

Pythagoras and Aristoxenos Reconciled

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THE PYTHAGOREAN and Aristoxenian viewpoints have represented poles of fundamental and irreconcilable conflict for some two thousand years. Pythagoras regards relationships among musical tones as manifestations of abstract number, signifying a pervasive cosmic principle. Aristoxenos ascribes the ordering of musical tones to the judgment of the ear, contingent therefore on mundane musical practice and its history.

Both viewpoints are partially supported by demonstrable facts, whence speculative dispute has engendered opposing metaphysical determinations of the nature of music, of man, and of the universe. But the relevant psycho-acoustic data are not so much opposed as complementary. Their positive indications therefore suggest how the more divergent philosophic implications may be reconciled.

The essentials of the Pythagorean formulae may be summarized thus:¹ if two sounding bodies—for example, stretched strings—are in simple proportionate lengths and all other aspects of the case are equal, together they will produce certain determinate musical intervals that are judged by the ear to be in harmonious agreement, or to be “consonant” in the ancient Greek sense. Conversely, all relationships that the ear accepts as consonant or in readily perceived harmonious agreement may be represented by the ratios of the small integers 1 to 4, so that these agreeable musical relations may be described as sounding embodiments of the respec-

tive numerical proportions. Present knowledge of acoustics further confirms the Pythagorean formulae by showing that reciprocals of these simple ratios appear among vibration frequencies of musical tones and also among the natural harmonic constituents of compound tones.² Thus harmony in music originates in number, while the fundamental laws of the universe as expressed in number achieve palpable form through musical harmony. And thus does rigorous scientific examination of the physical facts lead inescapably to the metaphysical doctrine of the harmony of the spheres.

The logic of the Aristoxenian position does not require denying the validity of these observed relations. It simply dismisses the Pythagorean formulae and their metaphysical extensions as extraneous to the art of music. The Pythagorean numbers are measures of physical magnitudes. But, Aristoxenos writes, “The mere sense-discrimination of magnitudes is no part of the general comprehension of music. . . . Mere knowledge of magnitudes does not enlighten one as to the functions of the tetrachords, or of the notes, or of the differences of the genera, or, briefly, the differences of simple and compound intervals, or the distinction between modulating and non-modulating scales, or the modes of melodic construction, or indeed anything else of the kind.”³

¹ Ratios of vibration frequencies and/or of harmonic partials: 1:1 = unison, 2:1 = octave, 3:2 = fifth, 4:3 = fourth.

² Aristoxenos, *The Harmonics*, ed. with trans. and notes by Henry S. Macran (Oxford, 1902), pp. 194-195.

³ The Pythagorean ratios: 1:1 = unison, 1:2 = octave, 2:3 = fifth, 3:4 = fourth.

Although relations among tones, viewed apart from music, may be submitted to objective analysis, such analysis cannot disclose their musical rôles, which hinge rather upon systematic considerations of a specifically musical order. Nature may show curiously consistent connections between harmony and number, and perhaps by strained analogy we may relate elemental aspects of harmony to remote cosmic mysteries. But music is not an affair of nature; it is a living human utterance. Hence the responses and judgments of people and the procedures evolved in the practice of the art are the primary facts before us when we examine music, and no mechanical measurements of externals will elucidate these. The proper explanation of musical harmony lies in the accumulated store of organized tone relations found in musical practice. So, writes Aristoxenos, "the most important and significant factor in the right constitution of melody is the principle of collocation in general as well as its special laws."⁴

But does not the principle of collocation itself arise from the harmonious numbers? Here philosophical speculation has led to a basic error of the music theorists, promulgated by the earliest writers of the Christian era and persisting unabated to the present day. The natural law of consonance was taken not only to illuminate the secrets of the cosmos but also to control inexorably the particular procedures of musical art. The Pythagorean formulae were stretched uncritically to encompass principles of collocation, and these no longer of ancient Greek music, but especially of the very different functional harmony of European practice. The history of music the-

⁴ *Ibid.*, pp. 177-178.

ory is littered with the wreckage of subsequent attempts to extract eternal principles of harmonic function from elemental acoustic data.

