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1906

HISTORY OF THE PLANETARY SYSTEMS

FROM

THALES TO KEPLER

BY

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CHAPTER IV.

THE HOMOCENTRIC SPHERES OF EUDOXUS.

Life and work of Eudoxus—His system of spheres—How far it
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CAMBRIDGE :
at the University Press

1906

CHAPTER IV.

THE HOMOCENTRIC SPHERES OF EUDOXUS.

THE examination of the astronomical doctrines of Plato has shown us that philosophers in the first half of the fourth century before the Christian era possessed some knowledge of the motions of the planets. No doubt astronomical instruments, even of the crudest kind, cannot be said to have existed, except the gnomon for following the course of the sun; but all the same the complicated movements of the planets through the constellations must have been traced for many years previously. That the moon, though its motion is not subject to very conspicuous irregularities, does not pursue the same path among the stars from month to month and from year to year must also have been perfectly well known, since Helikon, a disciple of Eudoxus, was able to foretell the solar eclipse of the 12th May, 361, for which he was rewarded by Dionysius II of Syracuse with a present of a talent¹. But the clearest proof of the not inconsiderable amount of knowledge of the movements of the heavenly bodies, which was available at the time of Plato, is supplied by the important astronomical system of his younger contemporary, Eudoxus of Knidus, which is the first attempt to account for the more conspicuous irregularities of those movements.

Eudoxus was born at Knidus, in Asia Minor, about the year

¹ Boeckh, *Ueber die vierjährigen Sonnenkreise der Alten, besonders den Eudoxischen*. Berlin, 1863, p. 153. For an account of the life of Eudoxus see *ibid.* p. 140, and about the geographical researches attributed to him see Berger's *Erdkunde d. Gr.* II. pp. 68-74.

408 B.C., and died in his fifty-third year, about 355¹. At the age of twenty-three he went to Athens and attended Plato's lectures for some months, but not content with the knowledge he could attain in Greece, Eudoxus afterwards proceeded to Egypt, furnished with letters of recommendation from the Spartan King Agesilaus to Nectanebis, King of Egypt. He stayed at least a year in Egypt, possibly much longer (about 378 B.C.), and received instruction from a priest of Heliopolis. According to Seneca² it was there that he acquired his knowledge of the planetary motions, but although this is not unlikely to have been the case, we have no reason to believe that Eudoxus brought his mathematical theory of these motions home from Egypt, in which country, as far as we know, geometry had made very little progress³. Diogenes of Laerte, who does not say a word about the scientific work of Eudoxus, does not omit to mention that the Egyptian Apis licked his garment, after which the priests prophesied that he would be short-lived but very illustrious. If this prophecy was really uttered it was a true one, as Eudoxus stands in the foremost rank of Greek mathematicians. Most, if not the whole, of the fifth book of Euclid is due to him, as well as the so-called method of exhaustion, by means of which the Greeks were able to solve many problems of mensuration without infinitesimals. We are told by Plutarch⁴ that Plato, on being consulted about the celebrated Delian problem of the duplication of a cube, said that only two men were capable of solving this problem, Eudoxus and Helikon; and if the story is apocryphal, it shows at any rate the high renown of Eudoxus as a mathematician. In the history of astronomy he is also known as the first proposer of a solar cycle of four years, three of 365 and one of 366 days, which was three hundred years later introduced

¹ Strabo (p. 119) mentions the observatory of Eudoxus (at Knidus) as not having been much higher than the houses, but still he was able to see the star Canopus from it.

² *Quæst. Nat.* vii. 3.

³ Cantor, *Gesch. der Math.* chap. 2. Whatever the Egyptians may have known of geometry, there is no doubt that the Greeks had long before the time of Eudoxus outstripped them completely.

⁴ *De genio Socratis*, cap. viii.

by Julius Caesar. He was therefore fully capable of grappling successfully with the intricate problem of planetary motion, which Plato (according to Simplicius) is said to have suggested to him for solution¹, and his labours produced a most ingenious cosmical system which represented the principal phenomena in the heavens as far as they were known in his time.

This system of concentric spheres, which was accepted and slightly improved by Kalippus, is known to us through a short notice of it in Aristotle's *Metaphysics* (A 8), and through a lengthy account given by Simplicius in his commentary to Aristotle's book on the Heavens². The systems of Hipparchus and Ptolemy eventually superseded it, and the beautiful system of Eudoxus was well-nigh forgotten. One historian of astronomy after another, knowing in reality nothing about it, except that it supposed the existence of a great number of spheres, contented himself with a few contemptuous remarks about the absurdity of the whole thing. That the system, mathematically speaking, was exceedingly elegant does not seem to have been observed by anybody, until Ideler in two papers in the *Transactions of the Berlin Academy* for 1828 and 1830 drew attention to the theory of Eudoxus and explained its principles. The honour of having completely mastered the theory and of having investigated how far it could account for the observed phenomena, belongs, however, altogether to Schiaparelli, who has shown how very undeserved is the neglect and contempt with which the system of concentric spheres has been treated so long, and how much we ought to admire the ingenuity of its author. We shall now give an account of this system as set forth by Schiaparelli³.

