

Placing Sectio Canonis in Historical and Philosophical Contexts

BARBERA A., *Placing Sectio Canonis in historical and philosophical contexts*: JHS CIV 1984 157-161. | The Pythagorean tradition must be kept in mind when reading the Ps.-Euclidean Sectio Canonis. The introduction provides a footing for Pythagorean musical theory, but this foundation needs to be supported with further Pythagorean dogma regarding the tetractys. The entire treatise must be read with the two-octave system in mind.

As the state of the sources will allow, it appears certain that Hieronymus used the name δ Λαμιακός πόλεμος for the war. On the other hand, it seems likely that Duris, writing within a decade earlier than Hieronymus, referred to it as δ Ἑλληνικός πόλεμος and had no knowledge of an alternative name. What little evidence we do have suggests that Hieronymus might well have been the first to use the name which later became standard for the war. That such a change in terminology could have occurred around the 260s has some support from epigraphy. The *Marmor Parium*, although not having an overall name for the war, does record the struggle at Lamia and the *naumachia* near Amorgos in the entry for 323/2. The reference to the events at Lamia reads:

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Ἀθηναίοις πρὸς Ἀντίπατρον.⁵⁷

Here, for the first time in the extant evidence, the military engagements at and around Lamia have been labelled a πόλεμος, an indication that in some quarters the Lamian events had been elevated in importance to a point from which it was no great step to identify the entire conflict with the 'πόλεμος' at that location. It is known from the prescript to fr. A of the *Marmor Parium* that the chronicle recorded selected events down to the archonship of Diognetus at Athens in 264/3,⁵⁸ which is virtually synchronous with Hieronymus' time of writing.

That the name δ Λαμιακός πόλεμος was in circulation in the second century BC seems confirmed by an odd reference to the war by Polybius:

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As it stands this account of what transpired is nonsense. Not only is it difficult to decide just what is meant by the μάχη περὶ Λαμίας, but Polybius also states that Antipater achieved a victory over the Greeks here. In fact, what battles were fought περὶ Λαμίας were certainly in favour of the Greek forces—the first resulting in Antipater being shut up in Lamia, and the later causing him to flee northwards following the death of Leonnatus and defeat of his cavalry. If it was Polybius' intention to refer to a decisive victory on land for Antipater, then only that near Crannon, fought some months later in 322, would fit the bill. Walbank, in his commentary on this passage, observes: 'What P. means by the "battle of Lamia" is not clear; the only

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noteworthy for the most memorable incident of the war as a whole...⁶⁰ The confusion in Polybius is explicable if it is understood that by the time this abbreviated account of the war was written, the name δ Λαμιακός πόλεμος was in circulation. Polybius has mistakenly assumed that the decisive land battle must have been near the city which had given its name to the overall conflict of 323 and 322, and by that error supplies the first indication of the time by which the name δ Λαμιακός πόλεμος had attained widespread recognition.⁶¹

If Hieronymus was the first literary figure to use the name δ Λαμιακός πόλεμος, it remains to ask why. Hornblower has argued that Hieronymus' final revision of the early sections of his work was undertaken in the 260s, after Athens had capitulated to Antigonos Gonatas in the Chremonidean War. Not only were there parallels to be drawn between the 'Hellenic War' of the 320s and the Greek struggle for freedom from Macedon in the 260s, but for a contemporary historian (with pro-Macedonian tendencies) the recording of the former revolt needed careful rewriting in view of the current developments.⁶² In particular the traditional name of Ἑλληνικός πόλεμος would have presented problems—both emotive and in the matter of precision. It is in that light, I would suggest, that Hieronymus decided to refer to the war of 323 and 322 BC as δ Λαμιακός πόλεμος.

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The construction of Pythagorean musical theory rests philosophically on the foundation provided by *Sectio Canonis*. Indeed, the treatise may have performed this role historically too. Andrew Barker has recently contributed to this journal a discussion of the methods and aims of the *Sectio*—JHS ci (1981) 1-16. In so doing he has pinpointed lapses in the theoretical reckoning of the treatise, especially in the case of proposition 11 (P11). I should like to reply to Barker's article. My remarks concern the authorship and date of the treatise, the introduction, a few propositions, and ultimately the historical and philosophical settings for the *Sectio*.

