerned to know how it fared in the great Catholic system of St. Thomas Aquinas and his intellectual descendants in northern Italy at the time of the Renaissance. One of the questions which the scholars, acting as subjects in the celebrated Würzburg experiments, were asked to reflect about and answer, was couched in the words: “Was the Pythagorean theorem known to the Middle Ages?” The most important question for us is whether it was known when modern science took shape. Part of the answer lies in the fact that at the University of Padua in northern Italy mathematics was flourishing in Renaissance times, and the great Galileo Galilei was Professor of Mathematics at Padua. So sure was he of the soundness of a mathematical approach to reality that he apparently belittled his own experiments as being necessary only for beginners who did not see the inner necessity of nature's laws. You will recall that Kurt Lewin opposed an Aristotelian to a Galilean principle in the sense that the latter looks for laws having “exceptionless validity,” that is, the necessity of mathematical logic. This does not cover all the historical facts; for Galileo, hearing of a telescope in the Netherlands, pushed telescopic observation about Jupiter's moons and other objects to a point where it intrigued and shocked the classicists of

his day. He and his pupil, Torricelli, exchanged messages with lanterns to see whether the speed of light was actually infinite, and as Conant has reminded us, he speculated on the failure of hand pumps which had to draw water more than about 25 feet, looking for empirical considerations (mistakenly as it happened) in the tendency of the column of water to break of its own weight. It is true that he had earlier begun his mathematical investigations by watching the swinging of a lamp in the cathedral at Pisa, and that his approach to the laws of the falling body is a clean-cut instance of an empirical determination of something which followed of necessity from the mathematics of the relations of velocity to acceleration, a straight question of arithmetic and geometry. Galileo, in short, married empiricism to mathematical deduction. But so had Pythagoras; so had the great astronomers of Egypt. Mathematics was a tool which operated when continually fed with good raw material. It is possible, but likewise it is idle, to say that the development of science could have been carried through without benefit of Pythagoras. The empirical fact stands clear: Pythagorean thinking was of the very essence of the applied mathematics which we find in the history of science. It was because number theory and its application to arithmetic and geometry was clear and exciting that empirical observations took the shape which we call science.

Historians of psychology have not forgotten that as Galileo's thought was causing an electrical storm in Western Europe, Descartes invented analytical geometry; that amazing science which showed that an orderly equation derived from the study of conic sections will lay out for you upon a plane the exquisite three-petal and four-petal roses which are inherent in the bare abstractions of an equation. Certainly nothing could more dramatically satisfy the triumphant Pythagorean prediction that the world of beauty, the beauty of music, of architecture, and indeed of the very structure and symmetry of human thought, lies in the ordered relations of the numbers themselves. Newton carried the torch further, and with the impact of his thinking the eighteenth century became the intellectualist century, or the century of the enlightenment. Voltaire, armed with mathematics in one hand and scepticism in the other, gave us the ordered, but essentially nonhuman universe in which Laplace could con-

ceive of "celestial mechanics," and of a "system of the world." The physician, David Hartley, having reference to Newton's Laws of the Pendulum, which in turn went back to Galileo's Laws of the Pendulum, created an "association psychology" based squarely upon the elementary mathematics of the sine wave incorporated in the functions of the "white medullary substance of the brain."

HERBART

But it was Herbart who saw the radical implications. It was he who conceived of elementary concepts, or psychic atoms, pushing upon one another like the molecules in a gas; he saw them colliding and interfering with each other's movements, and under other conditions coalescing or structuring themselves into larger wholes. He saw the mind as a dynamic system of energies running an ordered course; he saw the difference between the ideas at a conscious level upon which we can make direct observations, and those below threshold which act, in the darkness of unconsciousness, exactly as if they were in the light of consciousness. He saw how the phenomena of conflict, and likewise the phenomena of assimilation, integration, and the apperception mass realize, in an ordered and predictable way, the structural potentialities from which dynamic potentialities must emerge. All this he did, of course, with the aid of a well-defined conception of energies, and with the benefit of the new differential and integral calculus which came from Newton and from Leibnitz. A giant, indeed, he was, with but few equals in either ancient or modern times—who saw the necessity of formal assumptions about elements, their relations, their energies, their dynamic interdependence, their capacities for synthesis. He went on, in a life of intense usefulness, to build educational institutions to make more rational the ordered acquisition of facts and ideas in the mind of the child. It was to a large degree from Herbart that Fechner derived his conception of the threshold, and the rich system of psychophysical method and psychological measurement which dominated experimental psychology in its opening decades. It was from this kind of a systematic experimental quantitative psychology that a psychology of individual differences was built.

