

Pythagoras' Inheritance at Paestum in South Italy *Number is the Substance of All Things*

R. de Zwarte

Abstract

The discovery of Pythagoras' shrine at Paestum in South Italy will enrich our knowledge about geometry done with numbers of various geometrical and arithmetical progressions. The accuracy of the constructions is fabulous. The diagonal of a square with side 309 was found to be rational by measuring. A golden isosceles triangle has been laid off by using the telling numbers 618 and 1000. The Archimedean value for π was already known in the 6th century BC. The Delian problem is older than the time of Plato. The Pythagoreans solved this problem as well as the quadrature of the circle.

INTRODUCTION

In a recent paper on the oldest temple of Hera (fig. 1) I stated firmly that the Pythagoreans hardly could have been the architects of this temple as higher mathematics is of no use for an architect in setting out the dimensions of a temple.¹ However, it is now clear that I was wrong as the position of at least three columns - Cn2, Cn6 and Cn7 - in the naos (fig. 2) turns out to be chosen primarily for the sake of fixation of (secret?) mathematical wisdom, not on aesthetic or on structural grounds. Therefore we must accept the Pythagorean community as the creator of the temple. I cannot explain what the aim of the fixation was, but it is certain that the results could not be checked afterwards, when the antae and the wall between pronaos and naos had been erected.²

The mathematical knowledge embedded in the 'Basilica' at Paestum is astonishing. This temple is generally dated to about 530 BC. Certainly, this is something of a problem as Paestum is not mentioned in the myths and legends dealing with the period 531-500, i.e., the maximum lifetime of Pythagoras in south Italy. To sum up the events: in 531, Pythagoras emigrated from Samos to Kroton, perhaps to escape the tyranny of Polycrates. At Kroton he is said to have founded a philosophic school under the protection of Milo. He married Milo's daughter Theano, an outstanding mathematician, who was much younger than himself. He died at Kroton in 510 or at Metapontion in about 500. We have all heard of his doctrine, that 'Number rules the universe'. Unfortunately, we have no documents of his time setting forth his views explicitly. The list of Pythagorean mathematicians is almost empty for the period 500-450.³

We have only the name of Hippasos of Metapontion, who allegedly was put to death by drowning because of having divulged secret knowledge on the dodecahedron, a body whose pentagonal surfaces are intimately linked with the golden section.⁴ Paestum lies, as the birds fly, about 160 km from Metapontion and about 250 km from Kroton. Clearly, there is something wrong with the tradition or with the date of the 'Basilica'. At this point, it may be useful to remark that what I have said above on Pythagoras and the Pythagoreans rests on myths and legends and on vague sayings by mathematicians or philosophers⁵ who lived at least half a century after the time of Pythagoras as well as on explanations and assumptions upon it by a commentator of the 4th century AD and by modern scholarship. However, hypercritical scholars even denied the very existence of Pythagoras.

An interesting question is whether Pythagoras was a deceiver or an honest man. Let me explain this. If the side of a square is 1, then its diagonal is $\sqrt{2}$. In other words, the ratio of the diagonal of a square to its side is irrational as the ratio cannot be expressed as a numerical fraction. However, if one sets out a square with a side of 408 centimetres by using accurate steel measuring-tapes, the diagonal is found to be exactly 577 cm and the ratio 577/408 is rational, which is in contradiction with the theory set out here. But the algebraic calculation results in 576.9991.. cm, a difference of about 1/100th of a millimetre. This difference cannot be detected with geometrical means what makes the geometrical method liable to fraud. Thus, by geometry it is possible to hold a theory - the diagonal of a square is rational - which is incorrect. There exist more pairs of numbers of

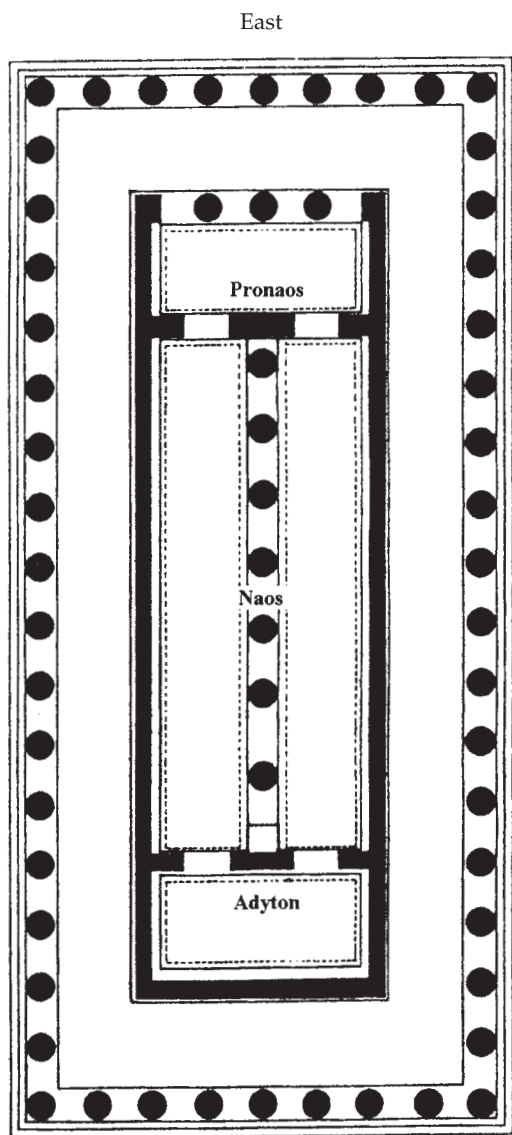


Fig. 1. Hera temple I ('Basilica') at Paestum
(after Mertens 1993, fig. 61c).

