



New Pythagorean Aspects of Music And Their Connection to Superstrings

by
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"In ancient times, music was something other than mere pleasure for the ear: it was like an algebra of metaphysical abstractions, knowledge of which was given only to initiates, but by the principles of which the masses were instinctively and unconsciously influenced. This is what made music one of the most powerful instruments of moral education, as Kong-Tsee (Confucius) had said many centuries before Plato."

G. de Mengel, Voile d'Isis

Abstract

The tetrahedral generalisation of the Platonic lambda discussed in Article 11 is shown to generate the tone ratios of the diatonic scale. Godname numbers define properties of ten octaves, which conform to the pattern of the Tree of Life. The latter is exhibited also in the 32 notes above the fundamental up to the perfect fifth of the fifth octave, which has a tone ratio of 24. Being the tenth overtone and therefore corresponding to Malkuth explains why this number is central to the physics of the superstring. The numbers in the tetractys form of Plato's lambda are shown to be — individually or in combination — the numbers of the various musical sounds that can be played with ten notes arranged in a tetractys. The number of melodic intervals, chords and broken chords is found to be the number of charge sources of the unified, superstring force. The 72 broken chords and 168 melodic intervals and chords correlate with the 72:168 division of such charges encoded in the inner form of the Tree of Life and manifested in the distinction between the major and minor whorls of the superstring described by Annie Besant and C.W. Leadbeater. The 90 musical sounds generated by a tetractys of ten notes correlate with the 90 edges of the five Platonic solids. Similarity between the root structure of the superstring symmetry group E_8 and the intervals and chords of the octave suggests that superstrings share with music the universal mathematical pattern of the Tree of Life, the eight zero roots of E_8 corresponding to the eight notes of the diatonic scale.

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1. The tetrahedral Platonic Lambda

In his *Timaeus*, Plato describes how the Demiurge measured the World Soul, or substance of the spiritual universe, according to the simple proportions of the first three

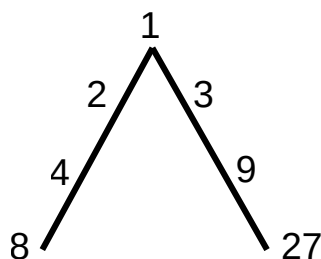


Figure 1. Plato's Lambda.

powers of 2 and 3. This is represented by his 'Lambda,' so-called because of its resemblance to the Greek letter Λ (Fig. 1). These numbers line but two sides of a tetractys of ten numbers from whose relative values the physicists and musicians of ancient Greece worked out the frequencies of the notes of the octaves of the now defunct Pythagorean musical scale. However, it was shown in Article 11 that, if we

ignore the speculative cosmological context in which this algorithm for generating the relative frequencies of the musical notes was presented and regard the Lambda and its

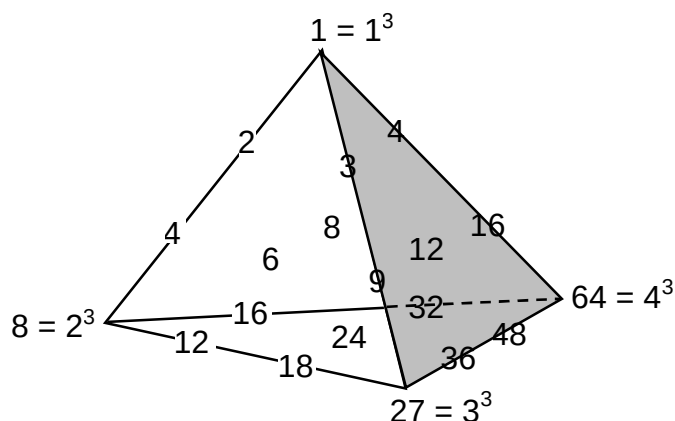


Figure 2. The tetrahedral Lambda tetractys.

underlying tetractys purely as a construction of Pythagorean mathematics, then it is incomplete. This is because the numbers 1, 2, 3 and 4 were the basis of Pythagorean number mysticism and its application to the study of natural phenomena such as musical sounds, whereas the number 4 is missing as a generative factor from the Lambda, which uses

only 1 (the monad), 2 (the duad) and 3 (the triad) to generate its numbers. The Pythagorean wholeness of the Lambda is restored naturally by realising that it is but two edges of a tetrahedron having 1 at its apex and a third edge with the first three powers of 4 arranged along it (Fig. 2). It may be argued that this three-dimensional figure is not consonant with the details of the cosmological theory that Plato presented in his *Timaeus*. This, indeed, is the case. Nevertheless, the value and universality of mathematics exist in their own right and do not have to be validated by the theories of any mathematician or philosopher, however renowned that person may be.* Properties of numbers are more important than how they may have been interpreted.

The tetrahedron of 20 numbers has the following musical virtue: the extended Lambda

* This is not intended as a criticism of Plato, who may have known about the tetrahedral generalisation.

tetractys generates the tone ratios of octaves along one side and perfect fifths along another side. But the numbers starting with 6 and generating the perfect fourths have to be added by hand, so to speak, following *ad hoc* rules of multiplication by 2 and 3 that

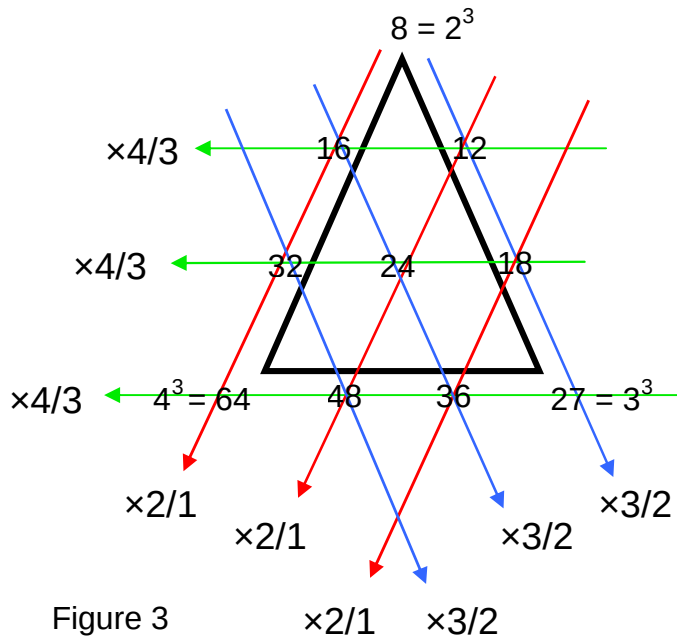


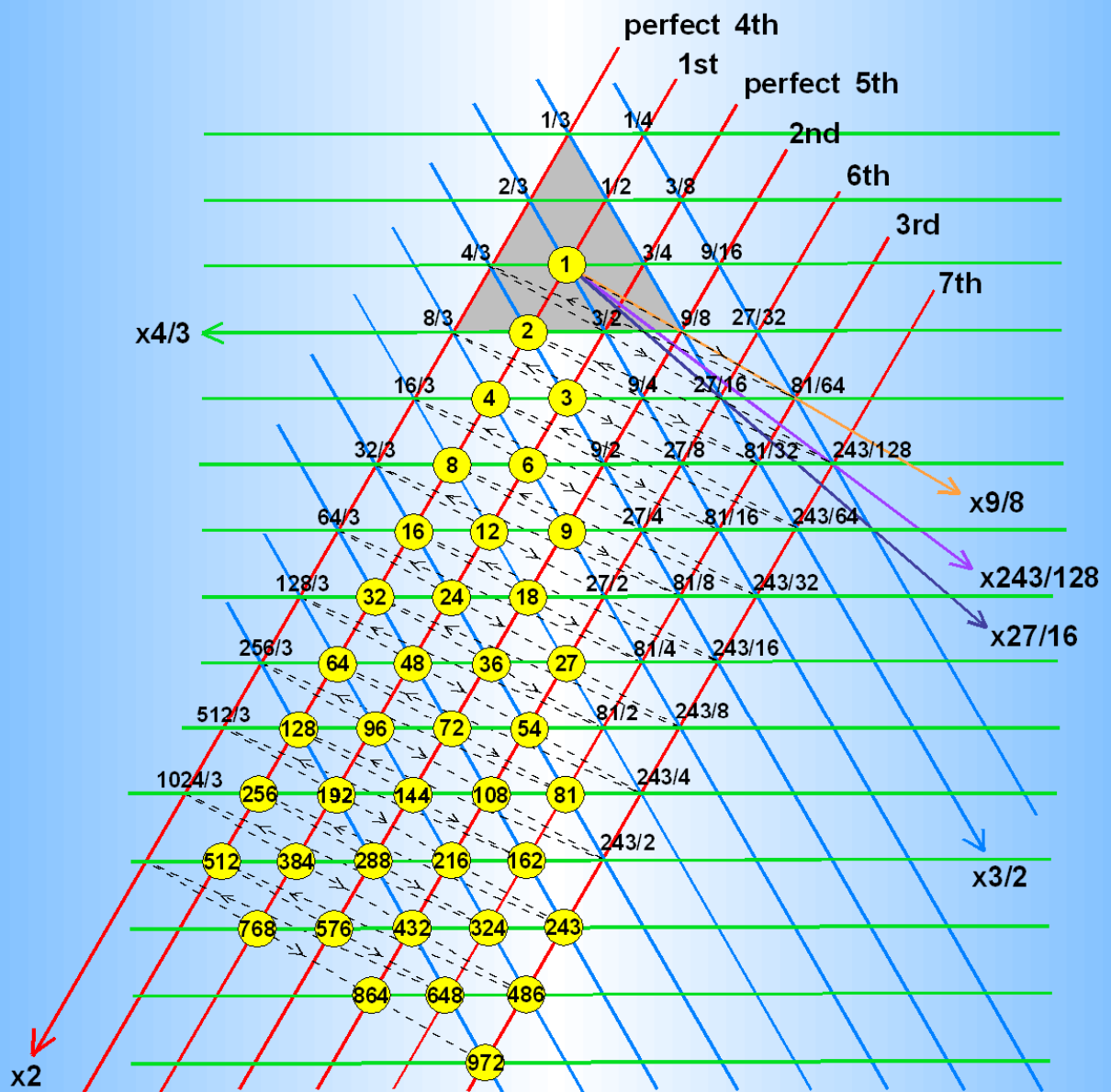
Figure 3

were not part of Plato's cosmological theory and whose justification is simply that they create the right numbers. Furthermore, whereas the pairing of numbers separated by octaves or intervals of the perfect fifth follows the natural geometry of the array of numbers set by the extended boundary of the Lambda, the pairing of successive perfect fourths does not respect the same symmetry because it occurs in *diagonal* fashion across the array.

Worse still, the other possible diagonal pairing of numbers whose tone ratios differ by a factor of 3 plays a relatively weak role in generating twelfths of the Pythagorean scale. The traditional construction of the tone ratios of the Pythagorean clearly lacks symmetry because the classical scheme is mathematically incomplete. On the other hand, the fourth face of the tetrahedron is a tetractys of numbers whose pairings *parallel to its three sides* create octaves, perfect fifths and perfect fourths with, respectively, the tone ratios, 2/1, 3/2 and 4/3 (Fig. 3). Its hexagonal symmetry means that, when extended in the traditional manner of the Lambda tetractys, every number becomes surrounded by six others that are octaves, perfect fourths or perfect fifths. The numbers may be divided by any one of them to generate the *same* lattice of tone ratios of the Pythagorean scale, e.g., the infinite, hexagonal lattice of numbers is invariant with respect to such division.

The number 24 ($=1 \times 2 \times 3 \times 4$) is at the centre of the fourth face*. Figure 4 displays the lattice of tone ratios, starting with 1, the fundamental, that are created by dividing every number in the tetractys and outside it by 24. Using any other number in or outside the tetractys as divisor would have created the *same* lattice of tone ratios. Overtones are shown in yellow circles, red lines connect octaves ($\times 2$), green lines connect perfect

* 6, the centre of the Lambda tetractys, is the *fourth* overtone and 24 is the *tenth* overtone. Integers 6, 8, 12 and 24 at the centres of the four faces have the ratios 1, 3/2, 4/3, 2, 3 and 4 of the integers 1, 2, 3 and 4.