¹ Simpl. *De Cælo*, p. 488 (Heib.).

² II, 12, pp. 493–506 (Heib.).

³ Schiaparelli: "Le sfere omocentriche di Eudosso, di Callippo e di Aristotele," *Pubblicazioni del R. Osservatorio di Brera in Milano*, No. IX. Milano, 1875. German translation in *Abhandlungen zur Geschichte der Mathematik*, Erstes Heft. Leipzig, 1877. Schiaparelli does not mention a paper by E. F. Apelt: "Die Sphärentheorie des Eudoxus und Aristoteles," in the *Abhandlungen der Fries'schen Schule*, Heft II. (Leipzig, 1849), which gives a fairly full exposition of the theory. Later than Schiaparelli's paper appeared one by Th. H. Martin in the *Mém. de l'Acad. des Inscr.* t. xxx. 1881. In this objections are raised to Schiaparelli's interpretation of the theories of the sun and moon, but they have been sufficiently refuted by Tannery in the *Mém. de la Soc. des sc. phys. et nat. de Bordeaux*, 2^e Série, t. v. 1883, pp. 129–147.

Although the various cosmical systems suggested by philosophers from the earliest ages to the time of Kepler differ greatly from each other both in general principles and in matters of detail, there is one idea common to them all: that the planets move in circular orbits. This principle was also accepted by Eudoxus, but he added another in order to render his system simple and symmetrical. He assumed that all the spheres which it appeared necessary to introduce were situated one inside the other and all concentric to the earth, for which reason they long afterwards became known as the homocentric spheres. No doubt this added considerably to the difficulty of accounting for the complicated phenomena, but the system gained greatly in symmetry and beauty, while it also became physically far more sensible than any system of excentric circles could possibly be. Every celestial body was supposed to be situated on the equator of a sphere which revolves with uniform speed round its two poles. In order to explain the stations and arcs of retrogression of the planets, as well as their motion in latitude, Eudoxus assumed that the poles of a planetary sphere are not immovable but are carried by a larger sphere, concentric with the first one, which rotates with a different speed round two poles different from those of the first one. As this was not sufficient to represent the phenomena, Eudoxus placed the poles of the second sphere on a third, concentric to and larger than the two first ones and moving round separate poles with a speed peculiar to itself. Those spheres which did not themselves carry a planet were according to Theophrastus called *ἀνάστροι*, or starless. Eudoxus found that it was possible by a suitable choice of poles and velocities of rotation to represent the motion of the sun and moon by assuming three spheres for each of these bodies, but for the more intricate motions of the five planets four spheres for each became necessary, the moving spheres of each body being quite independent of those of the others. For the fixed stars one sphere was of course sufficient to produce the daily revolution of the heavens. The total number of spheres was therefore twenty-seven. It does not appear that Eudoxus speculated on the cause of all these rotations, nor on the material, thickness,

or mutual distances of the spheres. We only know from a statement of Archimedes (in his *ψαμμίτην*) that Eudoxus estimated the sun to be nine times greater than the moon, from which we may conclude that he assumed the sun to be nine times as far distant as the moon. Whether he merely adopted the spheres as mathematical means of representing the motions of the planets and subjecting them to calculation thereby, or whether he really believed in the physical existence of all these spheres, is uncertain. But as Eudoxus made no attempt to connect the movements of the various groups of spheres with each other, it seems probable that he only regarded them as geometrical constructions suitable for computing the apparent paths of the planets.

Eudoxus explained his system in a book "On velocities," which is lost, together with all his other writings. Aristotle, who was only one generation younger, had his knowledge of the system from Polemarchus, an acquaintance of its author's. Eudemus described it in detail in his lost history of astronomy, and from this work the description was transferred to a work on the spheres written by Sosigenes, a peripatetic philosopher who lived in the second half of the second century after Christ. This work is also lost, but a long extract from it is preserved in the commentary of Simplicius, and we are thus in possession of a detailed account of the system of Eudoxus¹.

While all other ancient and medieval cosmical systems (apart from those which accept the rotation of the earth) account for the diurnal motion of sun, moon, and planets across the sky by assuming that the sphere of the fixed stars during its daily revolution drags all the other spheres along with it, the system of Eudoxus provides a separate machinery for each planet for this purpose, thereby adding in all seven spheres to the number required for other purposes. Thus the motion of the moon was produced by three spheres; the first and outermost of these rotated from east to west in twenty-four hours like the fixed stars; the second turned from west to east round the axis of the zodiac, producing the monthly

¹ Simplicius also quotes in the course of his account Alexander of Aphrodisias and Porphyrius, the Neo-Platonic philosopher (p. 503 Heib.).