Barker chooses to avoid the issue of authorship of the *Sectio*, stating: 'Whether or not they [introduction and twenty propositions] are by Euclid himself, there is no good reason to assign at least the first eighteen propositions to a date later than Euclid's, or to suggest

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Although that figure is open to question, it is certain that he lived long and that his history included events down to at least 272.⁵⁶

As far as the state of the sources will allow, it appears certain that Hieronymus used the name *ὁ Λαμιακὸς πόλεμος* for the war. On the other hand, it seems likely that Duris, writing within a decade earlier than Hieronymus, referred to it as *ὁ Ἑλληνικὸς πόλεμος* and had no knowledge of an alternative name. What little evidence we do have suggests that Hieronymus might well have been the first to use the name which later became standard for the war. That such a change in terminology could have occurred around the 260s has some support from epigraphy. The *Marmor Parium*, although not having an overall name for the war, does record the struggle at Lamia and the *naumachia* near Amorgus in the entry for 323/2. The reference to the events at Lamia reads:

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fighting there was like the sally which cost Leosthenes his life. . . . The likelihood is that P. has confused the name of the decisive land battle with that of the town noteworthy for the most memorable incident of the war as a whole. . . .'⁶⁰ The confusion in Polybius is explicable if it is understood that by the time this abbreviated account of the war was written, the name *ὁ Λαμιακὸς πόλεμος* was in circulation. Polybius has mistakenly assumed that the decisive land battle must have been near the city which had given its name to the overall conflict of 323 and 322, and by that error supplies the first indication of the time by which the name *ὁ Λαμιακὸς πόλεμος* had attained widespread recognition.⁶¹

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Barker chooses to avoid the issue of authorship of the *Sectio*, stating: 'Whether or not they [introduction and twenty propositions] are by Euclid himself, there is no good reason to assign at least the first eighteen propositions to a date later than Euclid's, or to suggest

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that they are the work of more than one hand' (p. 1). Barker's choice is not unique among modern scholars. The questions of 'who' and 'when', however, are critical for a formulation of an answer to the question 'why'. In other words, we could better evaluate *Sectio Canonis* if we could firmly establish its historical context.

The sectional nature of the treatise appears to indicate more than one hand. The *Sectio* comprises: an introduction; nine purely mathematical propositions, three of which rely on propositions contained in the eighth book of Euclid's *Elements of Geometry*; seven general acoustical propositions that relate to the introduction and first nine propositions; two propositions concerning the enharmonic genus; and finally two propositions that divide a string according to the diatonic genus. From these last two propositions the treatise apparently derives its name.

Two appearances of *Sectio Canonis* in late antiquity underscore the sectional nature of the treatise. In his commentary on Ptolemy's *Harmonics*, Porphyry presents P1–16 alone,¹ and gives a version of these propositions that is essentially the same as, but not identical to, the version ascribed to Euclid (see below). Where are the introduction and the last four propositions? Shortly before stating P1–16, Porphyry refers to Euclid's 'Division of a Monochord' (92.29–30 and 98.19 Düring), but without the last two propositions, the title is not applicable to the propositions stated.

Boethius, at the beginning of the fourth book of his *de Musica*, provides a Latin rendition of the introduction and the first nine propositions.² His version of the introduction differs in several ways from the Euclidean treatise.³ In the case of the propositions, Boethius interpolates numerical demonstrations that parallel the apparent geometric proofs of the original. At no point in this passage does Boethius, or the Greek author whom he is following—perhaps Nicomachus,⁴ cite Euclid or give a title such as '*Sectio Canonis*'. The fourth book of *de Musica*, however, is largely concerned with dividing the monochord. Thus the introduction and mathematical propositions are not entirely out of place there.