this kind, which give a rational expression for $\sqrt{2}$.⁶ Well then, Pythagoras was a deceiver if, first, he had heard of such pairs during his years of study in Egypt or Babylonia and, second, if he himself could find the length of the diagonal algebraically but ordered his students to find the length by geometry. Here it may be recalled that the Pythagoreans were divided into two groups, the (presumably) older group, the 'akusmatikoi', who flatly accepted what the Master said and a younger group, the 'mathematikoi'. According to Van der Waerden⁷ Hippasos was (about 500?) the first mathematician, but Pythagoras' wife Theano does not fit in the story, if we accept this. However,

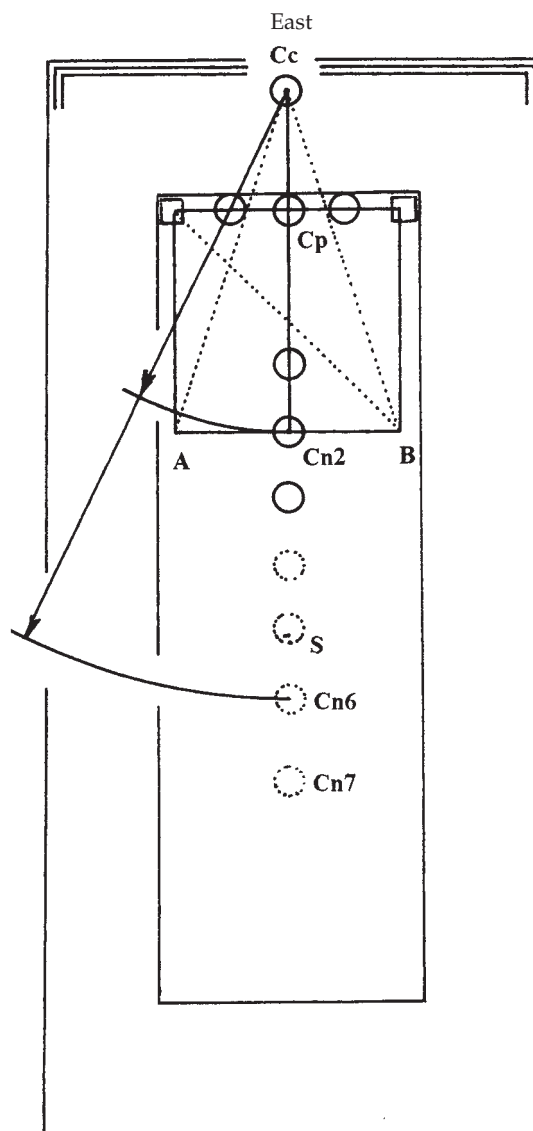


Fig. 2. Hera temple I at Paestum. Cc is the central column of the east colonnade, Cp the central column of the pronaos, Cn columns in the naos and S is a scratch. Columns Cn4-7 have been removed in the Middle Ages, but the centre points of Cn5-7 are known.

Pythagoras' famous theorem on the right-angled triangle (half the square is a special case of this theorem) might have been regarded originally only as a numerical proposition, which is the opinion of Seidenberg.⁸ This assumption holds good for right-angled triangles with unequal sides, e.g., the exact triples $3/4/5$, $5/12/13$, $7/24/25$, $8/15/17$, $12/35/37$ and many others, but never exactly for $a/a/b$, a right-angled triangle with two equal sides, a fact, says Seidenberg, 'supposedly realized by Pythagoras'. If so, Pythagoras was an honest

man who became disappointed when he himself or another member of the community made progress into the study of *algebra*, the computational branch of mathematics, not of geometry, the constructive one. But here I touch a delicate question among historians of mathematics.⁹ I leave it to the reader to judge which view of Pythagoras' character might be correct. As we shall see below, Pythagoras used the special number 309 (in double dactyls of his foot standard) as the side of his square. Indeed, geometrically, number rules the rational diagonal of the square.

RECAPITULATION

In my paper on the 'Basilica' of 2002 I have dealt with the progression of Pythagoras. In 1877, this progression was rediscovered by Édouard Lucas. It is a series of integral numbers, each of which is the sum of its two predecessors and any two of which will produce an approximation for the golden section. The same description holds for the well-known progression of Fibonacci. These series run as follows: Pythagoras 1, 3, 4, 7, 11, ..., 521, 843, 1364, etc. and Fibonacci 1, 1, 2, 3, 5, 8, ..., 377, 610, 987, etc.

The ratio of the real golden section¹⁰ is $1/2\sqrt{5} - \frac{1}{2} = 0.61803..$. In practice, successive high numbers of both series can be used for a very accurate construction of the golden section, called the section of mean and extreme ratio by Euclid (c. 300 BC) and the construction of the mean proportional - that is the major part of a line that consists of minor + major - by modern mathematicians. At the 'Basilica', in the middle step of the colonnade, on north side as well as on south side, the Pythagoreans have given a material character to the numbers 521 (minor), 843 (major) and 1364 (minor + major). Minor : major = major : (minor + major) or $0.6180.. : 1 = 1 : 1.6180..$. Most handbooks give this information accurate to three decimals, thus 0.618, 1, 1.618. Is it not marvellous that Pythagoras (below) did the same by using the numbers 618 and 1000?

PYTHAGORAS' FOOT STANDARD

The present paper is, in part, an attempt to carry my point of view that the introduction of a new fixed standard is useful if its introduction solves a problem that could not be solved before, but whatever inquiries based on assumed variations of known standards must be abandoned. I do not deny the existence of local foot standards or regional standards, e.g., the Attic foot of 32.66

