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CHAPTER 4

Cosmic Significance of Mathematics

While the world was still unaware of the critical developments of Pythagorean mathematics, the numerical theory of the Brotherhood was diligently applied to the various aspects of the cosmos. The initial step in this process was conditioned by the distinction between odd and even numbers. The Pythagorean assimilation of the odd with the limit and of the even with the unlimited or indefinite was probably connected with the theory of bipartition. As an odd number is not divisible by two, it sets a limit to bipartition and is therefore limited, while an even number is unlimited, as it does not set a limit to bipartition. Thus the limit and the indefinite become the ultimate principles of the universe. The one is identified with the limit; by drawing towards itself more and more of the indefinite, it sets a limit to the latter and transforms it into a definite thing.

The Pythagoreans developed this original distinction into a table of 10 fundamental principles:¹ the limit and the unlimited (or indefinite), odd and even, one and many, right and

¹ *Met.* 986^a 22.

left, resting and moving, straight and curved, square and oblong, light and darkness, male and female, good and bad. The Pythagoreans used the limit and the indefinite in a way which makes of these opposites an expression of form and matter, and indeed the elementary constituents of the world. They thought of matter as unlimited, and probably imagined it in much the same way as the indefinite of Anaximander or the air of Anaximenes. Such a view appears justified by the primitive belief of an endless expanse of air beyond the cosmos, from which the world draws its breath. The connection between the Pythagorean opposites and the Milesian doctrines seems to find a striking confirmation in Plato's cosmogony,² where mist and darkness are given as forms of air.

To the Ionian conception of a primary stuff, the Pythagoreans added the notion of the limit which plays a part similar to that of form. The generation of things out of Anaximander's "indefinite" becomes easier with a limit shaping the amorphous energy of the indefinite. Discussion of the views of Anaximenes about the rarefaction and condensation of air may have shown how these processes imply the quantitative ideas of more or less. The next step was to consider quantity and air as two separate principles producing the world when combined. This is precisely what the Pythagoreans did by assimilating air with the void, the boundless and abstract extension emanating from the even, and by identifying with the limit the principle of number, the one exemplifying the odd. Thus, under its dual aspect of odd and even, number was the principle of matter as well as of the form which limits and shapes it. Indeed, number was the essence of everything.

An interesting hypothesis about the generation of the Pythagorean monad is put forward by Enriques.³ The boundless and formless matter of the cosmos, as conceived by Anaximander, would produce the various elements by rarefaction and condensation, as imagined by Anaximenes. Condensation

² *Timaeus* 58D.

³ Frederigo Enriques, *Il Mondo Antico*, trans. Jerome Rosenthal (New York, 1929), II, p. 17.

of a gas in a vessel spreads all over its cold surface clusters of minute drops which are themselves many centers of condensation. This phenomenon might have led the Pythagoreans to consider each of these monadic centers as a small solid nucleus separated from the others by the surrounding rarefied medium. The monad would thus be formed by the mechanical variations of the shapeless matter. Enriques believes his suggestion to be consistent with the description of the Pythagorean doctrines given by Aristotle, and with the method used by Eurytus in identifying things with the number and position of material points.

The main difficulty of this explanation is that the Pythagorean monad is not the result of any material process, but the principle of all such processes. We may quote here the reported testimony of Alexander Polyhistor about the beliefs of the Brotherhood:

For them, the principle of all things is the monad; arising from the monad, the undetermined dyad acts as matter to the monad which is cause; from the monad and the undetermined dyad arise numbers; from numbers points; from these, lines out of which arise plane figures which produce in turn solid figures; from these, material bodies whose constituents are four—fire, water, earth, air. These elements interchange and turn into another completely; out of them arises a world which is animate, intelligent, spherical and has the earth as its center, a spherical body inhabited round about.⁴

These remarks summarize the relations established by the Pythagoreans between their number theory and their physical and astronomical observations, although very little is known about their method of generating the universe. Because Plato used the 5 regular solids for this purpose,⁵ some of the early commentators believed that the Pythagoreans held a similar opinion. Probably on the authority of Theophrastus, Aetius says that Pythagoras "considering the five solid figures, also called the mathematical figures, maintains that the earth arose

⁴ Diogenes Laertius viii, p. 24-25.