The mischief began when natural consonance was assigned a prescriptive rôle in harmony. One result was the hypothesizing of a parallel *organum* in which only the perfect intervals were admitted as simultaneous sounds. But no significant body of music supported such a theoretical construction. More serious distortion came about through the attribution of differential resolution values to isolated harmonic sonorities. The perfect intervals, principally the fourth and fifth, were held to induce of themselves the stable or consonant pole of the resolution relation. Other combinations were declared unstable or dissonant, requiring either to be hidden away among rapid divisions or to expiate their sin by motion or resolution. The fact that in practice the fourth was patently treated as dissonant, resolving to the theoretically inferior third, would have been enough to demolish this hierarchy, were it not a reflection also of anterior theological precepts. The question was hotly debated for centuries, and it was finally allowed to languish by the ingenuous elevation of the hitherto despised thirds to that transparent rationalization of permissible sin called imperfect consonance. Yet the error in the theory of consonant numbers did not lie mainly in the inconsistency of its predilections, but rather in the initial improper equation between acoustic norms and musical functions.

That fundamental error has remained with us, and the proliferation of more refined techniques for measurement has led to no better interpretation. The correlations between perfect consonances and simple relative

sizes of sounding bodies now appear also as ratios of vibration frequencies; ratios of harmonic overtones; ratios involving freedom from allegedly disturbing beats among these overtones;⁵ ratios resulting in coincidences of theoretical overtones;⁶ ratios within ratios among combination tones;⁷ ratios of postulated microrhythms;⁸ ratios of postulated tonal volumes;⁹ tonal brightnesses,¹⁰ tonal blendings,¹¹ tonal fusions,¹² tonicities and phonicities,¹³ and any number of comparable psycho-acoustic dimensions, some imaginary, some tautological, and a few capable of objective verification. All such in-

⁵ Hermann L. F. Helmholtz, *On The Sensations Of Tone*, 2nd English ed., trans. from the 4th German ed. (1877) with notes and appendix (1885) by Alexander J. Ellis (London, 1912), p. 194.

⁶ *Ibid.*, p. 182.

⁷ Felix Krueger, "Differenztöne und Konsonanz," *Archiv f. d. gesamte Psychologie*, Bd. 1 (1903); Bd. 2 (1904). Cf. critique by Carl Stumpf, "Differenztöne und Konsonanz," *Beiträge zur Akustik u. Musikwissenschaft*, Heft 4 (1909).

⁸ Theodor Lipps, "Das Wesen der musikalischen Konsonanz und Dissonanz," in his *Psychologische Studien*, 2nd ed. (Leipzig, 1905), pp. 115-240, also trans. by Herbert C. Sanborn (Baltimore, 1926), pp. 138-265. See also A. J. Polak, *Über Zeiteinheit in Bezug auf Konsonanz, Harmonie und Tonalität* (Leipzig, 1900); Joseph Achélik, *Der Naturklang als Wurzel aller Harmonien* (Leipzig, 1922); and critique by Carl Stumpf, "Konsonanz und Dissonanz," *Beiträge zur Akustik und Musikwissenschaft*, Heft 1 (1898), pp. 24-29.

⁹ Henry J. Watt, *The Psychology of Sound* (Cambridge, 1917), p. 195.

¹⁰ Géza Révész, *Zur Grundlegung der Tonpsychologie* (Leipzig, 1913), also trans. by G. I. C. De Courcy (Norman, Okla., 1954).

¹¹ Constantine Frithiof Malmberg, "The Perception of Consonance and Dissonance," *Psychological Monographs*, XXV, No. 2 (1918), p. 108.

