

Evidence of the so-called Golden Section in Archaic South Italy: the Hera Temple I ('Basilica') at Paestum *With an addendum on the Parthenon at Athens*

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INTRODUCTION

The Greeks knew it as the section of mean and extreme ratio. For Luca Pacioli, a theologian and mathematician, it was the divine proportion *par excellence*, implying that it is of a superhuman nature. Unfortunately, his book (*De Divina Proportione*, Venice 1509) also contains a treatise on architecture, which led many readers astray.¹ There is no proof, however, that Leonardo da Vinci, who made the drawings for Pacioli's book, ever used the expression *sectio aurea*. It is more recently (19th century), that this proportion has been better known as the golden section.

The golden section belonged exclusively to a world in which geometrical shapes and ratios were valued for their own sake. In the Middle Ages that meant architects and artists and the places to look for it are the dimensions of buildings or the frames of manuscript illuminations. They may choose to order their work with the help of the golden proportion, or may turn out to have done so unintentionally. In fact, critical inquiries never reveal mathematical precision.² Many authors hold that the golden section has been an aesthetic ideal since the days of Pythagoras. However, they failed to differentiate between mathematical romanticism and mathematical history. The idea is, indeed, attractive but must be corroborated by the analysis of some Greek temple plans which reveal the use of the golden section with *precision*, since the measurement predicted by a rule of 1 : 1.61803.. (the golden section) may well be very close to that predicted by a rule of 1 : 1.6 (= 5 : 8).³ There is no evidence that the golden section was ever used by Greek architects. However, 'number mysticism' was practised occasionally in designing the dimensions of rectangles ('Basilica' at Paestum and Parthenon at Athens): the difference in length of two sides of a rectangle measures a round number of a specific foot length, the Ionic foot of 29.86 cm.

This paper deals with the progression of Pythagoras⁴ laid down on the steps of an archaic temple, thus widening our knowledge of mathematical history. The ratio of two successive high numbers of this series is an accurate approximation for the golden section. However, there is no evidence that the architect based his design on this ratio to deal with the aesthetic form of the temple. Although this is therefore not the place to attempt a full reconstruction of the temple design, it is important to make some comment on the subject. The mathematical part of this study will be kept superficial as many implications are best left to specialists in mathematics.



Fig. 1. Map of southern Italy about 530 BC.

The golden section is described by Euclid (ca. 300 BC), but it is well known that much of his work is of Pythagorean origin. Pythagoras (born on Samos ca. 580) founded a philosophic school at Kroton in south Italy (*fig. 1*) in the second half of the sixth century. After being expelled from Kroton in 510, he settled down at Metapontion, where he died ca. 500. But here we are already in the realm of myths and legends. According to another legend Pythagoras and many of his followers died after the destruction of Sybaris by Kroton in 510, when the house of his patron Milo and the adjacent school was burnt by discontented Krotonians instigated by Cylon, a rejected candidate of the school. The oldest temple of Hera was built at Poseidonia (Paestum) in about the same period (ca. 530). If this date is correct, we may conclude that the Pythagorean brotherhood already expanded before the events at Kroton in 510.

THE GOLDEN SECTION

The golden section divides a line (*fig. 2*) in such a way that A (minor) is to B (major) as B is to A + B. In a formula: $B^2 = A(A + B)$. The terms A, B and A + B form part of an additive geometrical progression. The characteristic of this progression is the constant quotient of two successive terms. The ratio of the terms belonging to the golden section (0.61803.. or 1.61803..) is irrational, that is not expressible by whole numbers or vulgar fractions.

A	B
Minor	Major
0.61803..	1
1	1.61803..

Fig. 2. The golden section.

FIBONACCI (1180-1240) AND PYTHAGORAS

The golden section and the progression of Fibonacci (1, 1, 2, 3, 5, 8, 13, etc.) are closely related. Successive high numbers of Fibonacci give results which hardly differ from the true golden section quotient, the further one goes, the more accurate it becomes, e.g., 233 divided by 377 = 0.61803... The progression of Fibonacci becomes a progression of Pythagoras by omitting one number one (1, 3, 4, 7, 11, 18,, 521, 843, 1364,

etc.) or, for practical building purposes in the archaic period, in Ionic feet (IF) of 29.86 cm: $1/8'$, $3/8'$, $1/2'$, $7/8'$, $1\ 3/8'$, $2\ 1/4'$,, $65\ 1/8'$, $105\ 3/8'$, $170\ 1/2'$, etc.