cm,¹¹ but I hold that ancient Greek society had a common foot standard, in roughly the same way as we today have a metric system, the Ionic foot of 29.86 cm. At the 'Basilica', this standard is easy to find from the dimensions of the altar: length minus width = 50 feet, or $(2100 - 607)$ divided by $50 = 29.86$ cm.¹² Greek foot standards had a *fixed* length. Thus, previous to the full account on Pythagorean mathematics, let me first ride my hobby on fixed standards,¹³ as fresh evidence can be presented. As we have seen, there is an abundance of pairs of numbers that represent the ratio of the golden section and the line as the sum of these numbers. Naturally, these numbers are also suitable for the construction of golden triangles, rectangles, etc. Of special interest is the golden isosceles triangle because of its aesthetical appearance and from the mathematical point of view, of its vertical angle of 36° as this angle encloses $1/10^{\text{th}}$ part of all circles having the vertex of the triangle as centre. However, occasionally the numbers of the series of Pythagoras or Fibonacci do not fulfil all our requirements. I may point to the education of the youth in our times and the fixation of knowledge in antiquity. Then the architect or mathematician must resort to telling numbers. Thus, e.g., the modern day architect of a new science museum, working in metres or fractions thereof (fig. 3a, b) will inform the visitors by tables of the telling numbers which give the ratio of a full-sized golden isosceles triangle, hoping that they shall never forget its form and its ratio presented in such an imposing way. Pythagoras' reason to do the same (fig. 3c) cannot be guessed with certainty, but, in my opinion, the fixation of knowledge is more likely than an enduring lesson in aesthetics. Of course, the metrologist goes into ecstasies because of the combination of modern

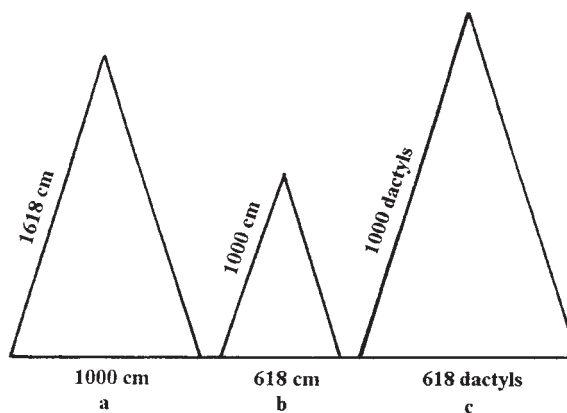


Fig. 3. Telling numbers of golden isosceles triangles: a and b modern (design) and c about 530 BC (executed).

measurement and archaic number that irrefutably guides him to Pythagoras' foot standard that was divided into 16 dactyls.¹⁴ The base of the triangle, which is also one side of the square, measures 1153.5 cm (below) and 1153.3 cm equals 618 dactyls of foot size 29.86 cm. Fortunately, all dimensions, which are relevant to the discussion of Pythagoras' mathematical knowledge, are accurate. With this information in mind, the reader has been prepared to see what Pythagoras already knew a few centuries before Euclid (c. 300 BC) and Archimedes (c. 250 BC) wrote their books.

THE RATIONAL DIAGONAL OF THE SQUARE

The square (fig. 4) and the isosceles triangle are situated in a horizontal plane, about two metres above floor level. It is something of a miracle that the information on side AB is still available as this is the inner width of the cella. In the Middle Ages, the walls have been demolished completely to its very foundations and not a single stone block is still *in situ*, but the antae are still there and the imprints of the former stone blocks are well visible. Mertens measured 1149.0 cm between the orthostates - that is the bottom course of the walls - and 1153.5 cm between the upper courses, the wall proper.¹⁵ It is the latter width that is of importance as the other side of the square, represented here by the distance between the columns Cp and Cn2, measures 1153.6 cm between the centres.¹⁶ In Ionic feet, $38 \frac{5}{8}' = 1153.3$ cm, in number (double dactyls) 309. The diagonal $\sqrt{309^2 + 309^2} = 436.991... (= 1631.07.. \text{ cm})$ and number 437 is 1631.10.. cm, a difference of about $3/10^{\text{th}}$ of a millimetre. As Pythagoras constructed the square, the diagonal was found to be rational

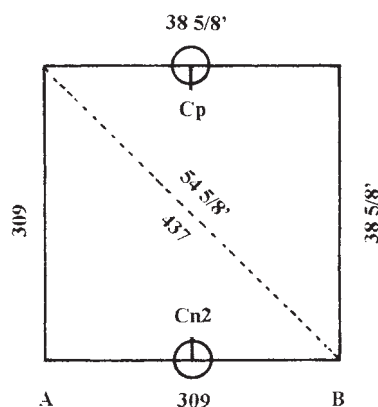


Fig. 4. Hera temple I at Paestum: a square selected by the Pythagoreans on account of its (practically) rational diagonal.

by measuring of the distance between two opposite points. I conclude that Seidenberg¹⁷ indeed was right in saying that Pythagoras held a numerical proposition on the square: the diagonal of a square with side 309 is 437. As the difference from reality is very slight, the approximation to $\sqrt{2}$ (1.414213..) is good, $437/309 = 1.414239...$ There are better approximations (side of the square number 408 and the ratio $577/408 = 1.414215..$) but the side AB of the square is also the base of the isosceles triangle and number 309 can also be combined with a proposition on the rectangle, which we find later in Euclid. The leading thread running through it, is the golden section.

THE PROBLEM OF SQUARING THE RECTANGLE AND A NEW PROGRESSION

In Euclid, Book II of *The Elements*, Proposition 14 is on the conversion of a rectangle into a square, or what is equivalent, as Smeur¹⁸ says, the construction of the mean proportional between two given line segments. As the mean proportional is involved, I gave it a trial looking forward to more precise information on what the mathematician must know if he wishes to turn a rectangle into a square by using this method. The result was: the long side of the rectangle can be chosen at will, then this side must be divided into minor and major (the mean proportional) and only minor can be taken as the short side of the rectangle. Or, the other way around, minor - the short side - can be chosen freely, major has to be calculated (minor $\times 1.618$) and minor + major becomes the long side of the rectangle. Fig. 5 gives the rectangle and its conversion into Pythagoras' floating square. Talking about minor and major, I introduced

