

1
Aufg. 2
C14
NUMBERS
<http://www.zahlenbedeutung.de/index.html>

Zu diesem Thema wurde die folgende Arbeit für die Fakultät des Studium
Fundamentale der Universität Witten-Herdecke verfaßt:

Die Zahlen – Idee und Wirklichkeit in der antiken griechischen Philosophie

Betreut durch Prof. Dr. phil. Rustemeyer

Geschrieben von Ruben Stelzner

Table of contents

<u>I. Introduction</u>
<u>II. Pythagoras and the Pythagoreans</u>
<u>1 (Monas)</u>
<u>2 (Dyas)</u>
<u>3 (Tryas)</u>
<u>4 (Tetras)</u>
<u>5-9</u>
<u>10 (Dekas)</u>
<u>III. Plato and Socrates</u>
<u>IV. Aristotle</u>
<u>V. In Conclusion</u>
<u>Bibliography</u>

I. Introduction

The first complete philosophical works handed down to us are from ancient Greece. The concept of philosophy as the teaching of the love of wisdom originates from that period. It was and is the aim of every philosopher to find general principles for understanding our world which are as comprehensive, i.e. as fundamental and interdisciplinary, as possible. These efforts are based on the very human desire for certainty and for values of general validity which testify to the meaningfulness of our existence.

From the writings passed down from the early thinkers we learn that, in earlier times, philosophers were always scientists and empiricists who saw it as their task to collect common principles from both areas of knowledge and derive certain basic rules of life from them. However, in the course of time, the sciences increasingly disassociated themselves from the humanities. In the Middle Ages, the humanities and religion, with which people were familiar from their religious doctrines, predominated. The religious view of the world gave the faithful meaning in life and moral guidance, but also resulted in an increased dependence on the Church which was often misused by the higher ranking clergy. Rapid development in science and scientific research methods soon led to conflicts between the newly discovered laws of nature and the traditional doctrines of the Church. Galileo Galilei, for example, had to revoke his revolutionary thesis of the heliocentric conception of the world because this would have necessitated a reinterpretation of the Bible. However, in the long term, the Church was unable to evade these scientific laws which were open to objective understanding and, with the historic division of spirit and matter (res cogitans and res extensa) by René Descartes, was rapidly deposed and became less and less important.

1620
STELZNER, R.
2003

Now, at the end of the 20th century, we are living in an era of the natural sciences. Until now the natural sciences have seen themselves as being exclusively descriptive, neither interpreting nor evaluating, have avoided teaching meaning and have provided no moral guidelines for using the knowledge they amass. Our knowledge of nature is growing steadily, but with it the inherent danger of misuse. Increasingly we are being forced to subject scientific potential to ethical values. This is really the job of philosophy and ethics, but the value attached to them is extremely low as they usually keep their distance from scientific findings. Stephen Hawking describes this dilemma as follows: *"Up to now, most scientists have been too occupied with the developing of new theories that describe what the universe is to ask the question why. On the other hand, the people whose business it is to ask why, the philosophers, have not been able to keep up with the advance of scientific theory."*¹ A terse sentence from Edward Teller² also reflects the distance between the natural sciences and philosophy with the connected questions of meaning, *"The scientist is responsible for knowledge and for the explanation, but not for how the knowledge is used."*³

The power of the natural sciences and the diminished importance of the humanities have led to a lack of orientation and ties among humankind. These result in division and separation, crime, the formation of sects, etc. The desire for interdisciplinary values of general validity to mediate between the natural sciences and the humanities as a common denominator is continually increasing. More and more scientists are now making it their task to search for such a connecting link, a kind of archetype, and are turning once again to classical philosophy. Thus, for example, Carl Friedrich von Weizsäcker, writes the following as a physicist and philosopher, *"As a physicist with a philosophical chair, I felt the double duty to interpret modern science philosophically and to teach classical philosophy. The more I become involved in these two tasks, the more inseparable they proved to be. All the basic terms which science naively uses have their origins in the efforts of thought of classical and, in the final instance, Greek philosophy."*⁴

