

The Alleged Fallacy of Hippocrates of Chios

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I

Hippocrates of Chios is recognised as a figure of cardinal importance in the early history of Greek mathematics. He is indeed the first Greek mathematician for whose work we have substantial evidence in the form of a detailed report and commentary. These are in Simplicius' *Commentary on Aristotle's Physics* — a report which itself draws extensively on earlier sources. If we discount, as we surely should, the generally unreliable testimonies in such sources as Diogenes Laertius and Proclus attributing a number of theorems and proofs to heroic figures in the legendary beginnings of Greek mathematics such as Thales and Pythagoras, we may say that Hippocrates provides the first definite evidence available to us by which to judge the aims, techniques and especially the proof procedures of early Greek mathematics.

The aim of this paper¹ is to review the evidence concerning one

¹ This paper is derived from one session of the Cambridge ancient philosophy seminars which were devoted to Simplicius' *Commentary on the Physics* during 1985-86. It benefited not just from the comments made on the occasion of that session, but also from the joint exploration, by all the members of the seminar throughout the year, of Simplicius' strategy and tactics in this work and especially of his recurrent criticisms of Alexander. What this paper owes to that last point will become obvious: I owe, in effect, the inspiration of the chief line of argument developed here to the seminar and wish to record at the outset my debt to all my fellow-participants. I should also like to thank those

specific aspect of Hippocrates' work on the quadrature of lunes,² namely the question of what fallacy, if any, his proofs contained. This question cannot be dissociated from five others. First, what is the fallacy that Aristotle has in mind in the passage in the *Physics*, 185a14ff., on which Simplicius is commenting when he gives his report on the work of Hippocrates? Second, what is the fallacy Aristotle accuses Hippocrates of at *Soph El* 171b13ff? Third, what fallacy, if any, does Eudemus ascribe to Hippocrates? Fourth, what fallacy, if any, does Alexander ascribe to Hippocrates? Fifth, what are Simplicius' views on these questions and on the substantive issue of the fallacy that Hippocrates committed? If the argument of this paper is sound, the conclusion is that Hippocrates himself is to be exonerated from any fallacy.

II

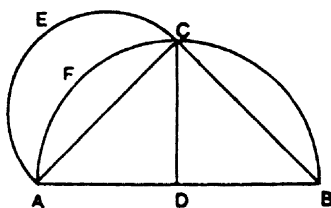
Although there is nothing that could be called an orthodox interpretation of Hippocrates' fallacy, certain assumptions are commonly made in commentaries on the issue,³ and I shall begin by setting out some points often taken to be secure or well grounded. Aristotle's text at *Phys* 185a14ff. distinguishes between quadrature by means of segments – which it is the business of the geometer to refute – and Antiphon's

who have been kind enough to let me have their detailed comments on an earlier draft of this paper, especially to Myles Burnyeat, David Fowler, Wilbur Knorr, Henry Mendell, Malcolm Schofield, Robert Wardy and Christian Wildberg.

- 2 The bibliography at the end of this paper sets out the main works of modern scholarship that have discussed aspects of the problem: they will be referred to in the notes by author's name and year of publication.
- 3 For what follows see, for example, Heath (1921), 183ff.; cf. Thomas (1939), 236f., 310f. There are, however, far more sophisticated recent analyses of Hippocrates' work in Knorr (1981), 151ff.; (1982), 127ff.; (1986), 26ff; and Mueller (1981), 146ff. Mueller in particular inclines to the view that Aristotle does not have Hippocrates in mind at *Phys* 185a14ff. and that we are in no position to say what the false proof ascribed to Hippocrates at *Soph El* 171b13ff. was. Knorr, who contemplates the possibility that Aristotle has misrepresented Hippocrates ([1986], 31), thinks Hippocrates himself may have presented the last of the lunule quadratures reported by Eudemus as a successful *reduction* of circle quadrature though not as a *resolution* of that problem (ibid., 36).

quadrature – which, because of its ungeometrical basis, the geometer has no duty to refute. On the basis of reports in Themistius⁴ and Philoponus⁵ as well as in Simplicius, the ‘quadrature by means of segments’ is attributed to Hippocrates and confirmation of this is often found in the reference to Hippocrates and to a squaring by means of lunes in connection with some *ψευδογράφημα* at *Soph El* 171b13ff. The question then arises of what Hippocrates’ fallacious quadrature was. On the basis of Simplicius’ report, Alexander is interpreted as ascribing to Hippocrates the following two quadratures: (1) that of a lune on the side of an inscribed square,⁶ and (2) that of three lunes on the sides of an inscribed hexagon plus a semi-circle.⁷ Alexander is further

Diagram 1



4 Themistius, *In Ar. Phys* 3.32ff., Schenkl

5 Philoponus, *In Ar. Phys* 31.3ff., Vitelli

6 Heath (1921), 185, paraphrases the argument as follows:

Suppose that AB is the diameter of a circle, D its centre and AC, CB sides of a square inscribed in it. On AC as diameter describe the semicircle AEC. Join CD. Now, since $AB^2 = 2AC^2$, and circles (and therefore semicircles) are to one another as the squares on their diameters, (semicircle ACB) = 2 (semicircle AEC). But (semicircle ACB) = 2 (quadrant ADC); therefore (semicircle AEC) = (quadrant ADC). If now we subtract the common part, the segment AFC, we have (lune AECF) = ΔADC , and the lune is “squared.”

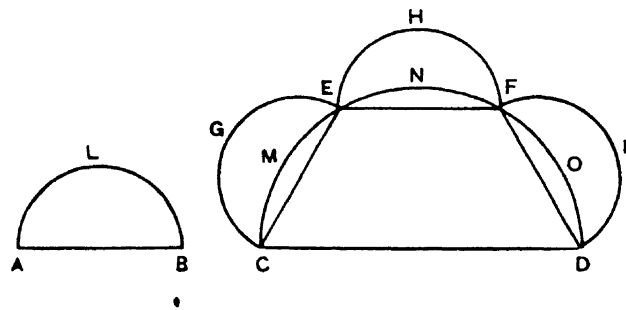
See diagram 1.

7 Heath (1921), 186, paraphrases the argument as follows:

Next take three consecutive sides CE, EF, FD of a regular hexagon inscribed in a circle of diameter CD. Also take AB equal to the

represented, on the basis of the same report, as having attributed to Hippocrates the fallacious claim that he had squared the circle. The grounds of this claim were (a) that having given a quadrature of one lune in (1) he could take it that he had given quadratures also of the three lunes in question in (2), and (b) that having given a quadrature also of the three lunes in question in (2), and (b) that having given a quadrature of those three lunes plus a semi-circle in (2) he could claim that the semi-circle itself (and so also a circle) had been squared, once the rectilinear figure equal to the three lunes is subtracted. But, it is

Diagram 2



radius of the circle and therefore equal to each of the sides. On AB, CE, EF, FD as diameters describe semicircles (in the last three cases outwards with reference to the circle). Then, since $CD^2 = 4AB^2 = AB^2 + CE^2 + EF^2 + FD^2$, and circles are to one another as the squares on their diameters, (semicircle CEFD) = 4 (semicircle ALB) = (sum of semicircles ALB, CGE, EHF, FKD). Subtracting from each side the sum of the small segments on CE, EF, FD, we have (trapezium CEFD) = (sum of three lunes) + (semicircle ALB).