¹² Carl Stumpf, "Konsonanz und Dissonanz," p. 35. See earlier statements by Jean Laurent de Béthizy, *Exposition de la Théorie et de la Pratique de la Musique* (Paris, 1754), p. 67; Jean Le Rond d'Alembert, *Éléments de Musique* (Paris, 1759), p. 11; Denis Ballière de Laisement, *Théorie de la Musique* (Rouen, 1765), p. 35.

¹³ Arthur von Oettingen, *Harmoniesystem in dualer Entwicklung* (Leipzig, 1866).

genious calculations and demonstrations bring us no closer to solution of the initial error. For the elusive quality of harmonic consonance does not and cannot reside in the sonorous dimensions of an isolated interval or combination at all. It must reside in the musical function of the combination, in its dynamics or motion within the framework of a given tonal system. The search leaves us with a simple but drastic test for any consonance criterion: namely, will it enable us to distinguish the consonant effect of an ordinary C major chord at a complete cadence in the key of C, from the dissonant urge toward resolution which the identical chord engenders when it serves as the dominant harmony in the key of F?

We submit that by this simple test, no psycho-acoustic criterion can be devised. Consonance and dissonance within the traditional tonal system are relationships among harmonies in motion, not qualities of sonorous constitution of the isolated harmonies themselves.¹⁴ The Pythagorean ratios, however exhaustively explored in depth, tell us nothing about the functions of musical tones or of their combinations or about the principles of their collocation. Here is precisely the critical position adopted by Aristoxenos, although he had another musical system in mind. The history of the consonance problem makes it clear that on this fundamental point Aristoxenos remains right.

It does not therefore follow that Pythagoras is wrong. The attributions of functional consonance in the later European tonal system to simple ratios formed no necessary or inherent part of the Pythagorean doctrine. And it remains the despair of the

¹⁴ Norman Cazden, "Tonal Function and Sonority in the Study of Harmony," *Jour. Res. in Music Educ.*, II (1954), pp. 22-27.

Aristoxenians that behind the extravagant concepts of natural consonance lies a compelling conviction that the Pythagorean measurements are indeed relevant to the problem, if not here then in some other connection not immediately evident. Because acoustic norms do not reveal the laws of musical art, the facts discovered through their measurement are not therefore incorrect, meaningless, or entirely accidental. We may not summarily dispose of the Pythagorean principle merely because some of its prominent deductions are invalid or because its core of reliable evidence is encumbered with tenuous speculation.

The Aristoxenian view may likewise be tinged with mysticism. Where it is not tempered by historical perspective, it seems to make a fetish of irresponsible subjective judgment, substituting arbitrary taste for natural law. Then sundry capricious and intuitive schemes for the conjunction of tones are proclaimed proper frameworks for art by the sovereign wills of their inventors. After all, who may question what the creative inner ear postulates?¹⁵ Thus electronic doodles and auditory kaleidoscope techniques, blithe happenstances and willful dissociations of tone relations win equal status with rational principles of harmony now outmoded. The novice at cacophony pushes Mozart aside on the grounds that all ears are created equal, no matter how long the ears. If there be no objective and natural criteria for the collocation of tones, then music, formless in the void, becomes a wild cry from the depths of anyone's unconscious, on his own verification. Here the Aristoxenian view would seem to predict lawless

¹⁵ Alois Hába, *Neue Harmonielehre* (Leipzig, 1927), p. 128.

disorder in the most highly organized of the arts.

Musicians and philosophers cannot escape the persuasion that musical harmony is not only beauty, but beauty become rational, as it is also reason become beautiful. Recoil from the fetish of private judgment strengthens the presumption that there must be some overriding natural, supernatural or at all events superhuman force external to man, restraining his individual follies and directing him toward a more exalted cosmic unity and harmony. And just this rationale is provided by the Pythagorean principle, its metaphysics included. Simple ratios may not account for the laws of harmony, but they must evidently account for some permanent natural element in music if that art is to have any roots in the real world.