I arrive at values in Ionic feet by dividing Pythagorean numbers by eight. This is not an odd method to introduce a new foot but a legitimate procedure to present fresh evidence for a standard measure of length whose existence I defend since 1994. Just as in the Fibonacci series, the quotient of two successive high numbers approximates 0.61803.

THE GOLDEN SECTION IN PRACTICE

Modern mathematicians who are satisfied with the algebraic approach of the terms, feel no need to construct the golden section. In practice, the geometric construction is difficult to draw accurately. For example, a golden rectangle with two unequal sides equal to 100 mm has sides of 38.196.. and 61.803.. mm. Using simple tools, as ruler and compasses, one is restricted to 38.2 and 61.8 mm giving a ratio of 0.61812.. instead of 0.61803... If an architect would use the golden proportion for aesthetic reasons he surely preferred high numbers of Fibonacci, which seems to have been done in the Middle Ages.⁵ Le Corbusier,⁶ whose starting-point for architecture on basis of the golden section was the height of an average European man of 183 cm (originally 175 cm), had to invent his own progression (5, 11, 16, 27, 43, 70, 113, 183, etc.) as the Fibonacci numbers 144 or 233 do not fulfil such requirements. Le Corbusier's progression is as to that less accurate than the Fibonacci progression, but he was satisfied with it. Clearly, it is only the aesthetic aspect that matters and of course, the design must fit in with the standard of length (metres and centimetres, cubit or foot and current fractions), i.e., in Greece, corresponding to the normal division of a foot into 16 dactyls.⁷ If restricted to golden rectangles, Greek architects might have used the progression of Pythagoras instead of another system of proportioning. The aesthetic aspect of such real or imaginary rectangles must please beholders from close by, e.g. the proportioning of the cella of the Parthenon (below) or from afar, e.g. the colonnade of the Parthenon (below) for which an entirely different system of proportioning was used.

Let us now return to the Pythagoreans. As the Pythagoreans were theorists in the first place, they almost certainly knew the progression of the golden section. Here we meet a problem that still exists, that is, a theorem is a position requiring

demonstration. How to demonstrate its validity by geometry as accurate as possible with the available means and restricted by the local foot standard and its current fractions? A practical solution to this problem is the use of high numbers of a progression which tally with the subdivisions of that standard of length. Thus, if we can show that, about 530 BC, successive high numbers of the above series materialize, then we have demonstrated that the mathematicians of that time knew the true progression of the golden section.

THE OLDEST TEMPLE OF HERA (SO-CALLED BASILICA) AT PAESTUM

The measurement of the temple of Hera has been executed by Dieter Mertens and his team. The results are stated in centimetres with accuracy to the mm. The work meets in every respect modern standards of graphic and metrical registration. Here we find no mean values of elements supposed to be identical. Every single stone has been measured, thus giving the opportunity for a profound study of the temple.⁸

The first impression left by Greek architecture is of extreme accuracy, but the steps of the temple of Hera on the north flank are about 5 centimetres longer than on the south flank, that is too much to be accounted for simply by inaccuracy of measurement. Indeed, the process of discovering the architectural design behind the remains of archaic temples is notoriously difficult. Vitruvius (IV 1. 3) talks of an earlier stage before the adoption of rules of proportion. Unless such a proportion is a simple arithmetical ratio, the rule cannot be discovered without knowledge of the foot size. Of course, if there are no proportional rules at all, the problems are almost too difficult to overcome if the modern investigator must resort to a discussion of the design based on the dimensions in centimetres. Thus, we have need of fixed foot standards which are certain to have existed. I will discuss these matters more fully in the addendum.

The middle step of the temple of Hera is of interest for several reasons. The initial results of our inquiry can be presented in centimetres, thus without presupposing the length of the foot used. At the end of this section the foot size presents itself by force of logic and a further proof is given below. Then, surprisingly, we find traces of mathematical knowledge embedded in the middle step of the 'Basilica' (530 BC?) at the time that Pythagoras is supposed to be at Kroton, i.e., about 250 km from Paestum. In the course of an inquiry into the design of this temple I discovered by chance that the middle step on both flanks of the temple platform revealed measurements which are of no use for the construction of the temple, but can easily be explained as to put on record the golden section on a very large scale. Beginning at the west end of both steps, the sum of a row of block measurements indicate that the Pythagoreans had something to do with this temple during its erection. This is not to say that the Pythagoreans were involved in the design process. The mathematical theories of philosophers perhaps may induce an idea as a starting-point for a design, but ancient Greek architectural design procedures had nothing to do with higher mathematics. Nevertheless, the possibility that the Pythagoreans have made an unsuccessful attempt at designing a temple must be kept in mind. The main point in favour of this hypothesis is the opinion of an experienced modern observer on the aesthetic aspect of the colonnade (*fig. 3*), which surrounds the temple.⁹ Anyhow, there was an agreement to construct the middle step in such a way that mathematicians could materialize their ideas on a geometric progression. The crucial measurements¹⁰ of the middle step have been summarized in table 1.¹¹