See diagram 2. Heath goes on to note: 'The author goes on to say that, subtracting the rectilinear figure equal to the three lunes ("for a rectilinear figure was proved equal to a lune"), we get a rectilinear figure equal to the semicircle ALB, "and so the circle will have been squared." This conclusion is obviously false, and, as Alexander says, the fallacy is in taking what was proved only of the lune on the side of the inscribed square, namely that it can be squared, to be true of the lunes on the sides of an inscribed regular hexagon. It is impossible that Hippocrates (one of the ablest of geometers) could have made such a blunder.'

then also commonly argued,⁸ Hippocrates could not have made such an obvious blunder as that in (a), namely that of thinking that because he had squared one type of lune in (1) he had also squared the lunes in question in (2). It is then observed that in the sequence of quadratures of lunes that, following Eudemos, Simplicius goes on to report,⁹ there is no paralogism. Not finding Simplicius' own attempted diagnosis, or rather diagnoses,¹⁰ of where the fallacy lay very convincing or helpful, modern commentators have offered several competing views of which the following are the principal:

According to Heiberg,¹¹ Hippocrates thought that because he had squared every kind of lune – viz., lunes with an outer circumference equal to, greater than, or less than, a semi-circle – he thought he had squared every lune. This is a version of Eudemos' interpretation as it is represented by Simplicius, as we shall see.¹²

According to Björnbo,¹³ Hippocrates knew what he had done – and what he had not – but deliberately described what he had done in language calculated to mislead, viz., to give the impression that every lune had been squared.

According to Heath,¹⁴ Hippocrates knew what he had done, but was trying to put what he had discovered in the most favourable light. Where Björnbo charges Hippocrates with a calculated ambiguity, Heath merely charges him with a foolish one.

According to Ross,¹⁵ Hippocrates undertook his quadratures of lunes in the hope of squaring the circle, but did not claim that he had

8 Van der Waerden (1961), 132, however, merely remarks: 'therefore, if it were possible to "square" the three lunules, it would also be possible to "square" the semicircle and hence the circle. This led Hippocrates to study the squaring of lunules, bounded by arcs of circles.'

9 Simplicius, *In Phys* 60.22ff., see below (section 5).

10 *In Phys* 68.32ff., see below (section 6).

11 Heiberg (1884), 337ff., especially 343f.

12 Cf below, p. 000 on *In Phys* 60.22ff. and 67.3ff.

13 Björnbo (1913), cols. 1792ff.

14 Heath (1921), 196f., n.1. Cf. Heath (1949), 35, where he agrees with Ross that Aristotle has misunderstood Hippocrates.

15 Ross (1936), 463ff., followed by Thomas (1939), 310f., note b.

done so, though he was misunderstood by Aristotle as having made that claim. Thus while the other three lines of interpretation leave Hippocrates' reputation in one or other way not unblemished, Ross resolves the problem of reconciling Hippocrates' fame as a mathematician with Aristotle's as a commentator on mathematics firmly in favour of the former. Hippocrates himself is blameless: the fault lies squarely with Aristotle who misunderstood him.

III

Some of the weaknesses in points commonly taken to be reasonably secure can be identified quite quickly. I shall deal with some relating to the evidence of Aristotle, Themistius, Philoponus and Eutocius, before turning to a detailed examination of the testimony of Simplicius.

First, Aristotle does not, of course, explicitly mention Hippocrates in *Phys* 185a14ff. Moreover when he there talks of a quadrature by means of segments, the question of whether it is to be identified with a quadrature by means of lunes is one that already exercised Alexander, according to Simplicius.¹⁶

Again, in another passage in Aristotle often adduced in connection with Hippocrates, namely the discussion of reduction at *Pr Anal* 69a30ff., where Aristotle uses the example of a circle 'with' lunes¹⁷ becoming equal to a rectilinear figure, Hippocrates is not mentioned by name, nor in any event is there any reference to a parallogism.

In fact, then, the *only* passage in Aristotle where Hippocrates is named in connection with faulty reasoning of some sort is that in *Soph*

16 In *Phys* 69.1f., see below (section 6). Although at *In Phys* 55.26ff. when about to give Alexander's version, Simplicius says that a lune is a segment of a circle, later, at *In Phys* 69.1f., he praises Alexander for hesitating over whether the quadrature by means of segments is the same as the quadrature by means of lunes and he goes on to note that by the definition in Euclid *Elements* III (definition 6) – where segments are said to be contained by a circumference and a straight – lunes are not segments.

17 μετὰ: whether what Aristotle has in mind is a quadrature by means of lunes, or whether μετὰ should be taken strictly and the reference is to a quadrature of a circle together with lunes (as in the fourth quadrature reported by Eudemus according to Simplicius), is disputed. In either case Aristotle here uses this as an example of an acceptable, not a fallacious, piece of reasoning.

El 171b13ff., though the interpretation of that text too is far from straightforward. The main difficulties are as follows. Aristotle draws a distinction between certain ψευδογραφήματα (where he may have either reasoning or construction or both in mind) that are eristic and/or sophistic, and others that are neither. He mentions Bryson's quadrature as an example of one that is sophistic.¹⁸ The non-eristic class includes some where the ψευδογράφημα is 'to do with what is true' and this is illustrated with the remark 'such as that of Hippocrates or (fi) the quadrature by means of lunes.'

Given that this sub-category is 'to do with what is true,' there is presumably some correct mathematical reasoning concerning the subject-matter where the ψευδογράφημα or ψευδογραφήματα occurred. However that is not the main problem, which lies rather in the ambiguity of 'or.' Are we given one illustration, or two? That is, is the 'or' epexegetic (the 'quadrature by means of lunes' explains which of Hippocrates' extensive geometrical studies is in question), or is it disjunctive (we have two examples: one from Hippocrates, the other the quadrature by means of lunes).¹⁹ If the latter, then Hippocrates cannot, on this score, be associated with an Aristotelian criticism of a quadrature by lunes at all. On that reading we are simply not told what, in Aristotle's view, Hippocrates' ψευδογράφημα was. The fact that Hippocrates is reported by Proclus to have been the first to have composed a book of *Elements*²⁰ means that there were plenty of different areas of Hippocrates' work where Aristotle might have found some paralogism. However, by the same token, the failure to specify the mistake in question is very surprising, for Aristotle leaves it to his reader to understand the illustration as if it were well known.

The criticism that Aristotle has not made the example clear has less force on the first, epexegetic reading of 'or' – generally favoured by modern commentators. Yet even on that reading the illustration must be acknowledged to be telegraphic. There are, of course, plenty of other

18 On the question of what Bryson's quadrature was, and how it may have differed from Antiphon's mentioned at *Phys* 185a17ff., see, for example, Heath (1921), 221ff.; Wasserstein (1959); Mueller (1982), 151ff., 160ff.; Knorr (1986), 26ff.