⁵ *Timaeus* 53C-55C.

from the cube, fire from the pyramid, air from the octahedron, water from the icosahedron, and the sphere of the universe from the dodecahedron."⁶ This opinion agrees with this fragment of Philolaus quoted by Stobaeus: "There are five bodies pertaining to the sphere, the fire, water, earth and air in the sphere and the vessel of the sphere itself as the fifth."⁷ This fragment does not mention specifically identification of the regular solids with the elements in the sphere, but it is consistent with this doctrine.

This view attributed to Philolaus does not differ greatly from the theory of Empedocles, who was the first to consider water, air, fire, and earth as the material principles of the universe. Empedocles may have taken the matter of his intuition from the philosophers of Croton, and his two principles of Love and Hate fit well in the Pythagorean table of opposites. But as number was the principle of things, the Pythagoreans had no need to stress the generating virtues of the 4 material principles as presented by the Milesians. Hence, they emphasized the geometrical or mathematical nature of the four elements, while Empedocles insisted on their material character.

Consequently we would not attribute to Plato, as does Heath,⁸ the original assimilation of material elements with regular solids. This identification was probably implied in the construction of the regular solids attributed to Pythagoras by Proclus and other commentators. But the early Pythagoreans were unable to construct the regular solids as systematically as Euclid did in Book XIII of the *Elements*, because the Euclidian method of constructing and calculating their sides in terms of the radius of the circumscribed sphere calls for a mathematical knowledge the Pythagoreans did not possess. But they could have "put together" the regular polygons in the manner Plato puts them together in *Timaeus*—by bringing

⁶ *Placita* ii, 6.5; Diels, *Vors.*, 44 B 12, III, pp. 412-413.

⁷ Diels, *Vors.*, *op. cit.*

⁸ Thomas L. Heath, *A History of Greek Mathematics* (Oxford, 1921), I, p. 158.

together several angles of equilateral triangles, squares, or pentagons at one point so as to make a solid angle, and then by completing all the solid angles in that way.

According to Proclus, the Pythagoreans put angles of certain regular figures around a point, and showed how only 3 kinds of such angles fill up the space in one plane around the point.⁹ The scholiast mentions "the five so-called Platonic figures which do not belong to Plato, three of the five being due to the Pythagoreans, namely the cube, the pyramid, and the dodecahedron, while the octahedron and icosahedron are due to Theaetetus."¹⁰ The last two solids mentioned were probably known to the Pythagoreans, as their construction is not difficult.

Some have questioned the Pythagorean origin of the dodecahedron with its pentagonal faces, as the construction of the regular pentagon entails the cutting of a segment in *extreme and mean ratio*. But this special problem is a simple case of the Pythagorean method of applying areas. Iamblichus even attributes this particular construction to Pythagoras when recounting the story of Hippasus, who perished by shipwreck for being "the first to divulge the construction of the sphere with the twelve pentagons; though he received credit for the discovery, it really belonged to Him, as they refer to Pythagoras whom they do not call by name."¹¹ This story recalls the one mentioned by Proclus about the Pythagorean who perished at sea for revealing the irrational. He may have been the same Hippasus, for the irrational is involved in the solids inscribed in the sphere. To be sure, the construction of the dodecahedron by means of 12 pentagons may be plausibly attributed to the earlier Pythagoreans, who were familiar with the star-pentagon. Both Lucian¹² and the scholiast to Aristophanes¹³ mention the "triple interwoven triangle" called the

⁹ *Commentary on Euclid*, p. 304.

¹⁰ Thomas L. Heath, trans., *The Thirteen Books of Euclid's Elements*, ed. Heiberg (Cambridge, 1908), V, p. 654 (repub. in New York, 1956).

¹¹ *De Vita Pythagorica*, p. 88.