The results are satisfying, in spite of the ruinous state of the temple: north-side $1943.5 : 3144.5 = 0.61806..$, $3144.5 : 5088.0 = 0.61802..$; south-side $1943.5 : 3145.4 = 0.61788..$, $3145.4 : 5088.9 = 0.61809..$

Table 1. The middle step of the flanks.

	Blocks	cm	Blocks	cm	Blocks	cm
North-side	W1 u/i 7	1943.5	W8 u/i 18	3144.5	W1 u/i 18	5088.0
South-side	W1 u/i 7	1943.5	W8 u/i 17	3145.4	W1 u/i 17	5088.9
If perfect		1944.6		3146.5		5091.1
In feet (IF)		65 1/8		105 3/8		170 1/2



Fig. 3. The oldest temple of Hera at Paestum from north-east.

Obviously, the Pythagoreans used a progression - since 1877 usually called the progression of Lucas - as a practical means of constructing the golden section. Here minor is $65 \frac{1}{8}'$, major $105 \frac{3}{8}'$ and the line $170 \frac{1}{2}$ Ionic feet of 29.86 cm. In numbers: 521, 843 and 1364. Everything is number, Pythagoras seems to have said. The explanation of the actual dimensions in numbers of the progression will only work for a specific value of the foot.

PEG-AND-CORD CONSTRUCTIONS

Van der Waerden¹² says: 'Die Pythagoreer haben selbstverständlich auch Konstruktionen ausgeführt. ... Im zweiten Buch (Euclid's *Elements*) ist immerfort von dem "von zwei Strecken aufgespannten Rechteck" die Rede ...'

Seidenberg¹³ says: '... we have tried to show that a number of points in Greek geometry are illuminated by the hypothesis that it started from a tradition of peg-and-cord constructions ...'

I refer the reader to Mertens' ground-plan.¹⁴ Instead of pegs and cords, pins and sewing-thread can be used or, avoiding damage on the published drawing, tracing-paper, ruler and pencil. Connect the Pythagorean points on both flanks (hypotenuse) at the 7th column from west (perpendicular). Connect zero and the point at the 17th column of the south flank to the end of the perpendicular on the north flank. At first sight you have constructed a huge triangle with a vertical angle of 90 degrees (fig. 4). In fact, this angle is about 89 degrees. Thus the perpendicular of 2520.5 cm is too long if we take the ends of the perpendicular at the edges of the second step.¹⁵ The true Pythagorean measuring points must be situated more inwards, that is, if symmetrically placed, about 23.4 cm inwards (the step width is about 36 cm) on both flanks as the exact length of the perpendicular has to be $82 \frac{27}{32}$ IF = 2473.7 cm. Then the length of the short side of the right-angled triangle is $105 \frac{3}{8}$ IF = 3146.5 cm, thus as long as the 'major' of the hypotenuse.

AGAIN THE MIDDLE STEP

Only on the south-side we find evidence for no fewer than four nearest lower terms of the progression by splitting up both groups of blocks already mentioned in table 1. Two terms appear twice (table 2), which seems superfluous for simple exposure of the progression. Perhaps, more 'golden' figures have been constructed. On this subject specialists in mathematics may decide what can be done with the available data.

For the present it seems more likely that architect and mathematicians acted together, rather than seeing a philosophic-mathematic community as the architects of the temple of Hera. An architect was presumably more interested in architecture than in pure mathematics. Perhaps more evidence for Pythagorean activity can still be found on the steps of archaic temples in Paestum or Metapontion.

