

11. PYTHAGORAS AND HIS FOLLOWERS

The early story of Greek geometry is strangely different from its founding in Miletus. Most of what we know is a mixture of myth and magic, shapes and rules, all revolving around the fabulous figure of Pythagoras.

The "divine" Pythagoras—that was what he was called, not only after his death but even in his own lifetime. For the latter part of the 6th century B.C. was still a time of superstition. The Ionian "physiologists" had only tried to find an orderly pattern in nature. Most men continued to believe that gods and spirits moved in the trees and the wind and the lightning. And cults were popular all over the Greek world—"mysteries," they were called—that promised to bring their members close to the gods in secret rites. Some were even headed by seers.

Pythagoras was one of these. A native of the island of Samos, not far from Miletus, he probably had a Phoenician mother and a Greek father, who was a stonecutter. But he gained such a reputation for wisdom and magical arts that people began to whisper that he was son of the god Apollo.

Actually, Pythagoras was Thales' contemporary, for a time at least. He was born about twenty years before Thales died, so his career spanned a later period. Parts of that career are a matter of history. The political situation in Samos became oppressive: a local tyrant, Polycrates, ruled harshly, and the neighboring Persian Empire demanded heavy tribute. So Pythagoras emigrated, as did many other refugees. He settled in Croton, a little island off the tip of Italy. There he founded a famous secret society that contributed a great deal to the development

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of geometry. We might call it the world's first mathematics club. But much of Pythagoras' life is enmeshed in legends—not just

amusing anecdotes, as with Thales, but wildly fanciful tales. And far too many discoveries are attributed to him. So we must pick and choose our way among facts and fables, in telling the story of Pythagoras and his followers.

To begin with, Pythagoras went on where Thales left off. Let us therefore accept the tradition that he was the older man's student.

Perhaps rumors of Thales exciting new game of string, straightedge, and shadows had spread throughout Ionia, and people came from the neighboring cities and islands to take part. Anyway, one visitor in particular was attracted to Miletus, to learn this new way of thinking and finding rules and tracing forms upon the ground—the youthful Pythagoras.

The aging Thales must have been pleased with the young man's keen interest; such penetrating questions showed a real thirst for knowledge. Thales taught Pythagoras all that he knew. Then he

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The aging Thales must have been pleased with the young man's keen interest; such penetrating questions showed a real thirst for knowledge. Thales taught Pythagoras all that he knew. Then he encouraged him to travel for himself in the ancient lands and study the development of learning at its source.

Pythagoras followed the advice, and his travels were even more extensive than Thales had been. Fired with enthusiasm by the stories of Babylon, he visited that fabulous city to absorb the learning of the Chaldean stargazers. Naturally, also, he wanted to see the ancient pyramids, obelisks, and temples of Egypt. There he studied the lore of the priests at Memphis and Diospolis.

In addition, he learned a great deal just by traveling to all the known parts of the Mediterranean world. During his long sea voyages, the Phoenician sailors taught him much about the

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importance of stars in navigation. And like Thales before him, he saw things in a way that men never saw them before.

On the open sea, he realized that the surface of the waters was not flat but curved. He could "see" this whenever another ship appeared in the distance. At first, only the top of its mast was visible over the horizon; then gradually the whole vessel would come into view as it sailed toward them. Surely then, he guessed, *the earth must be round!* And what about the other heavenly bodies?

The moon, when it was full, was a round disk in the sky, rosy or yellowish or silver white. As it waxed and waned, you could imagine that its surface was curved too, and partly in light and partly in shade. Doubtless the moon also was spherical.



And the radiant sun itself made a blazing circle in the heavens! Certainly, concluded Pythagoras, the earth and the sun and the moon and the planets were all spheres. That was the one perfect form: it must be so! In history he is given credit as the first person to spread this idea.

Observing and studying in this way, Pythagoras traveled for many years. Some say he got as far as India and was deeply influenced, for he took up Oriental dress, including a turban. And certain of his mystical ideas, such as number magic and reincarnation, were typical of the East.

Finally he came back to Samos. We don't really know how his countrymen received him, but a number of stories suggest that they were indifferent to all the knowledge he had brought home. This is borne out by the tale of Pythagoras' first pupil.

Tired of finding no one who would listen to his learning, Pythagoras bought himself an audience. He found an urchin, a poor and ragged little fellow, and offered him a bribe. He would pay three oboli for every lesson the boy mastered.