¹² *Pro Lapsu in Salutando* ii, 330.

¹³ *The Clouds*, 609.

pentagram or pentalpha, the symbol of health used by the Brotherhood as a sign of recognition.

The physical experiments of the Pythagoreans relating to acoustics are of particular interest. Since number ruled the world, it must explain the various phenomena of nature, especially the art of music, for which Pythagoras had a great predilection. We have no definite information about the discovery of the fundamental harmonic relations of a string vibrating over a resounding board. But it is probable that Pythagoras himself found the numerical ratios determining the concordant intervals of the scale. In those days, the most common instrument was the lyre with 7 strings; the eighth string was probably added after the Pythagorean discoveries. Yet Pythagoras did not use the lyre for his experiments, but the *monochord*, an instrument he made with one string which could be stopped at different intervals by a movable bridge.¹⁴ He could have also used some details of Eastern music he may have learned during his Egyptian travels.

Although Pythagoras could have been aware that the pitch of notes depends on the rate of vibrations communicating impulses to the air, he had no means of measuring the rate of vibration. But as the rate of 2 similar strings are inversely proportional to their length, the experiment could be reduced to a simple comparison of length along the single string of the monochord. He could discover in this way how the fifth and the octave of a note are produced on the same string by stopping at $\frac{2}{3}$ and $\frac{1}{2}$ of its length, respectively. This harmony may have suggested the name of harmonic proportion, since

$$1 : \frac{1}{2} = 1 - \frac{2}{3} : \frac{2}{3} - \frac{1}{2}$$

The interest of the Pythagoreans in number and music accounts readily for their wonder at this unexpected but intimate connection between number and sound. A simple experiment

¹⁴ An account of the Pythagorean discoveries in acoustics is given in Boethius *De Institutione Musica* i, chaps. 10-11.

with a primitive instrument had revealed the most remarkable operation of law in a field hitherto closed to systematic investigation. Intervals between sounds perceptible only to the fine ears of professional musicians, which could be neither explained to others nor referred to definite causes, were now reduced to clear and fixed numerical relations. The rule of spatial quantity was thus imposed on a most intangible and delusive phenomenon affecting the ear: sound was shown to be measurable in space, to be subject to number. Having established a basic principle of the mechanics of sound, Pythagoras may have thought that all other mechanical systems could be investigated according to similar principles. Hence, he may have sought to explain the motion of the heavenly bodies by means of some numerical regulative law.

The Pythagorean views on astronomy might be considered, indeed, as an extension of experiments with sound. Here again we may quote Aristotle, who recounts how the idea of harmony was applied to nature.

Some have supposed that the motion of the (heavenly) bodies of that size must produce a noise, since on our earth the motion of bodies far inferior in size and speed has that effect. When the sun, the moon and all the stars so great in number and in size are moving with such a rapid motion, they say, how should they not produce an immensely great sound? Starting from this argument and from the observation that their speeds as measured by their distances are in the same ratios as musical concordances, they assert that the sound produced by the circular movement of the stars is a harmony. And since it appears unaccountable that we should not hear this music, they explain that the sound is in our ears from the very moment of birth and is thus indistinguishable from its contrary silence, since sound and silence are discriminated by mutual contrast.¹⁵

A reference to this view, generally known as the *harmony of the spheres*, is found in Plato's myth of Er, where the whorls representing the spheres of the heavenly bodies "together form

¹⁵ *De Caelo* 290b 15.

one harmony,"¹⁶ and also in the formation of the world-soul,¹⁷ where ratios are given to the planets on the pattern of a musical harmony. Aristotle rejects this "melodious and poetical" theory, saying that any sound emitted by the heavenly spheres would be so great in proportion to their size that it would shatter any solid body. "If the heavenly bodies moved in a generally diffused mass of air or fire, as every one supposes, their motion would necessarily cause a noise of tremendous strength, which would necessarily reach and shatter us. Since this effect is evidently not produced, none of them can move with the motion either of animate nature or of constraint."¹⁸ Hence, there cannot be any noise, for sound is created by friction alone.