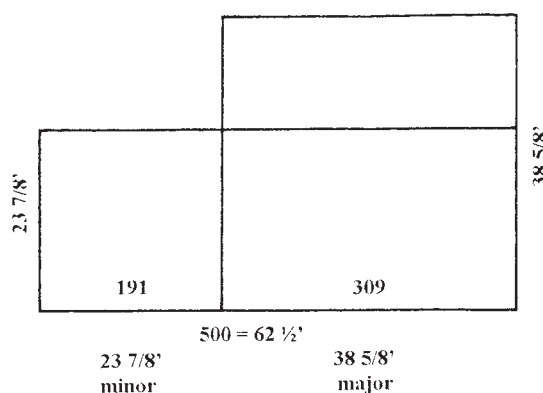


Fig. 5. The problem of squaring the rectangle (Euclid, *Elements*, II, Proposition 14).

again the golden section, but the numbers 191, 309 and 500 are not among the numbers of the series of Pythagoras or Fibonacci. Thus, we have found another progression of integral numbers, which runs as follows: 1, 5, 6, 11, 17, 28, 45, 73, 118, 191, 309, 500, 809, etc. We shall meet number 809 again, which almost certainly means that Pythagoras was familiar with this progression. This assumption is corroborated by the fact that Pythagoras uses some doubles, 618 and 1000 (below).

At this place an aside to modern architecture recommends itself. Since 1950, the famous architect Le Corbusier¹⁹ has designed buildings on basis of the golden section by using his own progression 5, 11, 16, 27, 43, 70, 113, 183, ... as, at the time, the height of an average European man was 183 cm. I have no exact figures to hand, but take it for certain that the average height of European mankind has gone up sharply in the second half of the past century, so the new progression, which includes the number 191, is a worthy successor of the obsolete series of Le Corbusier. If the trend carries on, the first series of Pythagoras can be considered for its number 199.

THE GOLDEN ISOSCELES TRIANGLE

If the base is 618 (fig. 6), the leg of a golden triangle has to be 1000. There is little further I can say about the triangle, but it may be useful to emphasize the degree of accuracy we find in the execution. We already know the base AB (measured 1153.5 cm), so half the base is 576.75 cm; if perfect 576.65 cm = 19 5/16 Ionic feet = number 309 (dactyls). The leg is unknown, but the height of the golden triangle is represented by the distance between the centres of the colonnade column Cc and the naos column Cn2:²⁰ 1774.9 cm; if perfect 1774.8 cm = 59 7/16 feet = number 951 (dactyls). The leg = $\sqrt{309^2 + 951^2} = 999.94.. (= 1866.13 \text{ cm})$; number 1000 = 1866.25 cm, a difference of 1.2 mm. By coincidence the slightly longer factual dimensions of half the base and the height just compensate for the difference: $\sqrt{576.75^2 + 1774.9^2} = 1866.25.. \text{ cm}$. By using modern equipment, Mertens has measured what Pythagoras has laid out with astonishing accuracy. Obviously, there was also no shortage of accurate measuring instruments in the archaic period. Unfortunately, nothing is known in detail about the measuring instruments used by the Greeks, for none have survived. In centimetres, the ratio of base to side is $1153.5/1866.25 = 0.6180...$ Clearly, we even do not need to know what foot was used to prove that mathematicians have worked in the

'Basilica'. For inquiries into fixed mathematical ratios or constants, e.g. π , supposed to have been known in archaic and classical south Italy, the cradle of the Pythagorean community, only two conditions have to be fulfilled. One is that a requisite length has been set out accurately by the ancient mathematicians, the other that the length has been measured accurately by the investigator in our times. In fact, an inquiry into mathematical knowledge is easier than an inquiry into the design of a temple, which certainly cannot be illuminated if we do not know what foot we are dealing with.

THE GREAT PYRAMID AT GIZEH AND THE GOLDEN RIGHT-ANGLED TRIANGLE

It becomes expedient to make an excursion to Egypt with Gizeh as the destination. By tradition, the Greeks believed that they had acquired their interest in geometry from the Egyptians. As Pythagoras should have studied mathematics in Egypt for more than twenty years, it may be useful to see what he could have learned there. Indeed, there are indications that the Egyptians attached importance to the ratio of the golden section. According to Wells²¹ a 'holy ratio' is mentioned in the Rhind mathematical papyrus (c. 1800 BC). Wells also says that the ratio at the Great Pyramid of Cheops at Gizeh, between the

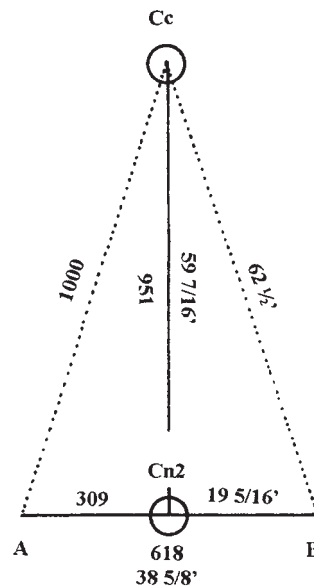


Fig. 6. Hera temple I at Paestum: the golden isosceles triangle constructed by the Pythagoreans using telling numbers for the sides.

height of a lateral face and half the base side, is almost exactly 1.618. Unfortunately, he gives no references. Berriman²² gives useful information as to the base side of the pyramid: the measurement by Petrie in 1882 gave a length of 9068.8 English inches (= 23034.8 cm = 440.82 royal Egyptian cubits)²³ and the measurement by Cole in 1925 resulted in 9069.4 English inches (= 23036.3 cm = 440.84 cubits). Concerning the height no information is given. All the pyramids have been despoiled of their casing stones that gave them smooth exteriors. Berriman²⁴ tells us that Vyse, when excavating at the base of the pyramid in 1837, uncovered some casing stones, but he gives no dimensions of these stones. I must resort to a plausible approach as the evidence is incomplete and make an allowance for the missing casing stones. Surprisingly, a reasonable proposal can be made (fig. 7) as 360 divided by $222\frac{1}{2}$ gives 1.61797.. or almost exactly 1.618. Of course, this is not a final proof, but the height of 360 cubits is interesting as in a problem in the Rhind mathematical papyrus the given base side of a pyramid is 360 cubits.²⁵

Let us return to Paestum to check upon the possibility that the Pythagoreans could construct a golden right-angled triangle by using numbers already known to us in a different position. In the present case this is easily done: the height of the Pythagorean golden isosceles triangle (fig. 6) becomes the hypotenuse of the golden right-angled triangle and half the leg of that triangle the short side (fig. 8). Using the theorem of Pythagoras we find the long side as 808.950.. or we can calculate the long side starting from the figures of the pyramid, thus $1000/445 \times 360 = 808.988.. \approx 809$. As wanted, the ratio $809/500 = 1.618$. For practical purposes the Egyptians used the cubit and Pythagoras the foot, but number is the basis both in Egypt and south Italy.