A detailed comparison of the three versions—?Euclid, Porphyry and Boethius—where possible (P1–9) prevents us from singling out one version as the model from which the other two were produced. For instance,

¹ Porphyrios Kommentar zur Harmonielehre des Ptolemaios, ed. I. Düring (1932; repr. N.Y. 1980) 99–103.25.

² Boethius, *De Institutione Arithmetica libri duo. De Institutione Musica libri quinque*, ed. G. Friedlein (1867; repr. Frankfurt 1966) 301.6–308.15.

³ For instance, in his definition of consonant notes as a blend, Boethius inserts the phrase 'struck at the same time', *simul pulsae*, referring to the individual notes that make up a consonance. The Greek equivalent, ἀμα κρούω, appears in most Pythagorean definitions but not in *Sectio Canonis*. See Calvin M. Bower's discussion of this matter, 'Boethius' *The Principles of Music*, An Introduction, Translation, and Commentary' (Ph.D. thesis, George Peabody College for Teachers 1967) 213, 440–3. Unlike the *Sectio*, furthermore, Boethius does not make the important connection between a 'single name' or 'one term' for multiple and superparticular ratios and the single blend of sound formed by two consonant notes (see below and also Aristotle, *de Sensu* 447a12 ff.).

⁴ See C. M. Bower, 'Boethius and Nicomachus: an essay concerning the sources of *De Institutione Musica*', *Vivarium* xvi (1978) 1–45, and U. Pizzani, 'Studi sulle fonti del *De Institutione Musica* di Boezio', *Sacris erudiri* xvi (1965) 5–164.

each version employs a unique sequence of alphabetic variables in P3. Distinct from Porphyry and *Sectio Canonis*, Boethius interpolates numerical demonstrations in P1–4 and P6–9. In P6 Porphyry omits a recapitulative phrase contained in the *de Musica* (306.3–4 Friedlein) and the *Sectio* (155.19–21 Jan). In fact, P6 receives the most varied treatment of the first nine propositions (see below). In P4, *Sectio Canonis* does not contain the culminating 'which is that necessary to prove' present in both Boethius (305.8 Friedlein) and Porphyry (100.10–11 Düring).

The three versions of the treatise along with its sectional nature—e.g. the introduction and first sixteen propositions minus the title would make a good, nearly self-sufficient musical treatise—invite questions regarding the number of hands involved. The disparity among the three versions at P1–9 and our inability to choose one version as the model here indicate a complex and probably protracted composition of the treatise we know as *Sectio Canonis*. That the entire treatise or any part thereof was written by Euclid is yet another matter.

In addition to a 'Division of a Monochord' by Euclid, Porphyry mentions an *Elements* of music (92.29 Düring). Both Proclus and his student Marinus also mention that Euclid wrote an *Elements* of music.⁵ These remarks and a confused manuscript tradition that combines the *Sectio* with an *Introduction to Harmonics*—an Aristoxenian work now ascribed to a certain, or perhaps uncertain Cleonides—constitute the external evidence for assigning the treatise to Euclid. There also exists some internal evidence for such ascription: with the exception of P19–20, the propositions are in the style of Euclid's *Elements of Geometry*. The style and contents of the *Sectio*, however, have divided modern scholarship on the issue of ascribing the treatise to Euclid. Karl von Jan, a modern editor of the *Sectio*, was convinced in part by the language of the treatise that Euclid was its author,⁶ whereas Paul Tannery held the contents to be unworthy of ascription to the famous geometer.⁷ Tannery concluded that the bulk of *Sectio Canonis* was written before the time of Aristoxenus and was probably a product of Plato's Academy. Noting Plato's famous remarks about harmonics in the *Republic* (530c–531c), Tannery suggests, as does Barker (p. 10), that the *Sectio* may be a response to Plato's criticism. I have shown elsewhere that Plato may not be directing his criticism at the Pythagoreans, and that if he is, his remarks are at best confusing and perhaps self-contradictory.⁸ Other scholars have viewed the *Sectio* as a reply of sorts to Aristoxenus' treatise on music. Thomas Mathiesen observes that the treatise, especially its acoustical propositions, may be an attempt 'to reconcile Pythagorean and what would later be called Aristoxenian schools, or the mathematical and the empirical'.⁹