The weight of tradition notwithstanding, it is not certain that Pythagoras believed in a celestial harmony. He probably developed his astronomical conceptions from the cosmic systems of the Milesians. Anaximander considered the sun, moon, and stars as 3 wheels of fire surrounding the earth and encased in air or mist, although we only see the single aperture through which the fire escapes "as through the nozzle of a pair of bellows." At this stage, Burnet suggests that "everything points to the conclusion that the Pythagoreans retained the rings of wheels of Anaximander"¹⁹ and improved on the arbitrary distances assigned by him between the earth and these 3 rings by making them correspond to the fourth, the fifth, and the octave. In such a natural explanation of the harmony of the spheres, there is no question of a musical harmony, but only of concordant intervals expressing a numerical law of the world. Furthermore, when the cause of eclipses was known, "it was natural to infer that the earth was a sphere; and we may probably attribute that discovery to Pythagoras, himself."²⁰

¹⁶ *Republic* x, 617B.

¹⁷ *Timaeus* 35B.

¹⁸ *De Caelo* 291a 18.

¹⁹ J. Burnet, *Early Greek Philosophy* (London, 1914), p. 56.

²⁰ *Ibid.*, p. 44.

A simpler view is put forward by Heath, who says that Pythagoras "attributed spherical shape to the earth as to the universe, for the simple reason that the sphere is the most beautiful of the solid figures. For the same reason, Pythagoras would surely hold that the sun, moon and the other heavenly bodies are also spherical in shape."²¹ Indeed, the astronomical conceptions of the Pythagoreans have a strictly mathematical character, as they do not involve any forces causing the respective movements of the heavenly bodies. Astronomy is geometry combined with arithmetic and harmony. All the stars are spheres, the most perfect solid figures, and they move in circles.

This is also the opinion of Aristotle, in whose view the Pythagoreans simply held the universe to be spherical, with fire at the center, and the earth as one of the stars creating night and day by its circular motion about the center. However, the view that the earth and the other heavenly bodies revolve about the central fire is probably due to Philolaus and other later Pythagoreans. Aristotle mentions this interesting addition to the revolving bodies: "they further constructed another earth in opposition to ours, to which they gave the name of counter-earth."²² This counter-earth was conceived in order to bring up the number of the moving bodies to 10, because the Pythagoreans liked to fit into their scheme "all the properties of numbers and scales they could show to agree with the attributes and parts and with the whole arrangement of the heavens; and if there were a gap anywhere, they readily made such additions as to make their whole theory coherent. For example, because the number 10 is thought to be perfect and to comprise the whole nature of numbers, they say the bodies moving through the heavens are ten; but as the visible bodies are only nine, they invent a tenth, the counter-earth."²³ The number 10 is said here to be perfect because it signifies the Decad, which has many mystical and numerical perfec-

²¹ Heath, *A History of Greek Mathematics*, I, p. 163.

²² *De Caelo* 293a 21.

²³ *Met.* 986a 3.

tions, and not in the numerical sense of being the sum of its aliquot parts, which is not the case for 10.

Before closing this discussion, it may be useful to indicate some early applications of number to psychology, as the mathematical analogies used by Plato in the construction of the soul and of the universe obviously display Pythagorean influences. Considering knowledge as a whole, the Pythagoreans used number as a cause in all the branches of their teaching. In fact, their mathematical conceptions enabled them to combine the naturalism of the Milesians, the mysticism of the East, and some of the religious practices of Orphism into one system.

The revival of the Orphic traditions introduced into Greek philosophy the germ of a dualism between matter and mind, body and soul, God and the world. These distinctions were unknown to the earlier generations, for whom nature was animate and every living creature somehow infused with mind. The incorporation of the Orphic doctrine of transmigration into a philosophic system showed the aim of life to be liberation from the circle of rebirths in order to enjoy the divine state of bliss: the road to salvation was purification from sensuality and renunciation of worldly interests. The ritualistic character of this Orphic purification was intellectualized and given a moral value by the Pythagoreans, who supplemented their ascetic observances with silence, daily self-examination, and mental effort. Hence, science, music, gymnastics, and medicine were studied systematically by members of the Brotherhood, who recommended them for the purification of the soul and the body.