GENERATION OF THE TONE RATIOS OF SUCCESSIVE OCTAVES OF THE DIATONIC SCALE

Figure 4

fourths ($\times 4/3$) and blue lines connect perfect fifths ($\times 3/2$). The tone interval of $9/8$ is also indicated by the orange line joining the centre of the tetractys (coloured grey) to one corner. The tone ratios $27/16$ of note A and $243/128$ of note B are similarly defined by, respectively, indigo and violet diagonals extending from the number 1 to corners of larger triangles. Successive notes of the scale for each octave are joined by dashed lines. They

Table of tone ratios of eleven octaves of the Pythagorean scale

	C	D	E	F	G	A	B	Number Of overtones
1	1	9/8	81/64	4/3	3/2	27/16	243/128	0
2	2	9/4	81/32	8/3	3	27/8	243/64	2
3	4	9/2	81/16	16/3	6	27/4	243/32	4
4	8	9	81/8	32/3	12	27/2	243/16	7
5	16	18	81/4	64/3	24	27	243/8	11
6	32	36	81/2	128/3	48	54	243/4	15
7	64	72	81	256/3	96	108	243/2	20
8	128	144	162	512/3	192	216	243	26
9	256	288	324	1024/3	384	432	486	32
10	512	576	648	2048/3	768	864	972	38
11	1024	1152	1296	4096/3	1536	1728	1944	39

(Red cells enclose integer notes up to end of tenth octave)

zigzag between an octave, the seventh note of the octave and its perfect fourth, i.e., between the extremities of the Pythagorean scale and its midpoint.

2. The first ten octaves

The tone ratios of the 71 notes in the first ten octaves are shown in the table above (red cells contain overtones and blue cells enclose notes beyond the tenth octave). The last column lists as a running total the number of overtones of the fundamental with a tone ratio of 1.

Comments

1. In the interval 243/128 between C and B, 243 is the **26th*** overtone and the 55th note after 1, where

$$55 = \begin{array}{ccccccc} & & 1 & & & & \\ & & 2 & 3 & & & \\ & 4 & 5 & 6 & & & \\ & 7 & 8 & 9 & 10 & & \end{array}$$

and 128 is the **21st** overtone and the **50th** note (the last note of the seventh octave). This shows how Ehyeh, Yahweh and Elohim, Godnames of the Supernal Triad, prescribe the range of pitch between the first seven notes (1). 256, which is part of the leimma 256/243 between notes E and F and between B and C of the next octave, is the **36th** note, counting from the beginning of the fourth octave. This shows how Eloha, Godname of Geburah with number value **36**, defines the 'leftover' between adjacent octaves. The **36th** note after 1 has a tone ratio of **36**;

2. the first ten octaves span 71 notes, of whose tone ratios

$$40 = \begin{array}{ccccccc} & & 4 & & & & \\ & & 4 & 4 & & & \\ & 4 & 4 & 4 & & & \\ & 4 & 4 & 4 & 4 & & \end{array} = 4 + 8 + 12 + 16$$

are integers and **31** are fractions. This shows how the Godname El of Chesed with number value **31** determines the number of notes whose tone ratios are not whole numbers, whilst Eloha prescribes the number of notes because 71 is the **36th** odd integer. It also demonstrates how the Pythagorean Tetrad (4) defines the number of integer tone ratios in *ten* octaves. The 71st note has a tone ratio of $1024 = 2^{10}$. This is the smallest number with *ten* prime factors (all 2), showing the Pythagorean character of the last note of the tenth octave — the 40th note that is an integer;

3. The 70th note is $972 = 36 \times 27$, where $27 (=3^3)$ is the largest integer in Plato's Lambda and **36** ($=1^3+2^3+3^3$) is the sum of the integers 1, 8, and 27 located at its apex and extremities:

$$\begin{array}{ccccccc} & & 1 & & & & \\ & & 2 & 3 & & & \\ & 4 & & & 9 & & \\ 8 & & & & & 27. & \end{array}$$

* The number values of the Sephiroth, their Godnames, Archangels, Angels and Mundane Chakras are written throughout the text in boldface.

Hence, $972 = 3^3 + 6^3 + 9^3$. This property is an example of the beautiful, mathematical properties of the first ten octaves (the reason for this will be given shortly). As $2700 = 3^3 + 6^3 + 9^3 + 12^3$ and $100 = 1^3 + 2^3 + 3^3 + 4^3$, the largest integer 27 can be expressed as the ratio:

$$\frac{3^3 + 6^3 + 9^3 + 12^3}{1^3 + 2^3 + 3^3 + 4^3}$$

Once again, it is the Pythagorean Tetrad that expresses a number important to the mathematics of the Pythagorean scale, for both the numerator and the denominator in the ratio are the sum of *four* cubes. It was pointed out in Article 11 that Yahweh prescribes the number 243 in the leimma because it is the **26th** overtone. $243 = 3^3 + 6^3$, i.e., it is the sum of the first two of the three cubes summing to the value of the tone ratio of the seventh note of the tenth octave. 3 and 6 are the integers in **36**, the number value of Eloha. The table indicates that $6^3 = \mathbf{216}$ is the tone ratio next smaller than 243. This is the number of Geburah whose Godname defines the number 256, as indicated in comment (1), as well as the number 243.

Because the tone ratios of corresponding notes in n successive octaves all increase by the same factor of 2, there are as many overtones in *any* such set of n octaves, taking their lowest tonic as the fundamental, as there are in the first n octaves; the first note of the first octave is set as 1 merely for convenience because the integers and fractions

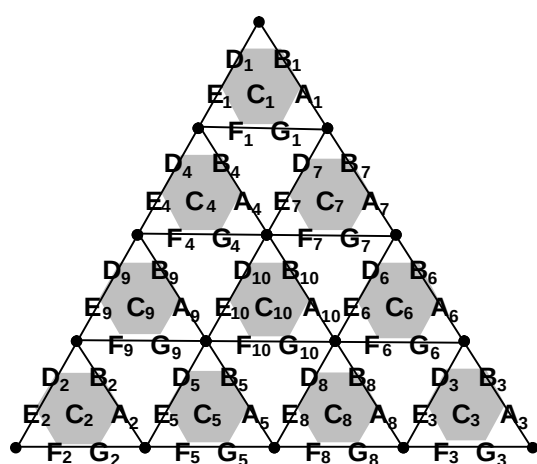


Figure 5

represent relative, not absolute, frequencies. The first seven octaves have **50** notes of which **21** are overtones up to 128. Counting from the first overtone with tone ratio 2, the number value of Elohim defines the next seven octaves whose last note B first becomes an integer (243). This is the **26th** overtone, which is therefore also prescribed by Yahweh. It is the $(7 \times 7 = \mathbf{49})$ th note and so is prescribed by El Chai, Godname of Yesod. The 70th note (972) represents the same note relative to 4, the tonic of the third

octave. It is, counting from this note, the **36th** overtone. This is how these four Godnames prescribe the 70th note of the first ten octaves.

Let us now represent the 70 notes of the ten octaves by what the author has called in previous articles a '2nd-order tetractys' (Fig. 5). The **21** notes of the first three octaves

are arranged at the corners and centres of hexagons at its three corners and the **49** notes of the next seven octaves are at the corners and centres of seven hexagons arranged at the corners and centre of a larger hexagon (2). The ten tonics C_n ($1 \leq n \leq 10$) are at the centres of the hexagons. The centre of the 2nd-order tetractys denotes the tonic of the tenth octave with tone ratio 512 (C_{10}). This kind of tetractys is a more differentiated version of the Pythagorean symbol of divine wholeness. It is why the 70th number, 972, exhibits arithmetic properties typical of the beautiful harmonies manifested by this pattern.

Figure 6 displays the equivalence between the 2nd-order tetractys* and the Tree of Life with its 16 triangles turned into tetractyses. The ten corners of these triangles correspond to the centres of the ten tetractyses (both shown as white circles). The tonics of the ten octaves can be assigned to the positions of the ten Sephiroth and the remaining 60 notes assigned to the 60 black yods. The first seven notes of each octave formally correspond to a Sephirah. This is the reason for our considering the first ten octaves of the

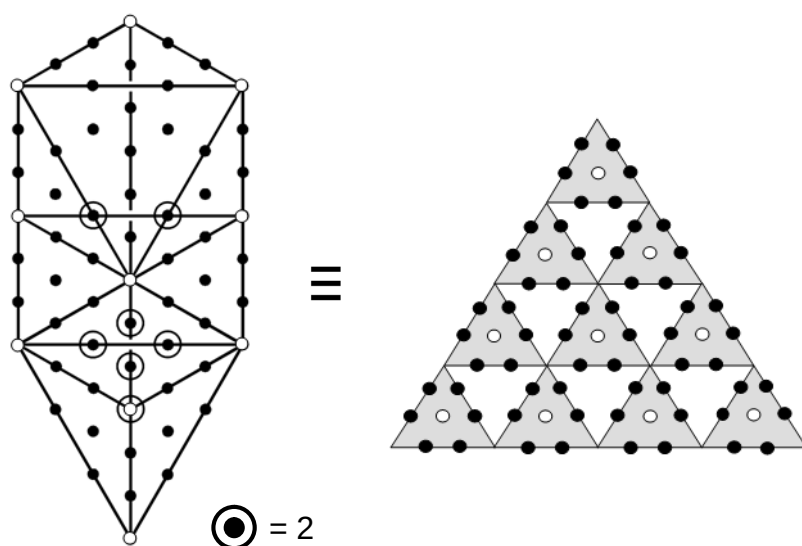


Figure 6

Pythagorean scale. The mathematical beauty of this Tree of Life pattern has already begun to show itself in the properties of the 70th note discussed above.

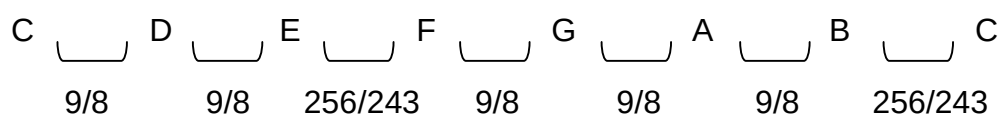
Let us now consider the integer tones in the ten octaves. As pointed out in comment 2 above, their number 40 can be represented as a tetractys array of the number 4. The sum of those 4's at its corners is 12, leaving 28 as the sum of the seven other 4's. Yods at corners of a tetractys correspond to the Sephiroth of the Supernal Triad and the seven other yods correspond to the Sephiroth of Construction. The 12:28 division of the 40 integer tone ratios therefore corresponds to the Kabbalistic distinction between the

* Actually, it is a slightly different version of that shown in Figure 4. The difference is immaterial.

subjective and objective Sephiroth of the Tree of Life. The largest number 27 in Plato's Lambda is the twelfth such tone ratio (A of the 5th octave). 4 is the fourth integer tone ratio and 12 is the eighth. The former is the **15th** note and the latter is the **26th**. This shows how the Godnames Yah with number value **15** and Yahweh with number value **26** that are assigned to the Sephirah Chokmah mark out notes that correspond to successive members of the Supernal Triad. Yahweh also defines the twelfth integer tone ratio because 27 is the **26th** integer after 1 (for the Pythagoreans, 27 would be the **26th true** integer because they regarded the number 1 not as an integer but as the source and principle of all numbers).

The 40 integer tone ratios comprise 11 octaves (note C) and 29 others. 29 is the **15th** odd integer after 1, showing how the Godname Yah with number value **15** defines the number of overtones in ten octaves that are not merely octaves.

There are five whole tone intervals (9/8) and two leimmas (256/243) that separate the eight notes of an octave*:



This means that the 70 intervals between the 71 notes of the ten octaves are made up of 20 leimmas and **50** tones. The Godname Elohim with number value **50** prescribes the

number of tones spanning ten octaves. This demonstrates *par excellence* the Tree of Life pattern formed by ten octaves. The correspondence in the Tree of Life of this 20:50 division of intervals is the fact that, when the former is constructed from tetractyses, there are 20 (●) yods on the faces of the tetrahedron and **50** (●) yods outside them (Fig. 7). The leimmas correspond to the yods on the tetrahedron and the tones correspond to the yods outside the tetrahedron. In general, n overlapping trees contain (50n+20) yods, of which 20n yods lie on

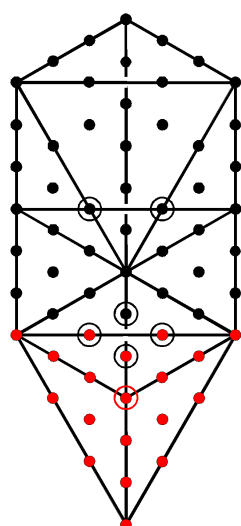


Figure 7

$$70 = 50 \bullet + 20 \bullet$$

* This 5:2 division corresponds in the Tree of Life to the lowest, five Sephiroth of Construction, which are always shared with overlapping trees, and to Chesed and Geburah, which are unshared.

n tetrahedra, so that this 5:2 correspondence exists only for a single Tree of Life.

Counting from the fundamental of the first octave, there are *ten* overtones up to the perfect fifth of the fifth octave with tone ratio 24:

2, 3, 4, 6, 8, 9, 12, 16, 18, 24.