⁵ Proclus, *Procli Diadochi in Primum Euclidis Elementarum librum Commentarii*, ed. G. Friedlein (Leipzig 1873) 69.3; Marinus, *Commentarius in Euclidis Data*, ed. H. Menge (Leipzig 1896) 254.20–7, vol. vi of Euclid, *Opera omnia*, ed. I. L. Heiberg and H. Menge.

⁶ K. von Jan, *Musici Scriptores Graeci* (Leipzig 1895; repr. Hildesheim 1962) 115–20. A new edition of *Sectio Canonis* is in order.

⁷ 'Inauthenticité de la "Division du canon" attribuée à Euclide', *CRAI* iv (1904) 439–45 = *Mémoires scientifiques*, ed. J. L. Heiberg and H. G. Zeuthen (Paris 1915) iii 213–19.

⁸ 'Republic 530c–531c: another look at Plato and the Pythagoreans', *AJP* cii (1981) 395–410.

⁹ T. J. Mathiesen, 'An annotated translation of Euclid's division of a monochord', *J. Music Theory* xix (1975) n. 34.

Barker, while not offering a reconciliation between Pythagorean and Aristoxenian musical theory, does view 'certain major disagreements' as reflecting 'an oblique rather than a direct confrontation' (p. 1). Although this position is possible, we should not lose sight of the direct repudiation provided by P15 of Aristoxenus' proof that a fourth equals two and a half tones. Boethius makes perfectly clear that in late antiquity Pythagorean and Aristoxenian theory remained unreconciled, at least for some.¹⁰

The scholarly debate has centered almost entirely on the fourth century BC as the time of composition. The sole evidence for this, I think, is the attachment of Euclid's name to the treatise. One wonders, then, why we receive the fragmented versions by Porphyry and Boethius. If the introduction and P1–18 sprang full-blown from the mind of a fourth-century author, then we must envision Boethius, or his source, with two versions of *Sectio Canonis* before him, dipping and choosing from both, giving credit to neither. This is so because we can not establish the archetypal version for P1–9.¹¹ One must further wonder why Nicomachus makes no mention of this treatise nor of Euclid in his *Manual of Harmony*. And why does Ptolemy, who carefully assigns musical theories to the likes of Archytas, Aristoxenus, Eratosthenes, and Didymus, link the fundamental principle of consonance contained in the *Sectio*'s introduction to the Pythagoreans rather than to Euclid?¹² Perhaps Porphyry and Boethius transmitted what they thought to be an entire little treatise. If so, the composition of *Sectio Canonis* may be a more fragmented affair than has been previously presumed by scholars and the date of composition may be much later than the fourth century. One might even speculate that the *Sectio* is a product of the Pythagorean or Neo-pythagorean revival of late antiquity and that the treatise assumed the form in which we know it only after the time of Porphyry.

In his discussion of the aims of the treatise as a whole, Barker observes that the *Sectio* translates a scalar system from one terminological framework into another, i.e. from the auditory realm into the numerical (14–15). He criticizes the work for relying *a priori* on the Greek scale in its attempt to give pitch relations a solid mathematical basis and for providing insufficient information to connect the principles of the introduction with the auditory propositions. I think it is fair to conclude that Barker views *Sectio Canonis* to be rather lax theory, at least in relation to, say, Euclid's *Elements* or even to Aristoxenus' treatises on music. Barker's assumptions about the historical context of the treatise, of course, shape his conclusions. As I point out below, reading *Sectio Canonis* from a contextual viewpoint different from Barker's can alter one's evaluation of the treatise.