THE PYTHAGOREAN VALUE OF π

The area of a circle is πR^2 . Consequently, the areas of two circles are to each other as the squares on their radii: $(R_2/R_1)^2$. It follows that the ratio can be adjusted for a special case, that is the fixation of knowledge on the value of π : $\frac{\text{area}_2}{\text{area}_1} = \frac{R_2^2}{R_1^2} = (\sqrt{\pi})^2$. The circumference of a circle can be expressed as $2\pi R$. So the circumferences of two circles are in the ratio R_2/R_1 . If the area increases π times, the circumference increases $\sqrt{\pi}$ times, a fact that might throw some light on modern misinterpretations of the value of π in interpreting ancient problems.²⁶

Now let us see how Pythagoras has fixed the value of π . R_1 is the distance Cc-Cn2, i.e., the height of the golden isosceles triangle, already discussed above: number 951 (measured 1774.9 cm). R_2 (fig. 9) is the distance between the centres of the colonnade column Cc and the naos column Cn6:²⁷ 3147.5 cm; if perfect 3146.5 cm = 105 $\frac{3}{8}$ Ionic feet, in number 1686 (dactyls). So $\pi = \frac{R_2^2}{R_1^2} = \left(\frac{1686}{951}\right)^2 = 3.143070.. = 22.001../7 \approx 22/7$. Calculating with the dimensions as measured, we find $22.01../7$, that is close enough to the mark to accept the result even without the support of the calculation in numbers. It is clear that Pythagoras already knew the upper limit of the Archimedean value for π . Archimedes (c. 250 BC) proved in his book *Measurement of a circle* that π is greater than $3 \frac{10}{71}$ and less than $3 \frac{1}{7}$.²⁸ In practice, the value $22/7$ was used by Archimedes himself and by later ancient and medieval mathematicians.²⁹

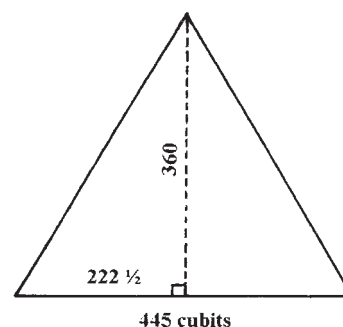


Fig. 7. The Great Pyramid at Gizeh: lateral face.

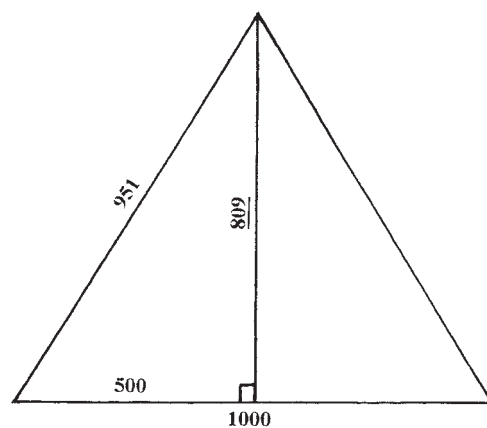


Fig. 8. The golden right-angled triangle constructed by using numbers already known in a different position.

Number 951 is related to the telling numbers 618 and 1000 of the golden isosceles triangle (fig. 6). As we have seen, these telling numbers are doubles of a progression that starts 1, 5, 6, etc. but 1686, or rather its half 843, is a number of the first progression of Pythagoras starting 1, 3, 4, etc. Pythagoras, who had clearly expert knowledge of the properties of progressions, certainly knew that such numbers were connected with the problem of squaring the circle. In the present case the area of the circle with radius 951 (fig. 9) is (almost) equal to a square with side 1686 as 1686^2 divided by $22/7 \times 951^2 = 1.00006...$

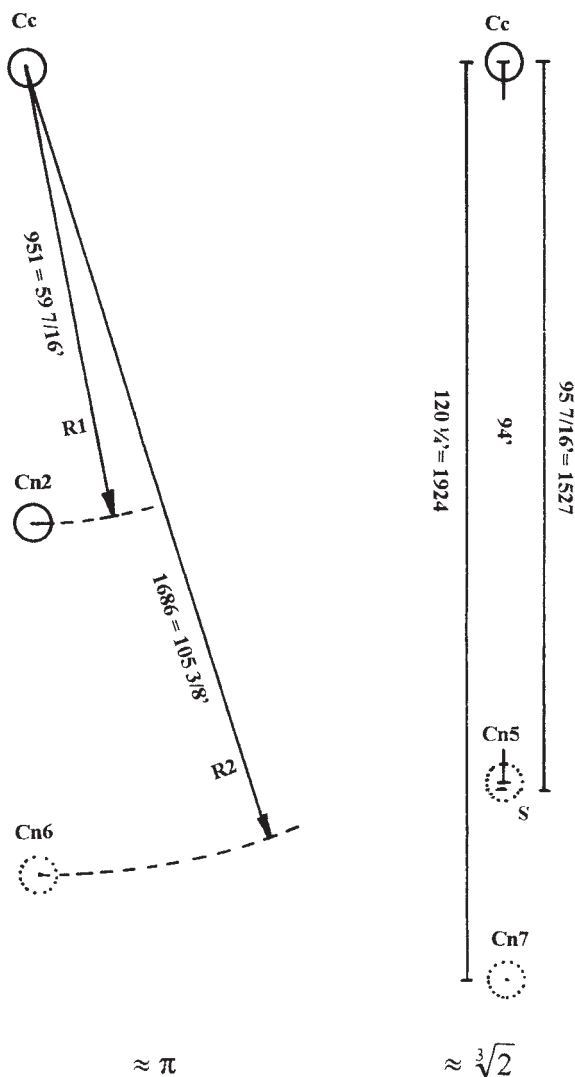


Fig. 9. Hera temple I at Paestum:
fixation of knowledge concerning π and $\sqrt[3]{2}$.