Of these, four (2, 4, 8, 16) are octaves, leaving six others. This 4:6 division corresponds in the Tree of Life to the four lowest Sephiroth at the corners of a tetrahedron and the uppermost six Sephiroth. The sum of the Godname numbers of the latter is

$$21 + 26 + 50 + 31 + 36 + 76 = 240 = \begin{array}{c} 24 \\ 24 \ 24 \\ 24 \ 24 \ 24 \\ 24 \ 24 \ 24 \ 24 \end{array},$$

that is, they sum to the number of yods in 24 tetractyses. This suggests something

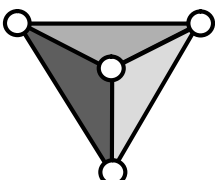
	Point	Line	Triangle	Tetrahedron	TOTAL
	1				1
	2	1			3
	3	3	1		7
	4	6	4	1	<u>15</u>
					TOTAL = <u>26</u>

Figure 8

special about the number 24. This might be suspected, because it is $1 \times 2 \times 3 \times 4$, i.e., the number of permutations of *four* objects, which shows its Pythagorean character. It is also the **26th** note and the tenth overtone, both counting from 2, the *first* overtone. This corresponds in the Tree of Life to the fact that the tenth Sephirah, Malkuth, is, as its lowest point, the **26th** and last geometrical element in its trunk (Fig. 8).

The significance of the perfect fifth of the fifth octave of the Pythagorean scale is that it is the seventh of a set of perfect fifths, the first five being successive:

G ₁	D ₂	A ₂	E ₃	B ₃		F ₄	C ₅	G ₅
3/2	9/4	27/8	81/16	243/32		32/3	16	24

(Subscripts denote the octave number)

i.e., 33 is the total number of permutations of four rows of 1, 2, 3 and 4 objects arranged in a tetractys. Of these notes, 11 are integers and $(33-11=22)$ are fractional, where

$$22 = 1^4 + 2^3 + 3^2 + 4^1.$$

The graph consists of 10 nodes and 24 edges. The nodes are labeled with red numbers: 2, 3, 4, 6, 8, 9, 12, 16, 18, 24. The nodes are arranged in a complex, symmetric pattern. The edges are labeled with black numbers representing ratios of products of primes. The graph is highly interconnected, with many nodes having multiple incident edges.

The Tree of Life pattern of the first ten overtones

the ten Sephiroth and the 22 fractional notes corresponding to the 22 Paths connecting the Sephiroth (Fig. 9). The ordering of notes in Figure 9 follows the traditional Kabbalistic

numbering of Paths. As the tenth overtone, the tone ratio 24 corresponds to the *lowest* Sephirah, Malkuth, which signifies the outer, physical form of anything embodying the universal blueprint of the Tree of Life. It is this correspondence that makes the number 24 significant vis-à-vis superstring theory, as will be explained in Section 6.

Arranged in a tetractys:

2
3 4
6 8 9
12 16 18 24,

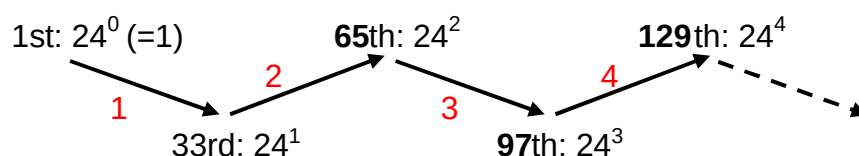
the ten overtones have 16 combinations of two or more notes selected from each row. They comprise ten harmonic intervals, five chords of three notes and one chord of four notes. These correspond in the trunk of the Tree of Life to, respectively, its ten Paths, five triangles each with three corners and the tetrahedron with four corners (see Fig. 9). Alternatively, the 16 harmonic intervals and chords correspond to the 16 triangles of the Tree of Life itself. The ten harmonic intervals correspond to the ten triangles below the level of the path joining Chesed and Geburah, and the six chords correspond to the six triangles either above or projecting beyond this line, which, according to Kabbalah, separates the subjective Supernal Triad from the objective aspect of the Tree of Life manifesting the seven Sephiroth of Construction. Of the six triangles, only *one* triangle is completely above this line. It corresponds to the *single* chord of four notes. The complete correspondence between the ten octaves and the Tree of Life is summarised below:

10 overtones	→	10 Sephiroth
22 fractional tone ratios	→	22 Paths
16 harmonic intervals and chords	→	16 triangles

Any set of ten successive octaves exhibits this same Tree of Life pattern because the tone ratios of corresponding notes in successive octaves differ by a factor of 2, which means that, *relative to the first note of any such set*, there are always ten overtones with the same set of values as that found above for the first ten octaves, starting with a tone ratio of 1 for the tonic of the first octave. The perfect fifth of the fifth octave, counting from any given octave, is still an overtone with the tone ratio of 24 relative to the tonic of that starting octave. The numbers in the table of tone ratios are not absolute pitches but frequencies defined relative to that of the fundamental, which is normally given the convenient value of 1. Their underlying Tree of Life pattern is, therefore, not dependent

on a particular starting point but holds for *any* set of ten successive octaves. The fact that most of their notes would fall outside the audible range of the human ear is irrelevant.

Counting from the tonic of the first octave, the tone ratio 24 ($=1 \times 2 \times 3 \times 4$) is the 33rd note ($33=1!+2!+3!+4!$) and the perfect 5th of the fifth octave. Counting from the latter, the 33rd note is $576 = 24^2$ and still the perfect 5th of the new fifth octave. This is the **65th** note from the tonic of the first octave, where **65** is the 33rd odd integer. The Godname Adonai



with number value **65** prescribes sequences of 33 notes whose last note has a tone ratio always 24 times that of the first note. Only the first sequence (1) has ten overtones. The second sequence (2) has 24 overtones in addition to the first overtone with tone ratio of 24. In general, the n th sequence terminates in the note with tone ratio 24^n . Notice that the Godname of Netzach, the *fourth* Sephirah of Construction with number value **129**, determines the end of the *fourth* sequence with tone ratio 24^4 , that is, $(1 \times 2 \times 3 \times 4)$ raised to the *fourth* power. This shows the principle of the Pythagorean Tetrad at work.

The 1680 coils of each whorl in the UPA superstring have been shown in previous articles to be due to 24 gauge charges of the superstring gauge symmetry group E_8 , the total number of 240 for all ten whorls corresponding to its 240 non-zero roots. The number of yods in the lowest n Trees of Life is given by

$$Y(n) = 50n + 30.$$

The lowest 33 trees have $Y(33) = 1680$ yods. This is the same number as 24 separate Trees of Life, each with 70 yods, because $1680 = 24 \times 70$. Just as the first 33 notes culminate with the tone ratio 24, so the first 33 overlapping Trees of Life have as many yods as 24 *separate* trees. This demonstrates the association of the numbers 33 and 24

Number of permutations	
A	$1! = 1$
B C	$2! = 2$
D E F	$3! = 6$
G H I J	$4! = 24$
	TOTAL = <u>33</u>

in the context of the Tree of Life. Ten objects arranged in a tetractys can be arranged in

their *separate* rows in 33 ways (see above). 24 of these are permutations of the last row of four objects. In this case, the 33rd permutation is the last of these 24 arrangements.

The tone ratio 24 is the perfect fifth of the fifth octave. As the 33rd note, it has its counterpart in the 33rd tree of what the author has called the 'Cosmic Tree of Life' — the 91 trees mapping all levels of consciousness (see Article 5). Counting upwards, the 33rd tree represents the fifth subplane of the fifth plane ($33 = 4 \times 7 + 5$). The fifth plane, called in Theosophy the 'atmic plane,' expresses the Divine Quality of Tiphareth, so that its fifth

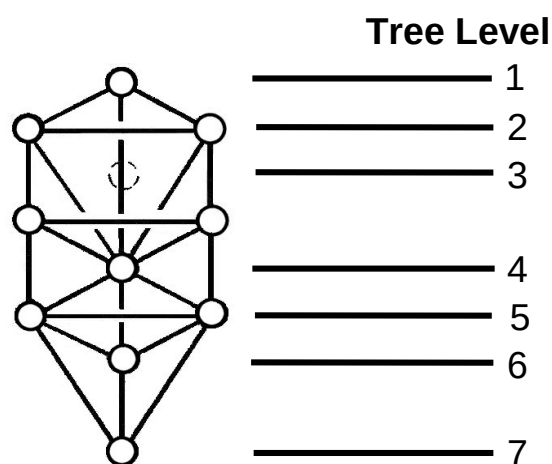


Figure 10

subplane also corresponds to this Sephirah.

We see that the 33rd subplane is the *most* characteristic of Tiphareth, namely "Beauty."

Little wonder then that it should determine the number 1680 characterising the form of a whorl component of a superstring, previous articles by the author having displayed its very beautiful properties.

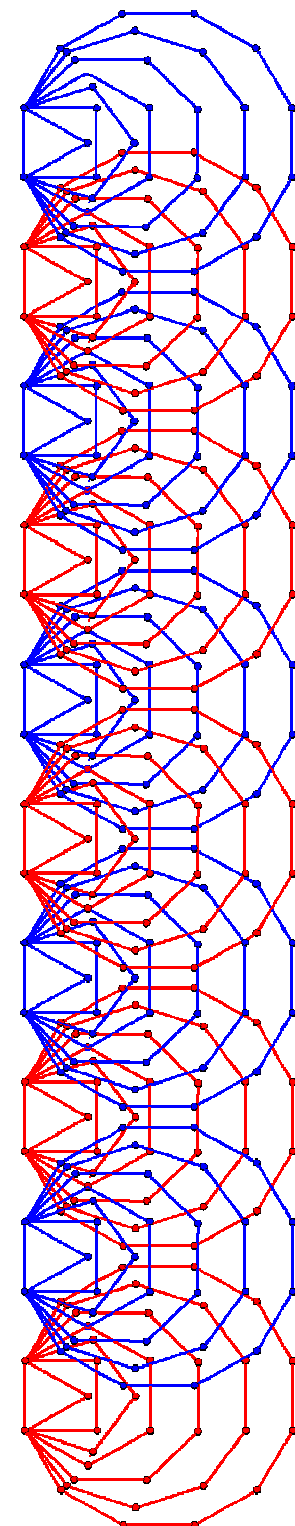
Confirmation that the number 33 represents a cycle of completion of a Tree of Life pattern

of which the ten overtones spanning 33 notes is an example comes from the concept of tree levels. The emanation of the ten Sephiroth takes place in seven stages (Fig. 10). Each Sephirah can be represented by a Tree of Life. Ten overlapping Trees of Life have 33 tree levels. This number thus parameterises the complete emanation of ten trees. Nine Tree levels extend down to the top of the seventh tree, marking the last of the 25 dimensions of space. Below them are a further 24 tree levels representing the 24 spatial dimensions at right angles to the direction in which the 1-dimensional string extends. This 9:24 differentiation in tree levels separating the purely physical plane from superphysical subplanes corresponds to the nine permutations of 1, 2 and 3 objects in the first three rows of a tetractys and the 24 permutations of the four objects in the fourth row. This is in keeping with the four rows of the tetractys symbolising the four fundamental levels of Divine Spirit, soul, psyche and body, i.e., the last 24 tree levels of the 33 tree levels determine the physical form of a superstring because they represent geometrical degrees of freedom as the dimensions or directions of space along which its whorls can vibrate.

