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Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations^{a)}

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(Received 26 February 1992; revised 3 September 1992; accepted 16 September 1992)

The purpose of this study was to determine which musical scale best models the solo performances of violinists when they play diatonic scales of C major very slowly and without frequency vibrato as accurately as possible. Eight professional violinists played without stopping three adjacent scales in ascending order (from C₄ to C₇), followed immediately by an analogous return to the initial note C₄ in descending order. Results show that when violin performances are analyzed *taking into account the context* of the scale in which they were played, Pythagorean and equally tempered scales are equally good models for the description of performances. Just intonation fits the data significantly less well. When the data analysis is limited to the intervals between separate pairs of notes, *not taking into consideration the context* of the scale in which they were played, observed interval sizes are almost identical to the arithmetic means of the corresponding interval sizes as defined in Pythagorean and equally tempered intonations. Octave intervals calculated by summing the sizes of performed adjacent major and minor seconds within octaves, show an average stretching of 3.9 cents. However, the arithmetic mean of computed interval sizes between two C's, deviates only 0.3 cents from the theoretical tonal extent of 1200 cents. Results suggest that when scales of C major are played, the tonic C is adopted as an absolute cognitive reference point.

PACS numbers: 43.75.Bc, 43.75.De, 43.75.St, 43.66.Hg

1577 pag
LOOSEN, F.
1993

INTRODUCTION

The great challenge a beginning violin student has to face is to play in tune. However, what is the standard? In beginners, a sharply defined internal template of relative pitch relations among the notes is not yet present. That knowledge must be progressively acquired from the directives and examples given by the string teacher. However, what model does he offer?

The main purpose of this study was to describe how professional violinists played the diatonic scale of C major, when they try to intone as accurately as possible. Playing the scale of C major is a typical exercise for violin students. Even more than 200 years ago the German pedagogue Leopold Mozart (1775), father of Wolfgang Amadeus Mozart, recommended to start with that scale.

Traditionally, pitch relations in musical performances are discussed with reference to a musical scale, that is, a physical model for pitch relations in a set of performed musical intervals. In the course of time, numerous musical scales have been proposed (Barbour, 1951; Devie, 1990; Ellis, 1954; Lloyd and Boyle, 1963; Partch, 1949). This study is restricted to equally tempered, Pythagorean and just intonations.

The equally tempered scale (the present standard scale in Western music) is unequivocally defined by 12 equal logarithmic steps, each step representing a frequency that is $2^{1/12}$ greater than the one below. In this paper the labels "Pythagorean" and "just intonation" stand for Ptolemy's diatonic ditoniaion and Ptolemy's diatonic syntonon, respectively. The major mode of the former is defined as one in which the ratios of the frequencies of adjacent notes from the tonic are 9/8, 9/8, 256/243, 9/8, 9/8, 9/8, and 256/243. Analogously, the major mode of the latter is defined by the ratios 9/8, 10/9, 16/15, 9/8, 10/9, 9/8, and 16/15 (for a detailed description of these tunings, see Barbour, 1951, Chap. II). The present study focused specifically on those two historical models because only those two have had any great influence upon modern music theory, and earlier investigators (i.e., Green, 1937a,b, and Nickerson, 1949) studied violin performance explicitly in relation to those two models.

Some authors claimed that violin performances are best described by Pythagorean intonation. They asserted that this is a consequence of practicing an instrument tuned by perfect fifths (i.e., intervals between two frequencies having a ratio of 3:2). Indeed, the tones of Pythagorean intonation can be easily derived from a sequence of adjacent perfect fifths. It suffices to iterate the tuning upward in perfect fifths from the frequency of a given note to include seven (or more) successive fifths, and to bring back the notes generated in this way to the same octave in order to obtain all the tones of the Pythagorean scale.

Others asserted that the practice of harmonic accompaniment has forced violinists to perform in a manner ap-

^{a)} Results of part of the present research have been presented at the 25th International Congress of Psychology held at Brussels, Belgium, 19-24 July 1992.

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proximating just intonation. This time, the conviction relied upon the association of two arguments: first, the knowledge that the tones of this intonation are obtained on a string instrument by dividing the string into 2, 3, 4, etc., equal sections (and bringing them back in the proper octave), and second, the belief that consonance gradually decreases as the frequency ratio between two tones becomes more complex.

Others still advocated that the conditioning influence of equally tempered tuning as employed upon fixed pitch instruments (especially keyboard instruments) had brought violinists to perform in a manner approaching the equally tempered scale.

There exist numerous other opinions about intoning by violinists in the musicological literature. However, most of them have never been verified empirically. Willemze (1973) for example, is convinced that violinists play ascending scales in Didymus' diatonic scale (i.e., a scale in which the ratios of the frequencies of adjacent notes from the tonic are 9/8, 10/9, 16/15, 9/8, 9/8, 10/9, and 16/15 in the major mode¹) and descending scales in just intonation. Lloyd (1940) went even further and rejected, explicitly, every rigidity of intonation. In his opinion, violinists are likely to be guided by the music itself as to what intonation to use.

In the past, a considerable amount of experimental research had been directed toward the investigation of the *perception* of isolated and synthesized tonal intervals in different intonations (e.g., Mathews and Sims, 1981; Roberts and Mathews, 1984; Vos, 1982, 1984; Vos and van Vianen, 1985; Ward, 1970; Ward and Martin, 1961). However, as far as I am aware, until now, only six attempts have been made to determine the intonation used by solo playing violinists during actual *performance*: Cornu and Mercadier (1869, 1871, 1872, 1873), Corso (1954a), Geringer (1978), Greene (1937a,b), Nickerson (1949), and Small (1937).

However, the sizes of the reading unit in the studies of Cornu and Mercadier, and Small were excessively large (10 to 20 cents²). Furthermore, the studies of Corso and Geringer were set up to investigate a psychophysical question and the impact of verbal inducement on intonation, respectively. Nickerson pooled the data of violins, altviolins, and cellos. Hence, only Green's investigation is concerned with the question of this study. Therefore, it appeared sensible to start by collecting new data of solo playing violinists. Some respects of the present study are different from the previous ones.

(1) The main objective of this study was to determine which musical scale best models the solo performances of

professional violinists when playing the diatonic scale of C major, very slowly, without vibrato, and as accurately as possible (i.e., the usual way of playing when a string teacher shows his pupil how a scale sounds).

(2) The present study focused on the relative size of the musical intervals between notes in performed diatonic scales of C major, considered *as a whole*. This approach was based on the finding that musical context may have a significant influence on intonation (Dowling, 1991; Francès, 1958; Krumhansl, 1990; Shackford, 1961, 1962; Small, 1937), and skilled musicians developed expertise for playing and perceiving typical musical sequences as a whole (Abraham, 1923; House, 1977; Krumhansl, 1979, 1990; and Watkins and Dyson, 1985).

(3) The intonation of violinists was studied as a function of two parameters: (i) *scale location*, that is, the location of the musical scale in the frequency range and, (ii) *scale direction*, that is, the ascending or descending direction of the musical scale.

(4) This study aimed to evaluate the size of inter- and intra-individual variability in performances.

(5) Instead of a stroboscopic analysis for measuring fundamental frequencies of violin tones an electronic technique was used.

I. METHOD

The subjects were eight highly trained professional violinists between the ages of 25 and 33 years. All of them had wide experience in solo performance. Moreover, all of them were members of an orchestra and were teachers in an academy of music. The musicians were asked to play without interruption three adjacent scales of C major starting in ascending order from C₄ up three octaves to C₇ (i.e., a range of fundamental frequencies from about 262 Hz up to 2096 Hz, immediately followed by an analogous return to the initial note C₄ in descending order. Subjects were given a printed form (without fingering) of the musical sequence to be performed (Fig. 1). Every violinist was given three trials. Between the trials there was a short pause of about 30 s. Musicians were prepared for the experiment by playing warm-up exercises and practicing diatonic scales of C major individually.

The violinists were asked to concentrate their attention on the accurateness of intonation. They were also instructed to sustain each note about 3 s and to play with an even tone

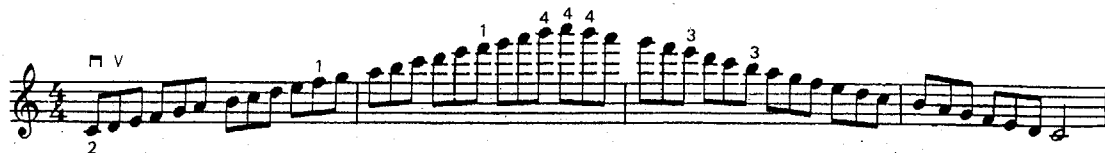


FIG. 1. Consecutive scales of C major played three times by eight violinists ($\text{♩} = 3 \text{ s}$). Each note was played on a separate bow strike. The numbers marked above the notes represent the finger that should be pressed on the string to produce the required note; 1, 2, 3, and 4 refer to the forefinger, the middle finger, the ring finger, and the pinkie finger, respectively.

without frequency vibrato. Each note was played on a separate bow strike. Open strings were not allowed. On loudness nothing explicit was mentioned because it was expected that the musicians would play mezzo-piano spontaneously, and because experimental data available indicate that intensity has no consistent effect on the pitch of violin tones or that the effect is so slight as to be counteracted by other factors in performance (Fletcher, 1934; Lewis and Cowan, 1936; Seashore, 1938).

Musicians were allowed (1) to tune their instrument at their own discretion and (2) to use the fingering best suited to them. The first option was based on the objective of this study to investigate the intonation of violinists in everyday conditions, and on the observation that no professional violinist uses a tuning fork or any other device to tune his/her instrument in daily practice. Furthermore, the differences in initial tuning were no disturbing factor in the present study because (1) open strings were not allowed, (2) the research was not interested in the frequency with which a given note was played, but in the ratios between frequencies of notes, (3) advanced players have acquired the skill to play in tune on a violin that is out of tune (Galamian, 1962), and (4) the subjects—due to the slow tempo—had the opportunity to make minor adjustments based on auditory cues all the time. Hence, generated tones were always the result of the violinist's tonal memory model and observed interval sizes between notes were not confounded to the initial tuning of the instrument.

It was not the purpose of the study to find out how a violinist intones in fast tempo. In that case, the frequency of a note depends completely on proprioceptive cues guiding finger placement, since the produced tone is so short that it can no longer be corrected. Neither did the present study want to give an answer to the question of whether the pitch of a tone emanates either from the memory for the "absolute" pitch of this tone or from its relation to the pitch of another tone produced before. However, due to the slow tempo, violinists were always able to rely upon their memory for absolute pitch (should they possess it). Therefore, it was desirable to allow the violinists to tune their instruments at their own discretion. In particular, possessors of absolute pitch can be annoyed by being forced into a pitch standard that is not their own (Risset, 1978).