The introduction to *Sectio Canonis* is perhaps the most important Pythagorean document on musical

theory, for there we read the Pythagorean creed on harmonics. The author of the introduction observes that rapid pulsations (*plēgai*) produce high pitches and rare pulsations produce low pitches. Barker points out, however, that if one assumes the proposition to refer to lengths of string, then a numerical inversion of sorts takes place (1–2), i.e. long strings produce low pitches and short strings produce high pitches. Except for the last two propositions, however, one need not assume strings to be the object of discourse. *Sectio Canonis* treats intervals, mathematical or numerical and musical. The line drawings that accompany the treatise in manuscripts represent strings and thus may be misleading. Although the last two propositions do concern the division of a string, they do not employ numbers in their demonstrations. Furthermore, these two propositions hold at best a tenuous relationship to the rest of the treatise, as nearly every scholar who writes on the subject has pointed out. Bartel van der Waerden summed up the situation by noting that, when dealing with Pythagorean documents, one should consider ratios to represent the essence of an interval and not necessarily a ratio of pulsations or a ratio of string lengths.¹³

This brings me to the most important general point that I have to make: the context in which *Sectio Canonis* should be read is a Pythagorean context, be it early or late. There can be little doubt about this matter given the rationalist nature of the treatise and its appearance in the works of Porphyry and Boethius. From this point of view, we should expect *Sectio Canonis* to be scientific and theoretically rigorous only to a rather limited extent. The writings of Nicomachus, Theon of Smyrna, Iamblichus, the fragments ascribed to Philolaus and Archytas, and the testimony of Aristotle, especially in the *Metaphysics*, verify that Pythagoreanism, both early and late, was a religion that incorporated some scientific empiricism with a profound appreciation for the mysteries of numerical truth.¹⁴ Therefore, Barker's criticism that the *Sectio* never proves that all intervals of the same size can be expressed by the same ratio, while not incorrect, is inappropriate. Such a theory of correspondence is asking a lot from any ancient treatise and would require that the treatise first establish how one determines that two intervals are or are not the same size. This is a tall order for any theory and extraneous to the Pythagorean method of demonstration. Barker criticizes further the author(s) of our treatise for assuming that the reader can distinguish which of two intervals is larger without explicitly basing the assumption on either knowledge of the musical system or sensory perception (13). It seems to me that in many cases one can determine which of two intervals is larger simply by referring to the names of the intervals: *diapente* is larger than *diatessaron*, *diapason* is larger than *diapente*, and so forth.

In addition to establishing numbers as parlance for the discussion of sound, the introduction sets down the fundamental Pythagorean principle of consonance: all consonant intervals are characterized numerically by either multiple or superparticular ratios (Barker's

¹⁰ Boethius devotes much of the third book of his *de Musica* to a repudiation of Aristoxenian theory. See also my 'Interpreting an arithmetical error in Boethius's *De Institutione Musica* (iii 14–16)', *Archives internationales d'histoire des sciences* xxxi (1981) 26–41.

¹¹ To some extent, this argument depends on the modern critical editions of the three versions, one of which—Porphyry's commentary—may be in need of considerable revision.

¹² Nicomachus, *Enchiridion*, in Jan, *Mus. Script. Gr.* 235–65. Ptolemy, *Die Harmonielehre des Klaudios Ptolemaios*, ed. I. Düring (1930; repr. N.Y. 1980) 13–14.

¹³ 'Die Harmonielehre des Pythagoreer', *Hermes* lxxviii (1943) 175.

¹⁴ In this respect, see W. Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. E. L. Minar, Jr (Cambridge, Mass. 1972).

second 'bridging principle').¹⁵ Since multiples and superparticulars are related by a single name or one term, and since two consonant notes form a single blend of sound, then two consonant notes must be related numerically by either a multiple or a superparticular ratio. The phrase 'a single name' or 'one term' has proved to be enigmatic for modern scholarship. Jan thought that Porphyry's 'superior' (*kreitton*) might be the one term for multiples and superparticulars. Edward Lippman argued, as does Barker (2–3), that since only multiple and superparticular ratios can be designated (in Greek) by a single word, 'one term' refers to such a word—*epitriton*, *displasios*, and the like.¹⁶ Mathiesen rejects this interpretation and offers Porphyry's 'consonant' (98.3–6 Düring) as the single term.¹⁷ Based on Porphyry's remarks, I prefer 'consonant' for the first part of the argument: multiple and superparticular ratios are related by the single name 'consonant'. Lippman's and Barker's suggestion, however, makes more sense out of the second part of the argument.