Musical procedures change in the course of history and also vary considerably from one culture area to another. But the objective acoustic and psycho-acoustic norms obtained by the Pythagorean numbers remain as constant and universal in scope as any law of nature. Accordingly, the Pythagorean ratios must pertain to some very general aspect of music, applying at once to all music and to none, transcending the mutability of all known and all possible musical systems, yet responsible for no single historically rooted feature of any.

There is little mystery here. The magic ratios are revealed to be nothing more than formulae for the tuning of musical instruments. The Pythagorean equations signify simply that certain musical relations, when correctly tuned by ear, can be expressed as ratios of low integers and conversely that musical intervals expressible as ratios of low integers can readily be tuned to a harmonious

agreement. Therefore musical instruments are best tuned by fourth and fifths and octaves, not normally by relations less easily determined. This is also the definition obtained for consonance by Aristoxenos, and he equates discord with indeterminate relations.¹⁶ Perfect consonance means, then, the standard by which a desired ordering of musical pitches may be established through direct measurement of their internal agreements.

The Pythagorean doctrine does not apply, therefore, to the art of music; it applies to what happens before music is sounded. The harmonious agreement which the ancient Greeks termed consonance had nothing to do with procedures of musical composition; it provided rather a scientific prescription for tuning, which we still follow. The law of nature is not the mystic source of the melody, the harmony, or the form of a Beethoven or a Schönberg string quartet; it merely explains why the players of the quartet tune their strings at distances of perfect fifths.

Thus, the positions of Pythagoras and Aristoxenos are not really in conflict here. Pythagoras correctly generalizes that standards for identifying harmonious agreement among musical tones are susceptible of numerical formulation. Aristoxenos correctly observes that this primitive level of the recognition of tones and their distances does not yet constitute the art of music, for music begins only when there is a musical system for the collocation of tones, and such a system is not given by external measurements but only by the ear of the musician nurtured in that system. In their facts and in their interpretations, Pythagoras and Aristoxenos are thus both right and re-

¹⁶ *The Harmonics*, p. 198.

main right to this day. But the complementary nature of their principles has also been obscured by ostensibly opposed prescriptions for intonation.

In European harmonic practice, the judgment of the ear which Aristoxenos upheld tended to accept the major triad as the model unit of the tonal system.¹⁷ But the official norms for tuning, approved by science and theology and mistakenly made accountable also for harmonic consonance, called without qualification for the pre-eminence in harmony of relations expressible as simple ratios. The conflict reached a critical state by the 16th century, resulting in a cleavage between the theory of music that was still dominated by misapplied Pythagorean involutions with their mystical authority and by the harmonic practice of musicians, which showed the working of wholly different laws. The solution, first obtained in general form by Zarlino, consisted in the hypothesis that the major triad, rightly conceived, could also be made to exhibit simple numerical ratios.¹⁸ Only, instead of the consonant numbers stopping at the ratio index 4, they would have to be extended to include also the index 5. The major triad would then be expressible as the compound ratio 4:5:6, the major third would henceforth be declared a proper natural consonance by virtue of its measure-

¹⁷ Knud Jeppesen, *The Style of Palestrina and the Dissonance* (London, 1927), p. 77. Cf. Walter Odington, "De Speculatione Musices" (c. 1300), and Anonymi 1, "Tractatus de Consonantibus Musicalibus," in Coussemaker, *Scriptores*, t. 1; Lionel Power, "Treatise on Counterpoint," from *Old Hall Manuscript*, in Sanford B. Meech, "Three Musical Treatises in English from a Fifteenth Century Manuscript," *Speculum*, X (1935), pp. 235-269; Bartolomeo Ramos de Pareja, *Musica Practica* (1482), ed. by Johannes Wolf (Leipzig, 1901), pp. 63, 98.

¹⁸ Gioseffo Zarlino, *Istitutioni Harmoniche* (Venice, 1573), pp. 176 ff.

ment by the ratio 4:5, and the minor third similarly as the ratio 5:6.

