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The Pythagorean Table

The Background

HARMONICAL PROPORTIONS
SPONTANEOUSLY EXPERIENCED
IN MUSIC
GIVE MEANING AND FORM
TO MANY PHENOMENA
IN OTHER FIELDS

IN 1868 AND 1876, two big volumes of a book by Albert Freiherr von Thimus were published in Cologne under the title *Die harmonikale Symbolik des Alterthums* ("Harmonical Symbolism of Antiquity").¹ The manuscript of a third volume never went to print and seems to have been lost. Thimus, who was born in Aachen in 1806, studied law in Bonn and Heidelberg and was called to the bar in Koblenz, where he later became a counselor. In 1862 he was appointed to the Court of Appeals in Cologne. He served as a member of the Prussian House of Representatives and of the Reichstag. In 1874 he retired from office; four years later, at the age of seventy-two, he died in Cologne.

An intimate friend, Dr. August Reichensperger—who played an important role in the process of writing and publishing the book—described Thimus as "an abyss of modesty and a wonder of intellect."² The erudition of Thimus was extraordinary. He was a practical amateur musician who knew all there was to be learned in his time about music theory and history. As an orthodox Catholic, he had studied theology. He thoroughly understood philosophy, mathematics, and archeology. He could read Latin, Greek, Hebrew, Arabic, and Chinese; he could interpret hieroglyphic and cuneiform writing. His book demonstrates that the thoroughness of his scholarship had not suffered from the immense scope of his learning.

Die harmonikale Symbolik des Alterthums was received by a handful of contemporaries with enthusiasm and admiration. A friend, Richard Hasenclever, published a fifty-page abstract of the first volume.³ Soon after Thimus's death, the stock was sold by the publisher to a second-hand bookdealer, and the remainder was destroyed. For several generations, the book was ignored equally by philologists and musicologists, historians and philosophers. Although Thimus offered, for the first time in modern history, a meaningful explanation of the *Timaeus* numbers, no Plato scholar of the late nineteenth and early twentieth centuries was aware of it. But the seed sown by Thimus at last brought ample harvest in the works of the German-Swiss philosopher Hans Kayser, who first called it to attention in the years following the First World War. Since then, the influence of Thimus has spread, and plans for an offset reprint of the two volumes have been announced.

Thimus's investigations were essentially historical. He was trying to relate the esoteric mathematics and harmonics of the Pythagoreans to older Greek and Oriental sources, in particular to ancient Semitic-Hebrew traditions. Build-

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ing on Thimus's findings, Kayser explored the theoretic and scientific possibilities of the Pythagorean tradition. His work serves the rehabilitation of hearing as a tool of cognition and understanding. In an article on the *Ur-Geräusch* ("Archisound"), Rainer Maria Rilke, with the prophetic intuition of a true poet, had regretted that the poet is generally only open visually and neglects the other senses, although they probably flow out of a common source that lies behind all.⁴ Rilke's biographer Katharina Kippenberg comments with specific reference to Hans Kayser that "Rilke would have enthusiastically embraced certain modern thoughts based on ancient knowledge: harmonical research. . . . Whereas modern science relies mainly on sight and touch, harmonics proceeds from ear and tone and develops from acoustical laws an interpretation of the world that includes value. Kayser seems to have answered Rilke's question 'whether research can significantly enhance the dimensions of the sensual world in the field accepted by us.' Of microscope and telescope Rilke expected little. . . . Harmonics would have shown him the possibility of such an enhancement."⁵

The starting point for Kayser's work was Thimus's reconstruction of the Pythagorean Table. We shall follow Thimus's own description without entering the argument concerning the historic age of the particular form he outlined.⁶

The Form of the Pythagorean Table

The general shape of the Table was known to Plato, who in turn probably employed much older symbolism, Egyptian or Babylonian. Crantor, Plato's successor at the Academy in Athens, knew that the numbers of the world soul mentioned in the *Timaeus* were arranged in the form of the Greek letter *lambda*, that is, an angle with the apex at the top. Nicomachus, writing important treatises on arithmetic in the second century A.D., also refers to the *lambdoma*, but his major work which contained an explicit diagram has been lost. Fortunately Thimus found a sketch of the diagram in a commentary on Nicomachus written in the early fourth century A. D. by the philosopher, mathematician and musician Iamblichus (who exercised a profound influence on the Emperor Julian Apostata). Iamblichus drew the Table as shown in Fig. 1.

To avoid any misunderstanding, Iamblichus added a lengthy explanation, the wordiness of which can be excused both by the unavailability at the time of an adequate mathematical notation and his concern for the importance of the Table. Translated into modern language, his instructions offer on one arm of the angle an arithmetic progression from 1 to 10 which moves toward the infinitely large; on the other arm, a harmonic progression from 1 to 1/10 which moves toward the infinitely small. Every horizontal line shows the "reciprocity of [things] equalizing each other."⁷ The product of any two corresponding items yields the monas, the One. Thimus criticizes the scribe of the Iamblichus manuscript for having forgotten to add the

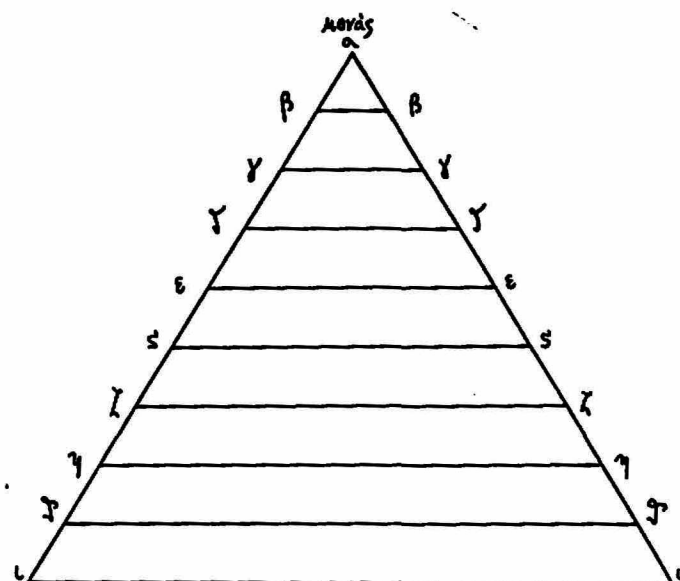


Figure 1. The lambdoma of Iamblichus.

apex to the letters on one of the arms, a kind of apostrophe that indicated the reciprocal value of any integer (if γ stood for 3, then γ' indicated 1/3).

Thimus calls the form of the diagram transmitted by Iamblichus *kenoma*, a hollow skeleton. It lacks, to use an ancient harmonical expression, *pleroma*, an appropriate filling. Thimus first added the diagonal consisting of 1/1, 2/2, . . . n/n, and then interpolated the resulting ratios for each crossing of the two rows—numbers called in classical terms *artioperisson* and *perissartion*.⁸ In Pythagorean tradition, the quantity expressed by a number corresponds to the quality expressed by a musical tone. The modern physical definition of pitch by either frequency or wavelength recognizes reciprocity in their rates of change: the product of their ratios in every case is always 1. The completed Table, as conveniently drawn for the modern reader, appears in Fig. 2.

The horizontal arm projects a harmonic progression from 1 to 1/∞, corresponding (in terms of wavelength) to the musical unfolding of the harmonic series above C, of which the first six partials form the major triad. The vertical arm projects an arithmetic progression from 1 to ∞/1, corresponding (again in terms of wavelength) to the musical unfolding of the reciprocal series below C or, as the first six partials are more commonly called, the F minor triad. Lines connecting identical pitches all meet in one point behind the monas 1/1. Thimus correctly identifies this point as 0/0. He gives no evidence for the awareness of ancient theorists of this 0/0 point; but he offers numerous, mainly philological, speculations for the antiquity of this "star of the pleroma," drawing examples from Chinese, Hebrew, Gnostic, and orthodox Catholic sources.⁹

Seeing that the frequency with which any ratio recurs in the Table sets up a hierarchy of consonance and dissonance in relation to the reference tone 1/1, Thimus is