Further confirmation of the cyclic nature of the number 33 in defining repeated Tree of Life patterns is that there are 33 corners outside the root edge of every successive set of

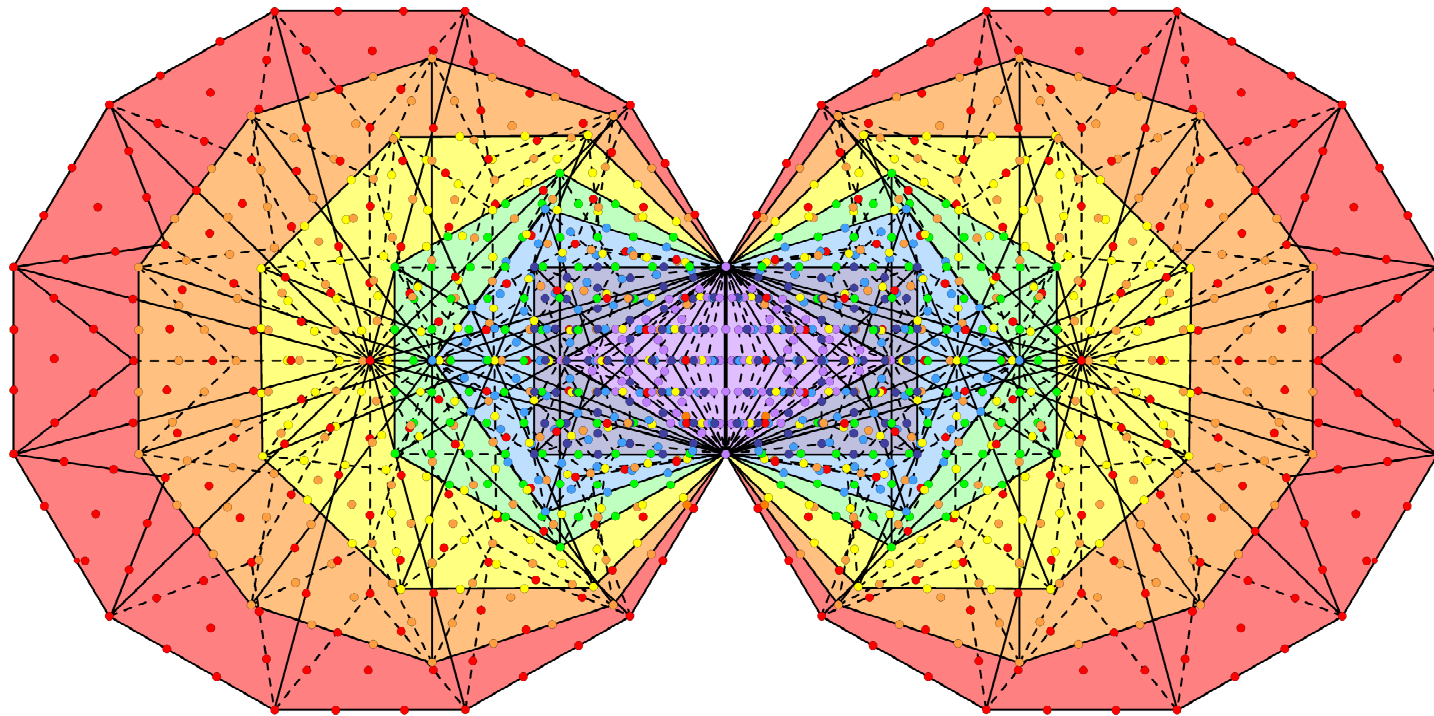
Ashim, the Order of Angels associated with Malkuth, prescribes ten overlapping Trees of Life because its number value 351 is the number of corners of the 70 polygons that are enfolded in them.

Figure 11.



351 (●) or (●)

1370 =



The (7+7) enfolded, regular polygons, whose 94 sectors are each divided into 3 tetractyses, contain 1370 yods. Of these, 70 yods are corners of polygons, leaving 1300 other yods, where $1300 = 26 \times 50 = 1^5 + 2^5 + 3^5 + 4^5$, **26** is the number value of the Godname Yahweh and **50** is the number value of the Godname Elohim. The number value **65** of the Godname Adonai defines the $(1300/2 = 650 = 65 \times 10)$ such yods associated with either set of polygons:

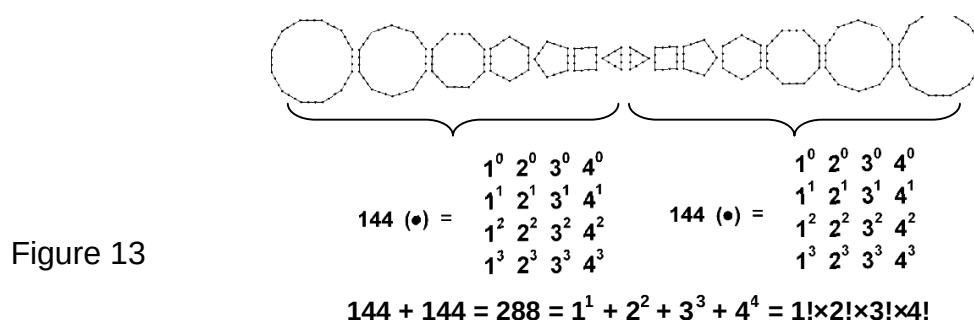
$$650 = \begin{array}{c} 65 \\ 65 \ 65 \\ 65 \ 65 \ 65 \\ 65 \ 65 \ 65 \ 65 \end{array} = \begin{array}{c} 1^2 \ 2^2 \ 3^2 \ 4^2 \\ 12^2 \ 5^2 \\ 11^2 \ 6^2 \\ 10^2 \ 9^2 \ 8^2 \ 7^2 \end{array}$$

The 94 sectors have 80 corners, leaving 1290 ($=129 \times 10$) other yods, where **129** is the number value of Yahweh Sabaoth. The Pythagorean tetrad (4) determines the number (137) of equivalent tetractyses as $137 = 33\text{rd prime number}$ and $33 = 1! + 2! + 3! + 4!$. The number of yods in 7 enfolded polygons other than their 7 centres $= 680 = 1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 13^2 + 15^2$, where **15** is the number value of the Godname Yah.

Figure 12

seven polygons enfolded in overlapping Trees of Life (Fig. 11). In general, the number of corners of the $7n$ polygons enfolded in n trees = $35n + 1$, '1' denoting the highest corner of the hexagon enfolded in the n th tree (the highest and lowest corners of each hexagon are shared with its adjacent hexagons). Hence, the 70 polygons enfolded in 10 trees have 351 corners (Fig. 11). This is the number value of *Ashim*, the Order of Angels of Malkuth. As a set of $(7+7)$ polygons has 70 corners, there are **64** corners outside their root edge that are unshared with hexagons in adjacent sets. This is the number of *Nogah*, the Mundane Chakra of Netzach, the Sephirah energising the music and art of the soul.

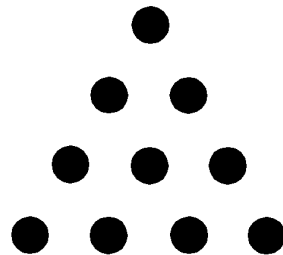
Another remarkable property of the number 33 is that the 33rd prime number is 137. This is one of the most important numbers in theoretical physics (3) because its reciprocal is almost equal to the so-called "fine-structure constant." This determines the probability that an electron will emit or absorb a photon and measures the relative strength of the electromagnetic force compared with the nuclear force binding protons and neutrons together inside atomic nuclei. As yet theoretically undetermined by physicists, the number 137 is encoded in the inner form of the Tree of Life as the 137 tetractyses whose yod population is equal to the yod population of the $(7+7)$ enfolded polygons when their sectors are each transformed into three tetractyses (Fig. 12). 1370 is also the number of yods in 27 overlapping Trees of Life (4), which gives another remarkable significance to the largest number in Plato's Lambda, which is prescribed by the Godname Yahweh as



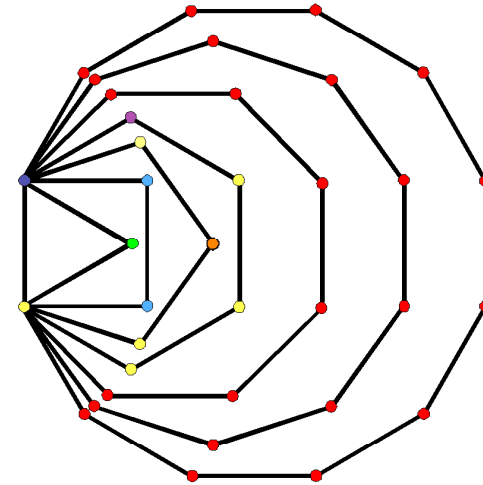
the **26th** number after 1. Indeed, 27 can be said to define the ten octaves of the Pythagorean scale in the sense that 972, the largest of their overtones, is the 27th overtone following the overtone 27, which is the 33rd note after the fundamental. Remarkably, 972 is also the 33rd *even* overtone.

A tetractys of ten objects has $(1!+2!+3!+4!=33)$ permutations of the objects in its rows. A tetractys of ten different notes would generate one note and 32 melodic intervals and broken chords formed from the other three rows of notes (see p. 22 for the definition of these musical terms). Suppose that we were to play one note, next a melodic interval,

THE 36 CORNERS OF THE 7 ENFOLDED POLYGONS DENOTE THE 36
MUSICAL ELEMENTS THAT A TETRACTYS OF 10 NOTES CAN PLAY



$$\begin{aligned}
 {}_1P^1 &= 1! = 1 \\
 {}_2C^2 + {}_2P^2 &= 1 + 2! = 3 \\
 {}_3C^3 + {}_3P^3 &= 1 + 3! = 7 \\
 {}_4C^4 + {}_4P^4 &= 1 + 4! = 25 \\
 \text{TOTAL} &= \underline{\underline{36}}
 \end{aligned}$$



$${}_2C^2 = 1 \text{ harmonic interval} = 1(\text{purple dot})$$

$${}_3C^3 = 1 \text{ chord of 3 notes} = 1(\text{green dot})$$

$${}_4C^4 = 1 \text{ chord of 4 notes} = 1(\text{orange dot})$$

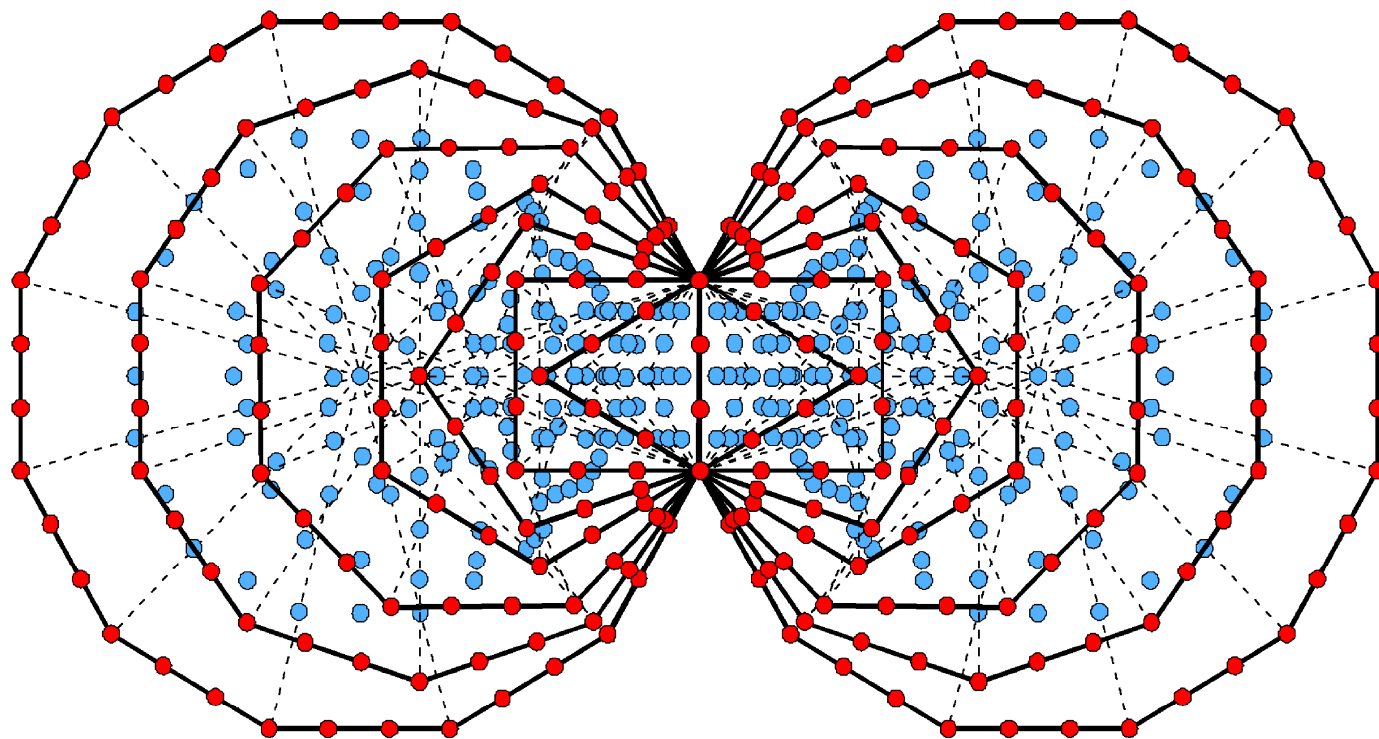
$$1! = 1 \text{ note} = 1(\text{purple dot})$$

$$2! = 2 \text{ melodic intervals} = 2(\text{blue dot})$$

$$3! = 6 \text{ broken chords} = 6(\text{yellow dot})$$

$$4! = 24 \text{ broken chords} = 24(\text{red dot})$$

Figure 14



236 (●)
288 (●)

Figure 15. 236 yods are on the edges of the (7+7) enfolded polygons and 288 yods are inside them.