One of the famous construction problems of antiquity is the duplication of the cube, the Delian problem, so called because the oracle at Delos is said to have given the doubling of Apollo's cubical altar as a means of ending a plague. In the legend, the temple architects are confronted with a geometrical problem. Here they needed a rational expression for $\sqrt[3]{2}$. It is further reported that the Delians sent emissaries to the geometers of Plato's academy to ask them for a solution. But the problem seems to be older than the time of Plato (c. 380 BC). If the edge of Apollo's cubical altar was a , the edge of the new altar had to be a $\sqrt[3]{2}$. The value $\sqrt[3]{2}$ is represented in the 'Basilica' by the ratio of the distances (fig. 9) Cc-Cn7 and Cc-s, a scratch under column Cn5, now missing. Cc-Cn7 = 3591.5 cm; if perfect 3590.7 cm = $120 \frac{1}{4}$ Ionic feet, in number 1924 (dactyls) and Cc-s = 2851.4 cm; if perfect 2849.8 cm = $95 \frac{7}{16}$ Ionic feet, in number 1527 (dactyls).³⁰ $\sqrt[3]{2} = \frac{1924}{1527} = 1.25998...$ and my calculator informs me that the real value of $\sqrt[3]{2} = 1.25992...$ In my opinion, Pythagoras was not a man to take things easy. So I wondered what happened with the man who put the scratch to demonstrate a better approximation to $\sqrt[3]{2}$ than Pythagoras' own elegant solution which can be found in fig. 6, that is $\frac{951+309}{1000} = \frac{1260}{1000} = 1.26$ exactly. However, having discovered this, I realized that an alternative explanation suggests itself about the first evaluation. That evaluation does not necessarily need to mean an improvement of Pythagoras' one but simply a confirmation of it by a pupil by using another pair of whole numbers. If so, 1924 was given and the place of the scratch calculated, $1924 \times 1000/1260 = 1526.98... \approx 1527$ and next set out from the same point as number 1924, the centre of the eastern colonnade column Cc.

PYTHAGORAS' METHOD OF FINDING THE SQUARE ROOT OF 2 AS A RATIONAL EXPRESSION

At first sight, Pythagoras' procedure of finding $\sqrt[3]{2}$ (fig. 6) seems odd. However, in fact the method is in line with a similar procedure for finding an approximation to $\sqrt{2}$ in whole numbers. This will be immediately clear when I present the hypotenuse of half the golden isosceles triangle as the diameter of a circle (fig. 10). The side of the added isosceles right-angled triangle is the square root of $1000^2/2 = 707.10... \approx 707$, which makes Pythagoras' value of $\sqrt{2}$ equal to $\frac{707+707}{1000} = \frac{1414}{1000} = 1.414$. This method, which seems to be obsolete nowadays, is in no way inferior to the approxi-

mative method current among modern number specialists ($1000/707 = 1.4144\dots$). The arithmetic mean of these two is $1.41421357\dots$, which is correct to seven places. Try it for yourself using the figures of Pythagoras' floating square (above). Fig. 10 gives a clear picture of geometry done with numbers, together with the associated algebra.³¹ Thus, returning to the Delian problem: the cubic root of $1000^3 \times 2 = 1259.92\dots \approx 1260 = 951 + 309$. Perhaps, Pythagoras has made progress in this field of mathematics during a stay - of course, it is a myth - with Thales of Milete (c. 624-543). The theorem of Thales says that an angle inscribed in a semi-circle is right.

Up till about 1930 the generally held view on the origin of mathematics was that it all started in Greece with Thales at about 600 BC. The theorem of Pythagoras was considered to be 'of Pythagoras'. However, the deciphering of cuneiform texts from 1928 onwards showed that the Old-Babylonians of 1800-1600 BC knew the theorem of Pythagoras. The view that it was the Old-Babylonian mathematics that stood at the beginning became dominant. But Seidenberg³² pointed to a difficulty: 'the only trouble with it is that many of the common elements of Greek and Indian mathematics and especially the geometrical constructions at issue, are not found in Old-Babylonia'.

THE CONFIGURATION OF THE COLONNADE: NUMBER MYSTICISM OR PYTHAGORAS' THEOREM?

Any student of Greek architecture would say that the oldest temple of Hera at Paestum is a building of 9×18 columns, which means that the colonnade has 9 columns on the fronts and 18 columns on the flanks, the corner columns *double counted*. Let me toy with these numbers:

$$\begin{aligned} 9^3 + 18^3 &= 81^2 \\ 9^3 &= 729; \quad 7+2+9 = 18 \\ 18^3 &= 5832; \quad 5+8+3+2 = 18 \\ 81^2 &= 6561; \quad 6+5+6+1 = 18 \end{aligned}$$

According to Wells³³ this property of number 81 is unique. Modern specialists in numbers do it, but the problem is that I cannot prove that Pythagoras did it and, of course, if he did it, what did it mean to him?