Although the introduction provides some basic tenets of Pythagorean musical theory, it could have provided more. Two important tenets not mentioned by the *Sectio* are the restriction of the musical system to two octaves and the reliance on the tetractys 1, 2, 3, 4 in establishing the numerical realm within which consonance is defined. With these two tenets in mind, some of the problems raised by Barker about the relation of the introduction to the propositions seem to be irrelevant (see P11 below).

Most scholars dealing with the *Sectio* have pointed out that three of the purely mathematical propositions (P2, P3, P9) rely on theorems proved in the eighth book of Euclid's *Elements*. Of these three, P3 is especially interesting because it appears in a slightly different form in Boethius' *de Musica* (iii 11). There Boethius attributes the proof to Archytas, disparages it, and promises a better proof of the same proposition: no integral mean divides a superparticular ratio. The promised better proof is P3. At no point here or in Bk iv where the better proof appears does Boethius mention Euclid. Archytas' version is a bit prolix, although not nearly as bad as Boethius and Burkert would have us believe.¹⁸ Both proofs begin by reducing a superparticular ratio to its lowest terms. The *Sectio* then observes that the difference between the two terms is the monad, which of course can not be divided. Rather than claiming that the difference between the two terms is unity, Archytas assumes that the difference is not unity and shows that such an assumption leads to a contradiction. The ingenuousness of Archytas' proof along with its distinction between numbers and unity may be signs of early Pythagoreanism that are absent from *Sectio Canonis*.

Regarding P6, each of the three versions is unique. *Sectio Canonis* gives two demonstrations of the proposition, only the second of which appears in Porphyry's version. Porphyry, however, supplements his single

proof of P6 with an additional proof that no multiple interval other than the duple can be formed from two superparticular ratios (100.26–101.8 Düring). Boethius gives only the second demonstration as it appears in the *Sectio*, and he interpolates numerical instances at the end of the demonstration. Thus the varied treatments of P6 argue for the unstable transmission of the *Sectio*. Such variety makes one suspicious of the traditional ascription of the treatise to Euclid.

Let us now consider P11, for it is here that Barker has pointed out a paralogism contained in *Sectio Canonis*. The proof of P11 rests on the observation that since the double fourth is dissonant, it can not be a multiple interval. As Barker notes (4–5), the introduction claims that all consonances are either multiple or superparticular, but not that all multiples are consonant. The latter notion is required for the proof of P11, and Barker observes that several other acoustical propositions depend on the verity of P11. I think that Barker is correct in centering so much attention on it, for with this proposition we may find unstated Pythagorean dogma in force.

First of all, the system under consideration by the *Sectio* is a two-octave system. Although the *Sectio* is not explicit on the matter, most musical theories of antiquity, especially the Pythagorean variety, restrict themselves to this two-octave system.¹⁹ Within this system, all multiple ratios are consonant. Therefore, if the doubled fourth is dissonant, it can not be a multiple. Second, in addition to the acoustical restriction to two octaves, the *Sectio* may operate under the numerical restriction of the tetractys 1, 2, 3, 4 when discussing consonance. A plethora of Pythagorean writings from antiquity and the Middle Ages define as consonant only those intervals that can be composed by relating any two terms from the tetractys.²⁰ The effect of this definition is to restrict the realm of consonances to the two-octave system, the number of consonances to five (fourth, fifth, octave, octave plus fifth, and double octave), and the categories of ratios to multiple and superparticular ($4:2 = 2:1$, $3:1$, $4:1$, $3:2$, $4:3$).