19 For the former view, see, for example, Heath (1949), 34: for the latter Mueller (1982), 150f.

20 Proclus, *In Euc. I*, 66.7ff.

highly elliptical illustrations in other Aristotelian texts (for instance the reference to Antiphon at *Phys* 185a17). At the same time in such cases there is an evident risk that the illustrative allusion is an interpolation, e.g., arising from a gloss: nor in this instance can that possibility be ruled out.²¹

The testimonies of Themistius and Philoponus need not detain us long since their value for the reconstruction of Hippocrates is in both cases minimal. When Themistius comments on *Phys* 185a14ff. he first mentions Hippocrates by name, as well as Antiphon, and his text contains some statement to the effect that Hippocrates squared *only* the lune on the side of the inscribed square.²² But then at the crucial point where the nature of Hippocrates' paralogism is explained, Themistius' text is lacunose.²³ The editors generally supply the lacuna with material derived from Simplicius: but then Themistius provides no independent corroboration of Simplicius' view. In any event, unless the statement that Hippocrates squared *only* the lune on the inscribed square was qualified in the passage where our text is corrupt, it suggests that Themistius is unaware of the other quadratures described in the report of Eudemus given by Simplicius,²⁴ and in that case Themistius would have to be said to be very poorly informed on the question.

In Philoponus' *Commentary* on the same passage in the *Physics*, the report on Hippocrates' work is extremely brief. It credits Hippocrates with a quadrature of the lune, but then simply accused him of thinking that from this could be inferred the quadrature of the circle.²⁵

21 This point was made forcefully at the Cambridge seminar by Myles Burnyeat. Diels, in his edition of the Presocratic philosophers, square-bracketed the phrase 'or the quadrature by means of lunes,' and Ross (1949), 491, took that to be a (correct) gloss, borrowed from *Soph El* 172a2. See also Mueller (1982), 151n. 20.

22 Themistius, *In Ar. Phys* 3.33ff., 4.2

23 See Schenkl's apparatus, at Themistius, *In Ar. Phys* 4.2

24 Eudemus, as we shall see, ascribes four quadratures to Hippocrates; see below (section 5). Alexander is quoted by Simplicius as discussing two, that of the lune on the side of the inscribed square, and that of three lunes on the sides of an inscribed hexagon plus a semi-circle; see below (section 2). However, so far as lunes on their own are concerned, Alexander discusses only one.

25 This would appear to be the force of the expression $\omega\eta\theta\eta$ συλλογίζεσθαι at Philoponus, *In Ar. Phys* 31.9; cf. also 31.24-7.

Finally there is a neglected passage in Eutocius' *Commentary on Archimedes' Dimension of the Circle* (264.12ff, Heiberg-Stamatis) to which Wilbur Knorr has kindly drawn my attention. Here Eutocius first shows that he is aware of the relevance of Archimedes' proposition on the area of the circle to problems of circle quadrature (cf. below n. 32). Then, like Simplicius (at *In Phys* 54.12ff., see below), Eutocius brackets Hippocrates and Antiphon as mathematicians who had produced paralogisms in this connection 'about which those who have examined Eudemus' *History of Geometry* and who are acquainted with the Aristotelian *Honeycombs* [viz., Sporus' work of that name] will have exact knowledge.' Unfortunately, although Eutocius' credentials as a mathematician are considerable, his testimony, like those of Themistius and Philoponus, is altogether too indeterminate to help to resolve the substantive questions that concern us here. In particular Eutocius does not distinguish Hippocrates' work from Antiphon's, nor does he actually specify what either Eudemus or Sporus had had to say on the subject. Like Themistius and Philoponus, Eutocius is clear that paralogisms of some kind are involved, but he is quite unspecific about their nature. Nor can we say whether the diagnosis of paralogism in the case of Hippocrates stems from Eutocius' own reading of Aristotle or from his study of the commentators beginning with Eudemus and continuing all the way down to Eutocius' own teacher, Ammonius. The possibility that Ammonius himself was the major formative influence on Eutocius here is an intriguing one, but no more than that. Ammonius' other pupil, Simplicius, tells us of the conversations he had with his teacher on this topic (*In Phys* 59.23ff., see below), but Simplicius' report does not retail Ammonius' interpretation of Hippocrates.

IV

It is, of course, on Simplicius himself that we have to rely not only for his view on our problem, but also for those of Alexander and of Eudemus. Simplicius' discussion falls into six main sections, and in the evaluation of his testimony insufficient attention has sometimes been paid to the articulation and overall strategy of his argument.

Section 1. In *In Phys* 53.28-55.24 Simplicius makes some general comments on Aristotle's text (53.28-54.11), remarks that many tried to square the circle and names Hippocrates as well as Antiphon among those who mistakenly believed they had succeeded (54.12-19) and then proceeds to discuss Antiphon's attempt (54.20-55.24). But after giving

his own account of Antiphon, Simplicius then records, expressly to disagree with, Alexander's interpretation on the point at which Antiphon breached the principles of geometry (55.12ff.). The issue on which Simplicius, claiming the authority of Eudemus behind him (55.23), disagrees with Alexander here, does not concern us.²⁶ But it is important to note that on this point, as so often elsewhere,²⁷ Simplicius is keen to establish a distance between his own line and Alexander's, in part, of course, to claim superiority for his own.

Section 2. In Phys 55.25-57.29. Picking up Aristotle's remark that it is the geometer's business to refute the quadrature by segments, Simplicius first notes that Aristotle is referring to the quadrature by lunes that was discovered by Hippocrates (55.25-8). Simplicius then gives, in the rest of this section, Alexander's version of this quadrature. Although Aristotle is the understood subject of λέγοι δὲ ἄν in 55.26, as of φησί in 55.25, the understood subject of φησί at 56.1 is probably, rather, Alexander, who is the source of the version that follows. That he is the source becomes clear – if there were any doubt on the matter – from a comparison between 57.25-9 (the diagnosis of the fallacy) and 60.18ff. where that diagnosis is expressly attributed to Alexander with the words 'as I said.'

As we have noted, the fallacy lies in Alexander's view (according to Simplicius) in taking the lune on the side of the inscribed hexagon to have been squared because the lune on the side of the inscribed square has been, or, as it is put at 57.25ff., in taking universally what has not been proved universally. It should, however, be remarked that from 56.1 to 57.29 *Hippocrates is not named*. It is clear, for example from 60.18-21,²⁸ that *Simplicius* represents Alexander as attributing the fallacy to Hippocrates. But so far as the exegesis of the fallacy at 56.1 to 57.29 goes, that could have come from a general commentary, in Alex-

26 Simplicius objects that the principle that Alexander says is violated (namely that a circle touches a straight at one point only) is not a geometrical hypothesis, but a theorem proved in Euclid *Elements* III (16). Simplicius prefers to believe that the principle breached is that geometrical magnitudes are infinitely divisible, citing Eudemus as support for this view.

27 Cf. note 1 above. Within his comments on *Phys* 185a14 alone we have further examples at 59.4ff., 59.17ff., 60.18ff., 60.22ff., 67.7ff. and 68.32ff., and possibly also, as I conjecture below, at 58.13ff.

28 See also 67.7ff. Cf., however, on 69.3f. below.

ander, on the paralogism Aristotle had in mind at *Phys* 185a14ff. *without* any specific attribution to Hippocrates. The style of the proof, and even of the diagnosis of the fallacy at 57.25ff., is impersonal throughout with the exception of two verbs in 56.19-21 which imply a reference to someone conducting the proof²⁹ – although he is not named. It seems possible that the explicit attribution of the fallacy to Hippocrates is a comment by Simplicius that has intruded itself into a report that otherwise follows Alexander closely.