then a broken chord of three notes and finally a broken chord of four notes. The number of possible ways of playing ten notes *in succession* by following the pattern of the tetractys is $1! \times 2! \times 3! \times 4! = 288$.^{*} This is the number of yods lying on the boundaries of the (7+7) regular polygons constituting the inner form of the Tree of Life (Fig. 13). Supposing that the notes are arranged in the tetractys in either ascending or descending order, there are 144 arrangements of ascending notes and 144 arrangements of descending notes. They have their parallel in the two similar sets of seven polygons that are shaped by 144 yods on their **48** sides. Now suppose that we were to play one note, then either a harmonic or a melodic interval, next either a chord or a broken chord of three notes and finally a chord or broken chord of four notes. The numbers of possible musical sequences created by playing 1, 2, 3 & 4 notes would be:

1. ${}^1C_1 = 1$
2. ${}^2C_2 + {}^2P_2 = 1 + 2 = 3$
3. ${}^3C_3 + {}^3P_3 = 1 + 6 = 7$
4. ${}^4C_4 + {}^4P_4 = 1 + 24 = 25$

The total number of musical elements is **36**, which is the number value of Eloha, the Godname of Geburah (Fig. 14). Playing 10 notes in the order of the rows 1, 2, 3 & 4 would generate $(1 \times 3 \times 7 \times 25 = 525)$ possible sequences of notes. One of them is the sequence of one note, one harmonic interval and two chords, i.e., four musical sounds. This is the minimum number of sounds created by playing the tetractys of 10 notes, leaving 524 sequences, each with 5-10 sounds drawn from the **36** musical elements. Compare this with the facts that the seven enfolded polygons in the inner form of the Tree of Life have **36** defining corners and that the (7+7) enfolded polygons have 524 yods. Moreover, there are $(1! \times 2! \times 3! \times 4! = 288)$ sequences of 10 successive sounds consisting of a note, one melodic interval and two broken chords, leaving $(524 - 288 = 236)$ sequences with 5-9 sounds. These correspond to the 288 yods *inside* the (7+7) polygons and to the 236 yods on their sides (Fig. 15)! We see that *the Tree of Life blueprint is inherent in the musical potential of 10 notes played in four steps according to the pattern of a tetractys, the symbol of the ten-fold nature of God (6)*.

3. The seven notes of the Pythagorean scale

The first seven notes of the musical octave:

C D E F G A B

^{*} $288 = 17^2 - 1 = 3 + 5 + 7 + \dots + 33$, where 33 is the 16th ($16=4^2$) odd integer after 1.

have $(2^7-1=127)$ different combinations, where 127 is the **31st** prime number. This indicates how the Godname El with number value **31**, which is assigned to Chesed, the first Sephirah of Construction, prescribes how many groups of notes can be played together to make basic musical sounds using the 7-fold musical scale. These combinations comprise the seven notes themselves and $(127-7=120)$ harmonic intervals and chords, where 7 is the fourth odd integer (also the fourth prime number) and $120 = 2^2 + 4^2 + 6^2 + 8^2$. This illustrates how the Pythagorean Tetrad, 4, determines both the numbers of notes and their combinations. The number of harmonic intervals is ${}^7C_2 = \mathbf{21}$, which is the number value of the Godname Ehyeh assigned to Kether. The number of chords is therefore $(120-\mathbf{21}=99)$. This distinction between intervals and chords is *arithmetically* defined as follows:

$$120 = 11^2 - 1 = \begin{array}{ccccccc} & & \mathbf{21} & & & & \\ & & 17 & 19 & & & \\ 11 & 13 & \mathbf{15} & & & & \\ 3 & 5 & 7 & 9 & & & \end{array} \left. \begin{array}{l} \} \mathbf{21} \text{ intervals} \\ \} 99 \text{ chords} \end{array} \right.$$

In other words, the Pythagorean character of the number 120 is shown by its being the sum of the first *ten* odd integers after 1, **21** being the tenth odd integer after 1 and 99 being the sum of the remaining 9 integers in this tetractys array. 99 is the **50th** odd integer, showing how the Godname Elohim with number value **50** defines the number of chords that can be played with the first seven notes of the Pythagorean scale.

Successive octaves comprise seven notes per octave and the eighth note beginning the next octave. N octaves therefore span $(7N+1)$ notes. The number of 'Sephirothic levels' (SLs) in the lowest, n overlapping Trees of Life is $(6n+5)$. For what values of N and n are the number of notes and SLs the same? The only solutions to:

$$7N + 1 = 6n + 5$$

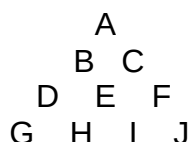
up to $N=10$ can be found by inspection to be $N = n = 4$ or $N = 10$ & $n = 11$, i.e., four octaves have as many notes (29) as the lowest four Trees of Life, whilst ten octaves have as many notes (71) as eleven such Trees of Life have SLs. Every eighth note in successive octaves is of the same type, whilst every seventh SL in successive Trees corresponds to the same Sephirah. The Pythagorean Tetrad and Decad define analogous successions of notes of the scale and the emanations of Sephiroth in overlapping Trees of Life. Excluding the highest note belonging to the next higher octave, four and ten octaves have, respectively, 28 and 70 notes, the same as the SLs in four and eleven overlapping Trees of Life. In general, the counterpart of the last note of the

Nth octave shared with the next higher octave is Daath of the nth tree, which is Yesod of the (n+1)th tree but which is not counted as an SL when the overlapping trees are considered as a separate set.

4. Tetractys of ten notes

A harmonic interval is two notes played together. A melodic interval is two notes played one after the other. A chord is three or more notes played simultaneously and a broken chord is a set of three or more notes played in succession. The following discussion will consider only melodic intervals and broken chords where the notes are all different.

Consider a tetractys array of ten *different* notes:



(Any notes can be considered here — the letters labelling them do not refer to the notes of the Pythagorean scale). The number of intervals and chords that notes *within the same row generate* when played will now be determined. A harmonic interval is a combination of two notes, whereas a melodic interval is two notes played with regard to their order in time, i.e. a *permutation* of two notes. A chord is a combination of three or more notes, and a broken chord is a pattern of three or more notes played in quick succession, i.e., a particular arrangement or *permutation* of these notes. The table below shows the numbers of harmonic intervals and chords (combinations of notes) and melodic intervals and broken chords (permutations of notes) for the notes in the four rows of the tetractys:*

Number of notes, harmonic Intervals and chords			Number of notes, melodic intervals and broken chords		
1	A	${}^1C_1 = 2^1 - 1 =$	1	${}^1P_1 =$	1
2	B C	${}^2C_1 + {}^2C_2 = 2^2 - 1 =$	3	${}^2P_1 + {}^2P_2 =$	4
3	D E F	${}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3 - 1 =$	7	${}^3P_1 + {}^3P_2 + {}^3P_3 =$	15
4	G H I J	${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 =$	15	${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4 =$	64
TOTAL =		26	TOTAL =		84

The number value **26** of Yahweh, Godname of Chokmah, is the number of notes, harmonic intervals and chords that can be played within the four rows of notes, the number value **15** of its older version, Yah, being the number that can be played from four

* Notation: ${}^nC_r = n!/r!(n-r)!$ and ${}^nP_r = n!/(n-r)!$

notes. There are ($26-10=16=4^2$) harmonic intervals and chords (10 intervals & 6 chords). The number of notes, melodic intervals and broken chords is

$$84 = 1^2 + 3^2 + 5^2 + 7^2.$$

This illustrates the defining role of the Pythagorean Tetrad because 1, 3, 5, & 7 are the first *four* odd integers. The number of melodic intervals and broken chords = $84 - 10 =$

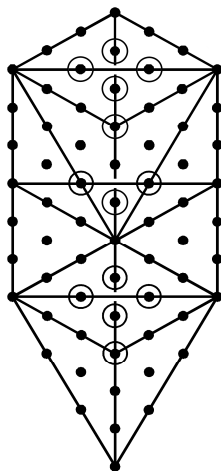


Figure 16

= 80

74, which is the **73**rd integer after 1. The number value **73** of Chokmah determines the number of basic musical elements (namely, melodic intervals and broken chords) that can be played from sets of 1, 2, 3 and 4 notes. The number of harmonic and melodic intervals, chords and broken chords = $16 + 74 = 90$, and the number of notes, intervals and chords of both types = $90 + 10 = 100 = 1^3 + 2^3 + 3^3 + 4^3$. This shows how the Pythagorean integers 1, 2, 3, & 4 express the *total* number of musical sounds created by playing notes from each row of the tetractys. These results can be represented by a tetractys array of the number 10:

$$\begin{array}{c} 10 \\ 10 \ 10 \\ 10 \ 10 \ 10 \\ 10 \ 10 \ 10 \ 10 \end{array}$$

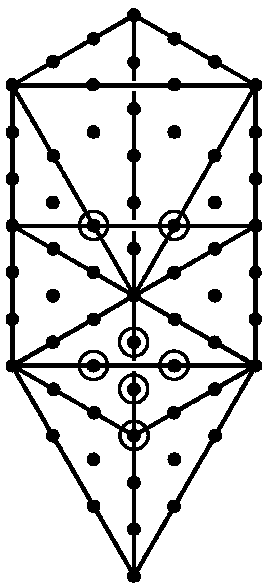
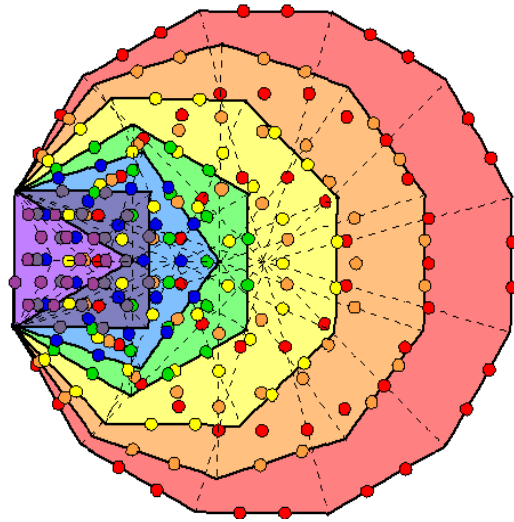
The central number 10 represents the ten notes and the sum 90 of the remaining 10's is the number of intervals and chords that they can create. This beautiful result demonstrates the power of the tetractys and the role of the Tetrad in defining its properties, whatever the nature of the things symbolised by its yods.

The number of melodic intervals, chords and broken chords = $90 - 10 = \mathbf{80}$, which is the number value of Yesod, the penultimate Sephirah. This is the number of yods in the lowest Tree of Life (Fig. 16). The meaning of Yesod is "foundation." It is appropriate, given that this tree is the base of any set of overlapping Trees of Life.

Now consider the three possible orientations of the tetractys of ten notes:

A	G	J
B C	H D	F I
D E F	I E B	C E H
G H I J	J F C A	A B D G

222 =



showing how the Tetrad determines this number, for it is the sum of the first *four* powers of 3. The number of notes and melodic intervals in the three arrays = $10 + 60 = 70$. Compare this with Figure 18, which shows that turning the 16 triangles of the Tree of Life into tetractyses generates 60 yods in addition to those at their ten corners. The ten Sephirothic points can be assigned the notes and the 60 other yods can be assigned the melodic intervals that they generate.

We have seen that the number of harmonic and melodic intervals and chords and broken chords is 270. The number of harmonic intervals, chords and broken chords = $270 - 60 = 210$

$$\begin{array}{c} 21 \\ 21 \ 21 \\ = \ 21 \ 21 \ 21 \\ 21 \ 21 \ 21 \ 21. \end{array}$$

The number value **21** of Ehyeh determines how many harmonic intervals, chords and broken chords the ten notes can create. The number of chords in each array = ${}^3C_3 + {}^4C_3 + {}^4C_4 = 6$, that is, $3 \times 6 = 18$ in the three arrays. The number of broken chords in the three arrays is therefore $210 - 18 - 30 = 162$ (54 per array). The number of chords and broken chords = $18 + 162 = 180$ (60 per array). The number of harmonic intervals and chords = $30 + 18 = 48$ (16 per array). In other words, the number of different combinations of the ten notes (i.e., new sounds) that can be played *simultaneously* when selected from their

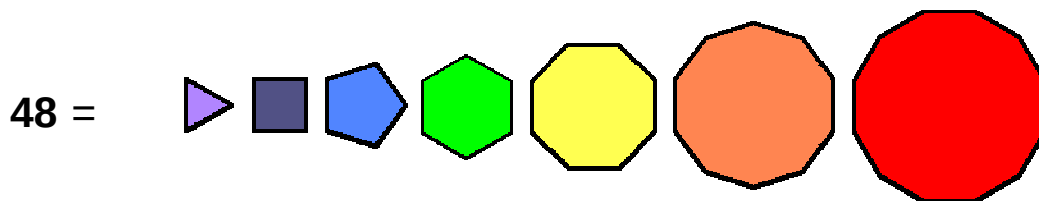


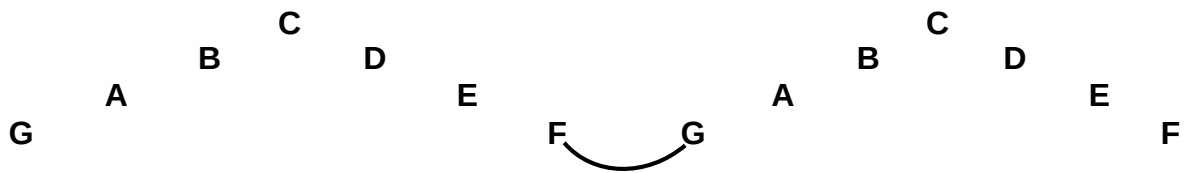
Figure 19. The seven separate polygons have **48** corners.

three possible tetractys arrays is the same as the number of corners of the seven, separate regular polygons (Fig. 19). This illustrates the character of the number **48*** (the number of *Kokab*, Mundane Chakra of Hod) in quantifying the most basic degrees of freedom making up a Tree of Life pattern — in this case the corners of the seven polygons.