The fingering was left free because accuracy of intonation depends primarily on maximum sureness in the shifting from one hand position to another, and this, in turn, is determined by the personal habit of the performer. Furthermore, fingering is based on personal characteristics of the individual performer and on the peculiarities of his technical equipment (Yampolsky, 1967; Flesh, 1979).

The performances took place in a slightly damped room, and were recorded on magnetic tape (Agfa PER 525) by means of a high-quality unidirectional microphone (Peerless-MBC 548) held approximately three feet above the bridge of the instrument. Recordings were made at a tape speed of 15 in./s on a Nagra tape recorder, model III-B. At this speed the flutter (DIN.45507) was 0.07%. This value corresponds to the most unfavorable case, where the periodic faults of the play back, were added to those of the record-

ing. The speed variation between the beginning and the end of a 7-in. reel was $\pm 0.1\%$.

The analysis of the recorded violin tones was based on the assumption that the pitch of violin tones can be related to the frequency of their fundamentals (Plomp, 1976; Rasch and Plomp, 1982). This assumption is plausible because it was found that the frequencies of the partials in the central part of violin tones are harmonic (that is, integral multiples of the fundamental frequency), and changes in the harmonic structure are small in that part of the tone (Fletcher *et al.*, 1965). Hence, the fundamental frequency of each of the 129 notes played by each violinist was measured in the following way.

First, two bandpass filters (Krohn-Hite, model 330N) were used to filter out higher partials. The resulting sinusoidal signal was controlled by means of an oscilloscope. Simultaneously, by means of an electronic digital timer-counter (Hewlett-Packard, model 5326B), as many samples of five consecutive periods as possible were drawn. For every sample, the fundamental frequency was calculated to the nearest 0.01 Hz and stored into a computer. However, samples at the beginning of each note were excluded because the change of bow generates important noise components which prevent a reliable sinusoidal signal to be extracted (see in this context also Saldanha and Corso, 1964). The number of samples that were excluded varied between notes and usually corresponded to the first 50 ms of the tone. However, for every note played, 150 to 300 raw frequency counts were retained. Next, the arithmetic mean of those frequencies was computed. In this paper, that mean is referred to as the *observed frequency* of the note.

In order to evaluate the reliability of the measurement procedure, the performance of two violinists (subjects 2 and 6) were analyzed twice. The greatest difference registered in repeated measurements of the observed frequencies of a single note was 0.35, 0.51, and 0.47 cents for, respectively, the lowest (C_4 - C_5), the middle (C_6 - C_5), and the highest (C_7 - C_6) scale location. For notes near A_4 (440 Hz), A_5 (880 Hz), and A_6 (1760 Hz), these differences correspond to a fundamental frequency change of about 0.09, 0.26, and 0.48 Hz, respectively.

The observed frequencies were analyzed from two points of view. The first one (method 1) assumed that the pitch of each note was derived by the violinist from the pitch of the immediately preceding note. The second point of view (method 2) assumed that when a violinist is performing a piece of music in the key of C, only the sequence C to D gives information about the major second. F and G, even when played in succession, indicate the intonation of the fourth and fifth, respectively, since the musician refers all tones to the keynote and thus is really judging C to F and C to G. Method 1 corresponds to the approach adopted by Greene (1937a,b), whereas method 2 follows the procedure used by Nickerson (1949). Hence, the observed interval sizes *between adjacent notes* in method 1, and the observed interval sizes *between each note and the keynote C* in method 2, were both expressed in cents. These interval scores in cents constituted the basic data for further statistical analysis.

Next, differences in cents were calculated between ob-

TABLE I. Example of the computation of the mean absolute difference (MAD) in cents between observed and theoretical musical intervals. Method 1: Intervals between adjacent notes.

	Note and interval														MAD in cents
	C ₄	C ₄ -D ₄	D ₄	D ₄ -E ₄	E ₄	E ₄ -F ₄	F ₄	F ₄ -G ₄	G ₄	G ₄ -A ₄	A ₄	A ₄ -B ₄	B ₄	B ₄ -C ₅	C ₅
Observed frequency in hertz															
Violinist 1—trial 1	261.88		295.18		331.00		349.68		392.42		442.99		497.76		524.57
Observed interval between adjacent notes in cents		207.21		198.30		95.05		199.64		209.85		201.79		90.85	
Theoretical interval in cents															
Pythagorean tuning		203.91		203.91		90.22		203.91		203.91		203.91		90.22	
Equal tuning		200.00		200.00		100.00		200.00		200.00		200.00		100.00	
Just intonation		203.91		182.40		111.73		203.91		182.40		203.91		111.73	
Observed interval minus theoretical interval in cents															
Pythagorean tuning		3.30		− 5.61		4.82		− 4.27		5.94		− 2.12		0.62	3.81
Equal tuning		7.21		− 1.70		− 4.95		− 0.36		9.85		1.79		− 9.15	5.00
Just intonation		3.30		15.90		− 16.68		− 4.27		27.45		− 2.12		− 20.88	12.94

served and corresponding theoretical interval sizes in Pythagorean, equally tempered, and just intonations. Finally, for each ($N = 144$) individual scale played, the arithmetic mean of absolute differences between observed and theoretical interval sizes was calculated as an "error" score for each of the three tuning models. The resulting means (in cents) are called *mean absolute departures* (MAD values) in this paper.

Table I and Table II present a summary of the computation of the MAD values according to method 1 and method 2, respectively. The same observed frequencies were analyzed in both examples. In the last column of Tables I and II, three MAD values are tabulated: one for Pythagorean tuning, one for equally tempered tuning, and one for just intonation. The smaller the MAD value, the better the performances correspond with the scale model.

Since the violinists played adjacent scales without interruption (Fig. 1), and the data were analyzed per scale, the C's on transition from one scale to another have always been incorporated into the MAD values of two scales. Thus the frequency of C₅ was incorporated in the calculation of the MAD value of the C₄-C₅ scale as well as in the calculation of the MAD value of the C₅-C₆ scale.

II. RESULTS

The arithmetic mean of the fundamental frequencies to which the violinists tuned the A₄ string of their instrument at the beginning of the experiment, was 441.01 Hz (s.d. = 0.70 Hz). The range extended from 439.82–442.23 Hz (i.e., an interval of 9.46 cents).

After the experiment the violinists were asked to dem-

TABLE II. Example of the computation of the mean absolute difference (MAD) in cents between observed and theoretical musical intervals. Method 2: Intervals from keynote.

	Note								MAD in cents
	C ₄	D ₄	E ₄	F ₄	G ₄	A ₄	B ₄	C ₅	
Observed frequency in hertz									
Violinist 1—trial 1	261.88	295.18	331.00	349.68	392.42	442.99	497.76	524.57	
Observed interval from keynote in cents	...	207.21	405.51	500.56	700.20	910.05	1111.84	1202.69	
Theoretical interval in cents									
Pythagorean tuning	...	203.91	407.82	498.04	701.95	905.86	1109.77	1200.00	
Equal tuning	...	200.00	400.00	500.00	700.00	900.00	1100.00	1200.00	
Just intonation	...	203.91	386.31	498.04	701.95	884.35	1088.26	1200.00	
Observed interval minus theoretical interval in cents									
Pythagorean tuning	...	3.30	- 2.31	2.52	- 1.75	4.19	2.07	2.69	2.69
Equal tuning	...	7.21	5.51	0.56	0.20	0.05	11.84	2.69	5.44
Just intonation	...	3.30	19.20	2.52	- 1.75	25.70	23.58	2.69	11.25

TABLE III. Arithmetic means (M) over trials and standard errors (SE) in cents of mean absolute departure from theoretical interval sizes in the Pythagorean (PT), the equally tempered (ET), and the just intonation (JI) scale systems. For every violinist the lowest arithmetic mean within each octave interval is printed in italics. Intervals were defined between adjacent notes (method 1). *Note:* $N = 3$ for cel entries, $N = 24$ for row totals, and $N = 18$ for column totals.

		Violinist								Over all violinists
		1	2	3	4	5	6	7	8	
Ascending scales										
C_4-C_5	PT M	6.7	4.3	8.8	6.1	5.6	12.3	9.5	5.2	7.3
	s.e.	1.7	0.2	3.9	0.7	1.0	1.5	0.1	1.3	0.7
	ET M	9.4	7.1	7.3	6.0	6.6	8.8	14.0	4.5	8.0
	s.e.	2.4	1.6	2.4	1.6	0.3	2.5	0.1	1.0	0.8
	JI M	17.3	14.7	13.1	14.1	14.2	10.2	21.0	12.7	14.7
	s.e.	2.4	1.6	0.8	2.1	0.3	1.9	0.9	1.3	0.8
C_5-C_6	PT M	10.3	7.7	6.9	3.7	6.8	14.0	8.8	6.3	8.0
	s.e.	0.6	1.6	0.9	0.2	0.7	0.8	0.1	1.2	0.7
	ET M	8.0	8.9	5.3	5.9	5.5	10.9	11.6	7.8	8.0
	s.e.	1.7	1.8	0.5	0.8	0.7	1.0	0.4	0.8	0.6
	JI M	14.0	14.6	11.3	13.8	10.0	12.4	18.2	15.2	13.7
	s.e.	2.1	2.3	0.2	0.5	0.6	1.2	0.2	1.1	0.6
C_6-C_7	PT M	11.1	7.7	8.9	8.5	6.0	19.2	9.0	6.7	9.6
	s.e.	1.0	1.3	0.4	2.0	1.4	3.7	0.8	2.0	1.0
	ET M	7.4	6.3	9.0	7.1	9.5	15.1	9.1	9.1	9.1
	s.e.	0.6	1.8	0.1	1.4	2.4	3.6	2.3	2.7	0.8
	JI M	10.1	12.2	14.2	12.8	17.0	10.2	14.4	14.3	13.1
	s.e.	1.9	2.5	1.6	0.8	2.0	2.6	2.2	2.0	0.7
Descending scales										
C_5-C_4	PT M	8.7	6.2	10.1	6.1	10.1	13.0	5.2	4.0	7.9
	s.e.	2.1	0.4	1.6	0.6	0.4	1.2	0.8	0.2	0.7
	ET M	9.1	5.7	9.4	6.8	7.1	9.7	7.6	6.4	7.7
	s.e.	1.9	0.9	1.8	0.5	1.8	0.6	1.6	1.5	0.5
	JI M	18.2	12.2	13.6	15.3	13.4	10.3	15.6	13.7	14.0
	s.e.	2.0	1.7	0.9	0.5	1.6	1.5	2.7	1.3	0.6
C_6-C_5	PT M	9.5	5.3	9.6	6.2	7.9	12.8	4.8	6.0	7.8
	s.e.	2.8	0.4	1.5	2.0	1.4	1.1	0.3	0.5	0.7
	ET M	9.4	5.9	7.2	9.2	9.6	11.3	7.2	8.1	8.5
	s.e.	3.1	1.4	0.4	1.2	1.6	0.4	0.5	0.5	0.5
	JI M	15.1	13.3	12.5	17.2	16.8	14.1	16.2	13.9	14.9
	s.e.	2.6	1.5	2.3	1.0	1.7	1.0	1.1	0.6	0.6
C_7-C_6	PT M	7.9	12.1	12.7	9.6	8.0	11.3	12.2	10.4	10.5
	s.e.	1.6	1.6	1.9	0.8	0.4	2.2	1.3	1.1	0.6
	ET M	10.2	12.7	15.7	10.3	10.4	13.9	16.1	10.8	12.5
	s.e.	0.8	2.7	2.3	0.4	1.2	1.7	1.8	0.9	0.7
	JI M	13.2	11.6	20.3	16.7	18.2	20.1	24.5	16.8	17.7
	s.e.	2.4	4.0	1.8	1.1	1.7	2.5	1.3	1.5	1.0
Over all scales										
	PT M	9.0	7.2	9.5	6.7	7.4	13.8	8.2	6.5	8.5
	s.e.	0.7	0.7	0.8	0.6	0.5	0.9	0.6	0.6	0.3
	ET M	8.9	7.8	9.0	7.5	8.1	11.6	10.9	7.8	9.0
	s.e.	0.7	0.8	1.0	0.5	0.7	0.9	0.9	0.7	0.3
	JI M	14.7	13.1	14.1	15.0	14.9	12.9	18.3	14.4	14.7
	s.e.	1.0	0.9	0.9	0.5	0.8	1.1	0.9	0.6	0.3