An alternative explanation came to my mind as we don't know at all whether double counting of corner columns was also the Greek way of looking at a temple. Perhaps, Pythagoras planned two times 5^2 columns, that is, on the fronts 3^2 columns (corner columns included) and on the

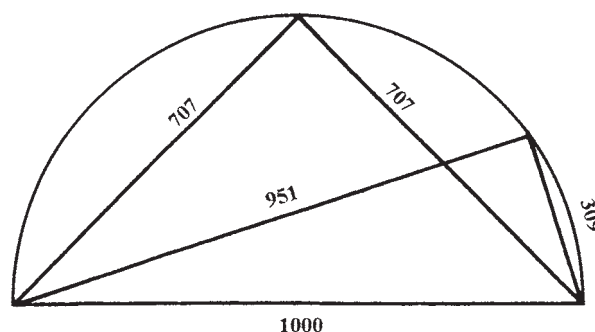


Fig. 10. An obsolete method of finding a rational expression for $\sqrt{2}$.

flanks 4^2 columns (corner columns *excluded*). In this way he visualized his theorem on the right-angled triangle covertly, certainly clear to the members of his brotherhood, but inconspicuous to outsiders. The 'Basilica' at Paestum seems to be unique. I know no other building of 50 columns in this configuration.

CONCLUSION

The history of mathematics has to be rewritten. According to Freudenthal³⁴ fractions were taboo in highbrow mathematics, because philosophy forbade the division of the unit. However, the work of Pythagoras does not seem to be based on just integers, but mainly on numbers of progressions. Pythagoras used at least three geometrical progressions, in which the terms increase or decrease by equal ratios, namely 1, 3, 4, ..., 521, 843, 1364, etc. and 1, 5, 6, ..., 191, 309, 500, 809, etc. (both already discussed above), but the quadrature of the circle also requires such a progression, which must be used together with the double of the first progression to match (between brackets): 15, 36, 51, ..., 588 (2×521), 951 (2×843), 1539 (2×1364), etc. The (almost) rational diagonal of the square rests on arithmetical progressions, in which the terms increase or decrease by equal difference. For the side 12, 111, 210, 309, 408, etc. (difference 99) and for the diagonal 17, 157, 297, 437, 577, etc. (difference 140).

Almost certainly more mathematical wisdom is contained in the dimensions of the 'Basilica'. Furthermore, some observations lack a satisfying explanation, but cannot be explained away as mere coincidence. Let me give one example to illustrate this. The axial distance on the flanks of the colonnade, $17 \times 10 \frac{3}{8}' = 176 \frac{3}{8}$ feet, seems quite normal. However, suspicion arises after the fixation in number, 1411 double dactyls, as 1411

can be explained in many ways in terms of the progression 1, 3, 4, ., e.g., 47+1364 or 521+123+123+123+521, but Pythagoras was concerned to show that ...? I shall end with the conclusion that the Pythagorean community was so anxious for secrecy of their knowledge that even the myths never hint at the place of their shrine, the oldest temple of Hera at Paestum.

ACKNOWLEDGEMENTS

In memory of the late (1988) Abraham Seidenberg (formerly Department of Mathematics, University of California) who made the history of mathematics comprehensible and fascinating for the outsider.

NOTES

- ¹ De Zwarte 2002.
- ² Antae: the almost square corner posts in line with the columns Cp of the pronaos. In the Middle Ages, about 90% of the building within the colonnade, including the wall between pronaos and naos, was demolished to its very foundations. Fortunately, the careful measurement of small traces by Mertens (1993) saved Pythagoras' inheritance.
- ³ Van der Waerden 1978, 355.
- ⁴ Van der Schoot 1999, 406 (after Iamblichos of Chalkis - a philosopher who struck noisily against Christianity by using Pythagorean notions as arguments - flourishing about 330 AD).
- ⁵ To find a philosopher or even a poet (Aristophanes, *The Birds*) dealing with mathematics is not unusual.
- ⁶ Wells 1986, s.n. 1,414.
- ⁷ Van der Waerden 1978, 355.
- ⁸ Seidenberg 1962, 499.
- ⁹ Freudenthal 1977.
- ¹⁰ Naredi-Rainer 1982, 185.
- ¹¹ The Attic foot is $1 \frac{1}{2}$ Ionic dactyls longer than the Ionic foot, what almost certainly means that the Attic foot is a derivative of the Ionic foot and, consequently, of later date. The Attic foot already existed at the time of Solon (De Zwarte 1998-1999, 25-27). See De Zwarte 1996 on the Hephaisteion for the Attic foot as building measure.
- ¹² Mertens 1993, 3, fig. 2; de Zwarte 2002, 16 for the origins of this standard; in my paper of 1994, 115, I explain why this standard foot has not been discovered by earlier metrologists.
- ¹³ Every bit of evidence is of importance as the majority of investigators of Greek temples still maintain that there were two basic standards in architectural use - long 29.4 and 32.7 cm - with a maximum variation of 1 mm. In my opinion, Greek foot units had a fixed length. The Ionic foot of 29.86 cm and the Attic foot of 32.66 cm were certainly used. The origin of the Roman foot of 29.393 cm is still unknown. Most probably, this foot was derived of the Ionic foot, i.e., the Roman foot is the Ionic foot reduced by a quarter of a Ionic dactyl, thus 63/64 of the Ionic foot. It is striking that Wesenberg (1983, 158-164) makes Vitruvian temple design comprehensible by introducing a module of 63/64 foot.