To the urchin this was indeed a bargain. By sitting in the shade for a few hours, and listening attentively to this wise man, he could make better wages than in a whole day's work in the hot sun. Naturally, he concentrated hard while Pythagoras introduced him to mathematical disciplines.

From the simple calculations of the rope-stretchers, to the methods of the Phoenician navigators, to abstract rules and reasoning, Pythagoras led his pupil on. Soon the subjects became so interesting that the boy begged for more and more lessons.

At this point, Pythagoras explained that he, too, was a poor man, and paying someone to listen was getting to be very expensive. So they reached another bargain. The boy

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had saved enough to pay Pythagoras for his future lessons. The story doesn't prove that Pythagoras began to collect a following this way. But it shows the fascination of the new game of string, straightedge, and shadows, and forecasts his great role as its teacher.

What we do know for sure is that Pythagoras left Samos and went to settle on a tiny island off the coast of Sicily, which was then swarming with new Greek colonies. This Isle of Croton has an immortal place in the history of mathematics. There Pythagoras finally gathered a group of students around him and founded his famous Secret Brotherhood.

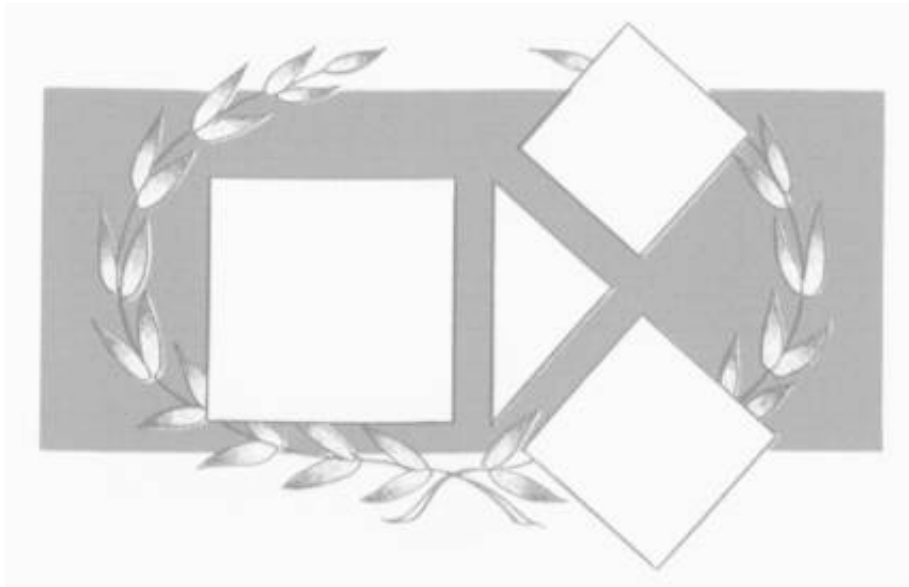
Like other mystery cults of that time, it was a religious order with initiations and rites and purifications.

These "Pythagoreans" had a special way of life. The members-women as well as men- shared all their simple belongings in common. Because Pythagoras taught the doctrine of the trans-migration of souls, they were respectful to animals and would eat no meat or fish because in those creatures might live the soul of some departed friend. Nor would they wear garments made of wool, nor kill anything except as a sacrifice to the gods. They bound themselves by great oaths to keep secret all their discoveries and teachings. So devoted were they to their leader that any argument was resolved by using the words of authority referring to Pythagoras: "He himself said it!"

But there was one great trait that set this Brotherhood apart from all the rest. Pythagoras taught that "knowledge is the greatest purification." So his followers were, above all, a study group, bent on gaining the knowledge that would free them from endless rebirths. And to the Pythagoreans-as we shall see

-this knowledge meant mathematics!

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12. A FAMOUS THEOREM

The most famous thing about Pythagoras is not his Brother-hood at Croton, nor the weird legend of his spending years in a cave and gaining magical powers. It is simply a theorem (or formal rule) of geometry.

The Pythagorean theorem says: In any right triangle, the sum of the squares of the two sides is equal to the square of the hypotenuse.

This theorem and its proof were a basic advance. It became a cornerstone of ancient geometry and had more influence on theory and more practical applications than any other. Later writers would call it "the measure of gold."

But perhaps Pythagoras ought to be most famous for something else. He was the first to teach mathematics as a liberal education-our very term "mathematics" originated from his course!