onstrate the fingering they had used. Without exception, they used the fingering indicated in Fig. 1.

The main results of the present study are summarized in Table III and Table IV, in which the arithmetic means (M) of the MAD values over trials and the associated standard errors (s.e.) are tabulated. For example, the value 6.7 in the left upper corner of Table III is the arithmetic mean of the three Pythagorean MAD values yielded by the three trials of violinist 1 for the ascending scale C_4-C_5 . Table IV is arranged in the same way, but here the intervals were measured from the keynote of the respective scales. The standard

errors in the row margins of Tables III and IV refer to the variability in MAD values between violinists. Analogously, the standard errors in the cell entries and column margins refer to the variability within violinists.

Visual inspection of Tables III and IV shows that violin performances always conform most closely with Pythagorean or equally tempered intonations, but for two exceptions in each table. From the analysis of the raw MAD values computed according to method 1, it appears that for 56.3% of the 144 scales played, the Pythagorean MAD value is smaller than the corresponding MAD values for the equally tem-

TABLE IV. Arithmetic means (M) over trials and standard errors (s.e.) in cents of mean absolute departure from theoretical interval sizes in the Pythagorean (PT), the equally tempered (ET), and the just intonation (JI) scale systems. For every violinist the lowest arithmetic mean within each octave interval is printed in *italics*. Intervals were defined from keynote (method 2). Note. $N = 3$ for cel entries, $N = 24$ for row totals, and $N = 18$ for column totals.

		Violinist								Over all violinists
		1	2	3	4	5	6	7	8	
Ascending scales										
C_4-C_5 PT	M	4.5	6.8	7.7	5.9	8.0	26.4	8.9	6.1	9.3
	s.e.	1.1	1.1	1.7	0.5	1.9	2.7	1.0	1.7	1.4
ET	M	6.4	10.4	7.6	7.4	7.1	22.5	12.8	4.3	9.8
	s.e.	0.6	1.1	1.9	2.0	0.4	2.7	1.7	0.5	1.2
JI	M	12.5	15.9	12.3	13.5	10.3	17.2	18.1	9.5	13.7
	s.e.	0.8	1.1	2.8	2.0	0.2	2.7	1.0	0.8	0.8
C_5-C_6 PT	M	7.3	7.3	6.4	5.1	7.1	11.7	14.4	6.2	8.2
	s.e.	0.4	2.2	0.8	0.3	0.8	1.9	3.3	1.4	0.8
ET	M	9.5	10.8	4.8	4.7	6.5	12.6	18.3	8.3	9.4
	s.e.	1.6	2.5	1.4	0.2	2.0	2.1	5.7	1.7	1.1
JI	M	15.0	15.9	8.7	10.6	11.0	16.6	23.6	13.8	14.4
	s.e.	1.7	2.6	1.2	0.5	2.5	2.0	3.3	2.1	1.1
C_6-C_7 PT	M	8.9	10.8	11.2	10.2	5.1	20.9	11.8	6.7	10.7
	s.e.	2.7	1.2	1.1	0.6	1.2	8.4	1.1	2.4	1.3
ET	M	7.9	9.5	11.4	7.4	7.3	21.9	14.3	7.6	10.9
	s.e.	1.2	0.6	1.7	0.5	1.1	10.0	3.4	0.9	1.5
JI	M	9.6	12.6	15.6	8.1	13.0	25.0	19.3	11.6	14.3
	s.e.	2.2	2.1	2.4	0.8	0.9	11.2	2.2	0.7	1.7
Descending scales										
C_5-C_4 PT	M	8.8	5.4	15.7	5.9	12.4	26.0	7.7	3.4	10.7
	s.e.	0.9	0.9	3.6	1.1	1.3	2.1	2.6	0.3	1.5
ET	M	10.7	5.2	11.9	6.8	8.5	22.1	5.5	6.0	9.6
	s.e.	1.9	0.3	3.4	1.1	1.3	2.1	0.4	1.0	1.2
JI	M	16.2	10.4	9.4	12.3	8.6	16.9	9.9	11.1	11.8
	s.e.	2.1	0.5	1.8	1.7	1.3	2.0	1.5	1.2	0.8
C_6-C_5 PT	M	10.8	9.5	13.2	5.4	5.3	12.3	11.0	7.2	9.3
	s.e.	2.8	2.6	2.3	1.3	1.0	2.3	0.4	1.4	1.0
ET	M	12.6	11.7	10.2	6.7	6.9	10.8	5.4	11.1	9.4
	s.e.	4.0	3.5	1.7	0.9	0.6	1.2	0.6	1.3	0.8
JI	M	18.1	17.6	8.1	12.6	12.4	13.1	9.1	16.4	13.4
	s.e.	4.2	3.2	1.3	1.0	0.6	1.3	1.9	1.4	1.0
C_7-C_6 PT	M	15.1	9.2	9.3	9.5	6.7	14.0	11.1	11.6	10.8
	s.e.	4.1	1.1	0.5	0.7	0.7	6.0	0.7	1.9	1.0
ET	M	18.3	11.3	11.4	7.8	9.8	17.6	13.6	14.7	12.9
	s.e.	4.8	1.9	1.7	0.8	1.3	6.3	0.6	2.7	1.2
JI	M	23.2	15.8	16.1	11.4	15.7	22.9	17.3	20.3	17.8
	s.e.	5.1	2.1	2.0	0.3	0.9	6.3	1.9	2.4	1.2
Over all scales										
PT	M	9.2	8.2	10.6	7.0	7.4	18.6	10.8	6.9	9.8
	s.e.	1.1	0.7	1.0	0.6	0.7	2.2	1.0	0.8	0.5
ET	M	10.9	9.8	9.5	6.8	7.7	17.9	11.5	8.7	10.4
	s.e.	1.3	0.9	1.0	0.5	0.5	2.1	1.3	1.0	0.5
JI	M	15.8	14.7	11.7	11.4	11.8	18.6	16.2	13.8	14.2
	s.e.	1.5	0.9	1.0	0.6	0.7	2.1	1.4	1.0	0.5

pered and the just intonations, respectively. On the other hand, the equally tempered MAD value is smaller than the corresponding MAD values for the other scale models in 38.2% of the scales only. The 95% confidence intervals for those two proportions do not overlap. Hence, it may be stated that more frequently the performed scales correspond better with Pythagorean than with equally tempered intonation ($p < 0.05$). The analysis of the raw MAD values, computed according to method 2 yielded a similar result. In this case the p value is even smaller ($p < 0.001$).

Figure 2 shows the arithmetic means (over trials and subjects) of the MAD values for the three scale models. As can be seen, methods 1 and 2 yield analogous results. For both methods, the arithmetic means of the Pythagorean and the equally tempered MAD values are significantly ($p < 0.01$) smaller than the arithmetic mean for the just intonation MAD values. The differences between the arithmetic means of the Pythagorean and the equally tempered MAD values are not significant ($p > 0.45$).

Figure 3 shows for each subject the number of scales

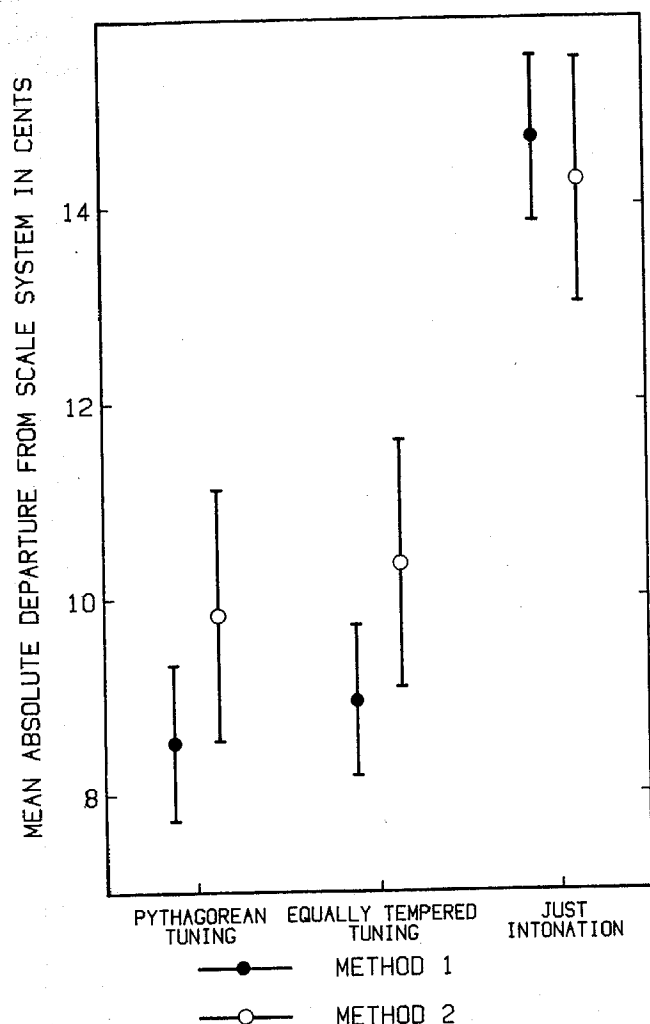


FIG. 2. Arithmetic mean of absolute departures from three scale systems by eight violinists playing diatonic scales of C major. Each data point represents the arithmetic mean of 144 MAD values expressing the arithmetic mean of absolute differences between performed and theoretical musical intervals within a single scale. In method 1 intervals were defined between adjacent notes; in method 2 intervals were defined from keynote. Vertical bars show the 99% confidence interval for the population mean.