The purgative function of music, which originated in the practices of the Corybantic priests, was fully recognized in ancient psychotherapy. The introduction of a mathematical element into music, through the connection between sound and number, encouraged the use of mathematics for purification of the soul. If ordinary music was a soul purge, a similar effect could be obtained by cultivating the "highest music," the name given to philosophy in Plato's *Phaedo* (86B). Just as Pythagoras discovered the means to blend such apparently elu-

sive things as the high and low notes of the octave, so could he determine numerically the blend of opposites in order to find a "mean" point fair to both, and to remove the "injustice" affecting the soul when one opposite encroaches upon the other.

Similarly, the health of the body must depend on the adequate blend of opposites, such as hot and cold, and wet and dry, traditionally considered as the principles of human life. According to Plato, the Pythagoreans held the body to be tuned to a certain pitch, like an instrument, the high and low notes in music identified with hot and cold, wet and dry. Consequently, health is just being in tune, and disease arises from the ill adjustment of hot and cold, wet and dry. The medical school founded by Alcmeon of Croton, which flourished at the same time as the Pythagorean Brotherhood, held similar views about health and disease and many associated topics, such as diet and climate. As friendly relations prevailed between their respective members, it is difficult to distinguish clearly what belongs to each school from the little evidence in our possession.

The proper function of the Pythagorean physician was to adjust an adequate blend of opposites in the human body, just as the curative function of music was to produce a proper blend of opposites in the human soul. The doctrine of mathematical means helped to determine their correct proportions and to combine efficaciously their various differences according to the constitution of individual patients. But such combinations depended ultimately on the restriction of the indefinite by the limit entailing number. Life and death themselves are thus ruled by number: if life is health, it is also harmony of the opposites; if death is the last phase of disease, it is also the result of the final elimination of the correct proportions of opposites in the human being.

In this primitive psychology, the principle of the harmony of the opposites is the soul, which "brings number and harmony into the body," according to Philolaus. An accurate account of the Pythagorean theory of the soul is not easy to

formulate, for the statements handed down by tradition imply differences of opinion between earlier and later Pythagoreans. The following passage of Aristotle seems to represent the views of the older members of the school:

There is yet another theory about the soul: for its supporters the soul is a kind of harmony, for harmony is a blend or composition of contraries, and the body is made of opposites. But harmony is a certain proportion or composition of the constituents blended, and the soul can be neither of these. Further the power of originating movements cannot belong to a harmony, while almost all regard this as a principal attribute of the soul. It is more appropriate to consider harmony as health or generally as one of the good states of the body, than to predicate it of the soul.²⁴

The Pythagorean theory of the soul is also connected with the doctrine of rebirth or transmigration, which Pythagoras may have learned from Orphism and the East. Xenophanes made fun of him for pretending to recognize the voice of a departed friend in the howls of a beaten dog,²⁵ and Empedocles seems to refer to him when he mentions a man who could remember what happened 10 or 20 generations before. The doctrine of transmigration may have inspired the Platonic doctrine of Reminiscence, which plays so great a part in *Meno* and *Phaedo*. Burnet suggests that Pythagoras was probably familiar with the idea of Reminiscence, for he must have noticed that "the realities he was dealing with were not perceived by the senses."²⁶ But such an interpretation is excessive, since the Pythagorean mathematical conceptions were less pure and abstract than those of Plato. Since material things seen, heard, or touched by the early Pythagoreans were essentially numbers, it was unnecessary for them to recall what their souls may have known before incarnation. The direct vision of a higher mathematical reality may be considered as a proper Platonic doctrine.

²⁴ *De Anima* 407b 30.

²⁵ Diels, *Vors.*, 21 B 7; I, p. 130.

²⁶ Burnet, *op. cit.*, p. 43.

IN GREEK MATHEMATICAL
PHILOSOPHY p. 38-48