Indeed, in his presidential address to the American Psychological Association in 1956, Cronbach

showed the massive place of quantitative method in the whole structure of modern psychology. Cronbach's analysis implied that systematic quantitative thinking was even more of the basic substance or essence of modern psychology than was experimental or other inductive methodology. As the slogan of the Psychometric Society expresses it, the trend and aim of scientific psychology is to create a "quantitative rational science," and of course, the realization of this whole trend in our own day lies in the preoccupation with the construction of mathematical models.

NUMBER IN MODERN PSYCHOLOGY

No form of psychology can escape this mathematical thrust. We may think of it as the very essence of the modern quest for truth, almost as Plato did; or we may think of it as a force which, hand in hand with materialist atomism and mechanism, debases our ideal and distorts our vision. But normative judgments of these types do not seem to impede too greatly the spread of mathematics which is so deeply ingrained in our modern way of thinking. Indeed Freud, whose thinking is as radically qualitative as any you could devise, made more and more of the economic principle with its quantitative approach to the resolution of conflict, and his invention of the polarities and of systems of three's may well represent the same cathexis upon numbers, the same love of abstract quantitative relationships, which is the very essence of Pythagoreanism.

Gestalt psychology began protesting against atomism and mechanism, yet found very early in Köhler's doctrine of the physical gestalt, the tendency of organismic operations to take the simplest mathematical form, leading on to the generalization of the principles of *prägnanz* and the law of closure, which tell us why, in a complex and heterogeneous field, the relatively simple, ordered, symmetrical, rhythmic, or structured will tend to press its way in, and to press out the components of randomness, confusion, or noise, which have the same stimulus intensity, but are doomed to vastly less effective roles in biological and psychological life. The mathematical obsession, or aspiration—however you view it—has seeped and percolated, pressed forward and invaded everything in psychology. Laws, however stated, soon became quantitative laws; as in Thorndike's dictum: "Whatever exists must

exist in some quantity, and therefore can be measured."

In Heinz Werner's magnificent developmental system, at first the global or undifferentiated world is not ready for number; at the second level it is a world of identifiable and measurable components; at the third level these components find articulate and structured relations, one with another, and we have the quantitative, the quasi-geometrical system which universal science would expect. We look everywhere now for isomorphism between physical, physiological, and psychological realities, assuring ourselves—possibly correctly, though we do not know for sure—that number and number systems apply equally to the object, to the subject, to the cosmos, and to the person, to the infinitely large and to the infinitely small.

I have moved ahead rapidly in praise of the Pythagoreans because I believe that we seldom realize the enormous dependence upon them which is characteristic of our work as psychologists, and the enormous strength of the preoccupation with numbers which feed in, together with a reality principle, to make psychology one special kind of number system. I would simply ask, as objectively as I know how, and without knowing the answer: "Are we overinvested in number?" There is not the slightest doubt that number theory and number preoccupation helps us towards the discovery of many kinds of reality. There is likewise no doubt that number leads into various types of mysticism which become so fascinating, so enriching, and so sustaining that one finds it difficult indeed to come back to the world of plain things and the immediate world to be dealt with. Number mysticism makes us believe that symmetry, order, rhythm have direct predictive power as to what will be actually observed. It has often led us astray. The colossal achievement of Fechner, for example, produced a colossal scientific and a colossal mystical output. Several decades of cautious inductive work have shown the great limitations in time and place upon Fechner's generalizations.

DISCONTINUITY

But I have another grave question to put regarding the adequacy of the formulation which I have used. I have not distinguished between number theory as such and quantitative theory in general. Of course, all of us recognize and use all the time

the principles of continuity and discontinuity. We know that people, for example, come in whole numbers, and we laugh when the statistician tells us that there are 2.3 children in the average family, or that you have a certain fractional chance of dying of cancer. The very nature of the mathematical operations, however, which have followed from the practical art of measurement, and likewise from the beautiful methods of Newton and his followers, have represented the real as continuous and continuity as the only modality of a true science. We teach our pupils about continuities, linear relationships, normal curves. Nature may hate a vacuum, but not more intensely than we hate the gaps which would appear from discontinuous distributions, or in particular from true quantum principles. The profound revolution in physics, coming from Max Planck's discovery of a constant "h" in black-body radiation, which has served as a prototype for unit thinking, whole number thinking in so much of modern physics, has almost completely passed psychology by. We have, in our devotion to the mathematics of the continuum, forgotten—or almost forgotten—the Pythagoreans' adoring attitude toward whole numbers. We have forgotten that nature—that is, the sidereal universe, and our own Mother Earth—and life, including amoeba and man, that psychology—including perception, memory, emotion, and thought—often come in chunks. Very often when properly observed, these identifiable chunks are discontinuous from other chunks. Some of these chunks happen to be called whole individuals, and we have idiographic as well as nomothetic methods to employ. But there are many discontinuities also *within* the individual, many potential "split brain monkeys" lurking within our own inner selves and sometimes becoming painfully manifest.