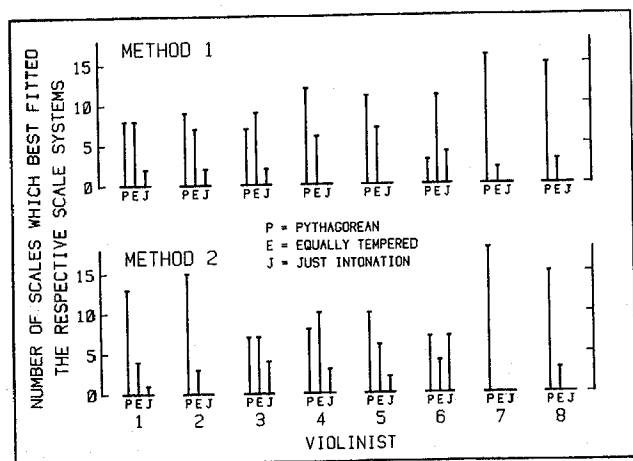


FIG. 3. Number of scales that best fit Pythagorean, equally tempered, and just intonation scales, respectively. For each individual scale played, the arithmetic mean of absolute differences between observed and theoretical interval sizes was calculated as an "error" score for each of the three scale systems. In method 1 intervals were defined between adjacent notes; in method 2 intervals were defined from keynote. Each violinist played 18 diatonic scales in C major.

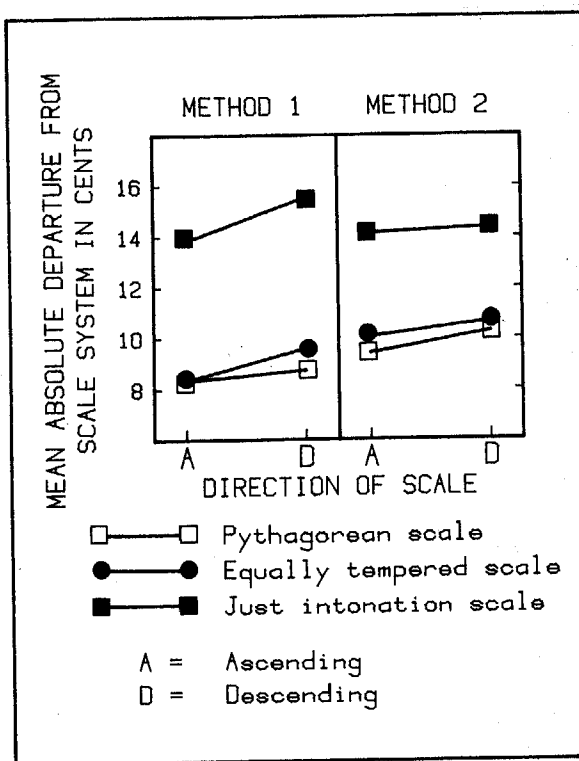


FIG. 4. Arithmetic mean of absolute departures from three scale systems by eight violinists playing diatonic scales of C major as a function of the direction of the musical sequence. Each data point represents the arithmetic mean of 72 MAD values expressing the arithmetic mean of absolute differences between performed and theoretical musical intervals within a single scale. In method 1 intervals were defined between adjacent notes; in method 2 intervals were defined from the keynote.

played which best fit the respective intonation models. Apart from one exception (viz., violinist 6), single scales always fit just intonation most poorly.

Figure 4 shows that descending scales always yield larger mean MAD values than ascending ones. Analogously, Fig. 5 shows upper scales always yield larger mean MAD values than lower and middle ones, respectively.

In order to investigate whether the trends in Figs. 4 and 5 are statistically significant, for each method of analysis, the raw MAD values were subjected to three analyses of variance: one for the Pythagorean data, one for the equally tempered data, and one for the just intonation data. The model was a 2 (scale directions) $\times$ 3 (scale locations) $\times$ 8 (subjects) mixed randomized block factorial design with three replications within each of the treatment combinations (Winer *et al.*, 1991). Since there was no evidence of the existence of any interaction between violinists and direction of scale, or between violinists and scale location, and there was also no evidence of a triple interaction between direction of scale, scale location, and violinists, these three interaction terms were pooled with the experimental error, thus making the *F* test for treatment effects more powerful.

Apart from one exception direction of scale never yields a significant ($p < 0.01$) main effect. Only the just intonation MAD values—for method 1—are significantly greater in descending scales than in ascending ones [$F(1,131) = 9.12$,

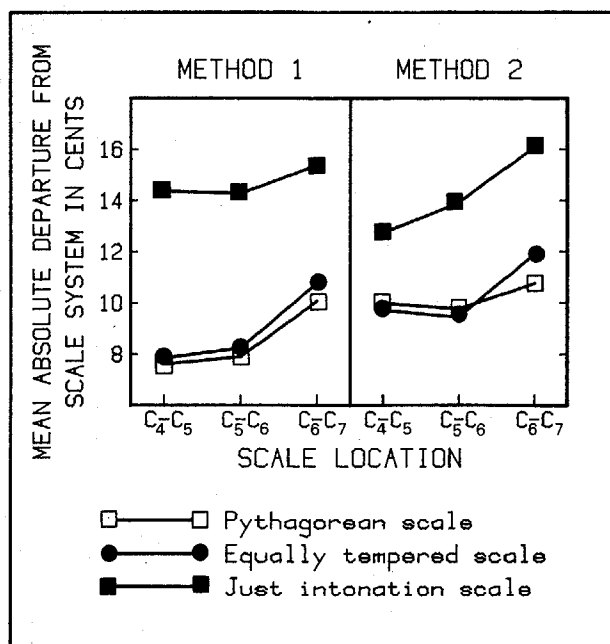


FIG. 5. Arithmetic mean of absolute departures from three scale systems by eight violinists playing diatonic scales of C major as a function of scale location. Each data point represents the arithmetic mean of 48 MAD values expressing the arithmetic mean of absolute differences between performed and theoretical musical intervals within a single scale. In method 1 intervals were defined between adjacent notes; in method 2 intervals were defined from the keynote.

$p = 0.003$]. Scale location, on the contrary, always yields a significant ($p < 0.01$) main effect, apart from two exceptions {the just intonation MAD values of method 1 [$F(2,131) = 1.66, p = 0.195$] and, the Pythagorean MAD values of method 2 [$F(2,131) = 2.16, p = 0.120$]}. It appears from Tuckey's studentized range tests that in all cases of a significant direction effect, the arithmetic means of the MAD values of the upper scale location are significantly larger than the corresponding arithmetic means of the lower and middle scale locations ($p < 0.05$). The interaction between scale direction and scale location (Fig. 6) is significant for the just intonation data of both methods and for the equally tempered data of method 1 [$F(2,131) = 7.11, p = 0.001, F(2,131) = 3.76, p = 0.026$, and $F(2,131) = 5.39, p = 0.006$, respectively].

In the above approach violin performances were analyzed, based on the assumption that a musical scale should be considered as an undivisible musical unit. The results of this analysis can be summarized as follows: (1) Violin performances approximate Pythagorean and equally tempered intonations better than just intonation; and (2) Pythagorean and equally tempered intonations are equally good models to describe performances. With reference to the second point, it could be objected that the scales played more frequently deviate less from Pythagorean intonation than from equally tempered intonation. However, the arithmetic means of departures from these two scale models does not differ significantly. Hence, according to measurement theory (Coombs,

1964), greater weight must be attached to the second result than to the first one, because it relates to a measure that utilizes more information which is relevant to the question.

Thus, when restricting to the two best fitting models (the Pythagorean and the equally tempered) it can be concluded that: (1) ascending and descending scales fit both models equally well; (2) apart from one exception, the upper scales (C_6-C_7) fit both models less well than the middle scales (C_5-C_6); (3) lower (C_4-C_5) and middle scales are equally well approximated by both scale models; and (4) apart from one exception, there is no significant interaction between scale direction and scale location.

However, analysis of violin performances can also be focused on the size of *specific intervals*. Table V shows arithmetic means of observed interval sizes between adjacent notes, and Table VI shows arithmetic means of interval sizes from the keynote. Since it appears from the previous analysis that performances are poorly modeled by just intonation, the

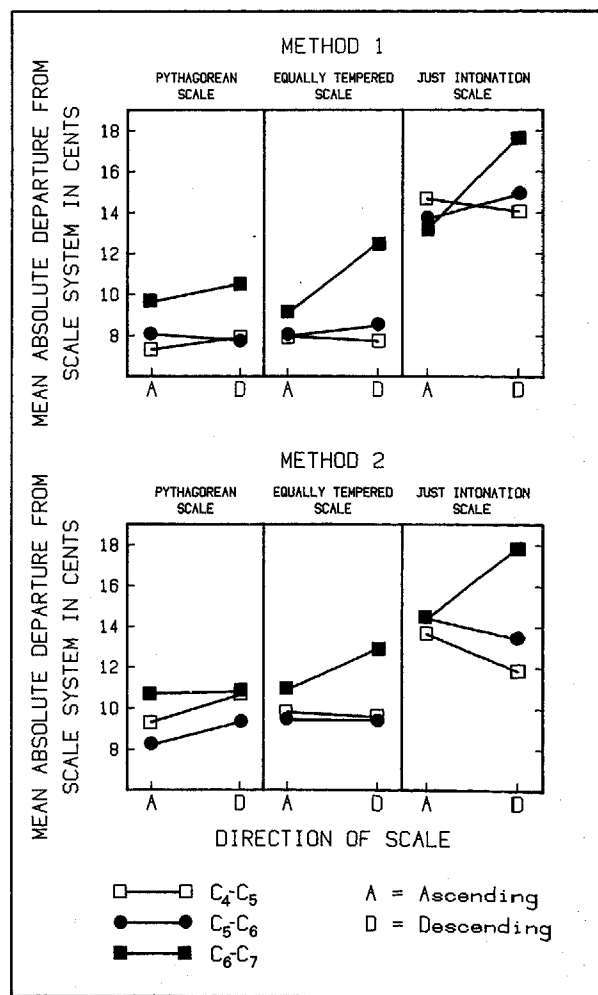


FIG. 6. Arithmetic mean of absolute departures from three scale systems by eight violinists playing diatonic scales of C major as a function of direction of musical sequence and scale location. Each data point represents the arithmetic mean of 24 MAD values expressing the arithmetic mean of absolute differences between performed and theoretical musical intervals within a single scale. In method 1 intervals were defined between adjacent notes; in method 2 intervals were defined from the keynote.