C.G. Jung worked all his life on the search for such archetypes which run as basic principles right through all fields of the sciences of mind and matter. Together with his friend, the scientist and Nobel Prize winner Wolfgang Pauli, he turned his main focus to mathematical ideas within the meaning of classical philosophy. Only shortly before his death he discovered in the basic components of mathematics, the numbers, an archetypal character still largely unknown today. His pupil and personal assistant, Marie-Luise von Franz, published a book entitled *"Zahl und Zeit"* in 1970 and reveals in the preface the importance numbers had for C.G. Jung at the end of his life, *"After completing his work on the synchronicity principle in 'Naturerklärung und Psyche', C.G. Jung expressed the suspicion, mentioned briefly in the work itself, that we could probably advance from here further into the area of uniform reality of the psyche and matter by examining the archetypes of the natural numbers. He even began to make notes on a piece of paper of several mathematical properties of the first five integers in the number series. But about two years before his death he gave me this with the words, 'I am too old to write this now, so I am giving it to you'. I didn't know for a long time whether I should really take on this task or just bear the subject in mind in order to pass it on to someone with a greater calling."*⁵ It is still largely unknown today that at the end of his life C.G. Jung attributed such a great importance to the numbers as qualities. He describes number as *"the archetype of order of which humankind has become aware"*⁶ and regarded it as *"a genuine symbol both in its arithmetical nature and in its content"*⁷. C.G. Jung sees in the numbers basic principles whose qualities are reflected in the humanities and natural sciences. However, he is not the only one to attribute such great importance to the principle of number.

When studying the classical Greek philosophers, it is noticeable how frequently the original texts mention numbers in the sense of qualities. As early as the presocratic thinkers, in particular by Pythagoras and his pupils, the numbers are described as the nature of all things. The presentation of the meaning of the numbers then runs through the works of Plato, Aristotle and of philosophers long after classical antiquity. It therefore seems interesting to take a more detailed look at the importance attributed to the numbers in the origins of our philosophy. This seems particularly important if we consider the influence the thought patterns of ancient Greek philosophy have had on the development of today's natural sciences and how strongly these basic principles are reflected in the modern humanities. This paper is intended to show the role the numbers play for the classical Greek philosophers, Pythagoras, Plato and Aristotle.

II. Pythagoras and the Pythagoreans

Pythagoras lived about 550 BC and was one of the Greek philosophers we today consider to be one of the presocratic philosophers. As with most thinkers prior to Socrates, only very few of his original sources still exist. His theories were mostly passed down by word of mouth and only written down later by his pupils and disciples. However, it is essential that we study his theories as he was the first of whom we know that he placed the meaning of the numbers at the centre of a Weltanschauung, a concept of the world.

According to tradition, Pythagoras was the first to use the word "philosophy" as we understand it today. It seemed to him presumptuous to call himself, in the manner usual until then, a "sophos" meaning a wise man and so, more modestly, he called himself a "philosophos", a friend or lover of wisdom. On the one hand, Pythagoras is seen as the religious and ethical reformer of his time but also, on the other hand, as a strict scientist and, in particular, as a mathematician. In Croton he founded the Pythagorean Brotherhood (named after him) to pursue his moral, religious, political, but in particular practical, scientific aims. The Pythagoreans were not working on an abstract, theoretical, specialised science; they saw science much more as an aid for finding very definite ideals of life with direct, practical application. Scientific findings resulted in practical guidance which helped human beings to act correctly and make decisions. What is and what should be thus formed a direct unity. The strict rules of this Brotherhood were based on the assumption that the aim of humankind lay in comprehending the divine order of the world and that this was of a mathematical nature. Pythagoras did not therefore study mathematics as an end in itself as a restricted, specialised science. He placed mathematics and in particular the mathematical elements, the numbers, at the centre of his philosophy. Pythagoras recognised numbers as the absolute principles of things and this certainly not only because of their quantitative, arithmetical properties, but due far more to their principle-like, qualitative characteristics. To him the numbers were individual entities. Pythagorean teachings see in them the real secret and the elements of this world. The harmony of the world is based on the fact that everything in it is arranged according to numeral relationships. The Pythagoreans started out from a basic order, a unity of the world based on the numerical principles. According to this, numbers do not conform to laws invented by man, but are very fundamental principles underlying all development and evolution of life. The interaction of the various elements which form a certain hierarchy create an all-embracing harmony which give the world and life rhythm and system. The Pythagorean Stobaeus writes in one of the works handed down to us, "*For the number also contains everything else and between all the numbers there exists a mutual, rational relationship (...) The One is the principle of the numbers*"⁸. Pythagoras often describes this harmony of the all-embracing One, particularly in art. He seems to have been the first to have attributed the harmony of the notes in music to their natural, whole number or integral proportions. By means of his monochord Pythagoras demonstrates that the intervals are based on simple, whole number proportions⁹. Harmonies to a particular note can only be achieved by playing a whole number multiple or a whole number part of the string for that note. Today this fact is taken for granted in music. However, his experiments confirmed Pythagoras in his conviction that the whole numbers could not be a thought construction invented by man, but must have existed a priori in nature.