Regarding the interval of the octave plus fourth, represented by 8:3, Barker notes correctly that *Sectio Canonis* is silent on the matter, but he errs when claiming that 'no one seems to have disputed' the consonant character of this composite interval (9). Barker claims further that the Pythagorean rejection of this interval from the category of consonance is 'plainly illegitimate' because the basis is numerical rather than auditory—8:3 is neither multiple nor superparticular, but rather multiple superpartient. Barker's claim rests on his assumption that the octave plus fourth sounds consonant. Since *Sectio Canonis* identifies consonances on an auditory basis, Barker criticizes the *Sectio* for not

¹⁹ See e.g. Nicomachus, *Enchiridion*, ed. Jan 255–65; Gaudentius, *Introduction to Harmonics*, ed. Jan 343–5; and Boethius, *De Musica* iv 3–13, ed. Friedlein 308–37.

¹⁵ For a detailed presentation of the Pythagorean order of ratios, see: Nicomachus, *Introductionis Arithmeticae*, ed. R. Hoche (Leipzig 1886) 44.8–72 and 119.9–144.19, and Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utilium*, ed. E. Hiller (Leipzig 1878). See also Barbera (n. 8) 406 n. 29.

¹⁶ E. A. Lippman, *Musical Thought in Ancient Greece* (New York 1965) 154.

¹⁷ Mathiesen (n. 9) n. 12.

¹⁸ Burkert (n. 14) 444–5.

²⁰ In late antiquity, for instance, see Theon of Smyrna, *Expositio* 58.13 Hiller. For the Pythagorean oaths involving the tetractys, see: Aëtius, *Placita* i 3.8, in H. Diels, *Doxographi graeci*⁴ (1879) 181; Iamblicus, *De Vita Pythagorica*, ed. L. Deubner (Leipzig 1937) 47.15–16, 85.4–5; Sextus Empiricus, *Adversus Mathematicos* iv 2, ed. J. Mau (Leipzig 1954) iii 133.16–17; and Theon, *Expositio* 94.6–7. See also: A. Delatte, *Études sur la littérature pythagoricienne* (1915; repr. Geneva 1974) 253 ff.; P. Kucharski, *Étude sur la doctrine pythagoricienne de la tétrade* (Paris 1952) 75–7.

treating the octave plus fourth. The consonance or dissonance of an interval, however, is a relative matter, dependent upon both sensory perception and reason or system. The consonant character of the octave plus fourth—essential for Barker's argument—is in no way certain, and the issue was hotly debated in musical treatises throughout antiquity and the Middle Ages. Certainly the Aristoxenians and Ptolemy deemed the interval to be consonant, but Ptolemy's criticism of the Pythagoreans for their rejection of this interval from the category of consonance indicates that ancient theorists were less than unanimous on the matter (Düring 13). Unlike the *Sectio*, both Plutarch and Boethius explicitly reject the octave plus fourth because it is dissonant.²¹ For some Pythagoreans, the interval sounded dissonant because its numerical characterization not only included a number, 8, not found in the tetractys, but also fell into the multiple superpartient variety of ratio, i.e. the variety furthest removed from the beauty of unity and equality. The orthodox Pythagorean position on the matter is exactly opposite Barker's: the octave plus fourth *sounds* dissonant because all consonances are either multiple or superparticular.²²

In conclusion, we see how important it is to keep the Pythagorean tradition in mind when reading *Sectio Canonis*. That the introduction provides a footing, albeit shaky, for the construction of Pythagorean musical theory goes without question. But as I have shown, this foundation needs to be supported with additional Pythagorean dogma regarding the tetractys. Furthermore, the entire treatise must be read with the two-octave system in mind.

The style and language of the *Sectio* are like those of Euclid's *Elements*, and there can be hardly any objection to calling the musical treatise 'Euclidean'. There is a great danger, however, in expecting from the *Sectio* a pure and general theory of acoustics similar to Euclid's treatment of geometry. In Euclid's *Elements* we find an abstract theory of geometric and arithmetic truth that can be applied impartially to the physical world. With the *Sectio*, the distinction between corporeal and incorporeal, be it between sound and number or sound and line, is not clear nor, I think, was it intended to be. The relationship between number and sound was both a miracle and a mystery that wowed the Pythagorean mind and ear. An appropriate response to this relationship was to demonstrate the mysterious rather than to deduce the obvious.