The same number appears in the context of what the ancient Greeks called ‘tetrachords.’

* **48** shows its Pythagorean character by being the smallest integer with *ten* factors, including 1 and itself.

They did not experience the musical octave as one complete whole but rather as a two-part structure (5). The octave evolved through the completion of two groups of four notes, or tetrachords. For example, the sequence of notes G, A, B, C below is a tetrachord.



They shared a central note that was always a perfect fourth with respect to the beginning of the first tetrachord (G here) and the endnote of the second tetrachord (here F). The number of permutations of four objects taken one, two, three and four at a time = ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4 = 4 + 12 + 24 + 24 = 64 = 4^3$. **64** is the number value of *Nogah*, the Mundane Chakra of Netzach (astrologically associated with the planet Venus). The number of permutations of four objects taken two, three and four at a time = $64 - 4 = 60$.

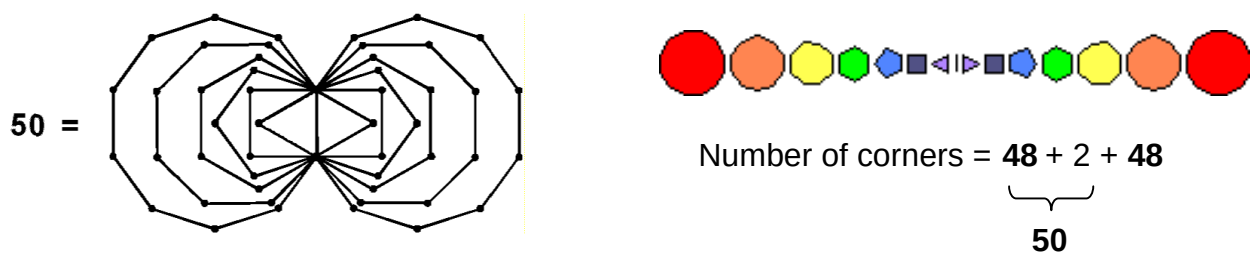


Figure 20

The number of permutations of four objects taken two at a time = ${}^4P_2 = 12$. Hence, each of the two tetrachords in an octave has 12 melodic intervals and $(60-12=48)$ broken chords, the latter comprising 24 $(=1 \times 2 \times 3 \times 4)$ broken chords of three notes and 24 broken chords of four notes. There are therefore two chords each of four notes and $(48+48=96)$ possible broken chords in an octave split up into two tetrachords. Compare these divisions with the fact that the $(6+6)$ enfolded polygons have two corners of their shared root edge and **48** corners outside their root edge, 24 on each side of it, whereas, when separated by the root edge, each set of all seven separate polygons also has **48** corners (Fig. 20). As this set of 12 polygons constitutes a Tree of Life pattern in its own right (see Article 8), we see that the ancient Greek depiction of the octave as two tetrachords conforms to the pattern of the Tree of Life. Elohim prescribes the seven polygons and root edge because, as Figure 20 shows, its number value **50** is the number of their corners (two belong to the root edge).

Of the 162 broken chords generated by the three orientations of a tetractys of ten notes,

six are descending and ascending tetrachords (two per orientation). Therefore, there are $(162-6=156)$ broken chords whose notes are *not* all in descending or ascending sequence. 156 is the **155**th integer after 1. This is how Adonai Melekh, the complete Godname of Malkuth with number value **155**, measures the number of sounds that can be made by playing the $[3 \times (3+4) = \mathbf{21}]$ notes in the rows of three and four of the three tetractyses one after the other but *not* in order of their pitch. **21** is the number value of Ehyeh, Godname of Kether.

5. The Platonic Lambda revisited

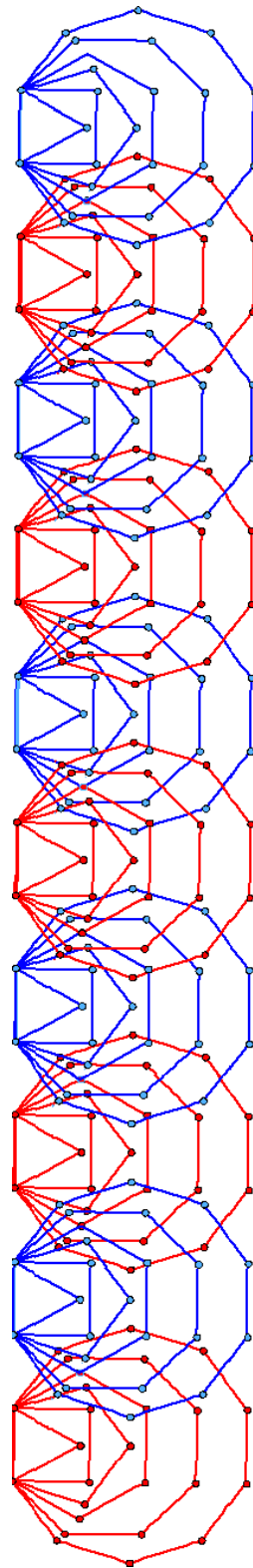
In Article 11, we found that the tetractys form of Plato's Lambda:

$$\begin{array}{cccc} & & 1 & \\ & & 2 & 3 \\ & 4 & 6 & 9 \\ 8 & 12 & 18 & 27 \end{array}$$

is but one face of a tetrahedron whose fourth face is a tetractys that generates in a symmetric way the tone ratios of the Pythagorean musical scale. Properties of this parent tetractys are compared below with the various numbers of intervals and chords generated from a tetractys array of ten notes.

1. Sum of 10 integers = 90 = number of both types of intervals & chords;
2. Sum of 9 integers surrounding centre = 84 = number of notes, melodic intervals & broken chords;
3. Sum of 7 integers at centre and corners of hexagon = 54 = number of broken chords;
4. Central integer 6 = number of chords;
5. Sum of 6 integers at corners of hexagon = **48** = number of harmonic intervals & chords in 3 arrays or number of broken chords in set of 4 notes;
6. Sum of smallest integer (1) and largest integer (27) = 28 = number of notes & chords in 3 arrays;
7. Sum of integers 1, 3, 9, 27 on side of Lambda = 40 = number of notes & harmonic intervals in 3 arrays.

We find that the numbers making up the Lambda tetractys do more than define the tone ratios of musical notes — a function known to musicians and mathematicians for more than two thousand years. They also *measure the various numbers of musical elements that can be played by using the four rows of different notes arranged in a tetractys.*



The number 90 is $^{10}P_2$, the number of permutations of two objects taken from a set of ten objects, i.e., in this context the number of melodic intervals that can be played with ten different notes without regard to their arrangement in a tetractys. In the context of the UPA superstring, a point on each of its ten whorls has (10×9=90) coordinates with respect to the 9-dimensional space of the superstring. Noting that Besant & Leadbeater states in their book *Occult Chemistry* (6) that none of the whorls ever touched one another as they observed them, this means that these ten, non-touching curves require 90 independent (but not necessarily all different) numbers as free coordinate variables. As discussed in Article 12, 90 is the number of trees above the lowest one in what the author calls the ‘Cosmic Tree of Life,’* i.e., the number of levels of consciousness beyond the most physical level represented by the lowest tree. This means that a musical sound containing up to four notes can be assigned to each of these levels of consciousness, with the tetractys of ten notes itself assigned to the 91st level. The counterpart of the latter for the superstring would be the time coordinate, the number that locates it in time.

Alternatively, a melodic interval generated from ten notes can be assigned to these levels. As $^7P_2 = 42$, there are 42 such intervals generated from seven notes sited at the centre and corners of the hexagon in the tetractys and (90–42=48) intervals generated by pairing either these notes with those at the corners of the tetractys or the latter themselves. This 48:42 division corresponds to the 48 subplanes of the cosmic physical plane above the lowest one and the 42 subplanes of the six superphysical cosmic planes. Whether or not this correlation may have deeper significance, it demonstrates a beautiful, mathematical harmony between the permutational properties of ten objects arranged in a tetractys and what the author has shown in previous articles to be the map of all levels of reality. The latter itself is a tetractys with the fractal-like quality that each of the nine yods surrounding its centre is a tetractys and that the central yod is the repetition of this on a spiritually lower but exactly analogous level.

The five types of musical elements are notes of the Pythagorean scale, their harmonic intervals, melodic intervals, chords & broken chords. Their numbers are shown below:

notes	harmonic intervals	melodic intervals	chords	broken chords
10	30 (3×10)	60 (3×20)	18 (3×6)	162 (3×54)

(The second number in each bracket is the number of musical elements per orientation)

* See Article 5 for details about this map of the spiritual cosmos.

There are ($2^5-1=31$) combinations of these elements, where **31** is the number value of El ("God"), the Godname assigned to Chesed, the first Sephirah of Construction. Some of these were discussed above. In order of increasing size, these numbers are:

10, 18, 28, 30, 40, **48**, 58, 60, 70, 78, 88, 90, 100, 108, 162, 172, 180, 190, 192, 202, 210, 220, 222, 232, 240, 250, 252, **280**.

Notice that the **21**st number is

$$210 = \begin{array}{c} 21 \\ 21 \ 21 \\ 21 \ 21 \ 21 \\ 21 \ 21 \ 21 \ 21 \end{array}$$

and that the 28th (last) number is

$$280 = \begin{array}{c} 28 \\ 28 \ 28 \\ 28 \ 28 \ 28 \\ 28 \ 28 \ 28 \ 28. \end{array}$$

The **26**th number is 250. Hence, the Godname Yahweh with number value **26** marks out the number of notes, melodic intervals, chords and broken chords. This significance of this is as follows: the number 250 comprises 10 notes, 60 melodic intervals and 180 chords and broken chords. This number was found in Article 11 to be the sum of the 16 ($=4^2$) tone numbers making up the four faces of the tetrahedron other than its corners. The significance of 250 for superstrings was discussed in this and earlier articles.* Its relevance to the Tree of Life is that the 60 polygons of the first six types enfolded in *ten* overlapping Trees of Life have 250 corners unshared with polygons enfolded in the next higher tree (Fig. 21). They comprise the 10 lowest and highest corners of the ten hexagons, the 60 corners of the triangle, square and pentagon that are outside their root edges and 180 corners of the hexagon, octagon and decagon. The (6+6) polygons constitute a Tree of Life pattern because they have **50** corners prescribed by the Godname Elohim with number value **50** (Fig. 20; see Article 4 for how the other Godnames prescribe their geometry). It is a remarkable demonstration of how Godnames are fundamentally connected to the tetractys that the number of Yahweh should pick out from all possible combinations of basic musical elements (themselves numbered by the Godname of the Sephirah lying below Chokmah) just that number which is also the number of geometrical degrees of freedom denoted by corners of polygons enfolded in

* In particular, see Article 5.

ten Trees of Life! Not five trees, nor nine trees, but precisely the Pythagorean and Kabbalistic measure of Divine Perfection, namely, the number 10!