In the written records of the Brotherhood handed down to us, the numbers are described individually in the form of symbols. The individual numbers reflect different, often apparently contrasting qualities and are thus attributed a kind of character. The meanings of the first integers ought to be explained here briefly. This appears necessary because other theories, such as Plato's theory of ideas, refer in essential features to the Pythagoreans, while no longer explaining the inner logic of the Pythagorean view of numbers¹⁰.

1 (Monas):

The One is the symbol for the unity of the world, the basis from which everything else follows. It is divine, without form and thus not physically tangible. The One is in everything which exists and is therefore to be found in all other numbers. Every subsequent number relates to Monas and illustrates this in its own way. The numbers

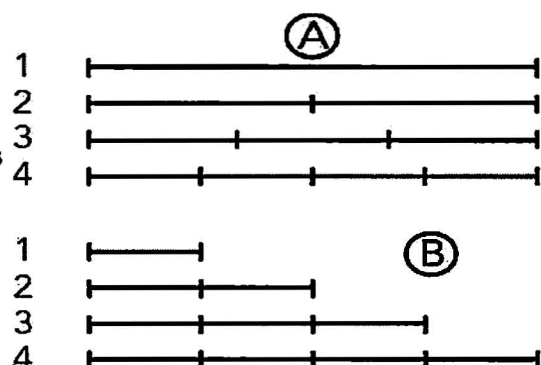


Figure 1:

A) Looking at the "hierarchical structure" makes the all-embracing Monas clear

B) Regarding the number as an "accumulation"

proceeding from Monas do not contradict the preceding unit, but can only be interpreted within its meaning; they are further manifestations of the very same prerequisite – the One.

For the Pythagoreans the One takes a superordinate position (see illustration 1 A). As the original number it “embraces” all the other numbers. All subsequent numbers are based on the One and originate in the frame of unity. As Monas is the first number and represents the principle of unity, the Pythagoreans always see the world as a part of this all-embracing unity. This explanation seems simple and plausible, but frequently causes misunderstandings which arise from reversing these elementary prerequisites. For example, many contemporary scientists no longer assume an all-embracing perfection; on the contrary, they take the apparently natural imperfection which they believe is reflected in the being of humankind as their highest principle. In the numerical understanding of the Pythagoreans, these scientists would place the Two, the principle of division and polarity, before the One.

2 (Dyas):

The Two symbolises the quality of duality. With regard to Monas it forms the apparent opposite of One and reflects the polarities existing in the world. According to Pythagoras everything which exists is a product of this duality. It symbolises the division and resulting development of opposites which form the basis of all existence. They have been compared with the poles between which our earth is “hung” and the polarised, symmetrical physical structure of all living beings. It is the quality of duality which makes life possible, by marking off within the unity a space for its existence. At the same time, its quality is linked to the emergence of life. The Two is the being of a different nature, the different, the special, which stands out as the antipole and provides tension. The Two is often misunderstood as the principle of the negative and imperfect, because it forms the opposite to Monas. In the Pythagorean, “structuring” view of numbers, the Two, like every subsequent number, is only one quality within the framework of the One and thus not a real opposite of the One, but rather a clarification of the principle of unity. The Pythagoreans list ten opposites to which all things can be attributed:

- 1.) Limit and the unlimited
- 2.) Odd and even
- 3.) One and many
- 4.) Right and left
- 5.) Male and female
- 6.) Rest and motion
- 7.) Straight and curved
- 8.) Light and darkness
- 9.) Good and evil
- 10) Square and oblong

Not much more is known about these ten original opposites of Pythagorean teachings. However, contrary to the interpretation of Hegel, evidently the only philosopher to have voiced an opinion on them, it may be assumed that these opposites are not to be regarded merely as an arbitrary list, but hold a deeper meaning. When considering the principle of the One to which all numbers relate, it becomes clear that each of these opposites forms a whole, something perfect, which provides a better understanding of the original One (Monas). The Two splits the original One into apparently irreconcilable polarities. However, these extremely different types of opposites can only be seen as a whole; one part makes no sense on its own. Thus, for example, one cannot think of >right< without >left<. “Right” can only be understood if the >left< is also known, i.e. the individual opposite can only be understood in view of the unity of both (Monas). Therefore Dyas, like every number thereafter, is only a further manifestation of Monas. With its dividing principle it serves to emphasise the One at the beginning which is illustrated more clearly in the number ten. The One is obviously elevated in the ten. Thus the division into ten opposites is resolved in the sense of the whole.