THE IONIC FOOT

Since 1994 I present evidence for the Ionic foot (IF) of practically 29.86 cm.¹⁶ For various reasons most scholars are reluctant to accept the widespread use of this foot standard. Till now, only its local existence has been admitted.¹⁷ In my opinion, this standard was almost universally used in designing Greek architecture of the sixth and fifth century. For only a few temples, the temple of Athena in Paestum, the Erechtheion and the Hephaisteion¹⁸ at Athens for example, the Attic foot of practically 32.66 cm has been attested. Although it is not yet possible to present a full reconstruction of the design of the Hera temple, it is imperative to prove that the architect indeed used a measure of 29.86 cm as his standard of length. Fortunately, this is an easy task. Let us look at the dimensions of the

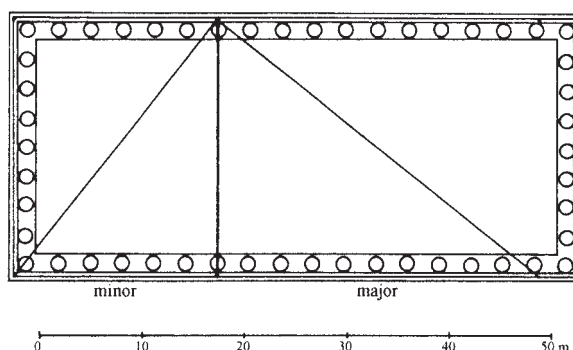


Fig. 4. General view of the oldest temple of Hera at Paestum. For details see Mertens 1993: ground-plan 1:100.

altar.¹⁹ With a length of 2100 cm and a width of 607 cm, the dimensions are in the ratio 13 : 45. This ratio was, I suppose, unknown to the architect as his intention was quite differently, namely width = length minus 50 feet. This equation gives again the length of the foot standard: $(2100 - 607)$ divided by 50 = 29.86 cm, precisely.

THE ALTAR AND 'NUMBER MYSTICISM'

By presenting the above data as evidence for the foot used, we have missed the clue to 'number mysticism'. A different arrangement of the facts will clear this matter up, at once indicating which elements were significant in the design of the altar. The altar has been erected at considerable distance to the east front with its short axis in line with the temple axis. More or less paraphrasing Vitruvius in his presentation of the rules for Ionic (*De Arch.* III 5. 5 and 5. 8), that is by relating each element to the one defined previously, we get: Altar, application of rules for finding its length

Table 2. The middle step: south-side.

Blocks	measured cm	If perfect cm	IF	Number of Pyth.
W 1	282.0	283.7	9 1/2	76
W 2 + 3	462.2	459.1	15 3/8	123
W 1 u/i 3	744.2	742.7	24 7/8	199
W 4 u/i 7	1199.3	1201.9	40 1/4	322
W 1 u/i 7	1943.5	1944.6	65 1/8	521
W 8 u/i 13	1943.6	1944.6	65 1/8	521
W 14 u/i 17	1201.8	1201.9	40 1/4	322
W 8 u/i 17	3145.4	3146.5	105 3/8	843
W 1 u/i 17	5088.9	5091.1	170 1/2	1364

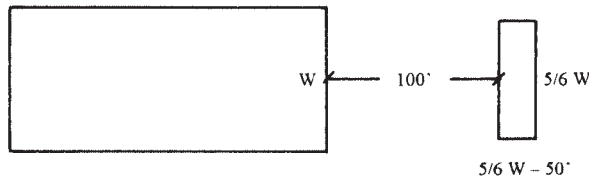


Fig. 5. Hera temple I at Paestum: middle step and altar. The way in which the dimensions of the altar were calculated; W is the width of the middle step.

and for relating the length to the width. Which rectangle is of primary importance to the design? (middle step of the temple). The middle step (fig. 5) having been laid out, the rule for the altar length will be as follows: divide the shorter side of the middle step into six parts. Five parts shall be the length of the altar. Henceforth number mysticism comes into play! What is the holy²⁰ measure (distance between middle step and altar) and what is its length? (100 feet). The rule for the altar width will be as follows: subtract half a holy measure from the altar length.

Middle step on east front: measured 2518.4 cm; 2519.4 cm = $84 \frac{3}{8}$ IF $\times \frac{5}{6} = 70 \frac{5}{16}$ IF = 2099.5 cm (length of the altar); total distance: $2952 + 37.5 = 2989.5$ cm on north-side of the altar and $2950 + 37.7 = 2987.7$ cm on south-side; 100 IF = 2986.0 cm; $70 \frac{5}{16}$ IF - 50 IF = $20 \frac{5}{16}$ IF = 606.5 cm (width of the altar).²¹

This way of calculating the proportion of length to width is very practical as there are no difficulties in handling fractions. Of course, the difficulties shift to modern investigators who try to analyse Greek architecture without knowing the architect's standard of length.