This theoretical achievement of a "consonant" form for the major triad served to dissipate the then existing impasse between musical theory and practice. But in this way the permanence of the Pythagorean principle was breached, and natural ratios were no longer self-evident to the ear. In our time the acoustician John Redfield belabors the ancient Greeks for their clumsiness in not discovering the just major triad and asserts that the musical development of the classical symphony would have occurred some two thousand years earlier but for this theoretical blind-spot!¹⁹ But more than an abstract absurdity is involved here. There remains the concrete question of the actual intonation of the major third. The new view requires this interval to be in the so-called just ratio of 4:5 (or we might say, 64:80), while the older Pythagorean calculations set its value at 64:81, noticeably larger. Since the Pythagorean and Aristoxenian principles alike accept the ear's judgment as final, can we not find a more factual basis for the correct value of the major third?

We can measure intonation by observing the practice of musicians during performance and by sampling preferences of normal listeners exposed to various sizes of intervals. The latter technique has proved somewhat faulty, because tests either have presented intervals wholly isolated from musical contexts or have introduced some successions too haphazardly for the results to be valid.

Critical principles of the relevant intonation values²⁰ affect not only the major third but the entire scale, including chromatic tones. As a fur-

ther complication, while the 64:81 ratio for the third is correctly called Pythagorean, the alternative just ratio 64:80 has been loosely termed Aristoxenian by some theorists, leading to utter confusion. The just intonation principle was indeed set forth by Aristoxenian-minded theorists, who were bent on justifying the judgments of their ears. But here they appealed with unconscious irony to the mathematical calculations of the Pythagoreans, while they loudly fulminated precisely against the fitness of such calculations and decried the Pythagorean thirds as harsh, unholy, false, intolerable, and like symptoms of fair evaluation. The mythical standard called just intonation may better be described as a theoretical compromise between Pythagoras and Aristoxenos reached by eliminating the facts supporting either view and combining the mystical efflorescences of both.

The concept of a just or naturally consonant scale remains alive only because some well-established facts have been consistently misrepresented, largely in such unlikely places as physics texts, following hasty thinking that musicians also fall into when they are not alerted to these subtleties. Let us observe how the values in column 1 of our table, Just Intonation, differ from those of Equal Temperament in column 2.²¹ Every violinist learns early in life that the compromise intervals of equal temperament are quite abominably out of tune, but fortunately whenever he

²⁰ Principles of intonation standards, with resultant values (in cents) for major and minor thirds:

	M3	m3
1. Just Intonation: Triads I, IV, V in ratios 4:5:6.	386	316
2. Equal Temperament: Half-step = $\sqrt[12]{2} = 1.059 \dots$	400	300
3. Pythagorean Intonation: Fifth = 3:2, Fourth = 3:4.	408	294

¹⁹ *Music: A Science and an Art* (New York, 1930), pp. 69-70.

is freed from the accompanying piano he can play the just intervals of the natural scale. Rarely then does the violinist notice that according to this just intonation every sharpened tone is supposed to be lower than the enharmonically equivalent flatted tone and the minor third notably larger, the major third smaller, than the equally tempered values. But here the violinist will object that the sharp is always higher than the flat, the true minor third is certainly smaller than the third of equal temperament, and the true major third larger! The facts are very simple. Our violinist, rightly convinced that he plays correct intervals, does not thereby observe just intonation; in-

stead he approaches closely the true natural scale of Pythagorean intonation, whose values are given in column 3 of our table (refer again to footnote 21).