- ¹⁴ De Zwarte 2002, note 7.
- ¹⁵ Mertens 1993, 87 and annex-drawing 8d.
- ¹⁶ Mertens 1993, annex-drawing 2 (ground-plan): Distance Cp-Cn1 812.5 cm and Cn1-Cn2 341.1 cm, in total 1153.6 cm.
- ¹⁷ Seidenberg 1962, 499.
- ¹⁸ Smeur 1970, 257. I disagree as it is just an elegant method for special rectangles, not for all rectangles. General procedure (Seidenberg 1975, 292): let a and b be the sides of a rectangle, then $\sqrt{ab}$ is the side of the square of equal area. General rule for the conversion of special rectangles into squares by using integral numbers: let a, b and a + b be successive high numbers of a geometrical progression, then a and a + b are the sides of the rectangle and b the side of the square of (practically) equal area, e.g., 89, 144 and 233 in the Fibonacci series.
- ¹⁹ Naredi-Rainer 1982, 103 (note 122), 187.
- ²⁰ Mertens 1993, annex-drawing 2, from east: 621.3 + 812.5 + 341.1 = 1774.9 cm.
- ²¹ Wells 1986, s.n. 1,618.
- ²² Berriman 1953, 72.
- ²³ De Zwarte 2002, 16: the royal Egyptian cubit = 28/16 Ionic foot = 52.255 cm.
- ²⁴ Berriman 1953, 77.
- ²⁵ Berriman 1953, 81. As an aside it seems useful to enter more extensively into the geometry of the Great Pyramid as I venture to differ from previous investigators. Let us connect the midpoint of the base side to the centre of the *square* base and drop a perpendicular from the top of the pyramid. In this right-angled triangle the vertical height of the pyramid is $\sqrt{360^2 - 222.5^2} = 283.008.. \approx 283$ cubits. The angle of slope is cosine $222.5/360 = 0.6180..$ (the golden ratio), that is, omitting seconds, an angle of 51° 50'. Vyse gave 51° 51' and Petrie 51° 52' (Berriman 1953, 77). Curiously, nobody observed that the golden ratio was involved. Instead of this, Berriman (following Taylor 1859 and Petrie 1883) thought that π underlay the design of the pyramid. According to these investigators π was equal to four times cotangent angle of slope, that is $4 \times 222.5/283 = 3.1448..$, but the Rhind mathematical papyrus informs us that the Egyptian value of π is $(16/9)^2 = 3.160..$ But it is noteworthy that cotangent and sine of 51° 50' are almost equal. It follows that 222.5×360 (conceived as the area of a rectangle) $\approx 283^2$ (area of a square), but I do not recognize any significance for this fact in the design of the pyramid.
- ²⁶ Smeur 1970, 252 (considering Cantor 1875).
- ²⁷ Mertens 1993, annex-drawing 2. Cc-Cn6 from east: 621.3 + 812.5 + 341.1 + 345.9 + 687.7 + 339 = 3147.5 cm.
- ²⁸ Smeur 1970, 253-254; Seidenberg 1988, 115-116.
- ²⁹ Smeur 1970, 254, note 18: Archimedes' proposition 2 'the area of a circle is to the square on its diameter as 11 to 14', is exact for the value $3 \frac{1}{7}$.
- ³⁰ Mertens 1993, annex-drawing 2. Distance Cc-Cn7 = Cc-Cn6 (see note 27) + Cn6-Cn7 = 3147.5 + 444.0 = 3591.5 cm. Distance Cc-scratch s = Cc-Cn6 - Cn6-Cn5 + Cn5-s = 3147.5 - 339 + 42.9 = 2851.4 cm. The distance Cn5-s has been calculated by myself from a drawing 1:50 (Mertens 1993, 73, fig. 58).
- ³¹ Seidenberg 1962, 500.
- ³² Seidenberg 1988, 102.
- ³³ Wells 1986, s.n. 81.
- ³⁴ Freudenthal 1977, 191.

BIBLIOGRAPHY

- Berriman, A.E. 1953, *Historical Metrology*, London/New York.
- Freudenthal, H. 1977, What is Algebra and What has it been in History?, *Archive for History of Exact Sciences* 16, 189-200.
- Gruben, G. 1976, *Die Tempel der Griechen*, München.
- Mertens, D. 1993, *Der alte Heratempel in Paestum und die archaische Baukunst in Unteritalien*, Mainz.
- Naredi-Rainer, P. von 1982, *Architektur und Harmonie*, Köln.
- Schoot, A. van der 1999, *De ontsteking van Pythagoras* (with English summary), Baarn.
- Seidenberg, A. 1962, The Ritual Origin of Geometry, *Archive for History of Exact Sciences* 1, 488-527.
- Seidenberg, A. 1975, Did Euclid's Elements, Book I, Develop Geometry Axiomatically?, *Archive for History of Exact Sciences* 14, 263-295.
- Seidenberg, A. 1988, On the Volume of a Sphere, *Archive for History of Exact Sciences* 39, 97-119.
- Smeur, A.J.E.M. 1970, On the Value Equivalent to π in Ancient Mathematical Texts. A New Interpretation, *Archive for History of Exact Sciences* 6, 249-270.
- Waerden, B.L. van der 1978, Die Postulate und Konstruktionen in der frühgriechischen Geometrie, *Archive for History of Exact Sciences* 18, 343-357.
- Wells, D. 1986, *The Penguin dictionary of curious and interesting numbers*, Harmondsworth.
- Wesenberg, B. 1983, *Beiträge zur Rekonstruktion griechischer Architektur nach literarischen Quellen* (AM 9th suppl.), Berlin.
- Zwarte, R. de 1994, Der ionische Fuss und das Verhältnis der römischen, ionischen und attischen Fussmasse zueinander, *BABesch* 69, 115-143.
- Zwarte, R. de 1996, Der ursprüngliche Entwurf für das Hephaisteion in Athen - Eine modulare architektonische Komposition des 5. Jhs. v. Chr., *BABesch* 71, 95-102.
- Zwarte, R. de 1998-99, Mass Metrological Arguments for Differential Weighing in the Eastern Mediterranean Bronze Age: Akrotiri on Thera, Ugarit, Tarsos, Katsambas on Crete and Athens - with a digression on Aristotle's Ath. Pol. 10, *Talanta* 30-31, 7-29.
- Zwarte, R. de 2002, Evidence of the so-called Golden Section in Archaic South Italy: the Hera Temple I ('Basilica') at Paestum. With an addendum on the Parthenon at Athens, *BABesch* 77, 9-18.

BULKSTRAAT 8

NL-4196 AW TRICHT