

The Interrelation between Philosophy and Mathematics in Special Context of Pythagoras

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Philosophy and mathematics are among the oldest academic disciplines, both dating back over 2500 years. In a variety of ways, their histories have been intertwined. Leading philosophers from all ages have both contributed to mathematics and been inspired by mathematics in formulating their philosophies. In antiquity, for instance, Plato (428-328 BCE) was heavily influenced by Pythagoras's views of abstract objects, such as numbers, and in trying to understand the nature of knowledge, used geometry as a model. In the early modern period, René Descartes (1596-1650) and Blaise Pascal (1623-62) each made lasting contributions to philosophy, mathematics and science. Gottlob Frege (1848-1925), the founder of modern mathematical logic, is also considered by many to be the founder of analytic philosophy. Logic and the foundations of mathematics are ancient subjects that originated in a joint enterprise between ancient philosophy and mathematics on which the philosopher Bertrand Russell (1872-1970) spent much of his early career producing significant work. The exactness and security of mathematical reasoning, the apparent abstractness of mathematical objects, and the contrast between mathematical and empirical knowledge have fascinated mathematicians and philosophers. In the 20th century, the logical foundations of mathematics have been extensively studied in both disciplines.

From the beginning, Western philosophers have relied upon two tools: logical and speculative reasoning. Aristotle aimed at constructing arguments that were both true and

1337 Pdx
DARBARI, M.
2005

valid by logical standards. Early Greek philosophers such as Pythagoras felt that mathematics was the key to understanding reality.

The union of mathematical genius and mysticism is common enough as seen in the case of Pythagoras, considered as the father of pure Mathematics and who was the first one to call himself “philosophos”, literally “lover of wisdom.” He founded a scholarly community in Crotona, Southern Italy in 539 BC, a commune somewhere between a religious order and a university. The philosophy of numbers that Pythagoras is associated with incorporates mathematical laws and geometric figures as proof that numbers are fundamental elements of the universe. To Pythagoreans, numbers are more than abstract figures. They are the first principles from which the universe is formed. The Pythagoreans saw mathematics and geometry as sacred tools for uncovering the true nature of the universe. The idea that "numbers" possessing the greatest virtue, produce always what is good and never what is evil, refers to justice, equanimity of temper, and everything that is harmonious. Pythagoras taught that the entire universe is one vast system of mathematically correct combinations:

He [Pythagoras] held that the ultimate substances of all things, material and immaterial, were numbers, which had two distinct and complimentary aspects. On the one hand, they had a spatial and dynamic existence, and, on the other, they were fundamental formulating principles, which were purely abstract. Thus, for example, the monad was understood by the Pythagorean both as the number one, which had physical properties that could be manipulated in nature, and as an idea, which embodied the original unity at the source of all creation.¹

Among the Pythagoreans the *monad* was the first thing that came into existence. On the plane below, the Monad or first number appears, and from this number the geometry of the universe emerges. Pythagoras called the Monad, or *One*, the first odd and therefore divine number. It was the number one, identified by the Pythagoreans with the unit point that was the epitome of exact objectification. From the unit point came all the numbers, and from numbers came the whole universe. The unit point for them, as for us, was indivisible. Their identification of the number one, however, with the unit point also made it indivisible. Ancient biographer, Diogenes Laertius, in *Lives of Eminent Philosophers*, writes:

The beginning of every thing is the monad, and from the monad the infinite diad is born, subjacent as matter to the monad which is the cause: from the monad and the infinite diad come numbers, and from numbers points, and from these lines, and from these the flat figures, and from these solids, and from these the perceivable bodies, whose elements are four: fire, water, earth, air, which mutate and move through the everything. ²

Pythagorean dualism is expressed in terms of ten contrarities (The Ten Pythagorean Principles): limit and unlimited, odd and even, one and plurality, right and left, male and female, resting and moving, straight and curved, light and darkness, good and bad and square and oblong. In Aristotle's day, the Pythagoreans traced all things back to their origin to two principles. He explains,

Evidently, then, these thinkers also consider that number is the principle both as matter for things and as forming both their modifications and their permanent states, and hold that the elements of number are the even and the odd, and that of

*these the latter is limited, and the former unlimited; and that the One proceeds from both of these (for it is both even and odd), and number from the One; and that the whole heaven, as has been said, is numbers.*³

The contrarities odd and even, square and oblong are related to the Pythagorean representation of numbers as patterns of dots. The odd series, as we can see plainly in FIGURE 1), has regularity to it while the even series does not have so (FIGURE 2).