The studies listed²² are in accord that Pythagorean intonation is consistently preferred to any other by performers and listeners, specifically for the crucial divergent values of the major and minor thirds. Further studies²³ remark also on the sharps' being higher than the enharmonically equivalent flats. Some others²⁴ also emphasize the importance to tonal music of standards permitting the clearest acoustic differentiation between major and minor qualities and between chromatic resolution tend-

²¹ Values of intervals in three intonation standards:

Int.	Ex.	1. Just		2. E. T.		3. Pythagorean	
		Ratio	Cents	Ratio	Cents	Ratio	Cents
m 2	c-d $\flat$	1.067	112	1.059	100	1.053	90
M 2	d-e	1.111	182	1.122	200	1.125	204
M tone	c-d	1.125	204	1.122	200	1.125	204
m 3	c-e $\flat$	1.200	316	1.189	300	1.185	294
M 3	c-e	1.250	386	1.260	400	1.266	408
p 4	c-f	1.333	498	1.335	500	1.333	498
a 4	c-f $\sharp$	1.406	590	1.414	600	1.424	612
d 5	c-g $\flat$	1.420	610	1.414	600	1.405	588
p 5	c-g	1.500	702	1.498	700	1.500	702
m 6	c-a $\flat$	1.600	814	1.587	800	1.580	792
M 6	c-a	1.667	884	1.682	900	1.688	906
m 7	d-e $\flat$	1.778	996	1.782	1000	1.778	996
M 7	e-d	1.800	1018	1.782	1000	1.778	996
M 7	c-b	1.875	1088	1.883	1100	1.898	1110
8ve	c-c	2.000	1200	2.000	1200	2.000	1200

²² J. Murray Barbour, "The Persistence of the Pythagorean Tuning System," *Scripta Mathematica*, I (1932), pp. 286-304; Auguste Cornu and E. Mercadier, "Sur les intervalles musicaux," *Comptes Rendus, Académie des Sciences, Institut de France*, t. 73 (1871), pp. 178-183 (this and other studies in the series are well summarized by Alexander J. Ellis in Appendix to Helmholtz, n. 5 above, pp. 486-488); C.-M. Gariel, "Acoustique Musicale," *Encyclopédie de la Musique et Dictionnaire du Conservatoire*, Partie 2, Tome 1 (Paris, 1923), pp. 405-518; Paul C. Greene, "Violin Performance with Reference to Tempered, Natural and Pythagorean Intonation," *University of Iowa Studies in Psychology of Music*, IV (1936), pp. 232-250; Arnold Small, "An Objective Analysis of Artistic Violin Performance," *ibid.*, pp. 172-231.

²³ J. Murray Barbour, "Bach and The Art

of Temperament," *Mus. Q.*, XXXIII (1947), p. 65 (see also Barbour, n. 22 above, p. 302); Ernst Nauman, *Über die Verschiedenen Bestimmungen der Tonverhältnisse* (Leipzig, 1858), p. v; Ottokar Cadek, "Problems of String Intonation," *Proc. MTNA*, XXXIII (1938), pp. 119-125.

²⁴ M. W. Dröbisch, *Nachträge zur Theorie der musikalischen Tonverhältnisse* (Leipzig, 1855), pp. 28-29; Carl Stumpf and Max Meyer, "Maassbestimmungen über die Reinheit consonanter Intervalle," *Ztschr. f. Psychologie u. Physiologie der Sinnes-Organen*, Bd. 18 (1898), pp. 321-404; Engelbert Röntgen, "Einiges über Theorie und Praxis in musikalischen Dingen," *Vierteljahrsschrift f. Musikwissenschaft*, Bd. 9 (1893), pp. 365-380; Cornu and Mercadier, series cited in n. 22 above, t. 76 (1873), pp. 431-434.

encies. Finally, it is well recognized²⁵ in finer detail that intonation practice does not simply match Pythagorean values but also is variable in accordance with musical context, as Aristoxenos predicts in a principle of latitude. Instructive is a summary of deviations in cents from the three standards of intonation, obtained in analysis of violin performance by Paul C. Greene.²⁶