TABLE V. Arithmetic means (M) of interval sizes and standard errors (s.e.) in cents over violinists and trials. Method 1: Intervals between adjacent notes.

		C-D	D-E	E-F	Interval F-G	G-A	A-B	B-C
Ascending scales								
	C ₄ -C ₅ M	198.0 ^a	204.2	95.0	201.4	203.5	204.7	92.0
	s.e.	1.5	2.0	2.4	1.6	1.6	1.5	2.0
	C ₅ -C ₆ M	206.2	201.3	96.9	200.7	202.0	202.2	94.8
	s.e.	1.6	1.6	2.1	1.8	1.7	2.7	2.2
	C ₆ -C ₇ M	204.8	198.1	101.4	201.7	198.4	199.7	96.3
	s.e.	2.7	2.5	3.5	2.3	1.8	1.9	2.5
	C ₄ -C ₇ M	203.0	201.2	97.8	201.3	201.3	202.2	94.4
	s.e.	1.2	1.2	1.6	1.1	1.0	1.2	1.3
Descending scales								
	C ₅ -C ₄ M	194.7	209.0	91.3	201.9	200.9	199.3	99.9
	s.e.	2.4	1.9	1.9	1.4	1.7	1.7	2.6
	C ₆ -C ₅ M	200.0	206.1	96.0	202.2	202.0	201.9	91.9
	s.e.	2.0	1.7	2.9	2.0	1.5	1.7	2.5
	C ₇ -C ₆ M	207.1	203.2	94.6	205.4	199.1	211.5	77.3
	s.e.	1.6	2.0	2.5	2.6	1.8	2.2	3.5
	C ₇ -C ₄ M	200.6	206.1	94.0	203.2	200.7	204.2	89.6
	s.e.	1.3	1.1	1.4	1.2	1.0	1.3	2.0
Ascending and descending scales								
	C ₄ -C ₅ M	196.4	206.6	93.2	201.7	202.2	202.0	95.9
	s.e.	1.4	1.4	1.6	1.1	1.2	1.2	1.7
	C ₅ -C ₆ M	203.1	203.7	96.4	201.5	202.0	202.1	93.2
	s.e.	1.4	1.2	1.8	1.4	1.1	1.6	1.7
	C ₆ -C ₇ M	206.0	200.7	98.0	203.5	198.8	205.6	86.8
	s.e.	1.6	1.6	2.2	1.7	1.2	1.7	2.6
	C ₄ -C ₇ M	201.8	203.7	95.9	202.2	201.0	203.2	92.0
	s.e.	0.9	0.8	1.1	0.8	0.7	0.9	1.2

^a For ascending and descending scales separately: $N = 24$ for single scales, and $N = 72$ for total note range C₄-C₇. For pooled ascending and descending scales: $N = 48$ for single scales, and $N = 144$ for total note range C₄-C₇.

TABLE VI. Arithmetic means (M) of interval sizes and standard errors (s.e.) in cents over violinist and trials. Method 2: Intervals from keynote.

		C-D	C-E	C-F	Interval C-G	C-A	C-B	C-C
Ascending scales								
	C ₄ -C ₅ M	198.0 ^a	402.2	497.2	698.6	902.1	1106.8	1198.8
	s.e.	1.5	2.9	1.8	2.1	3.1	3.4	2.8
	C ₅ -C ₆ M	206.2	407.5	504.4	705.1	907.1	1109.3	1204.1
	s.e.	1.6	1.8	2.3	1.8	2.0	2.4	2.3
	C ₆ -C ₇ M	204.8	402.9	504.3	706.0	904.4	1104.1	1200.4
	s.e.	2.7	2.4	3.6	3.4	2.6	2.5	3.1
	C ₄ -C ₇ M	203.0	404.2	502.0	703.2	904.6	1106.8	1201.1
	s.e.	1.2	1.4	1.6	1.5	1.5	1.6	1.6
Descending scales								
	C ₅ -C ₄ M	194.7	403.7	495.0	697.0	897.9	1097.2	1197.1
	s.e.	2.4	3.4	2.4	2.0	3.3	2.9	2.3
	C ₆ -C ₅ M	200.0	406.1	502.1	704.3	906.4	1108.3	1199.9
	s.e.	2.0	2.2	3.2	2.0	2.3	2.2	2.3
	C ₇ -C ₆ M	207.1	410.4	504.9	710.3	909.4	1120.9	1198.2
	s.e.	1.6	2.5	3.3	2.0	1.9	3.0	2.6
	C ₇ -C ₄ M	200.6	406.7	500.7	703.9	904.6	1108.6	1198.4
	s.e.	1.3	1.6	1.8	1.3	1.6	1.9	1.4
Ascending and descending scales								
	C ₄ -C ₅ M	196.4	403.0	496.1	697.8	900.0	1102.0	1197.9
	s.e.	1.4	2.2	1.5	1.5	2.2	2.3	1.8
	C ₅ -C ₆ M	203.1	406.8	503.2	704.7	906.7	1108.8	1202.0
	s.e.	1.4	1.4	2.0	1.3	1.5	1.6	1.6
	C ₆ -C ₇ M	206.0	406.6	504.6	708.1	906.9	1112.5	1199.3
	s.e.	1.6	1.8	2.4	2.0	1.6	2.3	2.0
	C ₄ -C ₇ M	201.8	405.5	501.3	703.5	904.6	1107.8	1199.7
	s.e.	0.9	1.1	1.2	1.0	1.1	1.3	1.1

^a For ascending and descending scales separately: $N = 24$ for single scales, and $N = 72$ for total note range C₄-C₇. For pooled ascending and descending scales: $N = 48$ for single scales, and $N = 144$ for total note range C₄-C₇.

discussion will refer only to Pythagorean and equally tempered intonations from now on.

The intervals that demand attention when contrasting Pythagorean and equally tempered intonations are the minor second and the major third. Indeed, the size of those intervals differs considerably in both intonations: 9.8 cents for the minor second and 7.8 cents for the major third.

Table V shows that, apart from one exception arithmetic means of observed interval sizes for minor seconds (E-F and B-C) are always larger than in the Pythagorean model (90.22 cents). Analogously, the arithmetic means of observed interval sizes of minor seconds are, apart from one exception, always smaller than in the equally tempered model (100 cents). Finally, over ascending and descending scales, arithmetic means of observed interval sizes for the minor second always lie *between* the normative interval sizes of the Pythagorean and the equally tempered intonations, apart from one exception. The analysis of major seconds, yielded similar results. Over all scale directions and scale locations, observed interval sizes of the major second always lie *between* the interval sizes of the Pythagorean and the equally tempered intonations. On the major third, this paper can only report indirectly. Indeed, in the diatonic scale of C major, the major third cannot be *observed* from two notes played immediately after each other. The size of the major third can only be *calculated* by summation of the sizes of two adjacent major seconds from the keynote C (or from the notes F or G). Table VI shows that computed mean interval sizes of the major third lay *between* the mean interval sizes of Pythagorean and equally tempered intonations. The results for the major thirds F-A and G-B are similar to those for the major third C-E.

The last column in the lower part of Table VI shows the arithmetic means of computed interval sizes for C octaves [i.e., the octave intervals C_i-C_{i+1} ($i=4,5,6$) and C_i-C_{i-1} ($i=5,6,7$)] over violinists and trials. As can be seen, computed interval sizes deviate only slightly from theoretical interval size (1200 cents). The upper part of Fig. 7 shows the distribution of differences between computed interval size and 1200 cents for all C octaves ($N=144$). The arithmetic mean of the distribution is -0.3 cent. The standard deviation (12.4 cents) reveals that the size of the computed interval sizes may vary considerably. The arithmetic means of the C-octave departures from 1200 cents for the eight violinists are: 2.9, 10.5, -6.7 , 1.3, -3.8 , -3.5 , -3.1 , and -0.8 cents, respectively.

A 2(scale directions) $\times$ 3(scale locations) $\times$ 8(subjects)-mixed randomized block factorial analysis of variance of the sizes of the C-octave departures from 1200 cents revealed that the mean departure score for ascending scales (1.1 cents) did not differ significantly from the mean departure score for descending scales (1.6 cents) [$F(1,131) = 2.23$, $p = 0.138$]. Furthermore, mean departure scores did not differ significantly for different scale locations [$F(2,131) = 1.66$, $p = 0.194$]. Neither was the interaction between scale direction and scale location significant [$F(2,131) = 0.80$, $p = 0.453$].

The distribution of departure scores of non-C-octave intervals from 1200 cents is shown in the lower part of Fig. 7.