The number of melodic intervals, chords and broken chords = $250 - 10 = 240$

$$\begin{aligned}
 &4! \\
 &4! \ 4! \\
 = &4! \ 4! \ 4! & (4! = 1 \times 2 \times 3 \times 4) \\
 &4! \ 4! \ 4! \ 4!
 \end{aligned}$$

Suppose that we consider a tetractys of nine notes with the central one missing:

A
B C
D • F
G H I J

The numbers of musical elements now become:

notes	harmonic intervals	melodic intervals	chords	broken chords
9 (3×3)	24 (3×8)	48 (3×16)	15 (3×5)	144 (3×48)

(The second number in each bracket is the number of elements per orientation)

The three tetractys arrays possess 240 notes, harmonic intervals, melodic intervals, chords and broken chords — the same number as the number of melodic intervals,

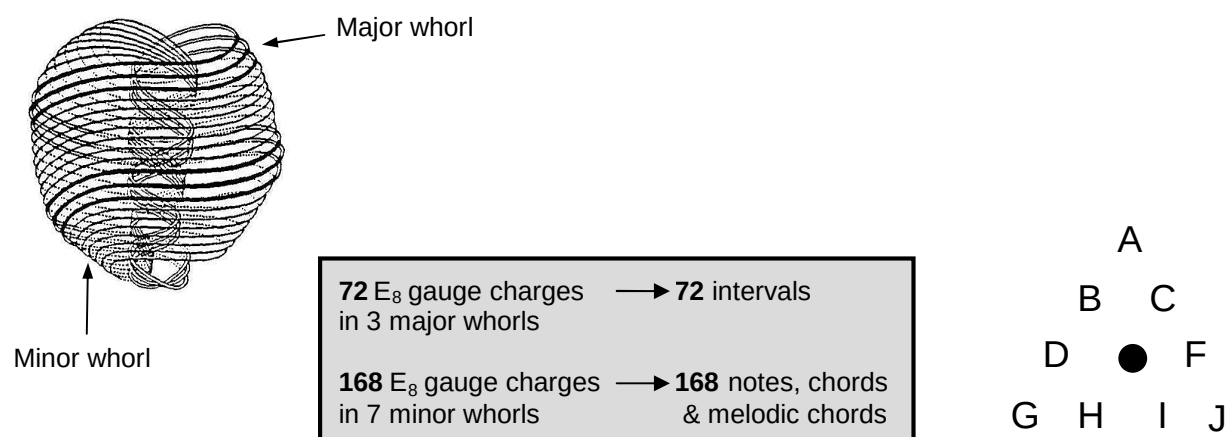
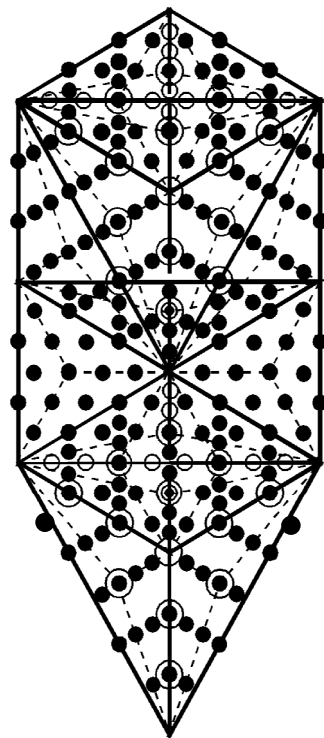
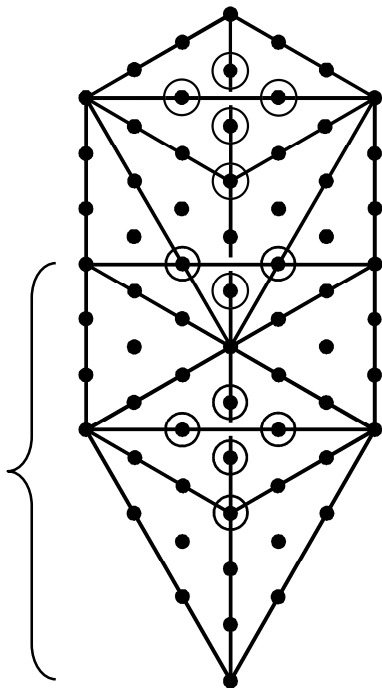


Figure 22. Gauge charges of superstring correspond to musical elements of tetractys.

chords and broken chords generated by a *complete* tetractys of ten notes. In terms of the correspondence between the tetractys and the Tree of Life, the central yod symbolises



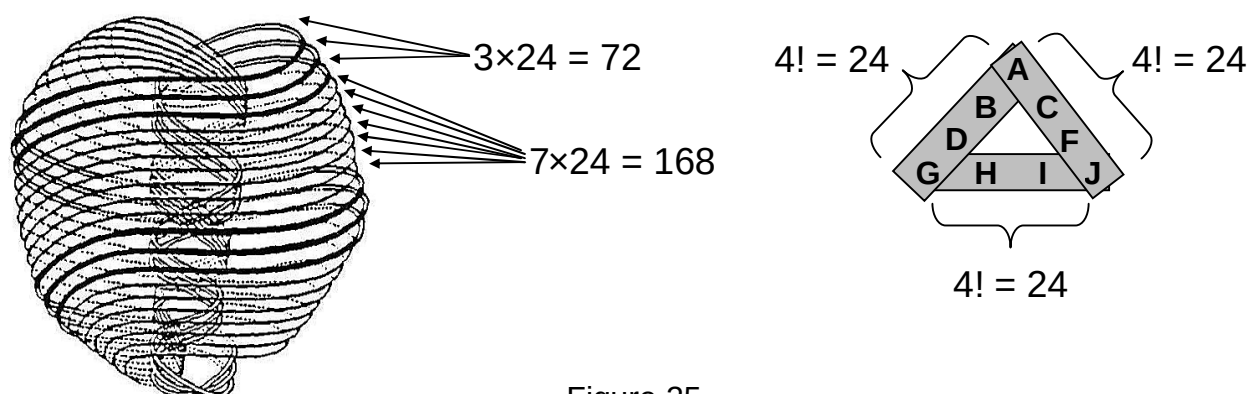
simple roots of E_6 , an exceptional subgroup of E_8 , there being **168** non-zero roots of E_8 that do not belong to E_6 . Remarkably, *the musical elements due to the notes corresponding to Sephiroth above Malkuth correspond precisely in number to the gauge charges mathematically associated with the non-zero roots of E_8 and its subgroup E_6 .*

Transformed into single tetractyses, the lowest Tree of Life contains **80** yods (Fig. 23). Using the numbers (shown above in brackets) of musical elements of each type generated by a tetractys of ten notes, the total number of such musical elements is

$$(3 + 8 + 16 + 5) + \mathbf{48} = 32 + \mathbf{48}.$$

$$= \mathbf{80}.$$

Each yod in the lowest Tree of Life can denote a musical element! Moreover, there are **48** yods up to the level of Chesed, the first Sephirah of Construction, illustrating once again the nature of this number in quantifying the number of *formative* degrees of freedom that are needed to express the part of the Tree of Life that manifests in an objective sense. The **48** yods up to Chesed denote the number of broken chords and the 32 yods above this level denote the number of notes, intervals and chords. In this remarkable way, the number of musical sounds (**80**) created by the notes corresponding to the nine Sephiroth above the objective one, Malkuth, and their division into melodic chords of three and four notes (the beginning of melody) reflects precisely the **48**:32 pattern of the lowest Tree. The number 240 is the number of yods *generated* in the lowest Tree of Life when its 19 triangles are transformed into three tetractyses, that is, yods in addition to its 11 Sephirothic points (Fig. 24). These 240 hidden or potential degrees of freedom



correspond to the potential of 240 musical elements that can be played with the set of nine notes corresponding to the Sephiroth above Malkuth. Notice that the correspondence is both qualitative and quantitative because the 11 Sephirothic points of

the lowest tree constitute its most basic outer form (its Malkuth aspect), so that the hidden 240 yods represent what is *beyond* this aspect, just as the 240 musical elements denote sounds generated by notes corresponding formally to Sephiroth *above* Malkuth.

A **72:168** pattern similar to that found above for three tetractyses with their central note missing also exists when the latter is present. The number of melodic intervals, chords broken chords with three notes. The $(3 \times 80 = 240)$ melodic intervals, chords and broken chords generated by the three orientations of the tetractys comprise $(3 \times 24 = 72)$ broken chords of four notes and $(3 \times 56 = 168)$ melodic intervals, chords and broken chords of three notes. It mirrors the **72** E_8 gauge charges carried by the three major whorls and the **168** such charges carried by its seven minor whorls (Fig. 25). In terms of the charge sources of all the forces other than gravity that superstrings of ordinary matter can exert on one another, superstring physics conforms to the same pattern as the melodic sounds generated by playing the ten notes of a tetractys in its three orientations.

7. The Platonic solids represent the 90 musical elements

In Article 3 it was shown how, when seen as constructed from the tetractys (the Pythagorean symbol of ten-fold Divine unity), the first four regular polyhedra embody numbers of significance to the mathematics of superstring forces. It will now be proved

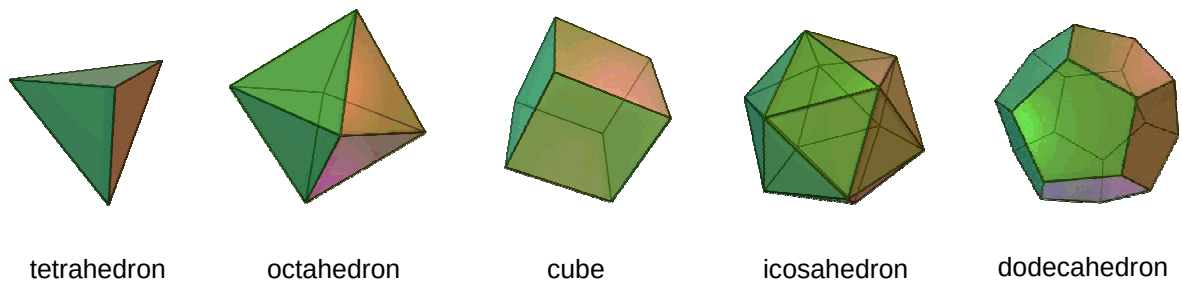


Figure 26. The five regular polyhedra, or Platonic solids.

that the five Platonic solids (Fig. 26) also embody in a geometrical way the numbers of different types of musical sounds that can be generated from a tetractys of ten notes.

The numbers of edges in the five Platonic solids are shown below:

tetrahedron	octahedron	cube	icosahedron	dodecahedron
6	12	12	30	30

The total number of edges is 90, which is the *same* as the number of musical elements *generated* by a tetractys array of ten notes, that is, *sounds other than the notes themselves*. They comprise six harmonic intervals generated from the row of four notes,

12 harmonic and melodic intervals generated from the rows of two and three notes, 12 melodic intervals generated from the row of four notes, 30 chords and broken chords of three or four notes generated from the rows of three and four notes (six chords, 24 broken chords of three notes from the row of four notes) and 30 broken chords (six from the row of three notes, 24 from the row of four notes). The correspondence between the edges of the Platonic solids and these musical elements is:

Tetrahedron	6 edges → 6 harmonic intervals from the row of 4 notes;
Octahedron	12 edges → 12 harmonic & melodic intervals from rows of 2 (3) & 3 (9) notes;
Cube	12 edges → 12 melodic intervals from the row of 4 notes;
Icosahedron	30 edges → 30 chords & broken chords from rows of 3 (1) & 4 notes (29);
Dodecahedron	30 edges → 30 broken chords from rows of 3 (6) & 4 notes (24).

(Numbers in brackets are the numbers of musical elements generated by the stated row)

Notice how the complexity of the musical elements builds up correspondingly with the number of edges of the Platonic solids. They start with the harmonic intervals (the simplest musical sounds), which correspond to the edges of the tetrahedron, the simplest Platonic solid, and finish with the broken chords (the most complex sounds), which correspond to the edges of the dodecahedron, the last of the regular polyhedra. This *natural* progression argues against the agreement between the various numbers of edges and musical elements being merely a coincidence or a concoction.

The Godname Elohim with number value **50** assigned to the Sephirah Binah prescribes both the form of the Platonic solids and the set of ten notes and their musical combinations because the former have **50** corners and the latter comprise 100 musical elements in one array, where 100 is the **50th** even integer. They include **26** *single* sounds (notes, harmonic intervals and chords), where **26** is the number value of Yahweh, Godname of Chokmah. The number of notes, melodic intervals and broken chords generated by a tetractys of notes is 84. As $84 = 3 \times 28 = 3(1+2+3+4+5+6+7) = 3 + 6 + 9 + 12 + \mathbf{15} + 18 + \mathbf{21}$, this is how the number value **21** of Ehyeh, Godname of Kether, determines the number of such musical sounds. The number of musical sounds made up of notes played in succession is 74, which is the **36th** even integer after 2. This is how

Eloha, Godname of Geburah with number value **36**, prescribes the number of sounds of this type. The number of broken chords of four notes generated by the fourth row of a tetractys = $4! = 24$. The three possible tetractys arrays therefore have ($3 \times 24 = 72$) such chords, where **72** is the number value of Chesed, the *fourth* Sephirah from the top of the Tree of Life. **72** is the **36th** even integer. It is therefore also prescribed by Eloha.