Among indications contrary to these conclusions are, first, the hesitant declarations by Cornu and Mercadier²⁷ that justly intoned major triads may be preferred when isolated from musical contexts, and similar claims by Helmholtz²⁸ and others. But arguments along such lines are ultimately futile, for with the slightest uncertainty over the accepted intonation of the thirds, it is already proved that they cannot be the automatically pure natural consonances they are claimed to be. No such indeterminacy ever appears for the fifth or octave. Second, in particular the psychologist Max Meyer mistakenly tested responses to major

²⁵ Barbour, Cornu and Mercadier, Gariel, Greene, Stumpf and Meyer, and Röntgen, cited above, n. 22 and 24; Aristoxenos, p. 217; D. Antonio Eximeno, *Dubbio sopra il saggio fondamentale pratico di contrappunto del Reverendissimo Padre Maestro Giambattista Martini* (Roma, 1775), pp. 75-86; Charles Meerens, *La Gamme Musicale, Majeure et Mineure* (Bruxelles, 1890), p. 13; Llewelyn S. Lloyd, *The Musical Ear* (London, 1940), pp. 72-74; Samuel Gardner, *School of Violin Study* (New York, 1939), p. 5; J. E. Orlov, "Über Täuschungen des Gehörs," *Archiv f. d. gesamte Psychologie*, Bd. 74 (1930), pp. 391-400.

²⁶ Deviations:

Int.	1. Just	2. E. T.	3. Pythagorean
m 2	-12	-6	-1
M 2 [9:8]	+1		
M 2 [10:9]	+12	+3	+1
m 3	-10	-2	-1
M 3	+10	+3	-1
p 4	0	-1	0

²⁷ Cornu and Mercadier, t. 68 (1869), 304-308 (see n. 22 above).

²⁸ *Op. cit.*, pp. 319-326.

and minor thirds in mixed successions²⁹ and thus proved unwittingly that the unparalleled beauty of the just minor third is correct only for the dissonant augmented second and that the equally unearthly consonant value of the just major third occurs only for the dissonant diminished fourth.³⁰

The just or "natural consonance" theory is thus contrary to the facts and to both the Pythagorean and Aristoxenian principles. It arose from a false extension of the Pythagorean numbers to the point of comprising wholly incommensurable properties of a harmonic system in which the central fact of the major triad seemed to require metaphysical justification. In positive fashion our intonation data show that proper derivatives of the Pythagorean norms are closely approximated in musical performance and in preferences for listening and that simultaneously the Aristoxenian ideal of variable magnitudes dependent upon collocation and function is achieved. Thus we may say that the Pythagorean norms for intonation describe correctly objective standards for the measurement and psycho-acoustic identification of the terms of musical relations, in precisely the same way as they define tuning standards, whereas the Aristoxenian principle correctly describes the treatment or transformation of these elementary natural materials for the purposes of art, operating on the much higher level of organized musical systems. The seemingly opposed views require each other for completion, and the conflict between them is resolved in favor of both.

We are dealing in fact with the

²⁹ Stumpf and Meyer, cited in n. 24 above, p. 342.

³⁰ Norman Cazden, "Musical Consonance and Dissonance" (unpubl. diss., Harvard University, 1947), pp. 420-423.

fundamental relation of man to nature. The Pythagorean principle, in its most general sense, states that man functions within limiting conditions set by the universal laws of nature. The Aristoxenian principle in the same sense declares that man imposes his own values and purposes on his natural environment, through methods determined by his own history, which is largely the history of his arts, taking these in their broadest sense. The lesson of the consonance problem is that the data of human activity may not be reduced to the one-dimensional data of acoustics, for arts that are demonstrably subject to

the exigencies of historical change in human societies cannot be comprehended by focussing attention on the unchanging natural conditions of their media. The study of music and its laws thus appertains properly to the sciences of the works of man and only incidentally to the contributory sciences of the works of nature. Music is not made by the strings of the lyre; it is made by the musician who both fashions and plays the lyre, and this musician cannot but reflect the historically evolved human world which forms the only setting for musical art.

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IN: AMERICAN MUSICOLOGICAL
SOCIETY " P. 97-105