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Pythagoras gave lectures on *mathemata*. In the language of his time, that was the word for studies; but his use of it came to mean mathematics. Pythagorean *mathemata* covered a very large field, but all the parts were interrelated. Perhaps the quickest way to understand this is just to imagine a poster at the entrance to the open-air meeting place where the lectures were held.

This was a curious fourfold range of subjects: music to elevate the soul, numbers and their properties, ancient Babylonian lore about the planets, and the abstract rules of the new theoretical geometry. Each topic was studied from a mathematical stand-point-and the whole course constituted the initiation into the Secret Brotherhood!



The very term "mathematician" meant one who was admitted to the inner mysteries, as distinct from a "hearer" or beginner. "Mathematicians" had to follow a rigorous course for several years, with a stern daily program of meditation, exercise, and study, before they were even permitted to hear Pythagoras intone some teachings behind a curtain. Only after full initiation might they attend his actual lectures.

But the wait may have been worth it. Great mathematical discoveries were attributed to this strange teacher, the most famous being the Pythagorean Theorem. So let us use our imaginations and mingle with the initiates at an exclusive closed lecture. Why not hear the great Pythagoras demonstrate his immortal proposition? Not that he did it in just these words, but these probably *are* the proofs that were used.

Perhaps the session began with an announcement: "I have found at last the solution to a problem that has long been puzzling us." A hush of awe fell on the gathering as "Himself"-in white robe and gold sandals, his head crowned with a golden wreath-took pointer and string and straightedge, and began to lecture.

"Listen to our baffling problem. You older members have already worked on it, but I will review it for the new initiates.

"Here is *the Egyptian right triangle*, the one used by the rope-stretchers, where the sides of the right angle are 3 units and 4 units, and the hypotenuse is 5 units." He drew it on a sandy space, and then added a square on each side, and inner squares. (See illustration.)

"There! You can see, by counting or by calculating the square units, that the total area of the squares on the two sides of the right angle is equal to the area of the square on the hypotenuse."

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He beckoned to the newcomers, who crowded close, multiplying and counting at the same time:

$$(3 \times 3) + (4 \times 4) = (5 \times 5)$$

$$9 + 16 = 25$$

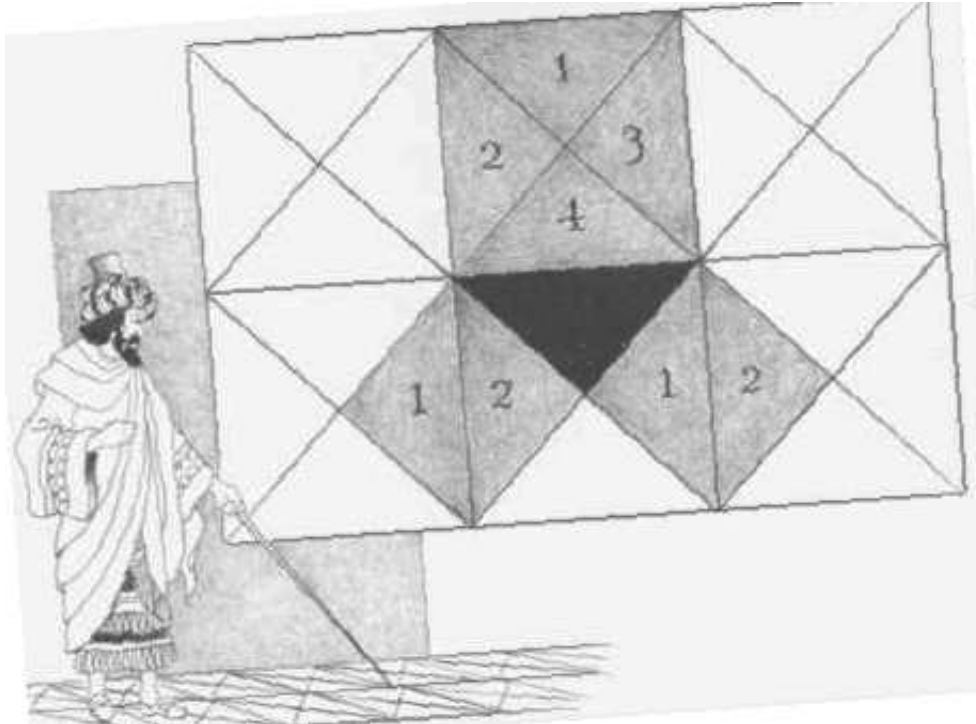
until all their heads nodded in agreement.