Alexander's text, which is Simplicius' source at this point, no doubt had as its chief goal the exegesis of Aristotle's *Physics* – where no mention is made of Hippocrates. Moreover, despite the confidence that Simplicius expresses at 60.18ff. and implies elsewhere, that Alexander attributed the fallacy he reports to Hippocrates, doubt is cast on that by a passage in the final section of Simplicius' commentary to which we shall come in due course. There, at 69.3-4, Simplicius raises as a possibility (though he does not follow it up) that the paralogism that Aristotle was thinking of was *not* the proofs of Hippocrates, but some others 'one of which Alexander also cited.' Yet if Alexander had expressly named Hippocrates as the author, it is surprising that Simplicius did not add that Alexander *mistakenly* attributed that proof to Hippocrates.³⁰

Section 3. In *Phys* 58.1-59.22. In this section Simplicius first gives (58.1-24) what he describes as an alternative 'simpler' proof of the quadrature of the circle by means of lunes which he attributes to certain unnamed 'these people too' (58.4). This simpler proof takes it that once the lune had been squared, viz., the lune on the side of the inscribed square, then so too had the circle, because a circle can be deemed to be constituted by the lunes into which it is divisible without remainder. But to this Simplicius then records, at 58.9-13, the objection that the

29 δείξας 56.19; πειράται 56.20.

30 I note, for what it is worth, that in the pseudo-Alexandrian commentary on *Soph El*, at 90.7ff., Wallies, a distinction is drawn between Hippocrates' quadrature and the quadrature by means of lunes which is there ascribed to Antiphon. Similarly the paraphrase of *Soph El* which is often ascribed to Sophonias distinguishes between the quadrature by means of lunes (also there ascribed to Antiphon) and Hippocrates' quadrature in its admittedly garbled report at 29.27ff., 29.34ff., Hayduck. (That Antiphon is the correct reading in the paraphrase at 29.34, not Hippocrates, becomes clear from a comparison with 30.35f.)

circle is not so divisible, for there is always a gibbous-shaped remainder in the middle contained by the lines of the lunes on either side. Simplicius does not here mention his source, either for the 'simpler' proof or for the objection, though we might conjecture Alexander (who had been used at 56.1-57.29, and who is to be cited expressly again at 58.25) at least for the proof and very possibly also for the objection. Whether or not that is the case, Simplicius himself goes on at 58.13ff. to raise his own objection to the objection that he has just recorded: Simplicius says that the key point is not that the whole circle cannot be divided into lunes, but that not every lune had been squared.

Picking up Alexander expressly again at 58.25ff. Simplicius then gives the view of some unnamed *τινὲς* who think that the circle can be squared if they give an arithmetic proof, viz., that a (so-called) circular number is equal to a square one. But this would be, according to Alexander (the subject of *φησὶν* at 59.2), a proof not from geometrical but from arithmetical principles. To this, yet again, Simplicius has two main objections: (1) at 59.4ff. on the way that the arithmeticians defined 'circular' numbers; that is, not (as Alexander supposed) by the addition of successive odd numbers, but as a number that ends in the same digit as its factors;³¹ (2) at 59.17ff. Simplicius says it is unlikely that anyone thought that if they had found the same number to be both circular and square, they had thereby squared the (geometrical) circle too. He concedes, however, that perhaps the discovery of that property in numbers stimulated the inquiry into the geometrical quadrature of the circle.

Section 4. In Phys 59.23-60.22. There is then an interlude during which Simplicius reports conversations he had with his teacher Ammonius relating to, among other things, such matters as the relationship between rectilinear angles and horn angles. To Ammonius' point that there are non-homogeneous geometrical magnitudes, Simplicius reports his own reply, the gist of which is the distinction between non-homogeneous magnitudes that are also incommensurable (such

31 Simplicius gives 25 and 36 as examples of numbers that are both square and circular by Alexander's definition, the first being both 5×5 and $1 + 3 + 5 + 7 + 9$, the second both 6×6 and $1 + 3 + 5 + 7 + 9 + 11$. But Simplicius gives 4, 9 and 16 as examples of numbers that are square but not circular by his own definition (though they are by Alexander's), and he gives as examples of numbers that are circular by his definition (though not by Alexander's) 125 (i.e., 25×5) and 216 (i.e., 36×6).

as the horn angle with the rectilinear angle) and those that are not. Simplicius is evidently impressed that *one* quadrature of a lune (on the side of the inscribed square) had been proved infallibly: given that the lune is of the same kind as the circle, what is there to prevent the circle also being squared, it too being a figure enclosed by circumferences (59.30ff.)? To emphasise the point that the quadrature of the circle is not to be rejected too swiftly Simplicius cites Iamblichus' *Commentary* on the *Categories* (at *In Phys* 60.7ff.). Iamblichus claimed that the Pythagoreans discovered the quadrature of the circle and noted that later further solutions (which Simplicius labels 'mechanical') were given to the problem by Archimedes, Nicomedes, Apollonius and Carpus. That is an impressive list of names, to be sure, but Simplicius gives no detailed account of their work, and some doubts might be expressed about precisely how clear Simplicius himself is on whether, or on what conditions, the circle could be squared.³²

Section 5. *In Phys* 60.22-68.22. The longest, and of course from many points of view the most interesting, section in Simplicius' commentary – and the one that has received most attention in modern scholarship – is the report, based on Eudemus' *History of Geometry*, of Hippocrates' sequence of four successful quadratures, that is of lunes with an outer circumference equal to, greater than, and less than, a semi-circle and finally of a lune together with a circle. Many questions, concerning for example the use of the term $\tau\mu\eta\mu\alpha$ at 61.14-19, have been elucidated by modern scholarship,³³ and especially the issue of

32 Although Simplicius twice cites exactly the same passage from Iamblichus' commentary on the *Categories*, that is to say both here in *In Phys* and in his own commentary on the *Categories*, *In Cat.* 192.18ff., Kalbfleisch, he gives no details of the works mentioned in either context, and in some cases, even with such other evidence as we have at our disposal, the reconstruction of their contents continues to defeat modern scholarship. Conversely, Simplicius does not here mention that in the *Dimension of the Circle* Archimedes demonstrated that the area of a circle equals that of a right-angled triangle where one side is equal to the circumference, the other to the radius of the circle. The circle can then be squared provided that the circumference can be rectified, that is that one can construct a straight line equal in length to it. The ancient commentators' views on the question of the conditions for solving circle quadrature are discussed by Knorr (1986), ch. 8.