$\frac{1}{1} \cdot c$	$\frac{1}{2} \cdot c'$	$\frac{1}{3} \cdot g'$	$\frac{1}{4} \cdot c''$	$\frac{1}{5} \cdot e''$	$\frac{1}{6} \cdot g''$	$\frac{1}{7} \cdot bb''$	$\frac{1}{8} \cdot c'''$	$\frac{1}{9} \cdot d'''$	$\frac{1}{10} \cdot e'''$
$\frac{2}{1} \cdot c$	$\frac{2}{2} \cdot c$	$\frac{2}{3} \cdot g$	$\frac{2}{4} \cdot c'$	$\frac{2}{5} \cdot e'$	$\frac{2}{6} \cdot g'$	$\frac{2}{7} \cdot bb'$	$\frac{2}{8} \cdot c''$	$\frac{2}{9} \cdot d''$	$\frac{2}{10} \cdot e''$
$\frac{3}{1} \cdot f$	$\frac{3}{2} \cdot f$	$\frac{3}{3} \cdot c$	$\frac{3}{4} \cdot f$	$\frac{3}{5} \cdot a$	$\frac{3}{6} \cdot c'$	$\frac{3}{7} \cdot eb''$	$\frac{3}{8} \cdot f'$	$\frac{3}{9} \cdot g'$	$\frac{3}{10} \cdot a'$
$\frac{4}{1} \cdot c_2$	$\frac{4}{2} \cdot c$	$\frac{4}{3} \cdot g$	$\frac{4}{4} \cdot c$	$\frac{4}{5} \cdot e$	$\frac{4}{6} \cdot g$	$\frac{4}{7} \cdot bb'$	$\frac{4}{8} \cdot c'$	$\frac{4}{9} \cdot d'$	$\frac{4}{10} \cdot e$
$\frac{5}{1} \cdot ab_3$	$\frac{5}{2} \cdot ab_2$	$\frac{5}{3} \cdot eb$	$\frac{5}{4} \cdot ab_1$	$\frac{5}{5} \cdot c$	$\frac{5}{6} \cdot eb$	$\frac{5}{7} \cdot gb''$	$\frac{5}{8} \cdot ab$	$\frac{5}{9} \cdot bb$	$\frac{5}{10} \cdot c'$
$\frac{6}{1} \cdot f_3$	$\frac{6}{2} \cdot f_2$	$\frac{6}{3} \cdot c$	$\frac{6}{4} \cdot f$	$\frac{6}{5} \cdot a$	$\frac{6}{6} \cdot c$	$\frac{6}{7} \cdot eb'$	$\frac{6}{8} \cdot f$	$\frac{6}{9} \cdot g$	$\frac{6}{10} \cdot a'$
$\frac{7}{1} \cdot d_3$	$\frac{7}{2} \cdot d_2$	$\frac{7}{3} \cdot g$	$\frac{7}{4} \cdot d$	$\frac{7}{5} \cdot b$	$\frac{7}{6} \cdot d'$	$\frac{7}{7} \cdot c$	$\frac{7}{8} \cdot d'$	$\frac{7}{9} \cdot e$	$\frac{7}{10} \cdot f$
$\frac{8}{1} \cdot c_3$	$\frac{8}{2} \cdot c_2$	$\frac{8}{3} \cdot g$	$\frac{8}{4} \cdot c'$	$\frac{8}{5} \cdot e'$	$\frac{8}{6} \cdot g'$	$\frac{8}{7} \cdot bb'$	$\frac{8}{8} \cdot c''$	$\frac{8}{9} \cdot d''$	$\frac{8}{10} \cdot e''$
$\frac{9}{1} \cdot bb_3$	$\frac{9}{2} \cdot bb_2$	$\frac{9}{3} \cdot f$	$\frac{9}{4} \cdot bb_1$	$\frac{9}{5} \cdot d'$	$\frac{9}{6} \cdot f$	$\frac{9}{7} \cdot ab'$	$\frac{9}{8} \cdot bb'$	$\frac{9}{9} \cdot c$	$\frac{9}{10} \cdot d'$
$\frac{10}{1} \cdot ab_3$	$\frac{10}{2} \cdot ab_2$	$\frac{10}{3} \cdot eb$	$\frac{10}{4} \cdot ab_1$	$\frac{10}{5} \cdot c$	$\frac{10}{6} \cdot eb$	$\frac{10}{7} \cdot gb'$	$\frac{10}{8} \cdot ab''$	$\frac{10}{9} \cdot bb''$	$\frac{10}{10} \cdot c'$

Figure 2. The Pythagorean Table.

tempted to write: "The main branches of a tree are thicker and stronger than secondary shoots growing out of these branches. . . . With every annual ring the development becomes more manifold and formally complex. If we let the organism of the Table grow, we see in the new rows first the dissonant diatonic intervals, then the chromatically altered steps between them, then the even finer distinctions of comma differences and gradations between the intervals. In this process, these ever smaller intervals and shadings get closer to the middle line (in terms of musical distance) the farther the two roots of the respective rows move away from the generating first monas 1/1 C and from the middle line. Thus gradually unfolds graphically before the eye, like a sound figure formed from numbers, the entire unlimited realm of tones. . . . In this play of positive and negative powers, to which correspond musically the opposition and mixture of the major and

minor modes, there emerges unmistakably the musical image of a more general dualistic law that permeates all nature and its creations."¹⁰

Before turning from the description of the Table, as reconstructed by Thimus, to applications in various fields, as developed by Kayser, we should like to insert a general comment. The Table is not a sacred mystery handed to man by God, like the Tables of the Law to Moses. It is rather a man-made arrangement of numbers and pitches—tone-numbers, as Kayser calls them. Now arrangements can be morphologically very revealing. Dmitri Mendeléeff's periodic table, for instance, is also a man-made arrangement.¹¹ Chemical elements appear nowhere in nature in the order determined by the table, yet the particular arrangement identifies properties that give deep insights, otherwise hidden, into chemical groups and families. In both the musical and chemical tables, a known

series of numbers yields significant laws because of a particular and artificial arrangement. In this sense, both tables are basically projections of the structure of the human mind and thus may be thought of as true "icons." They convey an inner form concept which, first, has an intrinsic value; second, proves to be applicable and thereby demonstrates an attunement between the inner and outer worlds; and, third, shows the existence of a higher kind of order that remains invisible under ordinary circumstances.

All of Hans Kayser's work was devoted to the development and interpretation of the Pythagorean Table. He was born in Württemberg as the son of a successful apothecary. When he was twenty, he went to Berlin to study music. He took lessons in composition with Schönberg and remained a passionate chamber-music player all his life, but he earned his doctorate in the history of art. After World War I he was active as editor and publisher while beginning his creative work in harmonics. In 1933 he received an offer from a reader of his first book to move with his family to Switzerland. Here the generosity of private admirers made it possible for him to devote the rest of his life to research and writing. Kayser settled near Bern and died in 1964. A recently published biography lists among his work fifteen books and forty-five articles, apart from lectures, editions, and musical compositions.¹²

Harmonics, as defined by Kayser, is based on scientifically treatable facts, on a sequence of correspondences, and on a system of value forms. The facts are the harmonical theorems of the tone-numbers. They are psychophysical realities demonstrable in nature and in our soul. In this regard, *harmonics is a science*. The correspondences, not just vague analogies, refer back to the harmonical theorems by which they are verified. This kind of thought and research, in addition to being scientific, operates with relationships between material, psychic, and spiritual forms which would seem unconnected were it not for the common harmonical theorem behind all of them. In this regard, *harmonics is a knowledge of correspondences*. The value forms, understood as being autonomous yet secured by harmonical theorems and achieving general significance by correspondences, may assume the character of symbols. Here the concern is with metaphysical, religious, and mythological forms. In this regard, *harmonics is a symbolism*.

The Pythagorean Table can be considered a visual aid of harmonics. Because the Table is a particular arrangement of tone-numbers, the immediate lessons to be learned from it are morphological. The importance of this potential is underlined by the conviction of some modern scientists, such as L. L. Whyte, that any future synthesis of our currently splintered knowledge will have to come from morphology.¹³

Harmonics in Various Fields

We shall now examine the Pythagorean Table by relating it to samples drawn from the following areas: mathematics

and music, philosophy and theology, architecture, physical science, and organic science.