The quantum principle, dating from the year 1900, may manifest itself in large units, or in small. Planck's discovery appears to indicate that action comes in unit packages, as we might expect from the new electron theory which just preceded it. I do not believe we have yet fully grasped the importance of this principle in behavior study, or in the analysis of our immediate awareness. The all-or-nothing law in neurophysiology and the related all-or-nothing law which applies to rods, cones, and receptors generally, gives discrete psychophysiological units which are concealed behind the misleading appearance of continuity. S. S. Stevens

won the Warren Prize for demonstrating the quantal basis of pitch. As Mary Shirley showed, growth is a matter of "saltatory" chunks, unit spurts, behind the maturation process. Affectivity, fears, rages, and other dynamic processes come in quantum terms when first excited, and it may well be that the affective life really comes in psychophysiological integer units too. There are, in personality structure and in the social order, stupid stubborn lines which say: "Thou shalt not cross." In fact, everywhere in society there are discontinuities so enormous that you can wonder, when you look at them, how a speaker could have taken two-thirds of his time assuming the universality of continuities.

The heart of the difficulty appears to lie in misunderstanding the process of abstracting. Whole numbers are an abstraction. The "threeness" which is present in three apples and three pears is not apprehended at the lowest sensory level, but at the level of cognition. If, by using the word *real* we mean that you can operate rationally, construct systems, and make predictions on the basis of whole numbers thus abstracted, the numbers are certainly real. They are, of course, one step away from sensory reality; one step towards some other kind of reality. If you speak of the square root of three, you are carrying out a further abstraction: You are asking what can happen if two equal quantities multiplied by one another give three; then you can perfectly well go on to emphasize that the square roots of negative numbers—which involve a still *further* abstracting process—can be multiplied by one another and come back giving the less puzzling reality of a whole negative number. There may be abstractions upon abstractions, and the abstracting process can itself be real. Sometimes after many such abstractions we come back and stub our toes upon simple sensory realities which have been predicted through these abstractions. I am not inveighing against the abstractions as unreal, or the process of manipulating them as scientifically unsound. On the contrary, I am endeavoring to justify higher-order abstractions by the same process of legitimization which applies to simpler, lower-order abstractions.

But there is always, in science, a further empirical test. If you come back from a simple factoring job with two answers, one of which is the square root of a positive number, and the other of which is the square root of a negative number, you have every reason to say that the intellect must

choose one as a real answer to your problem and reject the other. In exactly the same way if the real examination of evidence shows discontinuity as one possibility, and continuity as another, it is an empirical question which one will fit the facts. I am raising the question whether we are aware of this in contemporary psychology. We have a strong tendency to believe that continuities are in some sense real, that linear relations and normal distributions, multiple-factor analysis, and most of the machinery of calculation, and inherently the normalization, the linearization, the adjustment of data through the use of inverse squares, and of logarithmic restatement of bizarre distribution forms, are all carried out in the name of intelligibility, and ultimately in the name of truth. The result is, of course, that we force data sometimes slightly, but sometimes profoundly, into channels which are conceived to make them more real, but can only in fact make them less real in the sense of confronting nature as she is. My statistical friends frequently remind me that the fundamental methods of dealing with discontinuous data in the biological sciences are very primitive and grossly incomplete. The result is that we forget the quantum principle which Schrödinger emphasizes in his definition of life and of evolution; we turn away from the discontinuities and the pure numbers, the step functions, or jump processes with which nature is so full—so full indeed that the theory of the electron, the theory of the cell, the theory of the individual, would all be utterly confused if we insisted upon continuities. What we have done, in the light of mathematics since the seventeenth century, is to defy the fundamental sense in Pythagorean number theory and to rely almost wholly upon those higher-order abstractions which make use of continuities, linearities, and “normalities” which nature so often contradicts. It is not the mathematics that is intellectually crippling, nor can mathematics ever take the side of one metaphysical proposition against another; but the use of mathematics can become blind, as can any tool revealed by the sociology of knowledge.