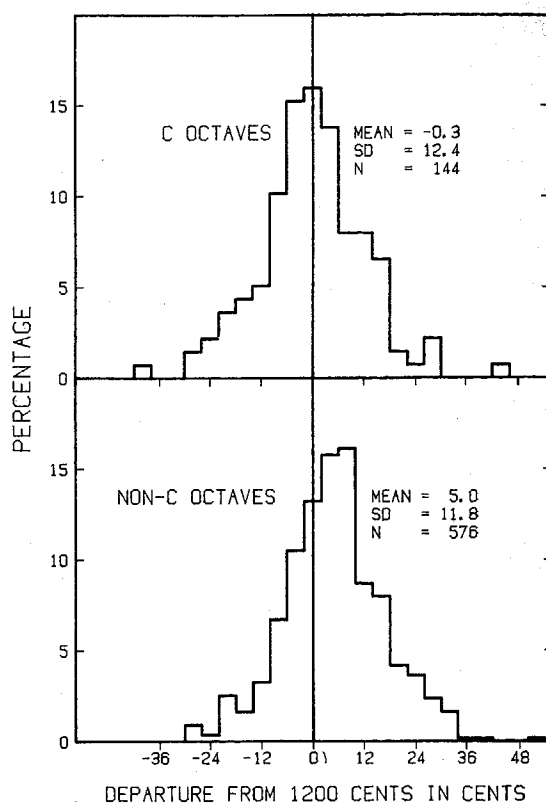


FIG. 7. Distribution of deviations from 1200 cents in C-octave intervals (i.e., intervals between two C's) computed from the performances of eight violinists playing diatonic scales of C major. The distribution in the upper part of the figure relies upon 144 C-octave intervals in the note range C_4-C_7 . The lower part distribution relies upon 576 non-C-octave intervals in the same note range. Cases of octaves contracted occur at the left of the 0 departure, cases of stretched octaves, at the right.

The arithmetic mean of the distribution is 5.0 cents (s.d. = 11.8 cents) and the arithmetic means of departures of non-C-octave intervals from 1200 cents for the eight violinists are: 2.0, 13.1, 5.8, 0.1, 1.4, 8.2, 5.3, and 4.1, respectively. An analysis of variance revealed that the difference between the mean stretch for ascending (3.3 cents) and descending (6.7 cents) non-C octaves is significant [$F(1,565) = 13.76$, $p = 0.0002$]. However, the difference between the mean stretch for non-C octaves localized between D_4 and B_5 and mean stretch for non-C octaves between D_5 and B_6 is not significant [$F(1,565) = 2.86$, $p = 0.091$]. Thus a shift in scale location is not accompanied by a change in physical octave extent. The interaction between direction of musical scale and location of scale is not significant [$F(1,565) = 1.89$, $p = 0.170$].

Finally, intra-individual variability in interval size between adjacent notes was analyzed. For each violinist and for each of the 42 intervals between adjacent notes played on each trial, the total range (in cents) of the interval sizes across the three trials was computed. In Table VII the arithmetic means of those total ranges across subjects are tabulated. For example, the value 8.5 in the left upper corner of the table was obtained as follows. First, the interval size of C_4-D_5 was computed for each subject on each of the three

TABLE VII. Arithmetic mean (M) of total ranges and standard errors (s.e.) in cents for sizes of musical intervals observed in scales of C major played three times by eight violinists.

		C-D	D-E	E-F	Interval F-G	C-A	A-B	B-C
Ascending scales								
	C ₄ -C ₅ M	8.5	13.1	14.2	10.8	9.1	13.4	17.9
	s.e.	0.4	0.8	0.7	0.4	0.9	0.5	0.7
	C ₅ -C ₆ M	10.5	11.3	8.1	13.3	8.3	9.9	11.7
	s.e.	0.5	0.7	0.4	0.6	0.7	0.5	0.5
	C ₆ -C ₇ M	18.3	12.9	13.3	13.7	10.0	12.8	15.5
	s.e.	1.5	0.6	0.6	0.4	0.4	0.8	0.6
Descending scales								
	C ₅ -C ₄ M	9.2	11.5	16.6	8.5	9.6	12.9	16.0
	s.e.	0.5	0.5	0.9	0.6	0.4	0.7	1.1
	C ₆ -C ₅ M	7.9	10.8	18.8	15.4	8.3	12.3	19.3
	s.e.	0.4	0.5	1.5	1.1	0.6	0.5	1.1
	C ₇ -C ₆ M	14.0	13.3	14.7	14.5	9.0	13.5	17.5
	s.e.	0.4	0.6	0.6	1.0	0.6	0.5	0.7

trials. Next, the total range of the three observed interval sizes was computed for each of the eight subjects. The arithmetic mean of these eight total ranges is 8.5 cents. The raw set of 336 total ranges was subjected to a 2(scale directions) $\times$ 3(scale locations) $\times$ 7(musical intervals) $\times$ 8(subjects)-mixed randomized block factorial design. Direction of scale and scale location yielded no significant effect. Neither was any interaction significant. On the contrary, the arithmetic means of total ranges for different musical intervals differ significantly [$F(6,299) = 3.74, p = 0.001$]. The arithmetic means of the total ranges for the minor second E-F and B-C are significantly larger than the arithmetic means of the total ranges for the major seconds (Tuckey's test, $p < 0.001$). There is also a significant difference between the mean total ranges for the eight violinists [$F(7,299) = 4.08, p < 0.001$].

III. DISCUSSION

The arithmetic mean of the starting frequencies of the open A₄ string (441.01 Hz) illustrates the actual custom to tune to a higher frequency than the current international standard tuning frequency for Western music (440 Hz). The total range of the starting A₄ frequencies over violinists (9.5 cents) corresponds almost perfectly with the arithmetic mean of the individual total ranges for the played A₄ during the experiment, namely, 9.8 cents (s.e. = 2.1 cents; $N = 8$). Corso (1954b), who examined a highly trained violinist obtained a root mean square of "error" scores (i.e., the intervals in cents between the frequency of a tuned violin tone and the frequency of a reference tone) of 6.13 cents. Analogously, Ward (1954) found that, for pure tones, octave judgments of trained musicians have a standard deviation averaging about 10 cents, and intersubject variability is two to five times as great. Furthermore, variability in starting A₄ frequencies shows that, in daily performance, a scale is not a sequence of fixed frequencies. Indeed, musical scales can be continuously slid up and down in pitch without distorting relative interval sizes (Siegel and Siegel, 1977; Burns and Ward, 1978). There is even experimental evidence that in-

terval information is stored in memory as an abstract entity and not as particular notes (Deutsch, 1978; Deutsch and Roll, 1974).

The conformity in the fingering is hardly surprising, considering the fact that the diatonic scale of C major is a standard exercise, and knowing that all subjects were educated in the same pedagogical tradition.

The results of the data analysis that makes use of *individual scales as units of analysis*, are in accordance with the previous literature. Indeed, one time it was found that unaccompanied violin performances were well approximated by Pythagorean intonation (Green, 1937a,b; Nickerson, 1949), and another time that performances corresponded fairly closely to equally tempered intonation (Ward, 1970). Analogously, Cornu and Mercadier (1871, 1873) found that the intonation of violinists did well approximate Pythagorean intonation, and was not modeled by just intonation at all. The findings of the present study are also in line with Mason's (1960) research on the solo performances of the members of two woodwind quintets. One of the quintets deviated least from Pythagorean intonation, whereas the other deviated least from equally tempered intonation. Analogously, the greatest departure occurred from just intonation. The results of the present study are also congruent with the finding of Moran and Pratt (1926) that average interval settings were more in agreement with interval sizes as defined in equally tempered intonation than with interval sizes as defined in just intonation. Clearly, the present study fails to support Willemze's (1973) claim that descending scales are played according to just intonation, and Benade's (1976, p. 295) assertion that just intonation is preferred when time and circumstances permit.

The result that departures from Pythagorean and the equally tempered intonations are not significantly different for ascending and descending scales corresponds also with the literature on wind instrumentalists, where no difference in intonation accuracy was found between ascending and descending directions (Mason, 1960; Duke, 1985).

The finding that the intonation on the highest scale location mostly deviates more from scale models than the intonation on the lower scale locations remains unexplained. The fact that it is harder to place fingers on the finger-board accurately in higher hand positions than in lower ones does not satisfy as an explanation. Indeed, the violinists played so slowly that they were able to make tone corrections at any moment, and obviously, those adjustments only influence the variability and not the arithmetic mean of raw frequency data. The greater deviation from scale models in the highest scale location cannot be explained by a higher differential threshold in that frequency range either. The power to discriminate between differences in frequency is quasiconstant in the interval 262–2986 Hz (Scharf and Buus, 1986). More specifically, Boring (1940) found that for a trained musician, the differential threshold for a tone of 261 Hz (C_4) is about 0.43 Hz (i.e., 2.85 cents) and for a tone of 1740 Hz (a flat A_6) it is about 2.76 Hz (i.e., 2.74 cents). Apparently, traditional scale models *as such* are less adequate to describe violin intonation in higher frequency ranges.

The finding that direction of scale was not associated with goodness of fit for scale models is in line with Greene (1937a,b).

The results of the analysis of *separate musical intervals*, *not considering the context of the scales in which they were played*, are consistent with Rakowski (1976, 1985), Abraham (1923), and Ward's (1970, p. 421) conclusion "that, on the average, the internal scale of musical pitch used by musicians corresponds fairly closely to equal temperament, but with a slight stretch that results in a tendency toward sharpening all tones relative to the tonic." Moreover, the present study allows one to specify further that the size of intervals played lies *between* interval sizes as defined in Pythagorean and equally tempered intonations. In the upper part of Fig. 8(a) intervals between adjacent notes in the diatonic scale of C major C_5 – C_6 are plotted on the abscissa, deviation in cents above and below theoretical interval sizes is plotted on the ordinate. The two curves in the upper part of Fig. 8(a) show the departures from Pythagorean and equal tempered scales. Intervals contracted relative to these scale systems, occur under the horizontal zero line, the expanded ones above it. If the intervals had been played in accordance with the scale models, all points would have fallen on the horizontal zero line, except for chance deviations. As can be seen, the departures from Pythagorean and the equally tempered tuning almost mirror each other. The lower plot of Fig. 8(a) shows mean departures of observed interval sizes from the arithmetic mean of the corresponding interval sizes in Pythagorean and equally tempered intonations. The largest mean departure from the mean interval size is smaller than 2 cents. Figure 8(b) is analogous to Fig. 8(a), but now the figure applies to the whole note range (C_4 – C_7) that was examined. In Fig. 8(c) and (d) musical intervals are defined relative to the tonic.