Mathematics and Music

Mathematicians and musicians were the first to draw the Table and give it meaningful shape, for to them it was a practical tool. The two disciplines supplement each other. Plato wrote repeatedly that training in music and mathematics was a prerequisite for higher thought. The quadrivium in the curriculum of the medieval university was filled by music and mathematics. The Table provided a simple visual aid. Ernest McClain has brilliantly suggested in some recent writings that Plato, while composing his baffling description of the soul in *Timaeus*, was not studying an unworkable metal model of the heavens, a kind of armillary sphere, but rather drawing a form of the Pythagorean Table in the sand at his feet.¹⁴

In its simplest aspect, the Table shows basic multiplications and divisions. To this day, *tavola Pitagorica* is every Italian schoolboy's designation of the multiplication table up to 10 x 10. Seen by modern eyes, the Table perfectly demonstrates concepts of group theory. The Table can be thought of as being generated from the monas via multiplication and division by the natural numbers. Multiplication by 1 provides identity operation. Each item of the set has its inverse. Each product with the *pleroma* has associative and commutative properties. Transformations are easily performed without damaging the invariant properties of any ratio. The system is closed, because the result of the combination of any two of its members is itself a member within the system, not out of it.

Each number corresponds to a tone. Each ratio corresponds to a musical interval. The quantities expressed by the mathematical symbols can thus precisely, spontaneously, and directly be experienced also as musical, aesthetic qualities. The idea of reciprocity that governs the morphology of the Table becomes phenomenized by specific musical behavior. The pitch of a tone is physically determined by either frequency or wavelength, the product of their ratios always yielding 1. The arithmetic row on one arm of the Table indicates frequency relationships, while the harmonic row on the other arm indicates the corresponding wavelengths. Pitch goes up the higher the frequency or the shorter the vibrating string. A string at 1/2 of the generator, for instance, will produce a pitch of the frequency 2/1.

Reciprocity further explains the relation of the major and minor modes. Whatever the generator, one mode is the inverse of the other. Taking the point 1/1 C as center, we notice two chords emanating from it: the major triad C-E-G in one direction, the minor triad C-A^b-F in the other. These two chords constitute the unfolding of the generator C. This unfolding, viewed as an entity, is motionless, ontic; but activity on only one side of the whole brings about a rupture of balance. The active chord will tend

toward its complement. The ear confirms this behavior, for a chain of alternating F minor and C major chords sounds endless: whichever chord one has reached demands resolution toward the other. The Table thus makes apparent the most fundamental rule of functional harmony: every major chord has the tendency to resolve to the minor chord one fifth below, or, every minor chord has the tendency to resolve to the major chord one fifth above. In technical terms, the musician says that every major chord has a dominant function, every minor chord a subdominant function. All other rules of traditional harmony derive from this polarity.

Any row in the Table read from left to right also shows the overtone series. They were scientifically described for the first time in the early years of the eighteenth century by Joseph Sauveur (who was born deaf-mute), but their ratios had always been visible in the Pythagorean Table for all to see. No secret was involved, merely an interpretation by uncovering, by discovery. A vibrating string divides naturally into a theoretically infinite number of constituent forces. It vibrates simultaneously as a whole and in sections of $1/2$, $1/3$, $1/4$, etc. of the entire length.

Pitch relations, finally, establish a hierarchy among intervals in regard to consonance and dissonance. One arrives at a definite order by reading off the Table, as a continuum from left to right, all tone values of our system (that is, ratios based on the first six numbers and their products) without repeating identical pitches. The intervals thus gained appear in the order $1/1$, $1/2$, $1/3$, $2/3$, $1/4$, $3/4$, $1/5$, etc.—in musical terms: unison, octave, fifth, fourth, major third, et cetera. Various consonance-dissonance theories starting from other premises have reached the same results.¹⁵

Philosophy and Theology

The concept of polarity as the main generative force pervading the universe has a long tradition. The Ancient Greeks were highly articulate about it. "Necessarily the principles of being are Two," said Plato's friend Archytas.¹⁶ In the *Symposium*, Plato lets Zeus create male and female by cutting a unified "round primeval man" in half. The idea of the monas had come from Egypt. Revealing is the central Jewish prayer admonishing the people to "hear that God is One." The emphasis is aural, unlike comparable admonitions in the New Testament to "see" and "behold." In the third century B. C., Diogenes Laërtius wrote: "The Oneness was considered by the Pythagoreans the beginning of everything. They say that out of the Oneness sprang the indefinite Twoness. The first, they say, is cause and motivation; the latter, effect and matter. Out of the Oneness and Twoness sprang the numbers."

This statement, which recurs in many versions, is well illustrated by the Table. The One at the apex C is indeed the origin of everything, for all number-quantities and hence all tone-qualities of the Table derive from division and multiplication of One. The two arms of the Table

spring from One and move toward infinity. If the monas is thus cause and motivation, the items of the *pleroma* are subsequent phenomenalizations—"effect or matter." Six centuries later, Sextus Empiricus expressed the same idea and the same image: "To the Oneness belongs the predicate of active motivation, to the Twoness that of passive matter."

The dialectic nature of the Table found verbal expression by many authors: Philolaos, who influenced Plato, wrote, "The similar and the like would not need harmony. The dissimilar and unlike, however, had necessarily to be united by harmony if it were to endure in the cosmos." Aristotle wrote, "According to the Pythagoreans, oppositions are the elements of that which exists." And, "Harmony is a mixture and union of opposites." The literal meaning of the Greek word *harmony* is a "joining together," originally a carpenter's term. Harmony always implies a synthesis out of thesis and antithesis, a joining of the dissimilar elements produced by the two rows of the *lambdoma*. For every number, there is an inverse. The product of both is always again the One. The Table as a symbol of the cosmos is created by the splitting of the One, and held together by the unifying force of the One.

Reconciliation of old Greek wisdom with the new Christian religion characterizes Gnostic thought. Bishop Hippolytus, writing in the third century A. D., attributes the following ideas to Valentinus and his followers: "The commencement of all things was the *monas*—unbegotten, incorruptible, inconceivable, incomprehensible, generative, and the cause of the origin of all existencies. They call this monas the *Father*. . . . But since [the Father] was generative, he resolved that whatever was most beautiful and perfect in himself should produce and bring forth. . . . He was altogether Love, and love is not love unless there is something beloved. The Father himself, therefore, produced and brought forth *Mind* and *Truth* (*Νους* and *Αλήθεια*), that is a dyad, which is the Mistress, and Sovereign, and Mother of all the Aeons."¹⁷ The relation to the Table is clear, but one discerns a new attitude and interpretation. The two primary series are called male and female—an identification not by number alone but by *quality*. Hippolytus himself, who refuted the Valentinian choice, yet upheld the Father-Mother image by characterizing the orthodox Logos as female. The dyad was also described as light and dark, ascent and descent—metaphors to which anyone can add others from his own experience of polarity. In the Table, the reciprocity of male and female, or light and dark, can be heard as that of major and minor. The German musical terminology, derived from Latin, is *dur* and *moll*, hard and soft.

The question of monas and dyas tore the new Church apart in the century following Hippolytus. Was God "One" or "Two in One"? The Table showed how to reconcile this apparent dilemma, for the One is actually $1/1$, unique yet containing the Two. By the fourth century, the concept of a Divine Trinity had permeated all Christian thought, and

the various systems accused one another of heresy and error. The seriousness of the issue can be measured by the intensity of emotions behind excommunications and bloodshed.

A statement by Iamblichus throws light on the implications: "But above the limited and unlimited, as prime cause of these two prime causes of the things that have come into being, as cause not come into being, stands God. He has set the limited and the unlimited. Before God, the limited and the unlimited, the One and the many, the monas and the indefinite dyas, appear as the same. . . . But through this, that the things have been limited and shaped by the One, they have become apprehensible to the soul, by virtue of the immanent number, and have become an object of intellectual comprehension. For the same is known by the same, and the soul is itself number as to its form, or at least is made according to a law of number." Here the concepts illustrated by the Table have been significantly widened. The text states that the One, the creator of the cosmos, is not the highest God; for above the two series of the *lambdoma*, above the monas, above these prime causes of the phenomenal world and its creator stands the unknown God who has not come into being, the real prime mover. We remember from the description of the Table the rays connecting identical pitches which all meet in a 0/0 point behind the monas. There is some doubt that pre-Christian thought knew how to operate with zero. Thimus brings philological proof that neither Greek nor Latin has an original word for zero; the etymological roots for this concept in all modern languages are Semitic. Discovery of the zero seems to coincide, as a cataclysmic psychic event, with the discovery of the Holy Spirit. The 0/0 and 1/1 points are One but they are also Three. As the Table shows symbolically, the world is created by the unknown God (0/0) through the demiourgos (1/1) who thus becomes the immediate creator of the material universe.¹⁸ The points of individuation—that is, the entities represented by tone-numbers—are part of that physical universe and thus under the authority of the One. At the same time, however, they are directly connected with the unknown spirit. In modern terms: we are part of the physical world, generated by the One, but at the same time psychic individualities directly connected with the ineffable spirit. The Table visually symbolized this complex theological theorem and thereby made it comprehensible. The image is stronger than one may at first concede. Painters trying to depict the Holy Spirit—throughout the Middle Ages and Renaissance into the Baroque—have instinctively resorted to a bundle of rays radiating across the painting from an idealized 0/0 point.