33 On the difficulties of the use of $\tau\mu\eta\mu\alpha$ at *In Phys* 61.7, 12, 13 and 15 – and the fluctuation in the sense, there, between 'segment' and 'sector' – see Rudio (1902), 41ff.; cf., e.g., Knorr (1982), 182f. The major attempts to distinguish be-

the additions and modifications Simplicius has made to Eudemus' account. The *broad* conclusions we may draw on that issue are clear. Simplicius *says* (60.27ff.) that he will quote Eudemus word for word, adding a few things from Euclid's *Elements* for the sake of clarity, because Eudemus set out his explanations in a concise form according to ancient practice. Yet Simplicius has clearly done more than just add the rather frequent references to the *Elements*, notably (1) at 63.19ff. where Eudemus omitted 'as clear' (so Simplicius supposed) the quadrature of the lune with an outer circumference greater than a semi-circle – where Simplicius then supplies what he thinks this proof would be, and (2) at 64.25ff. where Simplicius gives what he considers the handiest proof of a certain equality.³⁴

Nevertheless, enough of the substance of Eudemus' report has been transmitted to allow us to form a picture of the scope and quality of Hippocrates' work – at least as Eudemus reported it. As Knorr has put it in a recent article:

From the technical point of view, it reveals a full mastery of the materials presented in Euclid's Books I, III and VI. Eudemus's account indicates further that the work had a careful deductive structure. Not only are the constructions presented and their quadratures effected in fully systematic fashion, but preliminary theorems are proven on which these quadratures depend.³⁵

We may allow that some of this systematic quality may be due to Eudemus. But, unless we reject his account as a *total* fabrication (and there are no good grounds for doing that), it provides impressive evidence of Hippocrates' ability in the matter of quadratures of lunes – in that he has identified, and provided proof of, four of the limited number of such quadratures that are possible.³⁶

tween quotations from Eudemus and Simplicius' own comments have been those of Bretschneider (1870); Allman (1883); Heiberg (1884); Schmidt (1903); Rudio (1907); Tannery (1912), 339ff.; Heath (1921), 183ff.; Becker (1936); van der Waerden (1961), 131ff.

34 As Heath notes (1921), 194, Simplicius gives a lengthy proof of the fairly straightforward point in question.

35 Knorr (1981), 151

36 For modern accounts of the conditions on which Hippocratean lunes can be squared, see, for example, Tannery (1912), 366ff.; Heath (1921), 199f.; Steele (1936); Knorr (1986), 36ff.

The *broad* conclusions we may reach concerning the relationship between Simplicius and Eudemus, and between Eudemus and Hippocrates, are not in serious doubt. But we are concerned here with the specific question of Simplicius' account of Eudemus' *interpretation* of Hippocrates, that is of Eudemus' evaluation and criticism of Hippocrates' quadratures. One key point is that Eudemus is represented by Simplicius as offering an interpretation that *differs from Alexander's*. This is indeed the way in which Simplicius introduces his long account of Eudemus. At 60.18-21 he had just reported Alexander's view of Hippocrates' fallacy, namely that having squared only the lune on the side of the inscribed square he took it that he had done so also for the lune on the side of the inscribed hexagon. Simplicius continues at 60.22ff.;

Ὁ μέντοι Εὐδημος ἐν τῇ Γεωμετρικῇ ἱστορίᾳ οὐκ ἐπὶ τετραγωνικῆς πλευρᾶς δεῖξαι φησι τὸν Ἱπποκράτην τὸν τοῦ μηνίσκου τετραγωνισμόν, ἀλλὰ καθόλου, ὡς ἂν τις εἴποι. εἰ γὰρ πᾶς μηνίσκος τὴν ἐκτὸς περιφέρειαν ἢ 25 ἴσην ἔχει ἡμικυκλίου ἢ μείζονα ἢ ἐλάττωνα, τετραγωνίζει δὲ ὁ Ἱπποκράτης καὶ τὸν ἴσην ἡμικυκλίου ἔχοντα καὶ τὸν μείζονα καὶ τὸν ἐλάττωνα, καθόλου δὲν εἶη δεδειχῶς ὡς δοκεῖ.

However Eudemus in his *History of Geometry* says that it was not on the side of the square that Hippocrates showed the quadrature of the lune, but, as one might say, universally. For if every lune has an outer circumference equal to, or greater than, or less than, a semi-circle, and if Hippocrates squares the lune with a circumference equal to a semi-circle and the one greater and the one less, he would have proved it universally, as it seems.

Then again at 67.3ff., having given Hippocrates' first three quadratures corresponding to those three types of lune, Simplicius puts it:³⁷

οὕτως μὲν οὖν ὁ Ἱπποκράτης πάντα μηνίσκων ἐτετραγώνισεν, εἴπερ καὶ τὸν ἡμικυκλίου 5 καὶ τὸν μείζονα ἡμικυκλίου καὶ τὸν ἐλάττωνα ἔχοντα τὴν ἐκτὸς περιφέρειαν."

37 Tannery (1912), 349 and 365, suggested emending the text of Simplicius by excluding πάντα and εἴπερ καὶ at *In Phys* 67.3. Cf. however Rudio (1902), 59f.; Heath (1921), 196 n. 1; Knorr (1986), 34ff.

'Ἄλλ' οὐχὶ τὸν ἐπὶ τῆς τοῦ τετραγώνου πλευρᾶς μόνον, ὡς ὁ Ἀλέ-
ξανδρος ἱστορήσεν, οὐ μέντοι οὐδὲ τὸν κύκλον ἐπεχείρησε τετραγωνίσαι διὰ
τῶν περὶ τὴν τοῦ ἑξαγώνου πλευρὰν μηνίσκων, ὡς καὶ τοῦτο ὁ Ἀλέξαν-
10 δρός φησιν.

Thus Hippocrates squared every lune, if [he squared] the one that has an outer circumference of a semi-circle, the one greater than a semi-circle and the one less. But he did not [square] only the lune on the side of the square, as Alexander recorded: nor indeed did he try to square the circle by means of the lunes on the side of the hexagon, as Alexander also says.

Now doubt may arise on two points: first, on whether or how far in either passage Simplicius is offering verbatim quotations from Eudemos; and, second, on whether, at 67.4, 'every lune' is used loosely for 'every kind of lune' (viz., each of the three types mentioned in the succeeding εἴπερ clause, and cf. also 60.24ff.) or whether it is used strictly for every single lune. But, on the first point, while it is absolutely clear that the references to Alexander at 67.7f. and 9f. cannot have been in Eudemos since Alexander lived several centuries after Eudemos, Simplicius is certainly offering what he represents as Eudemos' interpretation of Hippocrates. Moreover it is also perfectly clear – and from our point of view crucial – that the idea that Hippocrates squared every lune/every kind of lune is an *inference*, not a report of a claim that Hippocrates himself made.³⁸

The questions (1) as to whose inference it was (Simplicius' or Eudemos') and (2) whether it was the strong claim (every single lune) or merely the weaker one (every kind) are, comparatively, of minor importance. It might look as if the weak claim, by itself, is innocuous enough: I shall return to that shortly. But that one or other or both of Simplicius and Eudemos made the strong claim as well is clear. The strong claim is certainly what Simplicius needs at 69.12ff. where he canvasses as one possible interpretation of Hippocrates' fallacy that it was based on the final quadrature given by Eudemos, namely that of a lune together with a circle. On this interpretation, the fallacy consisted (rather as it had done in Alexander's version) in arguing that

³⁸ Note, in particular, the expression ἂν εἴη δεδειχώς in the apodosis of the conditional sentence at 60.27.

because the circle and the lune together had been squared, and the lune itself had been squared universally (καθολικός, 69.13), it could be taken that the circle itself had been squared – that is, when the rectilinear area equal to the lune with which it had been squared is subtracted. Nevertheless a little later, at 69.23ff., Simplicius points out that there is no question of Hippocrates having actually squared every single lune. The essential point is, of course, that, however much lunes differentiated as to the extents of their *outer* circumference had been covered, lunes also differ indefinitely as to their *inner* circumferences. In each case in his quadratures Hippocrates took lunes whose inner circumference is defined as fulfilling certain strict conditions, notably in respect of the similarity of the segments they form to other segments of the outer circumference.