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* Etc.

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FIGURE 1

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* Etc.

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FIGURE 2

An Oblong number can be arranged in a rectangle whose width and height differ by one unit; for simplicity let them be wider than tall. So whereas the Square numbers have sizes 1 X 1, 2 X 2, 3 X 3, 4 X 4, etc., the Oblong numbers have sizes 1 X 2, 2 X 3, 3 X 4, etc. (FIGURE 3). The reconciliation of these opposites creates harmony. For human

FIGURE 3

FIGURE 4

behavior, this was harmony resulted from behaving temperately or by exhibiting moderation. One, the Monad, was the principle of Unity from which all things begin. Two, the Dyad, was Duality. It was both the beginning of strife and the possibility for the development of relationships between things, as everything is no longer the same. The Triad, Three, forms a bridge, allows relation between the two extremes. A graphic example of this is shown below (FIGURE 4). According to Porphyry,

*Things that had a beginning, middle and end the Pythagoreans denoted by the number Three, saying that anything that has a middle is triform, which was applied to every perfect thing.*⁴

Pythagoras discovered the Triangular numbers also, which played an important role in his study of science of numbers. The numbers, which can be arranged in a compact triangular pattern, are termed as triangular numbers. (FIGURE 5) The triangular numbers are formed by partial sum of the series $1+2+3+4+5+6+7...+n$.

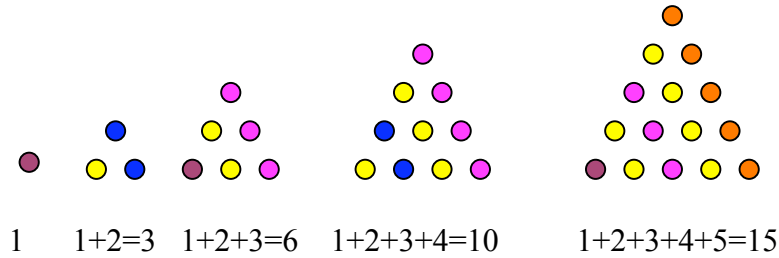


FIGURE 5

So, the n^{th} triangular number can be obtained as $T_n = n \times (n+1)/2$, where n is any natural number. Thus, the triangular number corresponding to 4 is 10. This is the explanation of the language of Pythagoras in the well-known passage in Lucian where the merchant asks Pythagoras what he can teach him. Pythagoras replies "I will teach you how to count." *Merchant*, "I know that already." *Pythagoras*, "How do you count?" *Merchant*, "One, two, three, four-" *Pythagoras*, "Stop! What you take to be four is ten, a perfect triangle and our symbol."⁵

The Pythagoreans held four, of which the Dodecahedron⁶ is the troika, sacred. Justice was the number four, because the number four was the first square number, symbolizing equality and requital. The Pythagoreans called the number *Four* the "Key-bearer of Nature." This was because, as mentioned before, numbers are an arrangement of points.

One point represented the number one, the beginning of all geometrical matter. A point has no length, breadth, or thickness (with zero dimensions). When the number one goes from the world of concept to the world of matter, it is extended and becomes divisible, thus allowing for two. The first extension into the first dimension is a line. The line has length, but no breadth or thickness (one dimension). It is the dyad, polarity. A triad is superfice with length and breadth. It is the visible (two) dimensions. Three points represent a surface. It is a trinity. The number four may be described geometrically as the first solid: It has length, breadth, and thickness. It takes us into the third dimension. (FIGURE 6) It has been described as the first descent into matter. "Hence in the realm of space, the Tetraktys represents the continuity linking the dimensionless point with the manifestation of the first body."⁷

Four is also, coincidentally the number of elements that exist: air, fire, earth, and water. From these elements come the world.