motion of the moon round the heavens; the third sphere turned slowly, according to Simplicius, in the same direction as the first one round an axis inclined to the axis of the zodiac at an angle equal to the highest latitude reached by the moon, the latter being placed on what we may call the equator of this third sphere. The addition of this third sphere was necessary, says Simplicius, because the moon does not always seem to reach its highest north and south latitude at the same points of the zodiac, but at points which travel round the zodiac in a direction opposite to the order of its twelve signs. In other words, the third sphere was to account for the retrograde motion of the nodes of the lunar orbit in $18\frac{1}{2}$ years. But it is easy to see (as was pointed out by Ideler) that Simplicius has made a mistake in his statement, that the innermost sphere moved very slowly and in the manner described; as the moon according to that arrangement would only pass once through each node in the course of 223 lunations, and would be north of the ecliptic for nine years and then south of it for nine years. Obviously Eudoxus must have taught that the innermost sphere (carrying the moon) revolved in 27 days¹ from west to east round an axis inclined at an angle equal to the greatest latitude of the moon, to the axis of the second sphere, which latter revolved along the zodiac in 223 lunations in a retrograde direction. In this manner the phenomena are perfectly accounted for; that is, as far as Eudoxus knew them, for he evidently did not know anything of the moon's changeable velocity in longitude, though we shall see that Kalippus about B.C. 325 was aware of this. But that the motion of the lunar node was known forty or fifty years earlier is proved by the lunar theory of Eudoxus.

With regard to the solar theory, we learn from Aristotle that it also depended on three spheres, one having the same daily motion as the sphere of the fixed stars, the second revolving along the zodiac, and the third along a circle inclined to the zodiac. Simplicius confirms this statement, and adds that the third sphere does not, as in the case of the moon, turn in the direction opposite to that of the second, but in the

¹ More accurately in $27^d\ 5^h\ 5^m\ 36^s$, the draconitic or nodical month.

same direction, that is, in the direction of the zodiacal signs, and very much more slowly than the second sphere. Simplicius has here made the same mistake as in describing the lunar theory, as, according to his description, the sun would for ages have either a north or a south latitude, and in the course of a year would describe a small circle parallel to the ecliptic instead of a great circle. Of course the slow motion must belong to the second sphere and be directed along the zodiac, while the motion of the third sphere must take place in a year¹ along the inclined great circle, which the centre of the sun was supposed to describe. This circle is by the second sphere turned round the axis of the zodiac, and its nodes on the ecliptic are by Eudoxus supposed to have a very slow direct motion instead of a retrograde motion as the lunar nodes have. The annual motion of the sun is supposed to be perfectly uniform, so that Eudoxus must have rejected the remarkable discovery made by Meton and Euktemon some 60 or 70 years earlier, that the sun does not take the same time to describe the four quadrants of its orbit between the equinoxes and solstices².

It is very remarkable that although Eudoxus thus ignored the discovery of the variable orbital velocity of the sun, he admitted as real the altogether imaginary idea that the sun did not in the course of the year travel along the ecliptic, but along a circle inclined at a small angle to the latter. According to Simplicius³, "Eudoxus and those before him" had been led to assume this by observing that the sun at the summer- and winter-solstices did not always rise at the same point of the horizon. Perhaps it did not strike these early observers that

¹ Strictly speaking in a period slightly longer than a tropical year, owing to the supposed slow, direct motion of the second sphere.

² This agrees with the statement in the so-called Papyrus of Eudoxus, that this astronomer gave the length of the autumn as 92 days, and that of each of the three other seasons as 91 days. This papyrus was written about the year 190 B.C., and seems to have been a student's note-book, perhaps hastily written during or after a series of lectures. See Boeckh, *Ueber die vierjährigen Sonnenkreise der Alten*, p. 196 and foll. It was published by F. Blass (*Eudoxi Ars astronomica*, Kiel, 1887, 25 pp. 4°), and translated by Tannery, *Recherches sur l'Astr. ancienne*, pp. 283-294.

³ p. 493, l. 15 (Heib.).

neither these rough determinations of the azimuth of the rising sun nor the observations with the gnomon were sufficiently accurate; they had without instruments perceived that neither the moon nor the five planets were confined to move in the ecliptic (or, as they called it, the circle through the middle of the zodiac), and why should the sun alone have no motion in latitude, when all the other wandering stars had a very perceptible one? This imaginary deviation of the sun from the ecliptic is frequently alluded to by writers of antiquity. Thus Hipparchus, who denies its existence, quotes the following passage from a lost book on the circles and constellations of the sphere, the *Enoptron* of Eudoxus: "It seems that the sun also makes its return (*τροπὰς*, solstices) in different places, but much less conspicuously¹." How great Eudoxus supposed the inclination of the solar orbit to be, or how long he supposed the period of revolution of the nodes to be, is not known, and he had probably not very precise notions on the subject. Pliny gives the inclination as 1° , and the point where the maximum latitude occurs as the 29th degree of Aries². On the other hand, Theon of Smyrna, who goes more into detail on this subject, states on the authority of Adrastus (who lived about A.D. 100) that the inclination is $\frac{1}{2}^\circ$, and that the sun returns to the same latitude after $365\frac{1}{8}$ days, so as to make the shadow of the gnomon have the same length, as he says, while the sun takes $365\frac{1}{4}$ to return to the same equinox or solstice, and $365\frac{1}{2}$ days to return to the same distance from us. This shows that the solar nodes were supposed to have a retrograde motion (not a direct one as assumed by Eudoxus) and in a period of $365\frac{1}{4} : \frac{1}{8} = 2922$ years³. Schiaparelli shows that with an inclination of $\frac{1}{2}^\circ$ between the axes of the second and third spheres the solstitial points would oscillate $2^\circ 28'$. This of course