If Eudemus made only the weak claim that Hippocrates had squared every kind of lune (as might be suggested, perhaps, by the expression that qualifies ‘universally’ at Simplicius 60.24, ‘as one might say’) then Simplicius is being highly disingenuous,³⁹ though even the weak claim in Eudemus allows Simplicius to score points against Alexander.⁴⁰ Yet even the weak claim, if it was all that Eudemus himself made, is itself a touch disingenuous, in that differentiation by the extent of the outer circumference is a totally inadequate way of classifying lunes, given that these differ also as to their inner circumferences. In either event, then, there is an element of potentially misleading overinterpretation of Hippocrates in Eudemus’ account as reported by Simplicius.

I return, however, to the fundamental point that Hippocrates is not said to have made either the weak or the strong claim himself.

Section 6. In Phys 68.32-69.34 [see Appendix (Ed.)]. In his concluding section, having finished his exposition of Eudemus’ account and

39 For by the time he gets to his concluding section, at 69.12ff., Simplicius is writing as if *every* lune had been squared (including the lune on the inscribed equilateral triangle which is in question in the final quadrature reported by Eudemus, where a lune together with a circle is squared). Yet, as already remarked, Simplicius immediately goes on to point out that there is no question of Hippocrates having squared every single lune.

40 That is to say the two points referred to at 67.7ff., namely that it was not just the lune on the side of the inscribed square that Hippocrates squared, and there was no question of attempting to square the circle by means of the second quadrature given in Alexander’s report, that of the circle together with three lunes on the sides of the inscribed hexagon.

noted that Eudemus' authority is to be respected because he was closer to Hippocrates' times (viz., than Alexander) and was a pupil of Aristotle, Simplicius returns to the question of the fallacy that Aristotle had in mind in the passage of the *Physics* at issue. He canvasses three possibilities (69.1ff.) but in the final analysis leaves a certain indeterminacy in what his own view of the matter was. (1) First, it may be that Aristotle's expression 'quadrature by means of segments' 'refers enigmatically' 'to the quadrature by means of lunes' (and Alexander was right to hesitate about whether these are to be identified). (2) Or, it refers not to the proofs of Hippocrates, but to some others, 'one of which Alexander also cited.' If, as appears to be the case, both these possibilities refer to Alexander's version of the fallacy (set out in section 2, 55.25-57.29), they differ only on the question of whether the author was indeed Hippocrates and on whether it was the only fallacy that Aristotle had in mind. (3) The third possibility is the one to which I have already referred in the last section, namely that Aristotle was censuring the quadrature of the lune together with a circle on the grounds that it was not a quadrature of the circle as such. Simplicius suggests that this might more strictly be called a quadrature by means of segments, since segments are indeed used in the proof.⁴¹ We then have his exploration of the possibility that Hippocrates' quadrature of lunes had been given 'universally,' followed by his remark that Hippocrates did not, in fact, square every lune.

V

We may now take stock of the results of our inquiry so far.

(1) The only direct reference in Aristotle to a paralogism by Hippocrates is that in *Soph El* 171b13ff., where the paralogism may, but need not, be identified with a quadrature by means of lunes.

41 Diels' text at *In Phys* 69.6 refers merely to the 'three segments in the lesser circle.' Heath's translation (1921), 198, following Tannery and Rudio, supplies what is needed (unless, that is, Simplicius has garbled his own point) namely 'the three [in the greater circle] and those in the lesser circle.' It should, however, be remarked that segments figure in one role or other in *all* the quadratures of both Alexander and Eudemus, not just the final quadrature of Eudemus: cf. Ross (1936), 464.

(2) Themistius, Philoponus and Eutocius provide no worthwhile testimony on the problem that is independent of and an addition to what we have from Simplicius.

(3) Alexander's version of the fallacy Aristotle has in mind at *Phys* 185a14ff. diagnoses a blatant error but it is open to doubt that Alexander ascribed it explicitly to Hippocrates.

(4) The evidence at Simplicius *In Phys* 54.12ff., and 58.1ff. confirms what we in any case know from other sources,⁴² namely, that there were plenty of attempts at squaring the circle before and after Aristotle, over and above any attributable to Hippocrates.

(5) The quadratures ascribed to Hippocrates by Eudemus as reported by Simplicius are all impeccable. They are quadratures of lunes, and of a lune together with a circle. They contain however, no attempt at a quadrature of a circle on its own, whether *by means of* lunes or any other method.

(6) Eudemus' interpretation, given by Simplicius, differs from Alexander's, for example in suggesting that Hippocrates squared lunes 'universally,' but that is an inference, not a report of what Hippocrates himself claimed.

VI

If we take the above as our starting-points, how far does it seem possible to answer our original questions, or at least to assess the likelihood of different combinations of answers?

First, five lines of interpretation look distinctly *unpromising*:

(A) It seems extremely unlikely that Hippocrates in fact committed the paralogism that Alexander thinks Aristotle had in mind at *Phys* 185a14ff. Anyone who showed such a mastery of the four quadratures reported by Eudemus could hardly have made the mistake of confusing lunes on the side of an inscribed square with lunes on the side of an inscribed hexagon or of believing that because he could square the first he could also square the second.

⁴² See, for example, Plutarch on Anaxagoras, *de exil* 17, 607F (DK A 38), and Aristophanes' *Birds* 1001ff. The evidence is discussed in full by Knorr (1986), 25ff.

(B) For the same reason – the evident high technical skill of Hippocrates' actual quadratures – it seems just as unlikely that Hippocrates took his quadrature of lune plus circle to justify any claim to have squared the circle on its own.

(C) Of the modern interpretations I referred to, Heiberg's again involves accusing Hippocrates of a gross confusion on the limits of the proofs he had given, the claim that he had squared every lune. But in addition to the argument from Hippocrates' mathematical ability, we may say that it appears that Eudemus at least knew of no such claim made by Hippocrates – for otherwise he would not have needed to infer it.

(D) That last point tells also against Björnbo's view, according to which, although Hippocrates knew what he had done, he deliberately described it in terms calculated to mislead: again, Eudemus appears not to know of any such description, though it corresponds to Eudemus' own interpretation.

(E) Equally, Heath's interpretation, that Hippocrates was a fool rather than a rogue, runs into similar difficulties. Putting it that Hippocrates 'was merely trying to put what he had discovered in the most favourable light,'⁴³ Heath did not specify further what Hippocrates' own self-misrepresentation was, but he was commenting on *In Phys* 67.3f., the remark about the squaring of *every* lune, and as we have seen, it is unlikely that Hippocrates himself claimed that.

One or other version of three other possible lines of interpretation seem more promising.