Architecture

The Renaissance architect Leo Battista Alberti wrote: "I confirm the saying of Pythagoras that Nature resembles itself in all things. . . . For it is obviously the same numbers by which the harmony of the voices pleases the ears of men

that also fill the eyes and soul with wonderful pleasure. We shall therefore take from musicians the entire rule of formation, because they know those numbers best. . . . Architects use all these numbers in the most comfortable manner by taking two measures for drawing a market or open place . . . and three for drawing an assembly hall or council room . . . in measured proportion."¹⁹

Throughout history, builders have used various devices for determining precise proportions and modules. For this purpose, the *mensa Pythagorea* (as Boethius called it) was one of the most common tools. The line to be divided extends at a right angle from the horizontal 0/0 axis to any intersection with the diagonal of the Table, that is, with the 1/1 identity ray. Each intersection of the line with an identity ray now corresponds (in accordance with an elementary Euclidian theorem) exactly to the division indicated by that ray. A musician knowing the interval corresponding to a certain fraction can actually perform this division by ear, provided the line to be divided can be represented by a vibrating string. For finding 3/5 of the total length, for instance, he listens for the musical interval of a major sixth; for 4/5, for that of a major third; et cetera. Vitruvius, fifteen hundred years before Alberti, suggested that the necessary equal tension of the two ropes operating a catapult be tested by bringing them to the same pitch.²⁰

Hans Kayser has devoted a special monograph to a harmonical dividing canon found in a sketchbook of the thirteenth-century French architect, Villard de Honne-court.²¹ The basic figure drawn by Villard solves in an elegant manner the problem of dividing simply and exactly a given line into any number of equal parts (Figure 3).

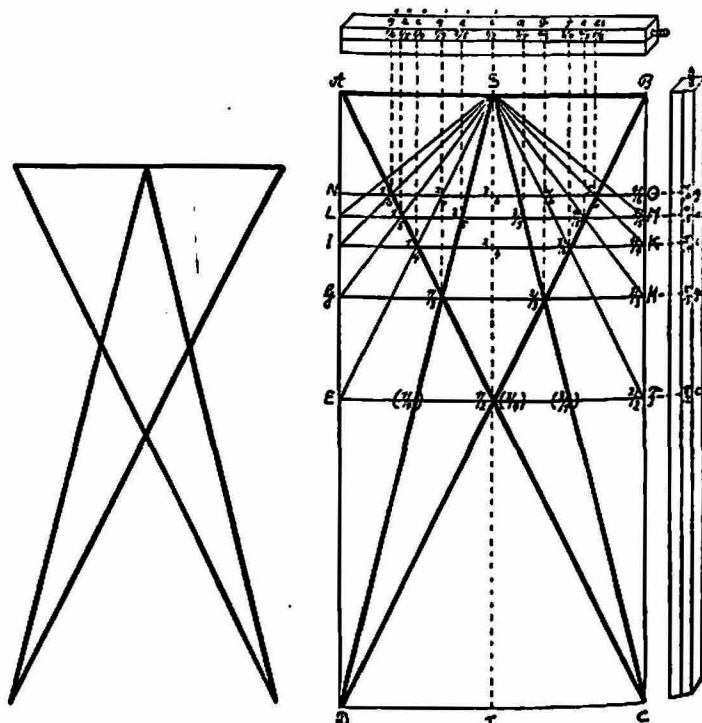


Figure 3.

a. Villard's basic sketch. b. Kayser's elaboration.

Gothic architects saw more in such a canon than a geometric trick; they rather looked upon it as a harmonical image. Number and form are to us a means to intellectual-scientific knowledge; to Villard's contemporaries they were symbols for spiritual experiences. Practical application and speculative interpretation of Villard's dividing canon both are developed in Kayser's monograph. We give one small example. Kayser draws Villard's canon in three modifications: 1:1/2, 1:1, and 1:2 (Figures 4-6.)

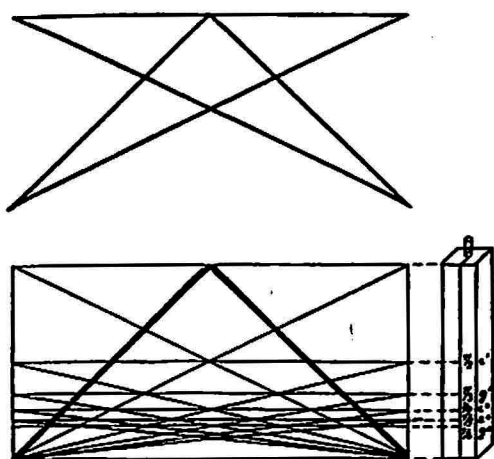


Figure 4. The "aegyptian" aspect of the harmonical dividing canon.

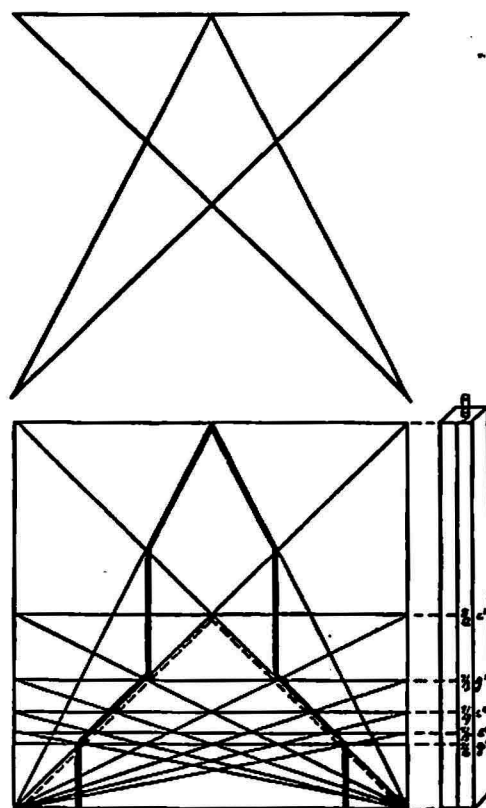


Figure 5. The "romanesque" aspect of the harmonical dividing canon.

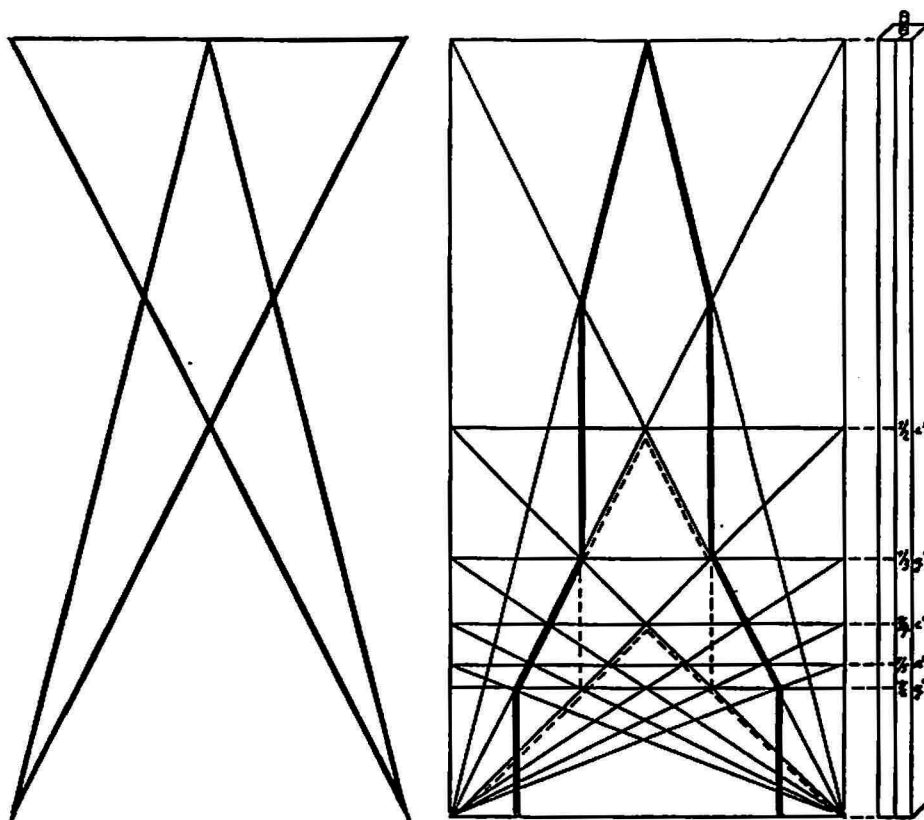


Figure 6. The "gothic" aspect of the harmonical dividing canon.