¹ That is, the maximum latitude is much less than that of the moon. Hipparchus adds, that observations with the gnomon show no latitude, and lunar eclipses calculated without assuming any solar latitude agree with observations within at most two digits. *Hipparchi ad Arati et Eudoxi Phenomena*, lib. I.; ed. Manitius, pp. 88–92.

² *Hist. Nat.* II. 16 (67). He has doubtless misunderstood his source and taken a range of 1° to mean an inclination of 1° .

³ *Astronomia*, ed. Th. H. Martin, pp. 91, 108, 175 (cap. XII.), 263 (cap. XXVII.), 314 (cap. XXXVIII.).

influences the length of the tropical year, and it is very possible that the whole theory of the sun's latitude originally arose from the fact that the tropical year had been found to be different from the sidereal year, the true cause of which is the precession of the equinoxes. To whom this theory in the first instance is due is not known. Notwithstanding the great authority of Hipparchus and Ptolemy the strange illusion is still upheld by the compiler Martianus Capella in the fifth century¹, who improves on it by stating that the sun moves in the ecliptic except in Libra, where it deviates $\frac{1}{2}^\circ$! The meaning is probably that the latitude of the sun was insensible to the instruments of the day except in Libra (and in Aries) where it reached $\frac{1}{2}^\circ$, and consequently the nodes must have been supposed nearly to coincide with the solstices. It is to be noticed that the precession of the equinoxes is unknown to all these writers².

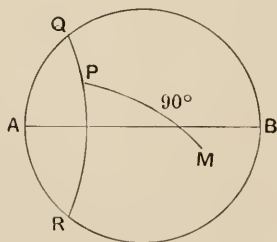
The solar theory of Eudoxus was therefore practically a copy of his lunar theory. But the task he had set himself became vastly more difficult when he took up the theories of the five other planets, as it now became necessary to account for the stations and retrograde motions of these bodies. Of the four spheres given to each planet the first and outermost produced the daily rotation of the planet round the earth in twenty-four hours; the second produced the motion along the zodiac in a period which for the three outer planets was respectively equal to their sidereal period of revolution, while it for Mercury and Venus was equal to a year. From the fact that the revolution of this second sphere was in all cases assumed to be uniform, we see that Eudoxus had no knowledge of the orbital changes of velocity of the planets which depend on the

¹ *De nuptiis Philologie et Mercurii*, lib. viii. 867, on the authority of a book by Terentius Varro.

² Schiaparelli (l. c. p. 17) shows that Theon's theory cannot have been designed to explain the motion of the equinoxes discovered by Hipparchus. He also gives a lengthy refutation of the assertion of Lepsius, that the third solar sphere of Eudoxus proves that Eudoxus knew precession and had received his knowledge of it from the Egyptians (l. c. pp. 20-23). This had, however, already been refuted by Martin, "Mémoire sur cette question: La précession des équinoxes a-t-elle été connue avant Hipparche" (*Mém. par divers savans*, t. viii. 1869, pp. 303-522).

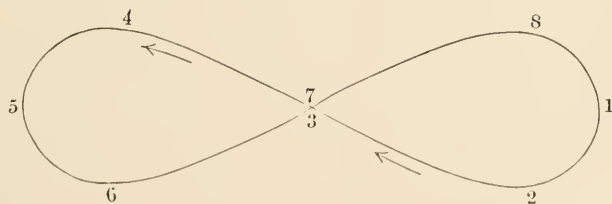
excentricity of each orbit, but that he believed the points of the zodiac in which a planet was found at successive oppositions (or conjunctions) to be perfectly equidistant one from the other. Neither did he assume the orbits to be inclined to the ecliptic, but let the second sphere of every planet move along this circle, while the latitudes of the planets were supposed to depend solely on their elongation from the sun and not on their longitude. To represent this motion in latitude, and at the same time the inequality in longitude depending on the elongation from the sun, a third and fourth sphere were introduced for each planet. The third sphere had its poles situated at two opposite points of the zodiac (on the second sphere), and rotated round them in a period equal to the synodic period of the planet, or the interval between two successive oppositions or conjunctions with the sun. These poles were different for the different planets, but Mercury and Venus had the same poles. The direction of the rotation of this third sphere is not given by Simplicius except as being from north to south and from south to north, but it turns out to be immaterial which of the two possible directions we adopt.