FIGURE 6

FIGURE 7

The Tetraktys of Pythagoras, which was composed of ten dots arranged in four rows to form a triangle (FIGURE 7), was the sacred symbol upon which the Pythagoreans took their most binding oath:

*I swear by him who the Tetraktys found,
Whence all our wisdom springs and which contains
Perennial Nature's fountain, cause and root.*⁸

Theon of Smyrna says that the Pythagoreans honored this symbol "because it appears to contain the nature of all things." According to Iamblichus, the Pythagorean *Tetraktys* had *eleven* forms, each one applying to some one particular phase of cosmic or terrestrial life.

The first quaternary is 1, 2, 3, 4 as $1 + 2 + 3 + 4 = 10$. The second is unity, the side, the square and the cube. The third is the point, the line, the surface and the solid. The fourth is fire, air, water, and earth. The fifth is the pyramid, the octahedron, the icosahedron and the cube. The sixth is the seed, the length, the width and the height. The seventh is man, the family, the village and the city. The eighth is thought, science, opinion and sense. The ninth is the rational, the emotional, the willful parts of the soul and the body. The tenth is spring, summer, autumn and winter. The eleventh is childhood, adolescence, maturity and old age. And the perfect world which results from these quaternaries is geometrically, harmonically and arithmetically arranged, containing in power the entire nature of number, every magnitude and every body, whether simple or composite. It is perfect because everything is part of it and it is itself a part of nothing else. This is why the Pythagoreans used the oath and through which all things are assimilated to number.⁹

Thus, in the Pythagorean system, the esoteric significance is derived from the mystic relation of every number to everything intelligible to the human mind. Concluding from *Divine Harmony*:

*For Pythagoras, mathematics served as a bridge between the visible and invisible worlds. He pursued the discipline of mathematics not only as a way of understanding and manipulating nature, but also as a means of turning the mind away from the physical world, which he held to be transitory and unreal, and leading it to the contemplation of eternal and truly existing things that never vary. He taught his students that by focusing on the elements of mathematics, they could calm and purify the mind, and ultimately, through disciplined effort, experience true happiness.*¹⁰

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2. Diogenes Laertius, *Vitae philosophorum* VIII, 24.
3. *Metaphysics*, 986a 15-21; see also 987a 15
4. The Pythagorean Source Book and Library, compiled and translated by Kenneth Sylvan Guthrie, (Grand Rapids, Mich.: Phanes Press, 1987), p. 29.
5. W.W. ROUSEBALL. *A SHORT ACCOUNT of the HISTORY of MATHEMATICS*. (4th ed.; New York: Dover, p. 26, 1960).
6. Another significant contribution of the Pythagoreans was the discovery of the fifth regular solid. Regular solids are any polyhedron that can be inscribed in (have its vertices on) or circumscribed about (have its faces tangent to) a sphere. The five regular polyhedrons are the regular **tetrahedron**, which has four equilateral triangles as faces; the **cube**, with six squares as faces; the regular **octahedron**, with eight equilateral triangles as faces; the regular **icosahedron**, with 20 equilateral triangles as faces; and the regular **dodecahedron**; with 12 regular pentagons as faces. Discovery of the fifth regular polyhedron, the dodecahedron, required the discovery of the equilateral pentagon. This discovery is credited to the Pythagoreans for this reason. No one has ever identified a sixth regular polyhedron.
7. The Pythagorean Source Book and Library, compiled and translated by Kenneth Sylvan Guthrie, (Grand Rapids, Mich.: Phanes Press, 1987), p. 29.

8. *The Pythagorean Science of Numbers*. Online. Cited on 31-07-2004.
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