The "symphonic perfection" projected by the *mensa Pythagora* contributes to good acoustical properties of a hall—a relation almost completely ignored by modern architects (who have generally paid the price for this neglect by a helpless struggle with acoustics). The measurements of older concert halls famous for good acoustics are almost always harmonical: 12:6:5 for Symphony Hall in Boston (emphasis on the fifth); 10:5:3 for Severance Hall in Cleveland (emphasis on the major third); 8:4:3 for the Gewandhaus in Leipzig (emphasis on octaves of the fundamental). As Goethe wrote: "A noble philosopher has spoken of architecture as 'frozen music' and has thereby caused much raising of eyebrows. We know of no better way to reintroduce this beautiful thought than by calling architecture 'music grown silent.'"²⁴

In 1901, the crystallographer Victor Goldschmidt established a congruence of crystal formation and musical

Let M (in Figure 7b above) be the center of the crystal, and A and B the directions of two primary surfaces. If a new surface is to grow between them, the forces of M-A and M-B produce the resultant M-C. There appears the new surface C (see Fig. 7a). The continuation of the growth process (for example, D between A and C) can now be easily imagined. Goldschmidt had the idea of presenting the numerical progression as shown in Fig. 8.

	Shell 0	I	II	III	IV
Step 0	(?)	0	0	0	0
1		1	1/2	1/3	1/4
2		2	1	2/3	1/2
3		3	3/2	1	3/4
4		4	2	4/3	1

Combination
4

His commentary runs as follows: "This quadratic arrangement shows remarkable qualities in the diagonals, horizontals, and verticals. Each preceding series is contained in the following series as a square. The new members of the following series envelop the old square like a shell. Development continues by the addition of such shells. We may consider these series and their quadratic projection as a function of number theory, and call them a combination function. It would be worthwhile to study the diagram from a viewpoint of number theory and to investigate the meaning of the function. Maybe this group has already

been developed by theoretical mathematicians."²⁶ The identity of Goldschmidt's diagram and the Pythagorean Table is obvious. Here as there, the formations can be explained by group theory.

Hans Kayser, who pursued Goldschmidt's suggestions, speculated on harmonical laws also in geology, although admittedly the evidence is much scantier. Proceeding from the generally accepted hypothesis that the globe of the earth is organized into shell-like layers, Kayser raised the question of a possible coincidence of these shells with harmonical divisions.²⁷ A first suggestion was provided by the work of the geologist Hans Cloos, who set a critical shell limit at approximately 3200 km in awareness of the average measure of the earth radius of 6366.5 km.²⁸ The implied octave relationship made Kayser wonder whether

further shell differentiation upward from the center would become elucidated by the concept of harmonical division, but he was quick to recognize that he needed more data. The results of recent research concerning the shell-like layers of the earth, that have since become available, should permit a new examination of Kayser's original idea (see Figure 9)²⁹.

Kayser felt more strongly about another, related speculation. Geologists generally assign to the center of the earth a rather simple, uniform substance which becomes increasingly differentiated toward the surface of the earth and finally reaches a maximum of differentiation in the minerals of the outer shell. The gradual individuation of an originally homogeneous substance corresponds in exact analogy to the quantum-like development of discrete tones from an archetypal beginning in the Pythagorean Table. The wealth of stones and minerals, the ninety-two elements, derives from the homogeneous substance in the center of the earth, as the wealth of musical tones derives from the One. From the viewpoint of harmonics, built on the prime phenomenon of tone-number and thus welding the domains of being and value into a unified epistemological method, Kayser summarizes: "It does not appear impossible that, in consequence of the harmonical structure of the globe and its constituent matter, there may have been first formed a prime substance, and out of it the elements and their combinations according to the apriority of the harmonical value-forms. This would have to be understood, not as a haphazard development of stray differentiations, but as one strictly bound to the spatial harmonics of the globe."³⁰

The most famous application of Pythagorean thinking to physical science concerns Johannes Kepler's formulation of his Third Law. In his *Harmonices mundi*, published in 1619, in which he announced his astronomical discovery, he writes: "To discover a suitable proportion in the phenomena means to reveal and perceive it, and to bring to light the similarity between this proportion and a certain archetype of harmony present within the soul. . . . That a certain proportion is harmonic is due to an act of the soul comparing the proportion with its archetype residing in the soul. The proportion could not be called harmonic, and it would not possess any power to move the soul, were there no archetype."³¹

This is a genuine Pythagorean position. Francis Warrain has summed up Kepler's doctrine by reducing it to the following syllogism: "Major premise: the world, being divine creation, must be a manifestation of intelligence and beauty. Minor premise: beauty is founded upon mathematical harmony, and the latter must be sought in the regular figures, for distances as well as for speeds. Conclusion: therefore the world is built on the geometric harmony of the regular polygons, wherefrom proceed, on one hand, the regular polyhedra and, on the other hand, the musical consonances."³²

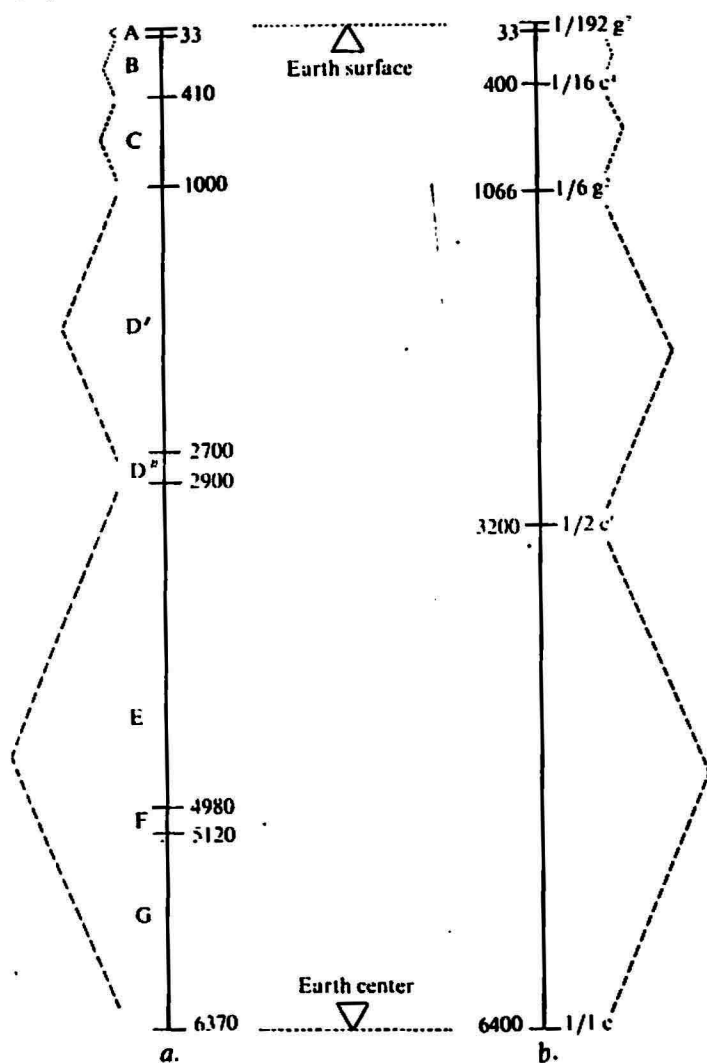


Fig. 9 Shell-like layers of the globe of the earth.

a. Recent estimates of the shells of the earth as derived from seismology (in kilometers). A, Crust; B, Upper Mantle (normal gradients); C, Upper Mantle (greater than normal gradients); D', Lower Mantle (normal gradients); D'', Lower Mantle (gradients near zero); E, Outer Core; F, Transition Region; G, Inner Core.

b. Harmonic division of the radius of the earth at nearest normative points.