On the surface of the third sphere the poles of the fourth were fixed, the axis of the latter having a constant inclination, different for each planet, to the axis of the third sphere. Round the axis of the fourth sphere the rotation of the latter took place in the same period, but in a direction opposite to that of the third sphere. On the equator of the fourth sphere the planet is fixed, and it is thus endowed with four motions,



the daily one, the orbital one along the zodiac, and two others in the synodic period. What effect will these two last-mentioned

motions have on the apparent position of the planet in the sky? In the appended figure a sphere (the third) rotates round the fixed diameter AB (we may leave the motion of the first, or daily sphere, altogether out of consideration, and for the present also neglect that of the second sphere); during this rotation round AB a certain point P , one of the poles of the fourth sphere, describes the small circle QPR , while this fourth sphere in the same period, but in the opposite direction, completes a rotation round P and its other pole P' . The planet is at M in the equator of the fourth sphere, so that $PM = 90^\circ$. The problem is now to determine the path described by M , projected on the plane of the circle $AQBR$. This is easy enough with the aid of modern mathematics, but was Eudoxus able to solve it by means of simple geometrical reasoning? This question has been admirably investigated by Schiaparelli, who has shown that the solution of the problem was well within the range of a geometrician of the acknowledged ability of Eudoxus. The result is that the projected path is symmetrical to the line AB , that it has a double point in it, and is nothing but the well-known "figure of eight" or lemniscate, the equation of which is $r^2 = a^2 \cos 2\theta$, or, strictly speaking, a figure of this kind lying in the surface of the celestial sphere, for which reason Schiaparelli calls it a spherical lemniscate. The longitudinal axis of the curve lies along the



zodiac, and its length is equal to the diameter of the circle described by P , the pole of the sphere which carries the planet. The double point is 90° from the two poles of rotation of the third sphere. The planet describes the curve by moving in the direction of the arrow, and passes over the arcs 1-2, 2-3, 3-4, 4-5, etc., in equal times,

So far we have only considered the motion of the point M under the influence of the rotations of the third and fourth sphere. But we must now remember that the axis AB revolves round the ecliptic in the sidereal period of the planet. During this motion the longitudinal axis of the lemniscate always coincides with the ecliptic, along which the curve is carried with uniform velocity. We may therefore for the third and fourth sphere substitute the lemniscate, on which the planet moves in the manner described above. The combination of this motion with the motion of the curve along the ecliptic gives the apparent motion of the planet through the constellations. The motion of the planet on the lemniscate consists in an oscillation forward and backward, the period being that of the synodical revolution, and during one half of this period the motion of the planet along the ecliptic becomes accelerated, and during the second half it becomes retarded, when the two motions are in opposite directions. Therefore when on an arc of the lemniscate the backward oscillation is quicker than the simultaneous forward motion of the lemniscate itself, then the planet will for a time have a retrograde motion, before and after which it is stationary for a little while, when the two motions just balance each other. Evidently the greatest acceleration and the greatest retardation occur when the planet passes through the double point of the lemniscate. The motions must therefore be so combined that the planet passes through this point with a forward motion at the time of superior conjunction with the sun, where the apparent velocity of the planet in longitude is greatest, while it must again be in the double point, but moving in a retrograde direction, at the time of opposition or inferior conjunction, when the planet appears to have the most rapid retrograde motion. This combination of motions will of course be accompanied by a certain amount of motion in latitude depending on the breadth of the lemniscate.

This curve was by the Greeks called the hippopede (ἵππου πέδη), because it was a favourite practice in the riding school to let the horse describe this figure in cantering; and Simplicius in his account of the planetary theory of Eudoxus expressly

states that a planet describes the curve called by Eudoxus a hippopede. This word occurs in several places in the commentary to the first book of Euclid written by Proklus, in which he describes the plane sections of the solid generated by the revolution of a circle round a straight line in its plane, assuming that the line does not cut the circle¹. A section by a plane parallel to the line and touching the inner surface of the "anchor ring" is by Proklus called a hippopede, and it is therefore proved that Eudoxus and his followers had a clear idea of the properties of the curve which represents the resultant motion of the third and fourth sphere. The curve and its application is thus alluded to by Theon of Smyrna in his account of the astronomical theory of the Platonist Derkyllides: "He does not believe that the helicoid lines and those similar to the Hippika can be considered as causing the erratic motions of the planets, for these lines are produced by chance², but the first cause of the erratic motion and the helix is the motion which takes place in the oblique circle of the zodiac." After this Theon describes the helix apparently traced by a planet in the manner of Plato in the *Timæus*; but the opinion rejected by Derkyllides is undoubtedly the motion in the lemniscate invented by Eudoxus³.

If we now ask how far this theory could be made to agree with the actually observed motions in the sky we must first of all remember that we possess no knowledge as to whether Eudoxus had made observations to ascertain the extent of the retrograde motions, or whether he was merely aware of the fact that such motions existed, without having access to any numerical data concerning them. To be able to test the theory we require to know the sidereal period, the synodic period, and the distance between the poles of the third and fourth sphere, which Schiaparelli calls the inclination. The length of this distance adopted by Eudoxus for each planet is

¹ Cantor, *Gesch. der Math.* i. pp. 229-30 (2nd ed.).

² Does this allude to the loops traced by the planets about the time of opposition, and not to the machinery used to produce them?