ADDENDUM: THE PARTHENON AT ATHENS

Again, I defend the position that Kallikrates, the architect of the Parthenon,²² used the Ionic foot (IF) of 29.86 cm. In the addendum to my paper of 1994 I have dealt with the colonnade of the Parthenon.²³ I repeat this here in part (table 3; fig. 6), notably the proportions of the stylobate rectangle (A, B) and the imaginary rectangle stylobate width to height of Doric order (B, E) to show in what way the proportional system differs from that of the cella, which is the subject of the present addendum.

We find simple proportions. A:B = 9:4 and B:E = 9:4 or A:B:E = $2 \frac{1}{4} \times 2 \frac{1}{4} : 2 \frac{1}{4} : 1$. But architects did not work only in simple arithmetic ratios. The relationship linking length to width can be expressed in various ways.

The temple of Hera at Paestum and the Parthenon have two things in common: the foot size and a proportional system that is based on numbers. Some Greek philosophers, notably the Pythagoreans, attributed an almost mystical significance to certain numbers. However, I do not know whether philosophers really had any influence upon the way in which temples were planned. Therefore, I hope for acceptance of the following notions. A 'holy number' is a round number of feet that pleases the architect or his principals and 'number mysticism' could be the explanation for a round number of feet that is related to the holy number. Here the latter round number of feet is the difference between length and width of various rectangles, both real and imaginary ones. By means of this procedure the architects set out the dimensions in order to obtain the aesthetic form of the cella.

The measurements of the cella have been published by Mertens.²⁴ The length of the east room - the largest room in the cella and called *hekatompedos naos* by Hesychios - clearly refers to a holy measure of 100 feet; measured 2987.1 cm; 2986.0 cm = 100 IF. Four rectangles have been designed by means of number mysticism (table 4; fig. 6): the stylobate rectangle (A, B), the imaginary rectangle between the axes of outer columns (C, D) and the vertical imaginary rectangles axial width to height of Doric order (D, E) and axial length to height of Doric order (C, E).

A - B and C - D = 125 IF, D - E = 25 IF and C - E = 150 IF. This design of the cella is easy to find

Table 3. Colonnade.

	measured cm	If perfect cm	IF
A	6953.9	6953.6	232 $\frac{7}{8}$
B	3089.2	3090.5	103 $\frac{1}{2}$
E	1371.8	1373.6	46

Table 4. Cella.

	measured cm	If perfect cm	IF
A	5904.8	5904.8	197 $\frac{3}{4}$
B	2171.5	2172.3	72 $\frac{3}{4}$
C	5723.6	5723.8	191 $\frac{11}{16}$
D	1991.4	1991.3	66 $\frac{11}{16}$
E	1245.1	1244.8	41 $\frac{11}{16}$

and the plan is very accurately executed. But generations of students did not find it.²⁵ Why not? Firstly, this method of proportioning was hitherto unnoticed as it cannot be detected by the modern investigator without knowing the foot used. Perhaps, scholars are unwilling to investigate other possibilities as the normal procedure - that is the division width by length or vice versa - is sometimes successful, e.g. on the colonnade stylobate of the Parthenon, also of course, with the wrong foot standard²⁶ or the measurements in centimetres. But the infrequency of simple arithmetic ratios in Greek temples is striking. It seems reasonable to conclude that the nature of a proportional system in many cases cannot be discovered unless the length of the foot is known. Secondly, where to find the measuring points of Hesychios' *hekatompedos* was a matter of debate, thus by suggesting another place for these points, the investigator will obtain another length of the foot. Of course, only the true standard can be demonstrated *everywhere* in the Parthenon. Therefore, I shall remain silent about suggestions that the architects of the Parthenon should have used two foot standards. Thirdly, Hesychios' remark upon the length of the naos was sometimes simply neglected, allowing full play to a man's imagination. Fourthly, a dogmatic point of view: the Athenian authorities should accept only the Attic foot as building measure.²⁷ Indeed, compared with the situation in our times no one would deny the possibility of such strict regulations in ancient Athens, however, such an assumption needs to be proved, not just accepted.

The Attic foot (AF) was used at Athens by the architects of the Erechtheion and the Hephaisteion. A foot size of 32.66 cm is certainly the *Attic* foot because it is at the basis of the Athenian system of measures of mass and capacity.²⁸ But the Ionic foot of 29.86 cm was adhered to over large areas of the Greek world (Ionia, *Attica*,²⁹ Aigina, south Italy and Sicily) through about seven centuries (temple of Hera on Samos 530 BC, temple of Zeus