By modern standards for theory, even by the standards set by Euclid's *Elements*, the *Sectio* falls flat on its face. I believe, however, that one must read the *Sectio* from a Pythagorean point of view. The Euclidean style of the treatise notwithstanding, one does better to approach *Sectio Canonis* with Nicomachus or Theon of Smyrna in mind rather than Euclid.

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²¹ Boethius, *De Musica* ii 27, and Plutarch, *On the E(psi)lon* at Delphi, *Mor.* 389d–e.

²² See my 'The consonant eleventh and the expansion of the musical tetractys: a study in ancient Pythagoreanism', *J. Music Theory* (forthcoming).

Signa tabulae priscae artis

The article 'Signa priscae artis: Eretria and Siphnos' in *JHS* ciii (1983) 49–67, by David Francis and Michael Vickers (hereafter 'FV'), is part of a programme of investigation of 'fixed points' in Archaic archaeological chronology, the tendency of which is to demonstrate that the conventional chronology is some half-century wrong. This broaches various problems of wider significance, not made explicit in the article and not considered here. The present Note considers the article alone, since some features of the content and manner of the arguments give ground for concern. It is written mainly as a guide to students who may have been puzzled or impressed that such radical new views could be published so confidently. Briefly, FV argue that the Temple of Apollo at Eretria should be dated to the 470s, and that, although Herodotus places the Siphnian Treasury at Delphi c. 525, we ought to be happy with a date in the 470s for this building also.

1. The Eretria Temple

The argument is simple. Herodotus says that the Persians burnt Eretria's temples in 490. The latest temple of Apollo Daphnephoros on the site is the marble one with the sculptures surviving from one pediment. Since inscriptions show the continuation of cult there after 490 the temple must have been constructed after 490 (in fact after the Persians left Greece finally in 479) and was destroyed only in the Roman sack of 198 when the attackers found little wealth but 'signa tabulae priscae artis ornamentaque eius generis' which they carried away. One of the temple pediment figures (an Amazon) has been found in Rome. I observe:

(i) 'Many scholars now accept a date c. 510' (FV 49). Their n. 5 shows that some would go later, as does the fullest recent publication of the pediment by E. Touloupa,¹ though no later than 490. So there is not that much in it and the question of construction, destruction and survival becomes of more moment than stylistic dating.

(ii) No inscription mentions a temple, and in the one so restored *ναοῖς* (quoted in FV 50 n. 11) is not the only solution suggested,² and, even if a *naos* were named, it gives no indication of its condition. All other instances in inscriptions cited (FV 50 n. 11) mention only a *hieron*, and as a location, not in a context of cult, although we might assume that the word implies cult. David Lewis has pointed out to me *IG* xii.9 191 lines 10 f., 43, which seem to imply that the *hieron* was spacious enough to accommodate the citizens of Eretria. For cult, of course, a temple is unnecessary: a *temenos* and altar are all required and often all available. Continuation of cult on the site is probable but there is no proof in inscriptions

¹ Τά ἐναέτια γλυπτὰ τοῦ ναοῦ τοῦ Ἀπόλλωνος Δαφνηφόρου στήν Ἐρέτρια (Ioannina 1983). And cf. Boardman in *The Eye of Greece*, *Studies* . . . Martin Robertson (Cambridge 1982) 9, where n. 29 should read 'later than 499', not '490'. FV cite (50 n. 10) Coulton's study of Doric capital proportions, placing the Eretria Temple with the Temple of Zeus at Olympia (and many others) in one group, without quoting his conclusion 'proportions must be used as evidence of date only with great caution', having reviewed evidence from Archaic to Hellenistic.

² Cf. A. Wilhelm, *ArchEph* 1892, 134.