³ Theon, ed. Martin, p. 328.

not stated either by Aristotle or Simplicius, and the periods are only given by the latter in round numbers as follow¹:

Star of	Synodic Period	Modern value	Zodiacal Period	Modern value
Hermes	110 days	116 days	1 year	1·0 year
Aphrodite	19 months	584 "	1 "	1·0 "
Ares	8 " 20 days	780 "	2 years	1·88 years
Zeus	13 "	399 "	12 "	11·86 "
Kronos	13 "	378 "	30 "	29·46 "

With the exception of Mars these figures show that the revolutions of the planets had been observed with some care, and Eudoxus may even have been in possession of somewhat more accurate figures, as the Papyrus of Eudoxus gives the synodic revolution of Mercury as 116 days, a remarkably accurate value, which he had most probably obtained during his stay in Egypt². If only we knew the inclination on which the dimensions of the hippopede depend, we should be able perfectly to reconstruct each planetary theory of Eudoxus. As the principal object of the system certainly was to account for the retrograde motions, Schiaparelli has for the three outer planets assumed that the values of the inclinations were so chosen as to make the retrograde arcs agree with the observed ones. The retrograde arc of Saturn is about 6° , and with a zodiacal period of 30 years, a synodic period of 13 months, and an inclination of 6° between the axes of the third and fourth sphere the length of the hippopede becomes 12° and half its breadth, i.e. the greatest deviation of the planet from the ecliptic turns out to be $9'$, a quantity insensible for the observations of those days. We have therefore simply a retrograde motion in longitude of about 6° between two stationary points. Similarly, assuming for Jupiter an inclination of 13° , the length of the hippopede becomes 26° , and half its breadth $44'$, and with periods of respectively 12 years and 13 months this gives a retrograde arc of about 8° . The greatest distance from the ecliptic during the motion on this arc, $44'$, was probably hardly

¹ p. 496 (Heib.).

² This papyrus gives the zodiacal periods of Mars and Saturn as two years and thirty years, in perfect accordance with Simplicius (Blass, p. 16; Tannery, p. 287).

noticeable at that time. For these two planets Eudoxus had thus found an excellent solution of the problem proposed by Plato, even supposing that he knew accurately the lengths of the retrograde arcs.

But this was not the case with Mars, which indeed is not to be wondered at, when we remember that even Kepler for a long time found it hard to make the theory of this planet satisfactory. It is not easy to see how Eudoxus could put the synodic period equal to 8 months and 20 days (or 260 days), whereas it really is 780 days, or exactly three times as long. All editions of Simplicius give the same figures, and Ideler's suggestion that we should for 8 months read 25 months seems therefore unwarranted; besides, it does not help matters in the least. For with a synodic period of 780 days and putting the inclination equal to 90° (the highest value reconcilable with the description of Simplicius), the breadth of the hippopede becomes 60° , so that Mars ought to reach latitudes of 30° . And even so, the retrograde motion of Mars on the hippopede cannot in speed come up to the direct motion of the latter along the zodiac, so that Mars should not become retrograde at all, but should only move very slowly at opposition. To obtain a retrograde motion the inclination would have to be greater than 90° ; in other words the third and fourth sphere would have to rotate in the same direction. And even this violation of the rule would be of no use, since Mars in that case would reach latitudes greater than 30° , and Eudoxus was doubtless not willing to accept this. On the other hand, if we adopt his own value of the synodic period, 260 days, the motion of Mars on the hippopede becomes almost three times as great as before, and with an inclination of 34° the retrograde arc becomes 16° long and the greatest latitude nearly 5° . This is in fair accordance with the real facts, but unfortunately this hypothesis gives two retrograde motions outside the oppositions and four additional stationary points, which have no real existence. The theory of Eudoxus fails therefore completely in the case of Mars.

With regard to Mercury and Venus, we have first to note that the mean place of these planets always coincides with the

sun, so that the centre of the hippopede always lies in the sun. As this centre is 90° from the poles of rotation of the third sphere, we see that these poles coincide for the two planets. This deduction from the theory is confirmed by the remark of Aristotle that "according to Eudoxus the poles of the third sphere are different for some planets, but identical for Aphrodite and Hermes," and this supplies a valuable proof of the correctness of Schiaparelli's deductions. As the greatest elongation of each of these planets from the sun equals half the length of the hippopede, i.e. the inclination of the third and fourth spheres, Eudoxus doubtless determined the inclination by observing the elongations, as he could not make use of the retrograde motions, which in the case of Venus are hard to see, and in that of Mercury out of reach. With a hippopede for Mercury 46° in length the half breadth or greatest latitude becomes $2^\circ 14'$, nearly as great as that observed. For Venus we may make the hippopede 92° in length, which gives half its breadth equal to $8^\circ 54'$ in good accordance with the observed greatest latitude. But, as in the case of Mars, Venus can never become retrograde, and no different assumption as to the value of the inclination can do away with this error of the theory. And a much worse fault is, that Venus ought to take the same length of time to pass from the east end of the hippopede to the west end and *vice versa*, which is not in accordance with facts, since Venus in reality takes 440 days to move from the greatest western to the greatest eastern elongation, and only about 143 days to go from the eastern to the western elongation, a fact which is very easily ascertained. The theory is equally unsatisfactory as to latitude, for the hippopede intersects the ecliptic in four points, at the two extremities and at the double point; consequently Venus ought four times during every synodic period to pass the ecliptic, which is far from being the case.