MODERN “NUMBER MYSTICISM”

My task has been to suggest our enormous emotional investment or cathexis in numbers; not just the danger of looking for discontinuities and quantum principles when they do not exist, but the

very much greater danger that we look for higher-order abstractions relating to numbers rather than respecting empirical stepwise or quantum numerical realities with which nature is shot through. It is only the empirical use of Pythagorean number theory, only the investigation of the tough problem whether nature is working in terms of units or continua, that can save us from the pitfalls inherent in the assumptions of modern psychology. Fifty years ago psychology unwittingly turned at the crossroads in the direction of continuity theory, but the issue is not really closed and if wrong decisions have been made they can be reconsidered.

Over and above the general cathexis upon numbers and ways of manipulating them, psychology is, of course, shot through with preference for specific numbers. A p value, for example, at a significance level of .05 is significant, but at .06 is not significant, I suppose because few of us have six fingers. Here we have misplaced the discontinuity, that is, where there is a real continuity we have made a gulf between the .05 and the .06, or indeed for certain problems, the .01 rather than the .02. When I was a graduate student at Columbia, a critical ratio had to be three and a doctor's dissertation had to report a nonsignificant difference between two means if the critical ratio was only 2.7. There had to be 30 subjects to make possible the use of a Pearson correlation coefficient. Despite the nonlinearities with which nature is so replete, we may not use the assumptions about eta or nonlinear relations unless we have really massive evidence against linearity.

It would, of course, take us further into metaphysics to ask whether there is more than our own human ways of thinking that produces this universality of mathematical laws to be found in science—whether, in short, the world is, in reality, ordered mathematically. If you are as much of a Pythagorean as I am, you have to believe that it is; but if you are as critical as you ought to be, you have to balk. Neither science nor metaphysics is ready to say anything final on this issue.

There is, however, one profound question from the time of the Pythagoreans, and very much alive today, which we do have to attempt briefly to answer in conclusion. This is the question of the isomorphism, the ultimate unity of the rhythms and symmetries, the quantitative laws of our life, and the rhythms and symmetries of stars and oceans, cells and electrical particles which emerge from

scientific study. Research in the biology and psychology of the last few decades has shown a rich abundance of quantitatively known behavior principles, principles of perception, memory, and thought which make up, as I said, a fair part of modern psychology. As in the case of Fechner's "outer psychophysics" and "inner psychophysics," and as in the case of Köhler's physical *Gestalten*, the laws which describe the *within* also describe the *without*.

ISOMORPHISM

It is not simply a question of one system of realities fitting another. It is a question, apparently, of their being one and the same. Isomorphism or identity of form forces us to the recognition that the same reality of adaptation, or learning, or forgetting, or generalization, or whatever it is, appears in different levels of observation because it is simply the same reality which is refracted through different media. That the curve of forgetting looks like the curve of declining gas pressure as more and more elements are lost is not an odd coincidence, but a statement that parabolic declines of this sort are a mathematical necessity for particles free from outside interference. James Miller and his collaborators have been undertaking, with some success, to show that the same basic laws apply at different levels in nature, and as in Herbert Spencer's and Heinz Werner's broad evolutionary schemes, we may say that the mathematical order is the same in all these different contexts for the simple reason that there are cosmic universals relating to observable realities in general, and to the mathematics of their change in time. Psychology will thus become isomorphic with physiology, with psychoanalysis, with linguistic and moral change and development insofar as there are observables subject to the same possibilities of measurement. If

this be mystical, make the most of it. If it be materialistic, make the most of it. This is not the first time in history that the mystical and the materialistic have both been suspected under the cover of a very broad groping generalization. Such ideas are fascinating, but, of course, likewise dangerous. Pythagoreans had to flee despots because their secret society was dangerous. But my point is that they really *were* dangerous. There is no deity that we worship more abjectly today than number theory in its two-theory form, the quantum and the continuity form. It would be wise to know at which shrine we are worshipping in each of our scientific endeavors, and to know, in the last analysis, what we think of the strange prophetic figure who defined both these deities, and apparently worshipped them both. I should like to conclude with a quotation from one of the great biologists of this century, D'Arcy Thompson (1952):

A "principle of discontinuity," then, is inherent in all our classifications, whether mathematical, physical or biological; and the infinitude of possible forms, always limited, may be further reduced and discontinuity further revealed by imposing conditions—as, for example, that our parameters must be whole numbers, or proceed by *quanta*, as the physicists say. The lines of the spectrum, the six families of crystals, Dalton's atomic law, the chemical elements themselves, all illustrate this principle of discontinuity. In short, nature proceeds *from one type to another* among organic as well as inorganic forms; and these types vary according to their own parameters, and are defined by physico-mathematical conditions of possibility. In natural history Cuvier's "types" may not be perfectly chosen nor numerous enough, but *types* they are; and to seek for stepping-stones across the gaps between is to seek in vain, for ever [p. 1094].

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