Finally, the octave intervals should be discussed. The set of all octave intervals that could be computed from the scales played shows a significant mean stretch of 3.9 cents ($N = 720$). It is interesting to note that Kolinski (1959) proposed a new system of temperament in which the musical

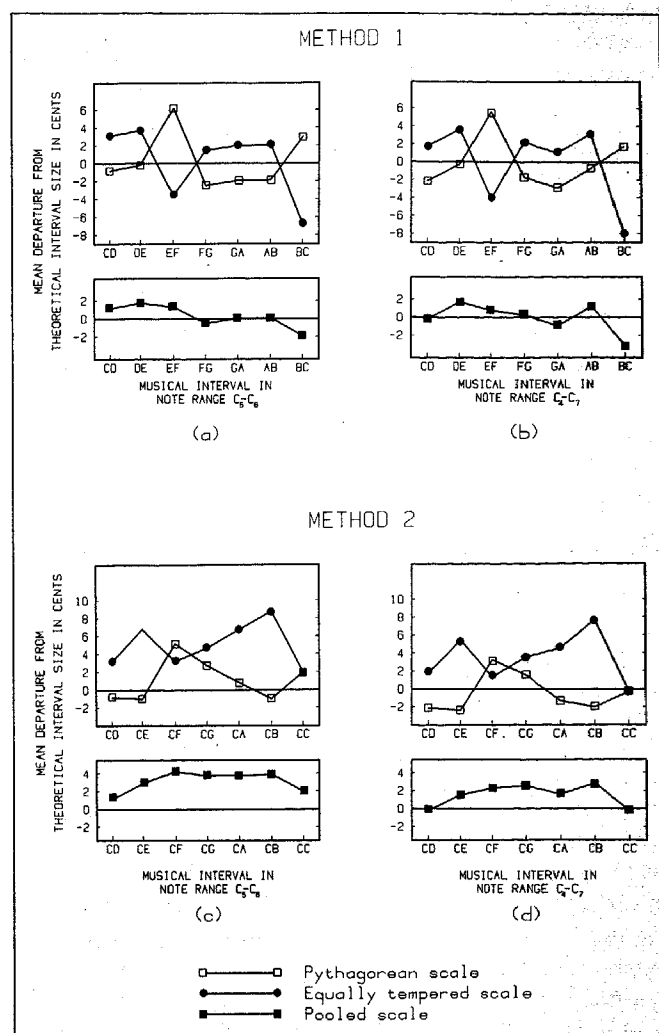


FIG. 8. (a) Upper plot. Arithmetic mean of departures from Pythagorean and equally tempered intonations by eight violinists playing diatonic scales of C major. Labels on the x axis represent ascending and descending intervals between adjacent notes in the note range C_5 – C_6 . Each data point indicates the arithmetic mean of 48 departure measurements. Lower plot. As upper plot, but departures are computed from the arithmetic mean of the corresponding interval sizes in Pythagorean and equally tempered intonations. (b) Same as (a) but labels on the x axis represent ascending and descending intervals in the note range C_4 – C_7 . Each data point indicates the arithmetic mean of 144 departure measurements. (c) Same as (a) for interval sizes defined from the keynote C. (d) Same as (b) but for interval sizes defined from the keynote C.

scale was deliberately stretched by about 3.5 cents per octave. The stretching of octave intervals has been known for a long time (Burns, 1974; Corso, 1954a; Railsback, 1938). Ward (1954) found that, through the midrange of up to frequencies of about 900 Hz, the frequency ratio of a subjective octave is about 2.02/1 (i.e., about 17.2 cents). This value is considerably larger than the 3.9 cents found in the present study. However, such a divergence is not exceptional. For example, Martin and Ward (1961) found that the mean stretch in the middle range of the piano scale is considerably smaller than that for subjective pure-tone octaves. Analogously, Sundberg and Lindqvist (1973) found a mean

stretch of only 4.5 and 5.2 cents for octave matching by a violinist for reference tones of 1050 and 2000 Hz (SPL of 65 dB). Moreover, for reference tones of 330 and 600 Hz, they found mean octave stretch values of -0.4 and -0.6 cents, respectively. Similarly, Corso (1954a) found that the size of the octave stretch by the violinist he observed did not fluctuate strongly and irregularly from one scale position to another, and was also around zero cents for more than one octave position.

The lack of octave stretching in C-octave intervals can be explained in the context of the literature on perceived tonal organization and absolute pitch (Deutsch, 1980; Deutsch and Feroe, 1981; Dowling, 1978, 1991; Dowling and Harwood, 1986; Harris and Siegel, 1975; Krumhansl and Kessler, 1982). This literature stresses that subjects use cognitive reference points based on their knowledge of the notes of the musical scale. Thus, in the present investigation in which diatonic scales of C major were played by highly trained musicians, it can be supposed that the keynote C was more important than the other notes of the scale. Of course, this speculative hypothesis should be investigated further for other keys. Moreover, in the present research, octave intervals were calculated by summing the sizes of adjacent intervals, whereas traditional studies on octave intervals always deal with situations in which the two notes delineating the octave interval are played immediately after each other, and without any musical context.

In the present study, the mean stretch for the lowest non-C-octave intervals was not significantly smaller than the mean stretch for the highest non-C-octave intervals. For violin performances also Corso (1954a) found that the magnitude of physical octaves did not increase with upward shifts in the location of octaves on the frequency continuum.

The phenomenon that non-C-octave intervals yield a significantly greater stretching in descending scales than in ascending scales is somewhat surprising. From the subjective experience of violinists one would rather have expected the opposite.

IV. CONCLUSIONS

The results of this study indicate that, when individual scales of C major are analyzed as a whole, (i) violin performances clearly fit Pythagorean and equally tempered intonations more precisely than the just intonation, and (ii) performances fit the Pythagorean and the equally tempered model almost equally well. When the analysis focuses on the size of the intervals between notes, not considering the context of the individual scales in which they were played, interval size is halfway between the interval sizes in Pythagorean and equally tempered intonations.

It does not seem meaningful to ponder any further about the question whether violin performances are more precisely approximated by Pythagorean intonation than by the equally tempered one. In special laboratory conditions, highly trained musicians may be able to discriminate with accuracy between isolated Pythagorean and equally tempered intervals synthesized on a computer. Experienced listeners can also have clear-cut preferences for musical sequences that

are presented for comparison in one of both tunings. Yet, in daily musical performance, those abilities are rather irrelevant, because violinists never play in exactly one tuning or the other, and because listeners pay attention to more important aspects of music. In addition, the total range of interval sizes for the same musical interval can easily reach 10 cents, which is larger than the greatest difference in interval size between adjacent notes in Pythagorean and equally tempered scales. Also, Ward (1970) showed that in real performances there is much variance in the frequency of notes being played, but that it is not usually noticed by listeners. Furthermore, it has been shown that a given interval may retain its character and identity in spite of considerable "mistuning" (Mursell, 1937). Finally, the acoustical properties of violin tones ensure that beats between their partials are almost inaudible, thus permitting the player great latitude in his choice of pitch (Benade, 1976, p. 299). It is generally said, that the vibrato reduces the demands on accuracy in violin playing even more. However, experimental evidence shows quite clearly that the accuracy with which the pitch of a vibrato tone is perceived is not affected to any appreciable extent by the vibrato (Sundberg, 1982).

Apparently, violin performances can be described equally well from intervals between the adjacent notes as from intervals from the keynote.

An intriguing finding of the present research is the lack of any stretching in C octaves within scales of C major. It would be of interest to know whether this phenomena can be generalized to other musical keys. Indeed, analogous results for other musical keys can provide further evidence for recent theories of tonal organization in music perception and more specifically for theories stressing the influence of musical knowledge on musical perception (see, i.e., Brown, 1988; Butler, 1983, 1989; Cross *et al.*, 1991; Krumhansl, 1990; Meyer, 1956, 1973; Narmour, 1990; Rosner and Meyer, 1986).

In the present study only scales in C major were played and these scales were performed very slowly without vibrato. Consequently, until further investigations are undertaken, it remains an open question whether the conclusions obtained for the diatonic scale of C major can be legitimately extended to cover other situations.

ACKNOWLEDGMENTS

The author is indebted to Noel Bovens and Rik Delabastita for technical support in measuring the frequency of violin tones, to Christel Snels for her assistance in drawing the figures, and to Godelieve Feyaerts for helping with collecting the data. The author would also like to thank Dr. Reinier Plomp, Dr. Robert W. Young, and three anonymous reviewers for their valuable suggestions and comments on an earlier draft of this paper. Finally, the author express special thanks to Monique Lioen for her significant support in getting this paper into its final shape.

¹ Burns and Ward (1982, p. 242) categorized this intonation as a variant of just intonation. Didymus' diatonic contains the same intervals as Ptolemy's diatonic syntonon, but with the minor tone (10/9) below the major tone (9/8) instead of the reverse.

² A cent is a unit of interval measure equaling the 1200th part of an octave. Therefore, if f_1 and f_2 are the frequencies of, respectively, the lower- and upper limit of an interval, then the interval size in cents is defined as $1200 \log_2 (f_2/f_1)$.