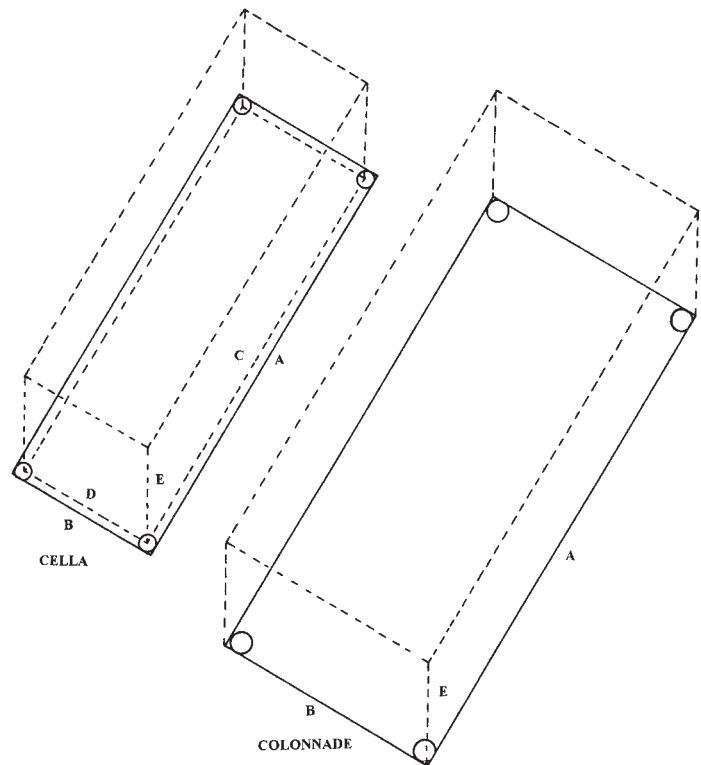


Fig. 6. Parthenon at Athens. Proportional system of the colonnade and the cella.

at Aizanoi in Phrygia 125 AD). Manolis Korres,³⁰ who is in charge of the restoration of the Parthenon, says that a foot size of 29.37 cm performs much better at the small dimensions (0.8, 1.75, 3.6, 5.6 and 11.0-11.1 cm or $\frac{1}{2}$, 1, 2, 3 and 6 daktyls) than a foot size of 32.7 cm. But in 1994 most scholars still held that there were only two basic standards in architectural use, thus it is not surprising that Korres agreed with the traditional view and did not test other values. A foot of 29.86 cm was not among the values proposed by earlier investigators of the Parthenon. Fortunately, the arithmetic ratio of length to width occasionally may give the clue to the foot used. Then it is only a matter of correct interpretation of the facts. Therefore, let us return to the cella and the dimensions as measured (*table 5*).

Table 5. Cella: in search of the Parthenon foot.			
Proportions in cm	Ratio	Numerical ratio	
		wrong	right
A:B = 5904.8 : 2171.5	2.719.. : 1	87 : 32	791 : 291
C:D = 5723.6 : 1991.4	2.874.. : 1	23 : 8	3067 : 1067
D:E = 1991.4 : 1245.1	1.599.. : 1	8 : 5	1067 : 667
C:E = 5723.6 : 1245.1	4.596.. : 1	23 : 5	3067 : 667

The lowest estimation of the foot is $2171.5 : 291 = 7.4621.. \times 4 = 29.848.. \text{ cm}$ and the highest estimation $1245.1 : 667 = 1.8667.. \times 16 = 29.867.. \text{ cm}$.

We may infer from table 5 that the foot of 29.86 cm is fallacious or that the simpler numerical ratios, which are equally accurate, have to be rejected. Thus, we must find some accurately known data to be sure that the proposed standard does not conflict with the facts. It is a great pity that the measurements in the main works on the Parthenon (Penrose, Balanos and Orlandos) do not agree on essential points.³¹ Fortunately, Korres informed Berger orally of three connected measures in the cella which were a matter in dispute previously:³² the column height in the pronaos and in the opisthodomos (1008 cm = A), the lower column diameters in the opisthodomos (171.6 cm = B) and in the pronaos (164.5 cm = C). It may be worth looking at the effect of doing the calculations in feet of 29.86 cm and in the rival feet of 29.37 an 32.7 cm to show clearly which foot size performs best. Of course, accuracy is important, but it is the simplicity of dimensions when expressed in feet which matters in the last resort (*table 6*).

We can, with some confidence, take the foot used in the Parthenon as 29.86 cm. Where does this foot come from? Briefly:³³ the Egyptians used a cubit of 52.25472 cm, which was divided into 28 digits. Herodotus (II 168) tells us that the Egyptian and the Samian cubits are equal. Since Greek architects seem to have worked in feet (of 16 digits) rather than cubits, the Samians - the famous local architect Rhoikos and Pythagoras for instance - preferred the foot of $16/28 \times 52.25472 \text{ cm} = 29.85984 \text{ cm}$ or practically 29.86 cm. The evidence for the metric value of the Ionic foot is overwhelming at Didyma in Ionia.