(F) A modified version of Heath's line of interpretation might be viable, that Hippocrates made *some* remark that helped to mislead Aristotle into thinking that he had claimed to square the circle. Though it seems that Eudemus found no statement in what he knew of Hippocrates to the effect that every lune had been squared, we cannot rule out the possibility that, in describing his work, Hippocrates *in some way* exaggerated its implications for the study of the quadrature of the circle. Knorr's suggestion (above n. 3) was that he may have presented the last quadrature of lune plus circle as a successful *reduction* (though not as a *resolution*) of the problem of circle quadrature. That by itself can hardly be said to help to exonerate Aristotle, who should

⁴³ Heath (1921), 196 n.1

have been quite clear, in general, on the difference between those two claims.

However, we might go further and conjecture that in introducing his work Hippocrates made some vague general claim – for example, to the effect that the study of the quadratures of lunes provided the key to the problem of squaring the circle – that helped to give rise to Aristotle's, still mistaken, accusation. For example where Hippocrates might have meant that his study contributed to *understanding* the problem of circle quadrature, Aristotle took him to mean that it *solved* it. It must, however, be emphasised that that would be pure conjecture.

(G) On the view favoured by Ross and others the blame for the misunderstanding lies squarely with Aristotle. It must, however, be pointed out that – if Aristotle was not in part misled by some ambiguous or rhetorical claim in Hippocrates – the misunderstanding would be gross. It would, indeed, be an egregious mistake if, on the basis solely of the Hippocratic quadratures that we have reported by Eudemus, Aristotle concluded that Hippocrates had claimed that he had squared, or even that he could square, the circle.

Both (F) and (G) assume that Aristotle charged Hippocrates with a fallacious quadrature of a circle, at least at *Soph El* 171b14ff. and probably also at *Phys* 185a14ff. if he has Hippocrates in mind there. In both (F) and (G) Aristotle's competence is, in one or other way, in question:⁴⁴ either he failed to see through Hippocrates' rhetoric, or he simply misunderstood what Hippocrates had done. On either (F) or (G) it remains an open question whether any of the later commentators suspected that a misunderstanding had arisen. Eudemus, who has a thorough knowledge of Hippocrates' quadratures, nevertheless (I suggested) overinterprets them with the inference that he had squared every lune/every kind of lune. The possibility arises that in so doing he was attempting to exonerate Aristotle: that is, that Eudemus saw that Aristotle would only have some justification for charging Hippo-

44 Yet the text of *Pr Anal* 69a30ff., mentioned above, may suggest (if 'with' is taken strictly) that Aristotle knows of a valid quadrature of a circle together with lunes. The extent of Aristotle's ignorance about, and lack of understanding of, mathematics has often been the subject of exaggerated comments: see Milhaud (1903), and contrast, for example, Heiberg (1884), 344.

crates with a fallacious quadrature if Hippocrates had, or could be represented as having, made some claim to have given a generalised quadrature of lunes. It is, however, notable that Alexander has an independent line. Although the first quadrature he reports is similar to the first Eudemus ascribes to Hippocrates,⁴⁵ thereafter Alexander's view of the paralogism Aristotle has in mind at *Phys* 185a14ff. differs. Moreover, Simplicius, though favouring Eudemus, does not straightforwardly endorse his interpretation. Neither Alexander nor Simplicius mentions the possibility of a misunderstanding of Hippocrates, although Simplicius is far from resolving all the puzzlement he expresses in the course of his discussion.

(H) As a third possible line of interpretation, the second suggestion that Simplicius offers in his final section, namely at 69.3f., might be followed up. Someone else, not Hippocrates, committed the paralogism reported by Alexander. It was that paralogism that Aristotle had in mind (possibly with others) at *Phys* 185a14ff. Alexander himself did not ascribe the paralogism to Hippocrates, though Simplicius initially represents him as envisaging Hippocrates (one reason why Simplicius might assume this is because of the similarity between the opening quadrature on the side of the inscribed square and the first quadrature expressly attributed to Hippocrates by Eudemus⁴⁶). Eudemus over-interpreted Hippocrates but did not diagnose any paralogism in the quadratures he reported. Simplicius introduces his long report from Eudemus in part to claim superior knowledge on the matter of Hippocrates' quadratures to Alexander. But while Simplicius uses Eudemus as a stick to beat Alexander with, his doing so in this context is, on this line of interpretation, unwarranted for two quite separate reasons. First, neither Aristotle at *Phys* 185a14ff., nor Alexander commenting on that passage, had Hippocrates in mind: so the introduction of a lengthy exegesis of Hippocrates' quadratures of lunes is a gratuitous display of learning. Second, both Aristotle at *Phys* 185a14ff., and Alexander on that passage, have some *paralogism* in mind: but (on this view)

45 Both, that is, deal with the same lune, on the side of the inscribed square: the proofs themselves, however, proceed differently.

46 See above n. 45. But the possibility that Hippocrates came to be associated with any quadrature by means of lunes simply because he was the most famous name connected with quadratures of lunes cannot be ruled out.

Hippocrates at no stage committed any paralogism, neither did he claim that he had squared, or could square, the circle.

The points in favour of (H) are considerable. On (H) there is no call (a) to consider Hippocrates' mathematical ability flawed, (b) to charge him with rhetorical exaggeration, (c) to charge Aristotle with grossly misrepresenting him at *Phys* 185a14ff., or (d) to accuse Alexander of misrepresenting Aristotle in turn in his comments on that passage. The chief source of confusion, on (H), is, of course, Simplicius. Taking Alexander to have Hippocrates in mind, Simplicius has a pretext to display his own greater learning: he can quote Eudemus at length to contradict Alexander's interpretation (supposedly) of Hippocrates. Nevertheless in Simplicius' favour it must be noted that although in the first five sections of his commentary the possibility that neither Aristotle at *Phys* 185a14ff. nor Alexander on that passage envisaged Hippocrates is not allowed to surface, it is Simplicius himself who mentions that possibility in his final section.

But if, over a very considerable stretch of the evidence, the superiority of (H) to its rivals is clear, on one item, namely *Soph El* 171b13ff., (F) and (G) at least adopt what may seem a more straightforward line, namely that that text provides evidence that Aristotle did, after all, charge Hippocrates with a fallacious quadrature of a circle. We saw that that is not the only possible interpretation of that text: for there are two others, (1) if the 'or' is taken disjunctively, Hippocrates' paralogism – whatever it was – is distinguished from the quadrature of the circle, though it is left quite indeterminate; nor (2) in such cases, we said, can interpolation be ruled out. It is only if neither of those lines of interpretation of *Soph El* 171b13ff. seems attractive that we shall be forced back, after all, to (F) or (G) – both of which have their high prices to pay, the one involving a conjectured ambiguity in Hippocrates that helped to mislead Aristotle, the other attributing to Aristotle an egregious misunderstanding.

We are faced, then, with a decision as to the balance of advantage and disadvantage between several competing lines of interpretation, but we should notice that the issue between (F), (G) and (H) is *chiefly* one that concerns Aristotle, not Hippocrates (for all three views have it that Hippocrates was too good a mathematician to have thought he had squared the circle). It is an issue between saying *either* that Aristotle misrepresented Hippocrates as making that claim (whether – as in [F] – aided and abetted in this misrepresentation by some unguarded remark by Hippocrates himself, or not – as in [G]), *or* that at *Soph El* 171b13ff. we have *at most* a quite indeterminate reference to *some* paralo-

gism in Hippocrates. And as between misrepresentation and indeterminacy, the lesser charge would clearly be the latter.