- Abraham, O. (1923). "Tonometrische Untersuchungen an einem deutschen Volkslied," *Psychol. Forsch.* 2, 1-22.
- Barbour, J. M. (1951). *Tuning and Temperament: A Historical Survey* (Michigan State College, East Lansing); reprinted 1972 (Da Capo, New York).
- Benade, A. H. (1976). *Fundamentals of Musical Acoustics* (Oxford U.P., New York), pp. 293-296.
- Boring, E. G. (1940). "The size of the differential limen for pitch," *Am. J. Psychol.* 53, 450-455.
- Brown, H. (1988). "The interplay of set content and temporal context in a functional theory of tonality perception," *Music Percept.* 5, 219-250.
- Burns, E. M. (1974). "Octave adjustment by non-Western musicians," *J. Acoust. Soc. Am. Suppl.* 1 56, S25.
- Burns, E. M., and Ward, W. D. (1982). "Intervals, scales, and tuning," in *The Psychology of Music*, edited by D. Deutsch (Academic, New York), pp. 241-269.
- Burns, E. M., and Ward, W. D. (1978). "Categorical perception—phenomenon or epiphenomenon. Evidence from experiments in the perception of melodic musical intervals," *J. Acoust. Soc. Am.* 63, 456-468.
- Butler, D. (1983). "The initial identification of tonal centers in music," in *Acquisition of Symbolic Skills*, edited by J. Sloboda and D. Rogers (Plenum, New York).
- Butler, D. (1989). "Describing the perception of tonality in music: a critique of the tonal hierarchy theory and a proposal for a theory of intervallic rivalry," *Music Percept.* 6, 219-242.
- Coombs, C. H. (1964). *A Theory of Data* (Wiley, New York).
- Cornu, A., and Mercadier, E. (1869). "Sur les intervalles musicaux," *C. R. Acad. Sci.* 68, 301-308, 424-427.
- Cornu, A., and Mercadier, E. (1871). "Sur les intervalles musicaux," *C. R. Acad. Sci.* 73, 178-183.
- Cornu, A., and Mercadier, E. (1872). "Sur les intervalles musicaux mélodiques," *C. R. Acad. Sci.* 74, 321-323.
- Cornu, A., and Mercadier, E. (1873). "Sur la mesure des intervalles musicaux," *C. R. Acad. Sci.* 76, 431-434.
- Corso, J. F. (1954a). "Scale position and performed melodic octaves," *J. Psychol.* 37, 297-305.
- Corso, J. F. (1954b). "Unison tuning of musical instruments," *J. Acoust. Soc. Am.* 26, 746-750.
- Cross, I., West, R., and Howell, P. (1991). "Cognitive correlates of tonality," in *Representing Musical Structure*, edited by P. Howell, R. West, and I. Cross (Academic, London), pp. 201-243.
- Deutsch, D. (1978). "Interactive effects in memory for harmonic intervals," *Percept. Psychophys.* 24, 7-10.
- Deutsch, D. (1980). "The processing of structured and unstructured tonal sequences," *Percept. Psychophys.* 28, 381-389.
- Deutsch, D., and Feroe, J. (1981). "The internal representation of pitch sequences in tonal music," *Psychol. Rev.* 88, 503-522.
- Deutsch, D., and Roll, P. L. (1974). "Error patterns in delayed pitch comparisons as a function of relational context," *J. Exp. Psychol.* 103, 1027-1034.
- Devie, D. (1990). *Le Tempérament Musical: Philosophie, Histoire, Théorie et Pratique* (The Musical Temperament: Philosophy, History, Theory and Practice) (Société de Musicologie du Languedoc, Béziers).
- Dowling, W. J. (1978). "Scale and contour: Two components of a theory of memory for melodies," *Psychol. Rev.* 85, 341-354.
- Dowling, W. J. (1991). "Pitch structure," in *Representing Musical Structure*, edited by P. Howell, R. West, and I. Cross (Academic, London), pp. 33-57.
- Dowling, W. J., and Harwood, D. L. (1986). *Music Cognition* (Academic, Orlando), Chap. 4.
- Duke, R. A. (1985). "Wind instrumentalist's intonational performance of selected musical intervals," *J. Res. Music Educ.* 33, 101-111.
- Ellis, A. J. (1954). *Additions to the Translation of the Book on the Sensations of Tone by Helmholtz 1855* (Dover, New York).
- Flesh, C. (1979). *Violin Fingering: Its Theory and Practice*, translated by B. Schwarz (Da Capo, New York) (original work published in 1960).
- Fletcher, H. (1934). "Loudness, pitch, and timbre of musical tones and their relation to the intensity, the frequency, and the overtone structure," *J. Acoust. Soc. Am.* 6, 59-69.
- Fletcher, H., Blackham, E. D., and Geertsens, N. (1965). "Quality of violin, viola, 'cello, and bas-viol tones I," *J. Acoust. Soc. Am.* 37, 851-863.
- Francès, R. (1958). *La Perception de la Musique* (The Perception of Music) (Vrin, Paris), pp. 53-61.
- Galamian, I. (1962). *Principles of Violin Playing and Teaching* (Prentice-Hall, Englewood Cliffs, NJ), p. 22.
- Geringer, J. M. (1978). "Intonational performance and perception of ascending scales," *J. Res. Music Educ.* 26, 32-40.
- Greene, P. C. (1937a). "Violin intonation," *J. Acoust. Soc. Am.* 9, 43-44.
- Green, P. C. (1937b). "Violin performance with reference to tempered, natural and Pythagorean intonation," *Exp. Res. Music: Stud. Psychol. Music* 4, 232-251.
- Harris, G., and Siegel, J. A. (1975). "Categorical perception and absolute pitch," *J. Acoust. Soc. Am.* 57, 511.
- House, W. J., (1977). "Octave generalization and the identification of distorted melodies," *Percept. Psychophys.* 21, 586-589.
- Kolinski, M. (1959). "A new equidistant 12-tone temperament," *J. Am. Musicol. Soc.* 12, 210-214.
- Krumhansl, C. L. (1979). "The psychological representation of musical pitch in a tonal context," *Cognit. Psychol.* 11, 346-374.
- Krumhansl, C. L. (1990). *Cognitive Foundations of Musical Pitch* (Oxford, U.P., Oxford).
- Krumhansl, C. L., and Kessler, E. J. (1982). "Tracing the dynamic changes in perceived tonal organization in a spatial representation of musical keys," *Psychol. Rev.* 89, 334-368.
- Lewis, D., and Cowan, M. (1936). "The influence of intensity on the pitch of violin and cello tones," *J. Acoust. Soc. Am.* 8, 20-22.
- Lloyd, L. S. (1940). "The myth of equal temperament," *Music Lett.* 21, 347-361.
- Lloyd, L. S., and Boyle, H. (1963). *Intervals, Scales and Temperaments* (Macdonald and Jane's, London).
- Martin, D. W., and Ward, W. D. (1961). "Subjective evaluation of musical scale temperament in pianos," *J. Acoust. Soc. Am.* 33, 582-585.
- Mason, J. A. (1960). "Comparison of solo and ensemble performances with reference to Pythagorean, just, and equi-tempered intonations," *J. Res. Music Educ.* 8, 31-38.
- Mathews, M. V., and Sims, G. (1981). "Perceptual discrimination of just and equal tempered tunings," *J. Acoust. Soc. Am. Suppl.* 1 69, S38.
- Meyer, L. B. (1956). *Emotion and Meaning in Music* (University of Chicago, Chicago).
- Meyer, L. B. (1973). *Explaining Music: Essays and Explorations* (University of California, Berkeley).
- Moran, H., and Pratt, C. C. (1926). "Variability in judgments on musical intervals," *J. Exp. Psychol.* 9, 492-500.
- Mozart, L. (1975). *A Treatise on the Fundamental Principles of Violin Playing*, translated by E. Klocker (Oxford U.P., London) (original work published 1756).
- Mursell, J. L. (1937). *The Psychology of Music* (Greenwood, Wesport), Chap. 3.
- Narmour, E. (1990). *The Analysis and Cognition of Basic Melodic Structures* (University of Chicago, Chicago).
- Nickerson, J. R. (1949). "Intonation of solo and ensemble performance of the same melody," *J. Acoust. Soc. Am.* 21, 593-595.
- Partch, H. (1949). *Genesis of a New Music* (Wisconsin U.P., Madison).
- Plomp, R. (1976). *Aspects of Tone Sensation: A Psychophysical Study* (Academic, London).
- Railsback, O. L. (1938). "A study of the tuning of pianos," *J. Acoust. Soc. Am.* 10, 86.
- Rakowski, A. (1976). "Tuning of isolated musical intervals," *J. Acoust. Soc. Am. Suppl.* 1 59, S50.
- Rakowski, A. (1985). "The perception of musical intervals by music students," *Bull. Council for Res. in Music Educ.* No. 85, 175-186.
- Rasch, R. A., and Plomp, R. (1982). "The perception of musical tones," in *The Psychology of Music*, edited by D. Deutsch (Academic, New York), pp. 1-24.
- Risset, J. C. (1978). "Musical acoustics," in *Handbook of Perception*, Vol. 4, edited by E. C. Carterette and M. P. Friedman (Academic, New York), pp. 521-564.
- Roberts, L. A., and Mathews, M. V. (1984). "Intonation sensitivity for traditional and nontraditional chords," *J. Acoust. Soc. Am.* 75, 952-959.
- Rosner, B. S., and Meyer, L. B. (1986). "The perceptual roles of melodic processes, contour, and form," *Music Percept.* 4, 1-40.
- Saldanha, E. L., and Corso, J. F. (1964). "Timbre cues and the identification of musical instruments," *J. Acoust. Soc. Am.* 36, 2021-2026.
- Scharf, B., and Buus (1986). "Audition I": Stimulus, physiology, thresholds," in *Handbook of Perception and Human Performance*, Vol. 1, edited

- by K. R. Boff, L. Kaufman, and J. P. Thomas (Wiley, New York), Chap. 14, pp. 54-59.
- Seashore, C. E. (1938). *Psychology of Music* (McGraw-Hill, New York), Chap. 18.
- Shackford, C. (1961). "Some aspects of perception, Part I," *J. Music Theory* 5, 162-202.
- Shackford, C. (1962). "Some aspects of perception, Part II," *J. Music Theory* 6, 66-90.
- Siegel, J. A., and Siegel, W. (1977). "Categorical perception of tonal intervals: Musicians can't tell sharp from flat," *Percept. Psychophys.* 21, 399-407.
- Small, A. M. (1937). "An objective analysis of artistic violin performance," *Exp. Res. Music: Stud. Psychol. Music* 4, 172-231.
- Sundberg, J. E. F. (1982). "Perception of singing," in *The Psychology of Music*, edited by D. Deutsch (Academic, New York), pp. 59-98.
- Sundberg, J. E. F., and Lindqvist, J. (1973). "Musical octaves and pitch," *J. Acoust. Soc. Am.* 54, 922-929.
- Vos, J. (1982). "The perception of pure and mistuned musical fifths and major thirds: Thresholds for discrimination, beats, and identification," *Percept. Psychophys.* 32, 297-313.
- Vos, J. (1984). "Spectral effects in the perception of pure and tempered intervals: Discrimination and beats," *Percept. Psychophys.* 35, 173-185.
- Vos, J., and van Vianen, B. G. (1985). Thresholds for discrimination between pure and tempered intervals: The relevance of nearly coinciding harmonics," *J. Acoust. Soc. Am.* 77, 176-187.
- Ward, W. D. (1954). "Subjective musical pitch," *J. Acoust. Soc. Am.* 26, 369-380.
- Ward, W. D. (1970). "Musical perception," in *Foundations of Modern Auditory Theory, Vol. 1*, edited by J. V. Tobias (Academic, New York), pp. 420-421.
- Ward, W. D., and Martin, D. W. (1961). "Psychophysical comparison of just tuning and equal temperament in sequences of interval tones," *J. Acoust. Soc. Am.* 33, 586-588.
- Watkins, A. J., and Dyson, M. C. (1985). "On the perceptual organization of tone sequences and melodies," in *Musical Structure and Cognition*, edited by P. Howell and R. Cross (Academic, London), pp. 71-119.
- Willemze, T. (1973). *Algemene Muziektheorie* (General theory of music) (Spectrum, Antwerpen), p. 122.
- Winer, B. J., Brown, D., and Michels, K. E. (1991). *Statistical Principles in Experimental Design* (McGraw-Hill, New York), 2nd ed.
- Yampolsky, I. M. (1967). *The Principles of Violin Fingering: Preface by David Oistrakh*, translated by A. Lumsden (Oxford, U.P., Oxford).