For the formulation of his Third Law, Kepler used a typically harmonical technique of thinking, namely, transposition by octaves, fifths, and so forth. The outcome, musically speaking, is the theorem that the time of revolution of any planet is the middle tone in a series of three tones with equal intervals, of which the lowest tone represents the mean distance from the sun, and the highest tone the cube of that distance and simultaneously also the square of the middle tone. To Kepler, light and tone were the two things that reveal to us the harmonies in nature. He transposed the distances of the planets from the sun, that is, he raised them to the 2nd, 3rd, and 4th powers, and compared the results of these calculations with parallel octave transpositions of the average speeds of the planets. We see in Fig. 10 that the second octave of the time of revolution produces the same ratio as the third octave of the distance. The Third Law states that the square of the period of any planet is proportional to the cube of the planet's mean distance from the sun.

Planet	Mean distance from the sun a	Cubes of the mean distance a^3	Period in earth years b	Squares of the periods b^2	Relation of $a^3 : b^2$
Saturn	9,510	860,085	29,3272	860,085	1
Jupiter	5,200	140,608	11,8578	140,608	1
Mars	1,524	3,539	1,8812	3,539	1
Earth	1,000	1,000	1,000	1,000	1
Venus	0,724	0,379	0,6156	0,379	1
Mercury	0,388	0,058	0,2408	0,058	1

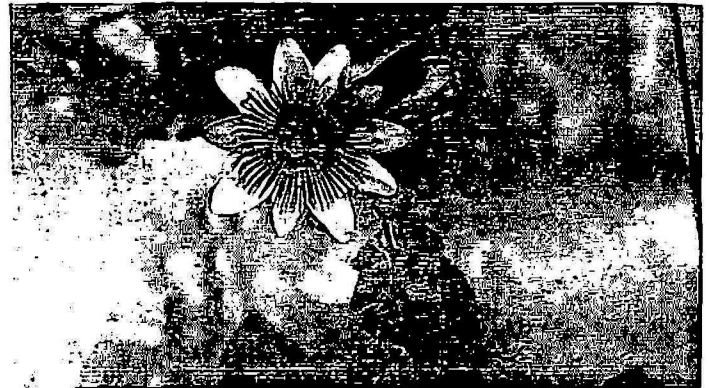
Figure 10. Data for Kepler's Third Law.

Organic Science

Except for isolated though valuable contributions, such as D'Arcy Thompson's Cartesian Transformations and investigations of the growth of shells and horns, and Julian Huxley's formulation of the allometric laws of growth, the work of Hans Kayser offers the first systematic attempt to apply harmonical analysis to the living world.³³ His book *Harmonia Plantarum* (the title taken from Goethe, who had originally intended it for *Die Metamorphose der Pflanzen*) deals in detail with forms, functions, harmonical value forms, and the "essence" of plants.³⁴

Morphology would remain merely descriptive unless it explained the individuation of a particular morphé as the result of an accentuation, a selection, of a particular degree in a development.

The flower shown in Fig. 11 shows organization by the ratios 3 and 5. The numbers 3 and 5 are distinct values heard by the musician as, respectively, intervals of the perfect fifth and major third. They have been selected, to the exclusion of all other intervals, in the version of the Pythagorean Table shown in Fig. 12. Many different formal variations are possible.



The receptacle is hollowed into a cup bearing on its margin 5 sepals (the calyx), 5 petals (the corolla), a dense mass of thread-like bodies (the corolla); 5 stamens spring from the base of the ovary, and there are 3 separate styles. [J. C. Willis, A Dictionary of Flowering Plants and Ferns (Camb. Univ. Press. 1957).]

Hans Kayser called this optical aspect of the acoustical constellation *Hörbild*, acoustical image. Such a diagram is not a portrait but a *scheme implying a form tendency*. Kayser suggests that the acoustical image provides a tool for the psycho-physical analysis of biological facts and forms.

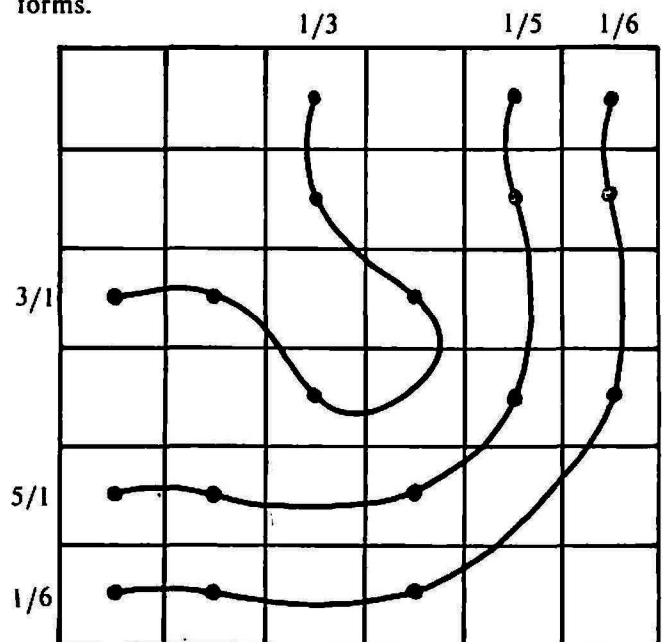


Figure 12. Pythagorean Table with Selection of 3 and 5.

An example of the morphological polarity of growth and limitation is provided by the diagram in Fig. 13. It is an acoustical image of an idealized archetypal plant (Goethe's *Urpflanze*), showing schematically plant growth above and below ground. The ratios, familiar to students of the *mensa Pythagorea*, are developed within a double logarithmic system of coordinates. The double horizontal line indicates the earth surface; a perpendicular to it, the middle axis of the plant. From the center 1/1, upward and downward, two "Tables" are developed in unidirectional logarithmic division. The upper shows the tendency to

approach a limit at its end, whereas the lower, revealing a tendency *ad infinitum* at its periphery, has the greatest density near the beginning at the central point. The diagram offers an exact plan of the inner growth dynamism of a plant. The system 1:10 is in balance at the index 10, that is, at the development of the partial tones of the Table up to the ratio 10. Any reduction of this ratio enlarges the part above ground while diminishing the part below ground