But with all its imperfections as to detail the system of homocentric spheres proposed by Eudoxus demands our admiration as the first serious attempt to deal with the apparently lawless motions of the planets. For Saturn and Jupiter, and practically also for Mercury, the system accounted well for the

motion in longitude, while it was unsatisfactory in the case of Venus, and broke down completely only when dealing with the motion of Mars. The limits of motion in latitude were also well represented by the various hippopedes, though the periods of the actual deviations from the ecliptic and their places in the cycles came out quite wrong. But it must be remembered that Eudoxus cannot have had at his command a sufficient series of observations; he had probably in Egypt learned the main facts about the stationary points and retrogressions of the outer planets as well as their periods of revolution, which the Babylonians and Egyptians doubtless knew well, while it may be doubted whether systematic observations had for any length of time been carried on in Greece. And if the old complaint is to be repeated about the system being so terribly complicated, we may well bear in mind, as Schiaparelli remarks, that Eudoxus in his planetary theories only made use of three elements, the epoch of an upper conjunction, the period of sidereal revolution (of which the synodic period is a function), and the inclination of the axis of the third sphere to that of the fourth. For the same purpose we nowadays require six elements!

If, however, the system was founded on an insufficient basis of observations, it seems that some of the adherents of Eudoxus must have compared the movements resulting from the theory with those actually taking place in the sky, since we find Kalippus, of Kyzikus, a pupil of Eudoxus, engaged in improving his master's system some thirty years after its first publication. Kalippus is also otherwise favourably known to us by his improvement of the soli-lunar cycle of Meton, which shows that he must have possessed a remarkably accurate knowledge of the length of the moon's period of revolution. Simplicius states that Kalippus, who studied with Polemarchus, an acquaintance of Eudoxus, went with Polemarchus to Athens in order to discuss the inventions of Eudoxus with Aristotle, and by his help to correct and complete them¹. This must have happened during the reign of Alexander the Great (336-323), which time Aristotle spent at Athens. From the

¹ Simpl. *De Cælo*, p. 493 (Heib.).

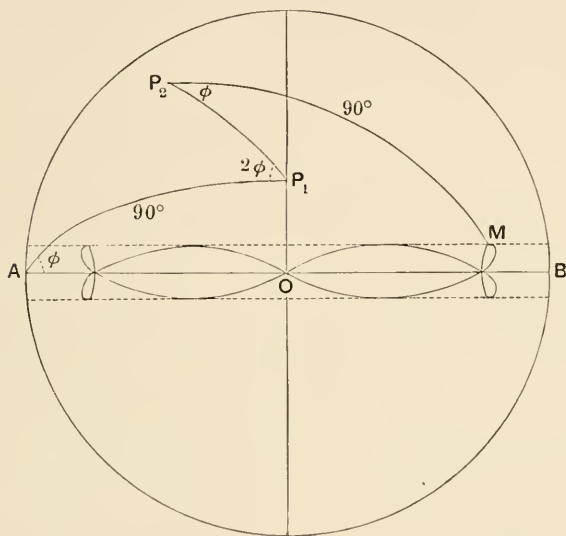
investigations of Kalippus resulted an important improvement of the system of Eudoxus which Aristotle and Simplicius describe; and as the former solely credits Kalippus with it, it does not seem likely that he had any share in it himself, though he cordially approved of it¹. Kalippus wrote a book about his planetary theory, but it was already lost before the time of Simplicius, who could only refer to the history of astronomy by Eudemus, which contained an account of it.

The principle of the homocentric spheres, as we shall see in the next chapter, fitted in well with the cosmological ideas of Aristotle, and had therefore to be preserved, so that Kalippus was obliged to add more spheres to the system if he wished to improve it. He considered the theories of Jupiter and Saturn to be sufficiently correct and left them untouched, which shows that he had not perceived the elliptic inequality in the motion of either planet, though it can reach the value of five or six degrees. But the very great deficiencies in the theory of Mars he tried to correct by introducing a fifth sphere for this planet in order to produce a retrograde motion without making a grave error in the synodic period. This is only a supposition, as we are not positively told why Kalippus added a sphere each to the theories of Mars, Venus, and Mercury², but Schiaparelli has shown how the additional sphere can produce retrogression without unduly adding to the motion in latitude. Let AOB represent the ecliptic, A and B being opposite points in it which make the circuit of the zodiac in the sidereal period of Mars. Let a sphere (the third of Eudoxus) rotate round these points in the synodic period of the planet, and let any point P_1 in the equator of this sphere be the pole of a fourth sphere which rotates twice as fast as the third in the opposite direction carrying the point P_2 with it, which is the pole of a fifth sphere rotating in the same direction and period as the third and carrying the planet at M on its equator. It is easy to see that if at the beginning of motion the points P_1 , P_2 , and M were situated in the ecliptic

¹ *Metaph.* xi. 8, p. 1073 b.