A view of the world in the form of polarities can be found time and again in many philosophical, artistic and scientific works since the teachings of the Pythagoreans, even if they are not consciously linked to

the quality of the number Two. Goethe's "Faust" is one well known and often quoted example: "*Zwei Seelen wohnen, Ach! in meiner Brust,...*" ("Two souls live, Oh! in my breast,...")¹¹

3 (Trias):

The Three is seen in the theories as the binding link between One and Two. It is the link connecting the first two numbers and thus becomes the number of a first whole, a triple. While Monas and Dyas could still be understood in a real sense – because they are objective and opposite – the number Three expresses the purely functional – binding – quality. It is seen as a functional and spiritual original element preceding every actual appearance. It is the first number to describe a symbolic process of development for it connects the One (1) with the opposite (2) at a higher level and is thus pure function. This first functional term in particular is said to be connecting and organising. The principle of the triple stands as a kind of "original formula" behind all actual and material things. For example, the abstract terms of space and time on which all existence is based are marked by the principle of the triple. Space determines the order of one thing next to another and consists of three dimensions, as does time, which determines the order of one thing after another and is structured in the present, past and future.

The Pythagoreans recognised that life in the abstract sense is development and function. Therefore they always saw science merely as the search for the principles of this function and did not try to search for final truths in the form of the smallest particle as does our modern science on the basis of the Epicurean "atomon".

The Pythagoreans also derive from this the three forms of address for the gods. For them the individual gods are not static. They represent a spiritual function rather than a "thing". Gods are principles in action, characterised by their various functions and embodying these in a human shape. They cannot therefore simply be described as a whole (Monas) or an opposite (Dyas), but rather mediate between these and are thus attributed to the functional principle and the triple. Many later philosophers see here, quite rightly, a connection to the Christian trinity. Most other religions, philosophical theories and systems of oracles are also based on such a underlying trinity. It appears to be an inspired Pythagorean presentiment that we meet this division into three classes in the modern natural sciences. Not only are the two parameters already mentioned, space and time, marked by the triad, but also many other fundamental unities such as the primary colours - red, yellow and blue, the physical aggregate conditions solid, liquid and gas, and many others.

4 (Tetras):

The Four is the physical product of the union of Monas with Dyas. If the Three was simply the connecting principle, i.e. the function, the Four is the result, the actual, the material, now tangible in its physical form. One can describe it as the coagulum of the preceding function.. Tetras is thus the specifically physical, the manifestation of the spiritual principle of the Three. The physical exists in space and in doing so makes space visible. Thus Pythagoras was also the first to establish that space is defined by four points. This is confirmed very clearly by the geometry of bodies: the simplest polyhedron in which all sides are symmetrical is a tetrahedron consisting of four points (corners) and four equilateral triangles. This simplest of the five Platonic bodies points with its triangular areas to the preceding triad from which it is created. Right through to present day mathematics, the fourth of the simple geometric symbols is allocated to the body (one to point, two to line, three to plane and four to body) in accordance with the findings of Pythagoras.

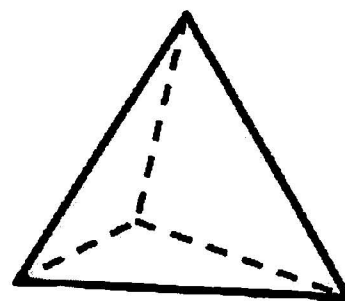


Figure 2: Tetrahedron

The first visible elements - fire, earth, water and air - described by the presocratic philosophers corresponded to Tetras. Today the consideration of these four elements is regarded as outdated. Impressively, today's "hard-core" sciences which deal with the physical and the objective are also

based on Tetras: the four quadrants in mathematics representing it, just as do the four basic arithmetical operations. The earth, the reflection of the physical on a global scale, reveals its secrets to the geometrician through the four points of the compass. It is impressive when the biologist discovers the four protein bases, adenine, guanine, cytosine, thymine as the actual basis of life. The latest discoveries in physics also seem to be subject to a basic quadruplicity: physicists have discovered four basic natural powers (gravitation, electromagnetism, weak and strong atomic power), in the combination of which they hope to find the key for discovering a global formula.