"Now let me show you a Greek design involving right triangles." He drew their attention to the tiled floor

on which they were standing, and then traced a similar pattern on the sand, outlining the important parts.

"Here the two sides of the right angle are equal, and the same relation holds." With his pointer, he indicated one triangle and the related squares, and they all counted together. (See illustration)

"Look! Two triangles plus two triangles equals four triangles. The total area of the squares on the two sides of this right triangle is likewise equal to the area of the square on its hypotenuse."



Again he waited until the newcomers nodded their assent, and then **continued**:

"In India the priests know other constructions that give similar results; they guard these numbers closely, but we have **found** some of them. In Babylon, a priestly astrologer whispered to me that there was a secret about this mystery that had never been penetrated."

Now the attention was almost breathless as Pythagoras in-toned solemnly: *"That secret is our problem! Would the same relation **always** be true of ANY right triangle, no matter what the length of its sides, and how could you show this?"*

At this dramatic moment, he withdrew behind a curtain, while attendants played on stringed instruments to indicate an intermission. Pandemonium broke out among the assembled initiates. All the newcomers began talking at once, making suggestions, arguing, and shouting. The older mathematicians, who had worked out this problem themselves, were less noisy **but even** more excited.

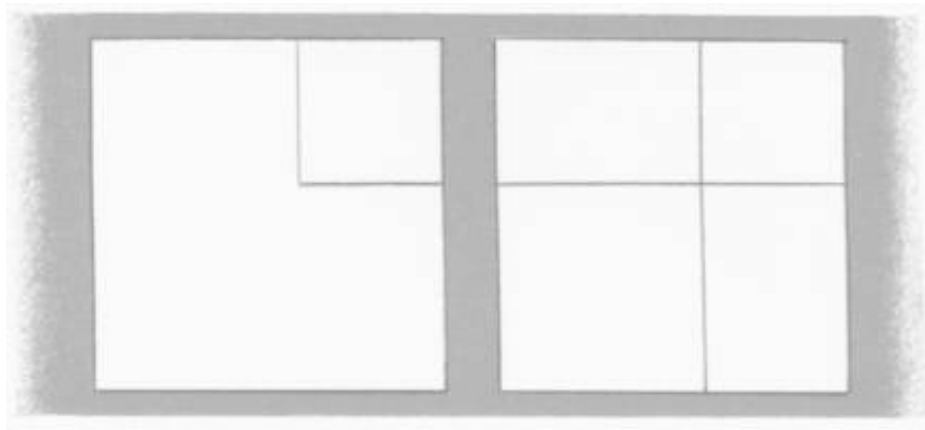
Finally Pythagoras reappeared. Silence instantly fell over the group as he resumed his lecture.

"I will now show you how to construct a wondrous figure which discloses that the answer is always yes! The older 'mathematicians' will realize that by slowly and carefully defining each step of the construction, and using a few simple theorems that you already know, this demonstration can be made into a rigorous proof. Today I will just draw it quickly, so you can all see my great discovery."

He signaled to attendants to smooth the sand, and began to draw, using his pointer to emphasize his words.

"Watch this beautiful construction! I make a square frame,

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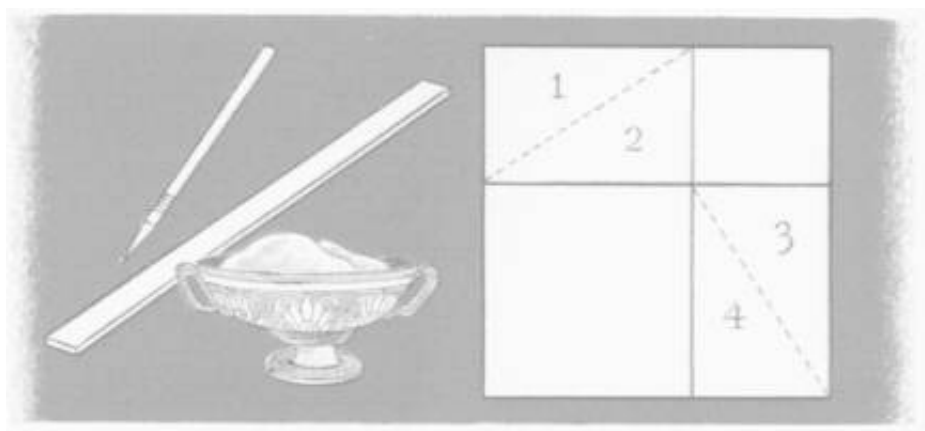
any size, and in its corner I place a small square, any size. Next I draw straight lines, continuing the sides of the small square to the edge of the frame.