Comparative metrology is an instrument of finding relationships. Here the theoretical value of the standard will come in very useful. However, the results of the comparative method must be used with caution. Romanticists run the risk of

inferring too much, e.g. that Egyptians ever visited England on the evidence that two Anglo-Saxon feet equal one Egyptian cubit. But identity of measures does not necessarily imply direct derivation. Comparative metrology is based on the theory of unbroken continuity. If a new measure was needed, the appropriate course of action would have been to adapt what was already at hand, not make a fresh start.

The following chain of figures may be useful for an attempt to connect data concerning measures of length which are certain to have existed locally, e.g. the Drusian foot, to the appropriate figure: $29.85984 \text{ (Ionic foot)} \times 35/32 = 32.6592 \text{ (Attic foot)} \times 9/10 = 29.39328 \text{ (Roman foot)} \times 8/9 = 26.12736 \text{ (Anglo-Saxon foot)}^{34} \times 7/6 = 30.48192 \text{ cm (English foot)}$. The legal value of the English foot is 30.48 cm, thus the figures approximate reality. But let us return to Attica by saying that the Attic foot is a derivative of the Ionic foot, as its length is exactly $1 \frac{1}{2}$ *Ionic* dactyls longer than the Ionic foot.

The Ionic standard also nicely fits in with the remains of the Older Parthenon (after 510, or 490, or 479 BC),³⁵ e.g., the dimensions of the stylobate of the colonnade, as given by Hill,³⁶ are $2351.0 \times 6688.8 \text{ cm}$, if perfect $2351.5 \times 6688.6 \text{ cm} = 78 \frac{3}{4} \times 224 \text{ Ionic feet}$. The ratio of width to length is 45:128. It so happens that 45 and 128 represent the dimensions in Egyptian cubits.

The date of introduction of the Attic foot is difficult to ascertain. To sum up: the building measure of the Hephaisteion (ca. 450 BC) and the Erechtheion (after 438 or 421 BC) was the Attic foot. The Ionic foot was used for the Older Parthenon, the present Periklean Parthenon (447/6 BC) and almost certainly for the temple of Nemesis at Rhamnous (436/2 BC). In my opinion, the Athenian authorities did not abolish the common Ionic foot after the introduction of their own longer Attic foot.

Table 6. Cella porches: column height and lower diameters						
	1' = 29.37 cm		1' = 29.86 cm		1' = 32.7 cm	
	F	cm	F	cm	F	cm
A	34 $\frac{5}{16}$	1007.8	33 $\frac{3}{4}$	1007.8	30 $\frac{13}{16}$	1007.6
B	5 $\frac{27}{32}$	171.6	5 $\frac{3}{4}$	171.7	5 $\frac{1}{4}$	171.7
C	5 $\frac{19}{32}$	164.3	5 $\frac{1}{2}$	164.2	5 $\frac{1}{32}$	164.5

ACKNOWLEDGEMENTS

It was the late Prof. J. de Waele who introduced the idea (1995, 513-518) of seeking for metrological significance into rows of stone blocks to provide insight into the way a temple was built. I am most grateful to Dr. M.D. de Weerd (Alkmaar) for reading a preliminary draft of this paper. He has at many points improved its clarity, but is not, of course, responsible for any errors it may contain. Dr.-Ing. D. Mertens kindly supplied a photo of the oldest temple of Hera.