The Pythagoreans emphasise Tetras and worship it as a holy number, regarding it as a basic component of the world and allocating it to nature, because it describes nature's basic principle. There is an analogy to the act of information in the entry of the spiritual and the principle (Trias) into the form of the physical (Tetras). Therefore each form is, for the Pythagoreans, the bearer of a spiritual principle, because the functional and spiritual (Trias) always coagulates and becomes visible in the form (Tetras). The Pythagoreans used to honour this in the following oath: *"With pure senses I swear to you by the holy Four, the original source of eternal nature and the very base of the soul."*

Tetras manifests itself by means of the connecting number Three, which now in turn finally moves the preceding, tension-producing original polarity (Dyas) into a more perfect light, because it makes the unity and perfection (Monas), which was at the very beginning, directly visible and tangible by its physicality. Each quality of the holy and the whole has already been mentioned in the ten opposites. The Four thus reveals the same wisdom expressed in the number Ten already mentioned. Four and Ten overcome the division in the same way. The Pythagoreans recognised that relationship and illustrated it in the tetraktys (see Dekas).

5-9:

From the Four the Pythagoreans moved on directly to the Ten. The numbers Five to Nine are not described in the written records handed down to us. This is probably not coincidence, but was intended by the Pythagoreans. The higher numbers, and particularly the Five, were obviously only revealed verbally and even then only to immediate pupils and initiates. This is supported by the story handed down about the number Five, that the Pythagorean Hippasus died because he was said to have divulged the regular 5-membered body (pentagram).¹²

Where does the prohibition on publishing knowledge of the Five or explaining it to the uninitiated come from? Although no direct knowledge of the Five has been passed down to us, it is possible to deduce indirectly from the logical structure of the preceding qualities, from our knowledge of Egyptian numbers to which the Pythagoreans are known to have had access and from Pythagoras's theorem:

The Five must develop out of its predecessors in the same way as all the other numbers. If therefore the Four is the actual, the material and therefore the objective, then – according to the model of opposites of Monas and Dyas – the objective and concrete must now be confronted by the subjective and individual as the next quality. How can the subjective, which by its very nature differs from one observer to the next, be recorded and explained in public? Logically, the unwillingness of the Pythagoreans to describe the subjective could be based on this paradox.

Subjective observation is a putting into relationship. From the logic of the previous numeral qualities, the Five connects spirit (Trias) and matter (Tetras). Pythagoras's theorem is impressive confirmation of this. According to indirect records, Pythagoras allocated it to the number Five. This relationship, which today is seen only mathematically, reveals the quality of the number Five on the simplest Pythagorean triangle with the sides 3, 4 and 5. The Five is allocated to the hypotenuse. This connects Three and Four, alias spirit and matter, in the right way - at a right angle - with each other. The Pythagorean triangle happens to enclose exactly the circle of unity, i.e. the circle with the radius of One. According to the Pythagorean wisdom of numbers this could easily be understood symbolically, but it is impossible to say to what extent it is speculative, as nothing has been handed down

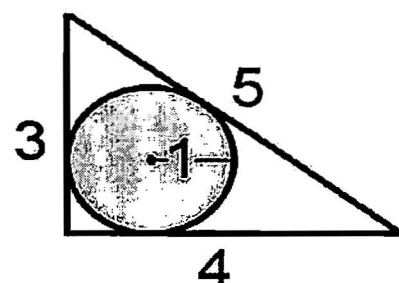


Figure 3: Pythagorean triangle

from the Pythagoreans in this respect. Pythagoras himself described this most simple of Pythagorean triangles as the wedding triangle, just as he called the theorem we know as that of Pythagoras the wedding theorem and allocated to it the quality of the number Five.

10 (Dekas):

The description of the Ten by the Pythagoreans follows immediately after that of the Four. Apart from the reason mentioned above for the Pythagoreans not having described the Five, the type of description (the Ten after the Four) makes the direct proximity of the number Ten to the number Four very noticeable. This is by no means a coincidence. The Pythagoreans give us a plausible and easily demonstrated explanation for it, which they call the "tetraktys" by which they mean the connection seen in calculation between the two numbers:

$1 + 2 + 3 + 4 = 10$ are known as triangular numbers. The sum of the first four numbers is ten, which is also the basis for our system of presenting numbers (the decimal system). The Four makes the unit and perfection (Monas) at the beginning visible and tangible by its physicality, which was the reason for its holiness. It is the unity at a further developed level. The Ten then reflects that principle at an even higher level. In the Ten, the One (unity) takes on a new dimension. The relationship of the unity (One) to the number Ten corresponds metaphorically to the relationship of the apple pip to the apple tree.