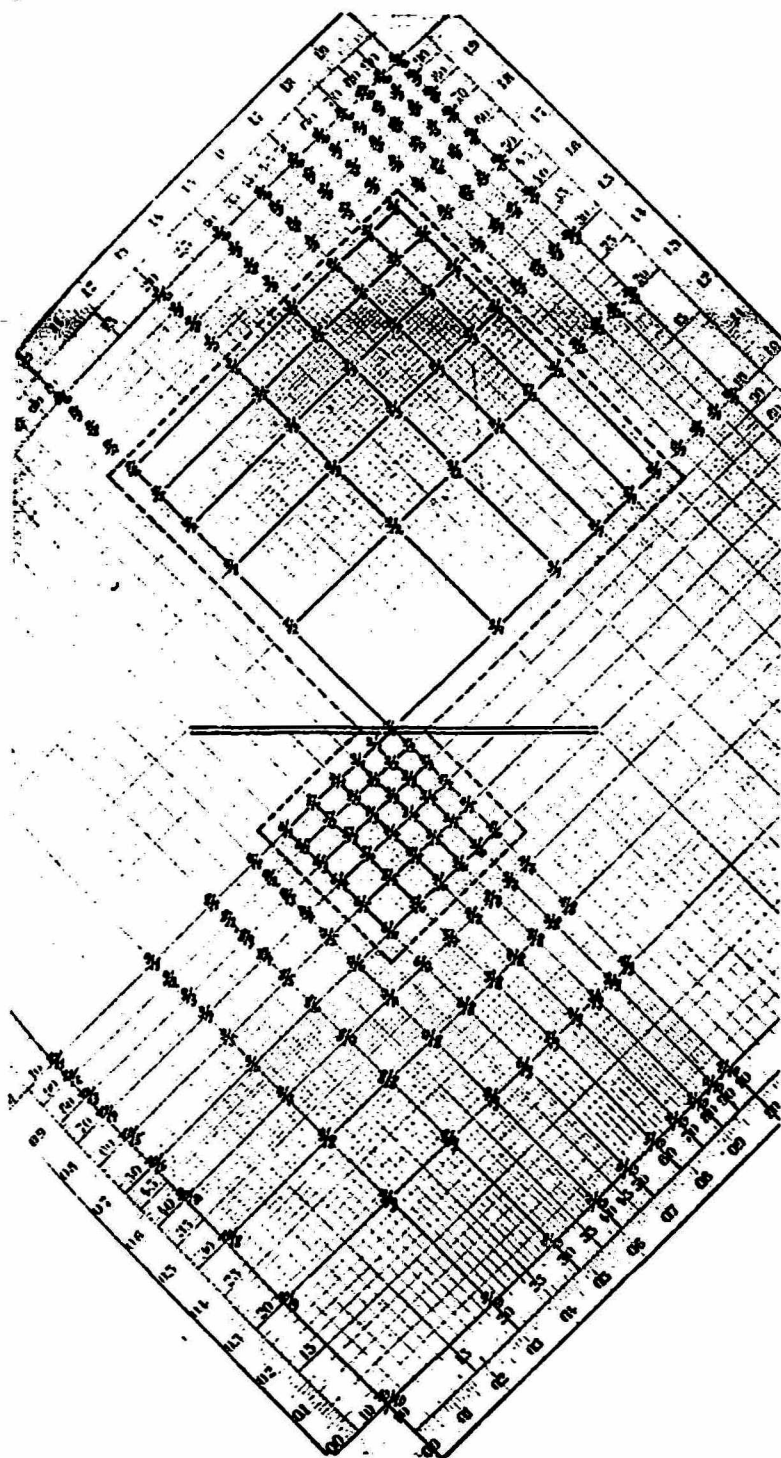


Figure 13. Acoustical Image of the Archetypal Plant.

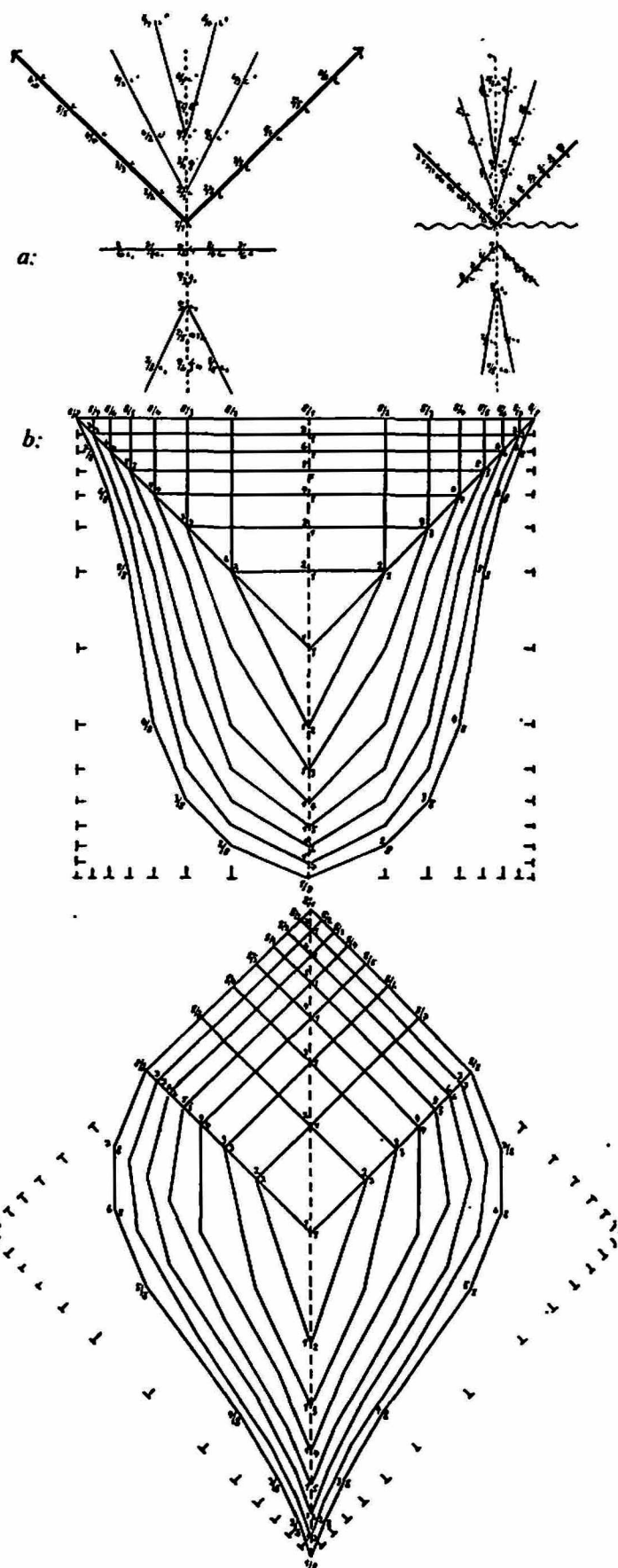


Figure 14. Acoustical Images of Plants.

(witness the dotted lines at the index 6). Inversely, any augmentation of the ratio 10 would enlarge the system of roots while diminishing that of stem and branches. Kayser summarizes: "In the course of its existence, every plant goes through a process of growth which passes through various harmonical indices, yet within any index always remaining a 'form.'"³⁵

In this spirit, Johannes Kepler writes, "If one inquires . . . why most fruit trees and berry bushes develop a blossom precisely according to a pentamerous system, . . . then I say that these things are accounted for by a contemplation of the beauty and particularly of the number 5 which characterizes the soul of these plants. . . . The fruit from a pentamerous blossom becomes fleshy, as in apples and pears, or pulpy, as in roses and cucumbers, the seed concealed inside the flesh or pulp. But nothing is born from a hexamerous blossom except seed in a dry cavity."³⁶

Harmonical analysis of plants makes possible a meaningful interpretation of the different participating value forms. Most significant is the morphological basis supplied by the tone-numbers of the Pythagorean Table for the generation and formation of branching, leaf, flower, and fruit. Fig. 14 shows two examples of plant structure, each with a corresponding blossom arrived at by a logarithmic "translation" of diagram *a* into diagram *b*. The exact analysis, too complex for the scope of this article, can be found in Kayser's *Harmonia Plantarum*.³⁷

We have stipulated all along that the Pythagorean Table, a man-made arrangement of tone-numbers, must correspond to some structural quality of our psyche in order to

explain satisfactorily events of the outside world. We are tempted to suggest a connection, yet to be studied, between the symbolism of the *mensa Pythagorea* and that of the mandala. C. G. Jung's thorough investigations of the latter, by analysis of dreams and of artistic representations, convinced him that he was looking at a symbolic expression of a psychic state.³⁸ A mandala usually takes the form of a flower, or cross, or wheel, with clear quaternary organization. Many of the forms reveal a harmonical basis. To Jung, reconciliation of opposites on a higher level was not a rational or willful affair but a process of psychic development expressed by symbols. "The psychic phenomenon cannot be grasped in its totality by the intellect, for it consists not only of *meaning* but also of *value*, and this depends on the intensity of the accompanying feeling-tones. Hence at least two 'rational' functions are needed in order to map out anything like a complete diagram of a given psychic content.

"If, therefore, in dealing with psychic contents one makes allowance not only for intellectual judgments but for value judgments as well, not only is the result a more complete picture of the content in question, but one also gets a better idea of the particular position it holds in the hierarchy of psychic contents in general. . . . The affective value gives the measure of the intensity of an idea, and the intensity in its turn expresses that idea's energetic tension, its effective potential."³⁹ If Jung defines mandala as a "symbol presenting an autonomous psychic fact, and marked by an ever recurring and everywhere identical phenomenology," we may well apply the identical terms to the Pythagorean Table.

Notes

- ¹ Köln: DuMont-Schauberg.
- ² Hans Kayser, "Albert von Thimus," in *Abhandlungen zur Ektypik harmonikaler Wertformen* (Zürich und Leipzig: Max Niehaus, 1938), pp. 23-37. The article contains valuable information about Thimus. This particular quotation, like most others from foreign languages, has been translated by the authors who occasionally sacrificed literalness for clarity.
- ³ *Die Grundzüge der esoterischen Harmonik des Alterthums* (Köln: DüMont-Schauberg, 1870).
- ⁴ *Sämtliche Werke* (Frankfurt a. M.: Insel, 1966), 6: 1085-93.
- ⁵ Rainer Maria Rilke (Leipzig: Insel, 1938), pp. 251-2.
- ⁶ Thimus, I: 129 ff.
- ⁷ *Ibid.*, pp. 133-4.
- ⁸ *Ibid.*, p. 136.
- ⁹ *Ibid.*, pp. 163-205. Yet the following statement by Aristotle makes one wonder: "The Pythagoreans too asserted the existence of a void, and that it enters the universe as it were breathed in from the infinite breath. This void delimits existents, it being a sort of separation and delimiting of things adjacent to one another. It is also primary in the case of numbers, the void delimiting their nature." (*Physics* 213b22-27)
- ¹⁰ *Ibid.*, p. 140.
- ¹¹ See his "Periodic Law of the Chemical Elements," in *The World of Mathematics*, ed. James R. Newman (N. Y.: Simon and Schuster, 1956), 2: 913-18.
- ¹² Rudolf Haase, *Hans Kayser: Ein Leben für die Harmonik der Welt* (Basel-Stuttgart: Schwabe, 1968).