NOTES

- ¹ Van der Schoot 1999, 406.
- ² Naredi-Rainer 1982, 196-197.
- ³ The distinction has been made because 5 and 8 - low numbers in the Fibonacci series (cf. infra) - give a poor approximation for the golden section.
- ⁴ See Wells 1986, s.n. 11. Since 1877, this series of integral numbers is usually attributed to É. Lucas (1842-1891). Progressions of Pythagoras/Lucas and Fibonacci: a series of numbers, each of which is the sum of its two predecessors and any two of which will produce an approximation for the golden section.
- ⁵ Naredi-Rainer 1982, 188.
- ⁶ Naredi-Rainer 1982, 101-103.
- ⁷ Haselberger 1983, 118 : e.g., $6 \frac{1}{4} \frac{1}{8} \frac{1}{16} \frac{1}{32}$, the specification of the intended diameter on a drum of an unfluted column of the temple of Apollo at Didyma (c. 250 BC).
- ⁸ Mertens 1993. Some errors arose from the process of converting field drawings into final drawings, but the author kindly answered my questions which are relevant for an inquiry into the design but not for the present subject.
- ⁹ Gruben 1976, 244: '... von wo aus immer man den Bau anschaut, man ihn nicht als einheitlichen Körper empfindet, daß stets die verwirrende Vielzahl seiner Säulen ins Bewußtsein dringt oder aber das Auge an der starken Erscheinung der einzelnen Säule haften bleibt.'
- ¹⁰ Mertens 1993, annex 2 (ground-plan). North-side: $101.5 + 340.0 + 322.5 + 299.5 + 249.5 + 312.0 + 318.5 = 1943.5$ cm; south-side: $282.0 + 232.0 + 230.2 + 289.3 + 240.5 + 303.0 + 366.5 = 1943.5$ cm; north-side: $220.5 + 376.5 + 254.0 + 314.0 + 280.0 + 302.0 + 304.4 + 333.1 + 328.0 + 258.0 + 174.0 = 3144.5$ cm; south-side: $324.0 + 394.0 + 313.2 + 287.8 + 305.0 + 319.6 + 385.8 + 309.2 + 240.0 + 266.8 = 3145.4$ cm.
- ¹¹ The abbreviation u/i means up to and including.
- ¹² Van der Waerden 1978, 356.
- ¹³ Seidenberg 1962, 497.
- ¹⁴ Mertens 1993, 82. The platform (= steps 1, 2 and 3) is not an exact rectangle. East-north: an exact right angle, east-south: 'fast ebenso genau'; length of middle step (north) 5500.4 and 5495.0 cm (south). If the worst comes to the worst zero on north flank have to be situated 5.4 cm west of a perpendicular erected in zero on south flank. As the evidence for Pythagorean activity goes from west to east, I cannot accept Mertens' supposition that the erection of the steps started from the east front.

- ¹⁵ Mertens 1993, 12: '2. Stufe (in Joch 4vW) 25.20,5', that is, between columns 4 and 5 from west, the nearest position with regard to the perpendicular, where this distance has been measured.
- ¹⁶ De Zwarte 1994: Temple of Apollo, Didyma; metrological relief in Oxford; temple of Hera, Samos; temple of Zeus, Aizanoi; temple of Nemesis, Rhamnous; temple at Segesta, Sicily; Parthenon, Athens. De Zwarte 1994-95: temple of Aphaia, Aegina.
- ¹⁷ Haselberger 1996, 165-168 and note 56 (temple of Apollo, Didyma; mausoleum, Halicarnassos; temple of Athena Alea, Tegea).
- ¹⁸ De Zwarte 1996.
- ¹⁹ Mertens 1993, 3, fig. 2.
- ²⁰ On 'holy' measures and numbers: Gruben 1976, 249 and 252; Naredi-Rainer 1982, 156-157.
- ²¹ Mertens 1993, 1 (distance altar to first step) and annex 2 (width of first step on east front) = total distance; annex 2: length of middle step on east front).
- ²² Wesenberg 1982.
- ²³ Dimensions in centimetres: Bankel 1983, 87 (after Penrose in English feet).
- ²⁴ Mertens 1984, 66-67 (presumably after Korres).
- ²⁵ Bankel 1983, 82-83: A list of previous investigators including the proposed foot or module.
- ²⁶ Wesenberg 1984, 547.
- ²⁷ Wesenberg 1995, 217.
- ²⁸ De Zwarte 1994, 127-128.
- ²⁹ De Zwarte 1994, 133: The temple of Nemesis at Rhamnous. In my opinion the Ionic foot was used, but I have left the question open for discussion. Those who are interested may judge the argument.
- ³⁰ Korres 1994, 63.
- ³¹ Mertens 1984, 58.
- ³² Berger 1984, 377, notes 7 and 8.
- ³³ In detail: De Zwarte 1994.
- ³⁴ The source for the Anglo-Saxon foot is a passage in the Old English Orosius in which Roman and early English measures of length are linked up. Philip Grierson (1972, 29) did not interpret the passage rightly, so he was not able to produce the required $70 \frac{1}{7}$ miles in a clear calculation.
- ³⁵ Wesenberg 1982, 124.
- ³⁶ Hill 1912, 544; the recalculation by Dinsmoor (2353.3×6694.0) has to be dismissed. Boersma (1970, 176) gives an excellent synopsis of the facts and the prevailing opinions.

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