The confrontation of Four and Ten – both of which describe in different dimensions the higher development in each case – and thinking about them at the same time together makes clear the box principle of that development. This is an important Pythagorean basic principle: unity and perfection develop into ever new dimensions and can be recognised as the constant principle in everything despite the changing forms.

Remarkably, this core idea of the tetraktys is found in the latest scientific theories today. For instance, in fractal geometry. Mandelbrot's fractals keep returning, apparently mystically, to form the same picture regardless of size, indicating a constant unity. The fact that an individual's entire hereditary disposition exists in every single body cell, regardless of its form and specialisation, indicates the same principle. The Pythagoreans described this apparent law of nature in the form of the number Ten.

The abstract idea of the tetraktys is reflected in Pythagorean geometry. They illustrated it as a triangle and the fundamental triangular form in connection with the Ten and Four was felt to be confirmation of the spiritual triple already described on which all things are based. The triangular tetraktys has on the one hand a side length of Four and on the other hand shows at the same time the possibility of constructing a triangle from one, two, three and four points. Such harmony fascinated the Pythagoreans, which is why they considered the tetraktys and also the tetras to be holy. They even swore oaths on them.

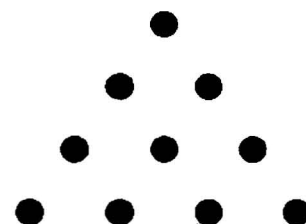


Figure 4: Tetraktys

We can summarize by saying that for the Pythagoreans Ten meant the "perfection" contained in the first four numbers and their stages of development: $1 + 2 + 3 + 4 = 10$. Ten is seen as the most important number, for it restores unity, although this time as the total of individual details, the completion of a development process. It unites the various, apparently contradictory qualities of the other numbers to a harmonious whole. It is accordingly the principle of unity (Monas) at a higher level. The special position of the Ten is often mentioned. The following words of the strict Pythagorean Stobaeus on the number Ten have been passed down to us: *"One must measure the performance and essence of the number by the power in the number Ten. For the power of the number and the Ten is great and perfect, omnipotent, divine and heavenly, the beginning of human life and sympathetic leader. For without this (power) everything is unlimited and unclear."*¹³

The Pythagoreans regarded the numbers as the essence of things. The numbers One to Four described simple, typical natural principles which stood behind all the appearances and life processes of the world like a smallest common denominator. Each number is seen as a special quality consisting

primarily not of a variety of individual details, but as a unity in itself, i.e. with an entity of its own. All numbers present the unity, each in its own way. This thought led the Pythagoreans to the realisation that unity is the basic principle of our world. On it is based the perfection of our world.

Pythagoras and his pupils are among the oldest philosophers known to us today. They are the first thinkers whose theories, or at least fragments of them, have been passed down to us. This is due not least to the eminent importance of Pythagorean theories for many great philosophers of the ancient world still known to us today. They continued to build upon the thinking of the Pythagoreans who, however, are not known to have been passed down any actual descriptions of numbers. At the same time, as we shall see, several later thinkers bear witness to the fact that they had been very involved with them. The influence of the Pythagoreans did not end with the death of their founder, but reached out, far beyond the circle of their immediate followers, throughout antiquity. In the centuries after the birth of Christ, the school of Neo-Pythagoreanism, which was based on Pythagoras, blossomed and earned recognition for a time. The significance of the numbers for this branch of philosophy and its interdisciplinary effect on other ancient thinkers is unknown to most modern philosophers today. However, it ought to awaken their interest that, on the one hand, a view of the world no longer discussed today possessed such impressive persuasive powers at that time and, on the other hand, that on a peak of the "objective sciences", the question of the quality of numbers is being placed again today (see page 25/Barrow).