² Simplicius merely says that Eudemus has clearly and shortly stated the reasons for this addition (*De Cælo*, ed. Heiberg, p. 497, l. 22).

in the order AP_2P_1MB , then at any time the angles will be as marked in the figure, and as $AP_1 = MP_2 = 90^\circ$, the planet M will in the synodic period describe a figure symmetrical to the



ecliptic which alters its form with the adopted length of the arc P_1P_2 , and, like the hippopede, may produce retrograde motion. And it has this advantage over the hippopede, that it can give the planet in the neighbourhood of O a much greater direct and retrograde velocity with the same motion in latitude. It can therefore make the planet retrograde even in the cases where the hippopede of Eudoxus failed to do so. Thus, if P_1P_2 is put equal to 45° , the curve assumes a figure like that shown; the greatest digression in latitude is $4^\circ 11'$, the length of the curve along the ecliptic is $95^\circ 20'$, and it has two triple points near the ends, 45° from the centre. When the planet passes O , its velocity is 1.293 times the velocity of P_1 round the axis AB , and as the period of the latter rotation is 780 days, the daily motion of P_1 is $360^\circ/780 = 0^\circ.462$, which number multiplied by 1.293 gives $0^\circ.597$ as the daily velocity of the retrograde motion on the curve at O . But as O has a direct motion on the ecliptic of $360^\circ/686 = 0^\circ.525$, the resulting daily

retrograde motion of the planet in the heavens is $0^{\circ} \cdot 072$, which is a sufficient approximation to the real motion of Mars at opposition. It must however be remembered that we have no way of knowing what value Kalippus assumed for the distance P_1P_2 ; but that the introduction of another sphere could really make the theory satisfactory has been proved by Schiaparelli's investigation.

In the same way an additional sphere removed the errors in the theory of Venus. If P_1P_2 is $= 45^{\circ}$, the greatest elongation becomes $47^{\circ}40'$, very nearly the true value; and the different velocity of the planet in the four parts of the synodic revolution is also accounted for; as in the curve depicted above the passage from one triple point to the other takes one fourth of the period, the same passage back again another fourth, while the very slow motion through the small loops at the end of the curve occupies the remaining time. In the case of Mercury the theory of Eudoxus was already fairly correct, and no doubt the extra sphere made it better still.

In the solar theory Kalippus introduced two new spheres in order to account for the unequal motion of the sun in longitude which had been discovered about a hundred years previously by Meton and Euktemon through the unequal lengths of the four seasons. The so-called papyrus of Eudoxus, to which we have already referred, gives us the values adopted by Kalippus for the lengths of the seasons (taken from the Parapegma, or meteorological calendar of Geminus), and though only given in whole numbers of days (95, 92, 89, 90, beginning with the vernal equinox), the values are in every case less than a day in error, while the corresponding values determined by Euktemon about B.C. 430 are from $1\frac{1}{4}$ to 2 days wrong¹. The observations of the sun had therefore made good progress in Greece during the century ending about B.C. 330. By adding two more spheres to the three spheres of Eudoxus, Kalippus had only to follow the same principle on which Eudoxus had represented the synodic inequalities of the planets, and a hippopede 4° in length and $2'$ in breadth gives in fact the necessary maximum inequality of 2° in a perfectly satisfactory manner.

¹ Boeckh, *Vierjährige Sonnenkreise der Alten*, p. 46.

Similarly the number of lunar spheres was increased by two, and though Simplicius is not very explicit, we can hardly doubt that he means us to understand the cause to be similar to that which he has just stated in the case of the sun. In other words, Kalippus must have been aware of the elliptic inequality of the moon. Indeed he can hardly have failed to notice it, even if he merely confined his attention to lunar eclipses without watching the motion of the moon at other times, since the intervals between various eclipses compared with the corresponding longitudes (deduced from those of the sun) at once show how far the moon's motion in longitude is from being uniform. A hippopede 12° in length would only be twice $9'$ in breadth, and would therefore not sensibly affect the latitude, while it would produce the mean inequality of 6° . The improved theory was therefore quite as good as any other, as long as the evection had not been discovered.

Such then was the modified theory of homocentric spheres, as developed by Kalippus. Scientific astronomy may really be said to date from Eudoxus and Kalippus, as we here for the first time meet that mutual influence of theory and observation on each other which characterizes the development of astronomy from century to century. Eudoxus is the first to go beyond mere philosophical reasoning about the construction of the universe; he is the first to attempt systematically to account for the planetary motions. When he has done this the next question is how far this theory satisfies the observed phenomena, and Kalippus at once supplies the observational facts required to test the theory and modifies the latter until the theoretical and observed motions agree within the limit of accuracy attainable at the time. Philosophical speculation unsupported by steadily pursued observations is from henceforth abandoned; the science of astronomy has started on its career.