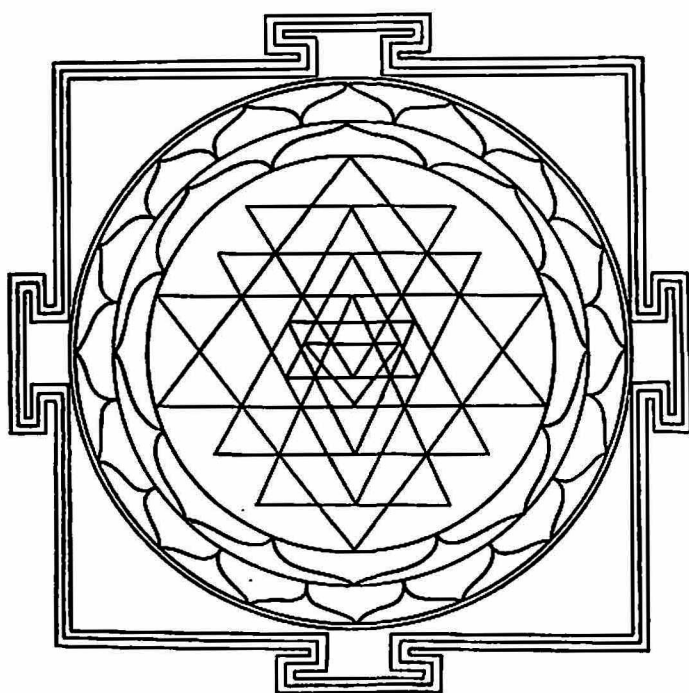


Figure 15. Mandala.

- ¹¹ "Atomism, Structure and Form," in *Structure in Art and in Science*, ed. Gyorgy Kepes (N. Y.: George Braziller, 1965), pp. 20-28.
- ¹² "The Pythagorean Plato," xeroxed. See also his article on "Plato's Musical Cosmology," in *Main Currents* 30, 1 (Sept.-Oct. 1973).
- ¹³ Cf. Helmholtz, Stumpf, etc.
- ¹⁴ Except where otherwise indicated, all quotations from Greek authors in this section are taken from Hans Kayser, "Pythagoras," in *Abhandlungen*, pp. 75-107.
- ¹⁵ W. Elfe Tayler, *Hippolytus and the Christian Church of the Third Century* (Hall, Virtue, & Co., 1853), pp. 91-4.
- ¹⁶ Comparable thoughts have been described by Seyyed Hossein Nasr in *An Introduction to Islamic Cosmological Doctrines* (Cambridge, Harvard Univ. Press, 1964). He quotes an esoteric manuscript of the fourth century in which "it is implied . . . that Being (*al-wujūd*) corresponds to one, and the Infinite, or the Divine Essence, to zero. Zero, therefore, symbolizes the Divine Ipseity, which is above all determinations including Being." (p. 46) See also Carl B. Boyer, "Zero: the Symbol, the Concept, the Number," in *National Mathematics Magazine* 18:8 (May 1944).
- ¹⁷ Here quoted from Hans Kayser, *Bevor die Engel sangen* (Basel: Schwabe, 1953), pp. 48-9.
- ¹⁸ *The Ten Books on Architecture* (New York: Dover, 1960), pp. 308-9.
- ¹⁹ *Ein harmonikaler Teilungs-Kanon* (Zürich: Occident, 1946).
- ²⁰ *Ibid.*, p. 26.
- ²¹ *The Gothic Cathedral* (New York: Pantheon, 1956), pp. 36-40.
- ²² *Maximen und Reflexionen*, no. 1207.
- ²³ *Ueber Harmonie und Complication* (Berlin, 1901). Figure 7 is taken from Hans Kayser, *Abhandlungen*, pp. 244-5.
- ²⁴ *Ueber Harmonie und Displcation* (Heidelberg, 1921), p. 34.
- ²⁵ *Abhandlungen*, p. 241.
- ²⁶ *Einführung in die Geologie* (Berlin, 1936).
- ²⁷ Data from Bullen, K. E., "Basic Evidence for Earth Division," in T. E. Gaskell, *The Earth's Mantle* (N. Y.: AP, 1967), pp. 11-39.
- ²⁸ *Abhandlungen*, pp. 238-40.
- ²⁹ Leipzig: Insel, 1925, pp. 115-16.
- ³⁰ *Essai sur "l'Harmonices Mundi"* (Paris: Hermann, 1942), p. 138.
- ³¹ D'Arcy Thompson, *On Growth and Form* (Cambridge: University Press, 1966), pp. 172-220; and Julian S. Huxley, *Problems of Relative Growth* (London: Methuen, 1932).
- ³² Basel: Schwabe, 1943.
- ³³ *Der hörende Mensch* (Berlin: Schneider, 1932), pp. 218-19.
- ³⁴ *Strena*, in *Gesammelte Werke*, ed. Max Caspar and Franz Hammer (München, 1941), 4:259-80.
- ³⁵ *Ibid.*, pp. 170-2.
- ³⁶ See particularly *Das Geheimnis der goldenen Blüte* (Zürich: Rascher, 1944); and *Psychologie und Alchemie* (Zürich: Rascher, 1952).
- ³⁷ *Aion* (Princeton: University Press, 1968), pp. 27-8.

Ernst Levy and Siegmund Levarie were colleagues at the University of Chicago, and later at Brooklyn College. Together they have published Tone: A Study in Musical Acoustics (Kent State Univ. Press, 1968), and have completed the yet unpublished manuscript of A Dictionary of Musical Morphology. Twenty-five years ago, Ernst Levy gave a series of unpublished lectures on "The Pythagorean Tradition" which, to a great extent, form the basis of the present essay; in 1965 he wrote an article for Main Currents (21, 3) on "The Pythagorean Concept of Measure." Prof. Levy, composer and pianist, is now living in retirement in his native Switzerland; Siegmund Levarie is professor of music at the City University of New York.

THE PYTHAGOREAN TABLE
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CITY U. OF NEW YORK, USA
MAIN CURRENTS IN MODERN THOUGHT XXX (3-4 1974) 117-29. ILLUS.
LANGUAGE: ENGLISH
DOC TYPE: ARTICLE IN PERIODICAL OR YEARBOOK

HARMONICAL PROPORTIONS SPONTANEOUSLY EXPERIENCED IN MUSIC GIVE MEANING AND FORM TO MANY PHENOMENA IN OTHER FIELDS. IN THE 19TH C. ALBERT VON THIMUS RECONSTRUCTED A FORM OF THE ANCIENT PYTHAGOREAN TABLE FROM A COMMENTARY ON NICOMACHUS. IN THE 20TH C. HANS KAYSER DEVELOPED THIMUS'S FINDINGS. SPECIFIC EXAMPLES SHOW THE APPLICABILITY OF THE PYTHAGOREAN TABLE TO MATHEMATICS AND MUSIC; PHILOSOPHY AND THEOLOGY; ARCHITECTURE; PHYSICAL SCIENCE; AND ORGANIC SCIENCE. (SIEGMUND LEVARIE)

DESCRIPTORS: MATHEMATICS - PYTHAGOREAN TABLE; PHILOSOPHY - PYTHAGOREAN TABLE APPLICATION; ARCHITECTURE - PYTHAGOREAN TABLE APPLICATION; TEMPERAMENT AND TUNING - PYTHAGOREAN TABLE RECONSTRUCTION BY A. VON THIMUS; THIMUS, ALBERT VON - RELATION TO NICOMACHUS AND PYTHAGOREAN TABLE; KAYSER, HANS - INFLUENCED BY THIMUS'S PYTHAGOREAN TABLE; THEORY - PYTHAGOREAN THEORY

THEORY, ANALYSIS, AND COMPOSITION - GENERAL (60)