8. The octave as analogue of the rank-8 gauge group E_8

The number of combinations of the eight notes of the musical scale = $2^8 - 1 = 255$

$$= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8.$$

The (${}^8C_7 = 8$) combinations of seven notes comprise the combination (let us call it ' 1_7 ') of its first seven notes, C, D, E, F, G, A, B and seven combinations (call this ' 7_7 ') of the octave with six of the first seven notes. (${}^8C_8 = 1$) denotes the single group of eight notes (call this ' 1_8 ').

$$255 = 8 + 28 + 56 + 70 + 56 + 28 + (1_7 + 7_7) + 1_8.$$

$$255 - 7_7 = \mathbf{248} = 8 + 28 + 56 + 70 + 56 + 28 + 1_7 + 1_8$$

'8' denotes the first seven notes and the octave.

$$\mathbf{248} - 8 = 240 = 28 + 56 + 70 + 56 + 28 + 1_7 + 1_8.$$

$$= (56 + 56) + 28 + 70 + 28 + 1_7 + 1_8$$

$$= \mathbf{112} + 128,$$

where $\mathbf{112} = 56 + 56$ and $128 = 28 + 70 + 28 + 1_7 + 1_8$. **112** is the number value of *Beni Elohim*, the Order of Angels assigned to the Sephirah Hod; **248** is the number of *Raziel*, the Archangel of Binah (see ref. 1). Compare this with the fact that the superstring gauge symmetry group E_8 is defined by its **248** simple roots, which comprise eight zero roots (seven of one kind and one of another kind) and 240 non-zero roots. The latter comprise **112** non-zero roots of one kind and 128 non-zero roots of another kind. The following correspondence emerges between the Pythagorean scale and the root structure of E_8 :

- | | |
|---|--|
| 1. 248 combinations of 8 notes other than groups of 7 containing the octave; | → 248 roots of E_8 ; |
| 2. 8 notes of the octave (first 7 notes + octave itself); | → 8 zero roots of E_8 (7 of one kind, 1 of another); * |

* It is unnecessary to explain here their difference in the technical terms of group theory.

3. 240 combinations of 8 notes other than these —→ 240 non-zero roots;
notes & groups of 7 containing the octave;
4. 240 combinations comprise **112** groups of 3 —→ 240 non-zero roots comprise **112**
& 5 notes & 128 groups of 2, 4 & 6 of one kind & 128 of another.
notes, the first 7 notes & all 8 notes.

The number 240 can be also written as:

$$240 = (70 + 1_7 + 1_8) + (28 + 56) + (56 + 28) \\ = \mathbf{72} + \mathbf{168},$$

where

$$\mathbf{72} = 70 + 1_7 + 1_8,$$

and

$$\mathbf{168} = (28 + 56) + (56 + 28) \\ = 84 + 84.$$

72 (the number of Chesed, the first Sephirah of Construction) is the sum of the number (70) of chords of four notes created from the eight notes of the Pythagorean scale, the chord of the first seven notes (1_7) and the chord of all eight notes (1_8). **168** is the number of harmonic intervals and chords of three, five and six notes. The point of this exercise is that the composition of musical sounds created from the eight notes of the Pythagorean

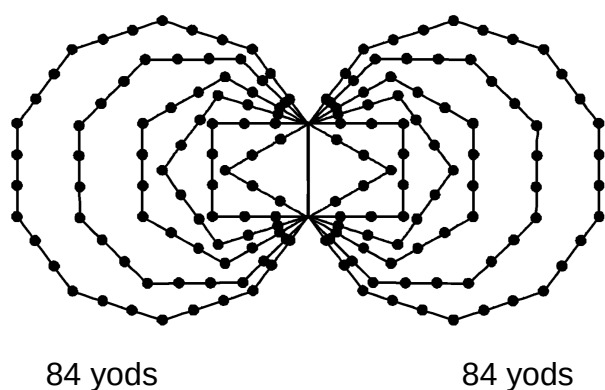


Figure 27

scale can be viewed as analogous to the **72:168** division of the non-zero roots of the superstring symmetry group E_8 and their corresponding gauge charges, which previous articles by the author have shown to be encoded in the inner form of the Tree of Life. As discussed earlier, this difference corresponds to the distinction between the

three major whorls and the seven minor whorls of the UPA superstring, that is, to the distinction in the Tree of Life between the Supernal Triad (the triple Godhead) and the seven Sephiroth of Construction.

168 is the number value of *Cholem Yesodeth*, the Mundane Chakra of Malkuth. The 84:84 division of the number **168** that emerges naturally above from the rearrangement of combinatorial numbers is reflected in its encoding in the first (6+6) enfolded, regular

polygons of the inner form of the Tree of Life (Fig. 27) because there are 84 yods in each set of polygons outside their shared root edge*. This splitting into *equal* numbers is reflected in the UPA itself as the spiralling of each whorl with 1680 coils $2\frac{1}{2}$ times around the axis of the UPA, returning to its top by making $2\frac{1}{2}$ narrower twists (see Fig. 25). Each half of the whorl comprises 840 coils, which is the number of yods shaping the 60 polygons of the first six types enfolded on each side of the central pillar of ten overlapping Trees of Life that represent each of the ten whorls of the superstring. The division of the structural parameter **168** into two halves, as physically manifested in the inner and outer halves of the superstring, is due to the mathematical fact (referring to the equation at the beginning of this section) that ${}^8C_2 = {}^8C_6 = 28$ and ${}^8C_3 = {}^8C_5 = 56$. In terms of the octave, this means that 28 harmonic intervals and 56 chords of three notes (making 84) can be played with an octave, as can 28 chords of six notes and 56 chords of five notes (also making 84). In terms of the first six polygons, namely, the triangle, square, pentagon, hexagon, octagon and decagon, there are, respectively, 5, 8, 11, 14, 20 and **26** yods along their edges outside their root edge. The square and octagon has (8+20=28) yods and the triangle, pentagon, hexagon and dodecagon have (5+11+14+**26**=56) yods. For one set of polygons, the harmonic intervals can be represented by the yods of the square and octagon, whilst the chords of three notes can be represented by the yods of the triangle, pentagon, hexagon and decagon. For the other set, the square and octagon represent the chords of six notes, and the four other polygons represent the chords of five notes. No alternative combinations of polygons are possible. *The musical potential of the Pythagorean octave finds its counterpart in the sacred geometry of the inner Tree of Life.*

9. Conclusion

The numbers of Plato's Lambda tetractys should not be seen simply as generating the tone numbers of the Pythagorean scale and all its octaves. Their absolute, as well as their relative, values have a meaning that is both metaphysical and musical. In the former case, the central number (6), the sum (**48**) of the six numbers at the corners of the hexagon, the sum (84) of the numbers surrounding the centre of the tetractys and the sum (90) of all ten numbers measure, respectively, the levels of physical consciousness above the most rudimentary, the cosmic counterparts of these levels, all superphysical levels and *all* levels of consciousness. In the latter case, these numbers refer to the

* See Article 4 for how the Godnames prescribe this set of polygons.

various musical sounds that can be played with a tetractys of ten notes. They are, respectively, the chords, next, the broken chords of three or four notes played from the base set of four notes, then, the notes, melodic intervals and broken chords, and, finally, the harmonic and melodic intervals, chords and broken chords. The numbers of the Lambda refer to higher levels of consciousness (*analogous* levels in the case of the above combinations) that bear a correspondence to the basic music sounds playable with a tetractys of notes. This is the real meaning of the music of the World Soul. The number value 280 of the Hebrew name of the Archangel of Malkuth is the number of musical sounds that can be played with a tetractys of notes in its three possible orientations. The Pythagorean scale conforms to the pattern of the Tree of Life, which maps each of the 91 levels of consciousness. As the perfect fifth of the fifth octave, the tenth overtone and the 22 fractional notes that precede it complete a Tree of Life pattern. Its tone ratio 24 is central to the structure and dynamics of superstrings. Music, the study of number in time, and geometry, the study of number in space, come together in the five Platonic solids. Their 90 edges correlate with the 90 harmonic and melodic elements, chords and broken chords that can be played with a tetractys of ten notes. This number also characterises the superstring because each of its ten string components extends in nine dimensions, their oscillations described by 90 independent variables. Music, the vibrations of superstrings and the spectrum of consciousness are interrelated in universal correspondence and harmonious proportion through the Pythagorean symbol of the ten-fold nature of Divine Unity.

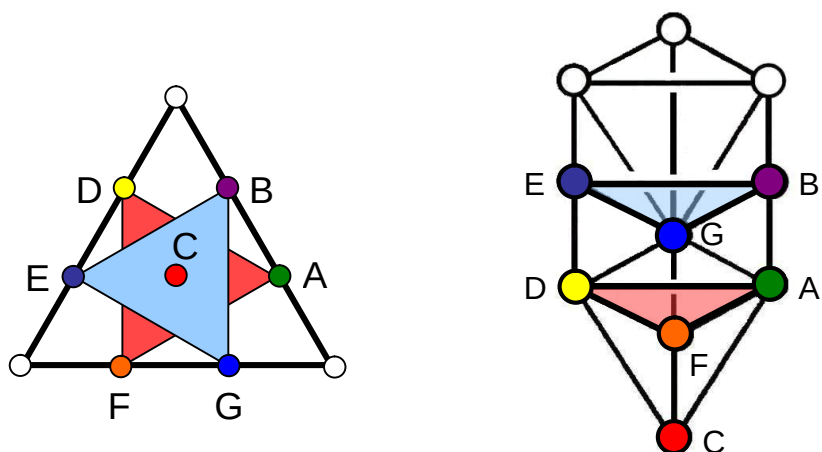
“Intellectual and celestial music, finally, was the application of the principles given by speculative music, no longer to the theory or the practice of the art pure and simple, but to that sublime part of the science which had as its object the contemplation of nature and the knowledge of nature and the knowledge of the immutable laws of the universe. Having then reached its highest degree of perfection, it formed a sort of analogical bond between the sensible and the intelligible, and thus afforded a simple means of communication between the two worlds. It was an intellectual language which was applied to metaphysical abstractions, and made known their harmonic laws, in the way that algebra, as the scientific part of mathematics, is applied by us to physical abstractions, and serves to calculate relationships.”

Antoine Fabre D'Olivet, 18th century Pythagorean.

[illegible]

- | | | | | |
|-----|------------|------------|-------------------------|--------------------------------|
| 7. | DFB | EGA | $9/8 : 4/3 : 243/128$ | $\neq (9/8)^2 : 3/2 : 27/16$ |
| 8. | DGA | EFB | $9/8 : 3/2 : 27/16$ | $\neq (9/8)^2 : 4/3 : 243/128$ |
| 9. | DGB | EFA | $9/8 : 3/2 : 243/128$ | $\neq (9/8)^2 : 4/3 : 27/16$ |
| 10. | DAB | EFG | $9/8 : 27/16 : 243/128$ | $\neq (9/8)^2 : 4/3 : 3/2$ |

Only the chords DFA and EGB listed in (6) made up of alternate notes and spanning the interval of a fifth exhibit the same proportions of their tone ratios, their corresponding notes being each separated by a whole tone. Musical harmony between a pair of chords exists in this sense only for *one* of the ten pairs. All seven notes form ${}^7C_3 = 35$ chords of three notes, that is, $(35-20=15)$ more chords than the other six notes. The number value **15** of the Godname Yah is therefore the number of chords of three notes containing the tonic. The chord EGB is the chord DFA lifted by a whole tone. Just as the six Sephiroth of Construction above Malkuth consist of two



triads, so the six notes above the tonic, which is assigned to the central yod symbolising Malkuth, form uniquely two harmonious chords. The pairs of notes (E, D), (G, F) and (B, A) have the same relative tone interval of $9/8$. Note E corresponds to note D, G corresponds to F and B corresponds to A. This is consistent with their assignment:

- | | | |
|---|---|-----------|
| B | → | Chesed |
| E | → | Geburah |
| G | → | Tiphareth |
| A | → | Netzach |
| D | → | Hod |
| F | → | Yesod |
| C | → | Malkuth |

This is because Netzach, which is below Chesed on the Pillar of Mercy, is its

counterpart in a new cycle of differentiation, whilst Hod similarly is the counterpart of Geburah below it on the Pillar of Judgement and Yesod, the psyche, is the *human* reflection of the spiritual individuality represented by Tiphareth lying above it on the Pillar of Equilibrium.

3. *The God Particle*, Leon Lederman (Bantam Press, 1993), pp. 28-29.
4. The number of yods in n Trees of Life $\equiv Y(n) = 50n + 20$. Hence, $Y(27) = 1370$.
5. *Cosmic Music*, Joscelyn Godwin (ed.) (Inner Traditions, Vermont, 1989).
6. *Occult Chemistry*, Annie Besant and C. W. Leadbeater (Theosophical Publishing House, Adyar, Chennai, India, 1951).
7. *The Universal Nature of Sacred Geometry and its Scientific Meaning*, Stephen M. Phillips (to be published).