"Do you see what my frame now contains? A *small square* and a *medium square*, and *two equal rectangles*."

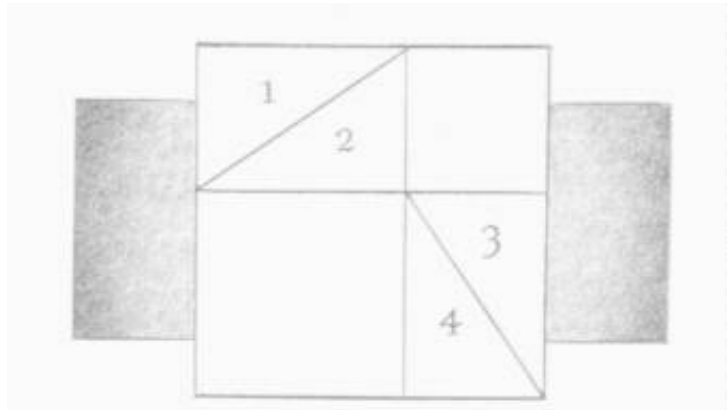
"And next-we are almost there-I simply add diagonal lines across the rectangles!"

"This is the figure I need. My frame now contains a *small square*, a *medium square*, and *four equal right triangles*. Now I will ask you to look more closely at this figure."

Pythagoras beckoned to the attendants, who poured colored sand from jars onto the parts of the drawing, so the pattern showed plainly.



"Look again!" He used his pointer and spoke with care. "All the triangles, you know, are equal; each is the same triangle in a different position. Now, notice how the triangles touch the squares, especially Triangle 2. You can see that the *same square* is the *square on the short side* of the triangle. And the *medium square* is the *square on the long side* of the triangle. So my frame is completely filled by *four equal right triangles* plus the *square on the short side* and the *square on the long side*!"

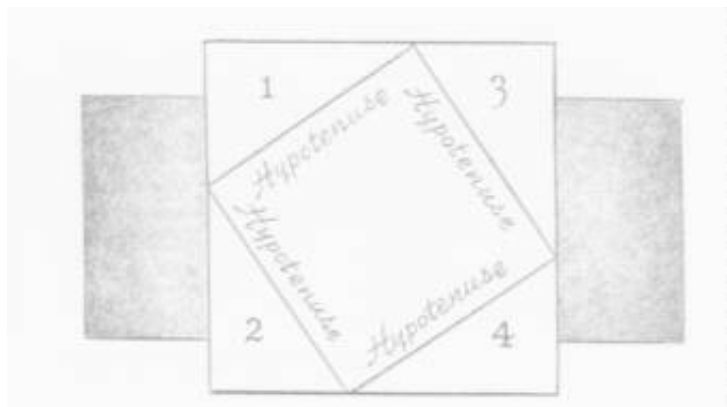


Pythagoras paused while a low murmur of awe rose from the initiates.

"Now watch!" he intoned. And while they all craned their necks to see, and the attendants poured more colored sand, Pythagoras drew his final masterful figure.

"Watch well! I have only to swing and push these four triangles around, like this, so that they fit perfectly into the four corners of the frame, and my frame is now completely filled by the same *four equal right triangles* plus *the square on the hypotenuse*!"

"Therefore, in any right triangle, the area of the square on one side plus the area of the square on the other side will add up to the area of the square on the hypotenuse!"



A mighty shout-we can imagine-went up from the assembled inner group of the Secret Brotherhood. For this theorem was a true landmark in the development of geometry by the Pythagoreans. Almost all later geometric work involving lengths, and measurement was based upon it. And this style of solving problems, especially equations, by *diagramming* them, would remain a chief trait of Greek geometry.

But to the initiate who first heard it, the theorem also partook of a mystical revelation. Tradition says that Pythagoras himself celebrated the occasion only a noble sacrifice-an ox, or a hundred oxen-to his "divine father," Apollo. Some ancient writers dispute this, as the Pythagoreans were vegetarians. Whatever the offering, we can easily picture the festivities described in the verse of legend. Doubtless the "mathematicians" chanted, torches waved, and smoke rose from the sacrificial altar,

The day Pythagoras the famous figure found

For which he brought the gods a sacrifice renowned!