Working back, now, through the questions we posed at the beginning: we have seen (on question 5) that while preferring Eudemus to Alexander, Simplicius is in some doubt himself, in the final analysis, about what fallacy to attribute to Hippocrates; (on question 4) it is not necessary, and does not even seem likely, that Alexander attributed any fallacy to Hippocrates; (on question 3) that Eudemus, while at points over-interpreting Hippocrates, found no fallacy in his quadratures; (on question 2) that opinion may remain divided on *Soph El* 171b13ff., but on one view, at least, we simply have no means of telling what paralogism; (on question 1) that there are several possibilities for the fallacy he has in mind at *Phys* 185a14ff., but they may be taken to include that recorded by Alexander.

While no general line of interpretation can be said to have everything in its favour and to be completely free from problems, on each of the three most probable ones (F, G and H) there is no question of attributing to Hippocrates a fallacy in his work on quadratures.

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Appendix

τὰ μὲν οὖν περὶ τοῦ Χίου Ἱπποκράτους μᾶλλον ἐπιτρεπτέον
 Εὐδῆμῳ γινώσκειν ἐγγυτέρῳ τοῖς χρόνοις ὄντι καὶ Ἀριστοτέλους ἀκροατῇ.
 ὁ δὲ διὰ τῶν τμημάτων τετραγωνισμὸς τοῦ κύκλου, δν ὡς ψευδογραφοῦντα
 αἰτιᾶται ὁ Ἀριστοτέλης, ἥ τὸν διὰ τῶν μηνίσκων αἰνίττεται (καλῶς γὰρ
 καὶ ὁ Ἀλέξανδρος ἐνεδοίασεν εἰπὼν “εἰ ὁ αὐτός ἐστι τῷ διὰ τῶν μηνίσκων”)
 ἢ οὐκ εἰς τὰς Ἱπποκράτους δείξεις ἀποβλέπει ἀλλὰ τινὰς ἄλλας, ὧν μίαν
 καὶ ὁ Ἀλέξανδρος παρέθετο, ἥ τὸν μετὰ τοῦ μηνίσκου τετραγωνισμὸν τοῦ
 5 κύκλου αἰτιᾶται τοῦ Ἱπποκράτους, δν τῷ ὄντι διὰ τῶν τμημάτων ἀπέδειξε
 τῶν τριῶν ἐν τῷ ἐλάττονι. τάχα γὰρ καὶ κυριώτερον αὕτη ἢ ἀπό-
 δεῖξις ῥηθείη ἢ διὰ τμημάτων ἥπερ ἢ διὰ τῶν μηνίσκων. τμήμα γὰρ κύκλου
 καὶ ὁ Εὐκλείδης ἐν τῷ τρίτῳ τῶν ἑαυτοῦ Στοιχείων ὠρίσατο “τὸ περιεχό-
 μενον σχῆμα ὑπὸ τε εὐθείας καὶ κύκλου περιφερείας” οἱ οὖν μηνίσκοι
 10 οὐδὲ κυρίως τμήματά εἰσι. καὶ εἴη ἂν ψευδογράφημα ἐν τούτῳ τὸ μετὰ
 τοῦ μηνίσκου τετραγωνίζειν τὸν κύκλον, ἀλλὰ μὴ καθ’ ἑαυτόν, ἐπεὶ πάντα
 τὰ ληφθέντα εἰς τὴν ἀπόδειξιν ἀπὸ γεωμετρικῶν ἀρχῶν εἴληπται. ἀλλ’ εἰ
 ὁ τοῦ μηνίσκου τετραγωνισμὸς καθολικὸς ὑπὸ τοῦ Ἱπποκράτους δοκεῖ παρα-
 δίδοσθαι (πᾶς γὰρ μηνίσκος ἦτοι ἡμικυκλίου τὴν ἐκτὸς ἔχει περιφέρειαν ἢ
 15 μείζονος ἡμικυκλίου τμήματος ἢ ἐλάττονος), δυνατόν φαίη ἂν τις ἐκ τοῦ
 ἴσου τετραγώνου τῷ τε μηνίσκῳ καὶ τῷ κύκλῳ, ἀφαιρεθέντος τετραγώνου
 ἴσου τῷ μηνίσκῳ, τὸ λοιπὸν εὐθύγραμμον τετραγωνίσαντα ἴσον τετράγωνον
 τῷ κύκλῳ ποιῆσαι μόνῳ. πῶς οὖν ἔτι ψευδογραφεῖσθαι δόξει ὁ τοῦ Ἱπο-
 κράτους τετραγωνισμὸς, εἰ μήπω εὐρῆσθαι ὑπὸ τοῦ Ἀριστοτέλους ἐνομισθῇ
 20 λέγοντος ἐν Κατηγορίαις. “οἷον ὁ τοῦ κύκλου τετραγωνισμὸς εἰ ἔστιν ἐπι-
 στητός, ἐπιστήμη μὲν αὐτοῦ οὐκ ἔστι πῶς, τὸ δὲ ἐπιστητόν ἐστι”, καίτοι
 τοῦ Χίου Ἱπποκράτους πρὸ Ἀριστοτέλους ὄντος, ὥστε καὶ τὸν Εὐδῆμον
 ἐν τοῖς παλαιότεροις αὐτὸν ἀριθμεῖν. μήποτε οὖν οὐ καθόλου πᾶς μηνίσκος
 ὑπὸ τοῦ Ἱπποκράτους ἐτετραγωνίσθη. καὶ γὰρ ἡ ἐκτὸς τοῦ μηνίσκου περιφέ-
 25 ρεια ὀρίσθη, ἀλλ’ ἐκείνης κειμένης τὰς ἐντὸς τοῦ μηνίσκου περιφερείας ἀπείρους
 ἦτοι ἐπ’ ἄπειρον ἄλλην καὶ ἄλλην γράφειν δυνατόν ἐπ’ ἄπειρον διαφου-
 μένου τοῦ ἐπιπέδου, ὥστε τῆς ἐκτὸς τῆς αὐτῆς μενούσης τοὺς μὲν μείζονας
 τοὺς δὲ ἐλάττονας εἶναι τῶν μηνίσκων. αὐτὸς δὲ τὴν ἐντὸς περιφέρειαν
 ὠρισμένην ἔλαβεν. ὅμοιον γὰρ αὐτὴν τμήμα ἀποτέμνουσαν τοῖς πρὸς τῇ
 30 ἐκτὸς περιφερείᾳ συνισταμένοις τμήμασιν ἔλαβεν. ὅτ’ αὖ μὲν τοῦ πρώτου
 θεωρήματος ἐπὶ τετραγωνικῆς ἦν πλευρᾶς, τὰ δὲ ἐν τοῖς ἄλλοις ἐπὶ ἀο-
 ρίστων. ὥστε οὐ πᾶς ἐτετραγωνίσθη μηνίσκος, ἀλλ’ οἱ τὴν ἐντὸς περι-
 φέρειαν ὁμοίαν ἴσχοντες τοῖς πρὸς τῇ ἐκτὸς συνισταμένοις τμήμασι καὶ
 αὐτοῖς ὠρισμένοις πῶς οὖσιν.