III. Plato and Socrates

Today Plato is considered to be the most important personality of ancient Greek philosophy. We find his fundamental ideas not only in modern humanities but especially in metaphysics. They also form the foundation of the classical natural sciences, particularly mathematics.¹⁴

In his thought patterns Plato was strongly influenced by the philosophy of Socrates, who had always been concerned with the essence of generality significant in the many specific cases. He linked up with the theories of Heraclitus that everything of the senses is constantly in flow and subject to change.¹⁵ But if knowledge is to exist at all, there must also be other permanent entities apart from the objects of the senses. Therefore Plato saw the causes of things experienced not like some of the presocratic thinkers in a (quasi) material substrate, which is how Aristotle also interpreted the Pythagoreans, but in a spiritual original image he called an idea. An important criterion for the acceptance of an idea is that its generality can be recognised, whereby the different details viewed together refer to an idea. According to Plato the senses do not perceive anything permanent and therefore give not certainty, but only a deceptive opinion. Only the notions, once correctly formed, can always be changed and represent true knowledge. The notions are the reflections of the ideas.

The idea of the good, the beautiful, the One in its purest form, is the basis of all ideas and at the centre of Plato's theory of ideas. Plato places the idea of the good beyond anything existing and compares it to the symbol of the sun, the source and the origin of all becoming and being. Plato's ideas are described as the original images on which all appearances are based. However, they are not just simply general notions formed in our minds from looking at the individual objects and summarising their common characteristics. They are actually real, they are even according to Plato the only true reality (-> the cave parable). Whereas the individual objects are transitory and pass away, the ideas live on as their immortal original images. At this point, the very fundamental, common base which Plato's theories share with those of the Pythagoreans becomes clear. The idea of a comprehensive oneness (the idea of the good, the beautiful, the One) as the essential principle behind all the details of the world and the processes of life is common to both theories. The idea of the good, the One is the only idea which Plato actually describes, otherwise he only speaks generally of the original image-like ideas to which all appearances can be referred. This original idea in the theories of Plato therefore apparently means the same as the principle of Monas (One) to the Pythagoreans. It may be assumed that Plato also studied the wisdom of the numbers intensively. A close examination of the writings of Plato shows clear evidence of this. In the following dialogue with Glaucon, Plato's point of view of the wisdom of a number becomes very clear:

" Counting, however, and calculating is entirely concerned with the number. This is shown as leading the way to truth. In a very excellent way. And it is part of the knowledge we are seeking. It is

necessary for the warrior to understand this for his lists; for the philosopher, however, because he wishes to rise above what is visible and is coming into existence and must grasp the essence or he is never really the actual calculator. (...)

It is therefore up to us, dear Glaucon, to introduce this as a legally compulsory subject and to press those who wish later to share in the greatest honours of the state, to start studying mathematics, to become involved and to take an interest in it, not in the general way but until, by pure thought alone, they arrive at the nature of the numbers, not for buying and selling like merchants and shop-keepers (...) but for their very soul and for the lightness of their turning back from becoming to being and the truth(...)

And now, I said, I understand, after the knowledge of mathematics has been thus described, how magnificent it is and how useful to us in many ways for our purposes if pursued for the sake of knowledge and not for the sake of trade. You see therefore, my dear, I said, how necessary this knowledge must indeed be for us as it leads the soul up into the heights, compelling it to become involved with the numbers themselves and not to be satisfied when somebody shows numbers with visible, tangible bodies and talks about them. (...)

For all these reasons therefore we must not relinquish this knowledge, but the most noble natures must be instructed in it."¹⁶

This dialogue follows almost directly after the well-known cave parable. It almost looks as though knowledge of the meaning of the numbers were prerequisite for understanding and interpreting the parable.

Like Pythagoras, Plato also studied mathematics in detail and made great discoveries, particularly in the field of geometry¹⁷, which we today consider to be the basic pillars of this science. Apart from its importance as the art of calculation, he was also deeply involved with its content matter and with the symbolic side, with the numbers as archetypes. Plato saw a double nature in the numbers; for him they had both a quantitative and a qualitative character. He adopted this insight from his teacher Socrates whom Plato quotes as saying, "that there are two kinds of theories of numbers and two of the art of measurement and in consequence still many others of this kind which have a dual nature while having one name in common."¹⁸

In the written records handed down from Plato we do not find a direct allocation of qualities to the various numbers beyond the number Two, as we do in the case of the Pythagoreans. He basically studied the One and the Two, from which the qualities of the other numbers develop. Each number consists of ones, which are each one in itself and is itself, depending on the number of ones gathered together, not many, but a certain how many, the unit of a collective manyness. The numbers are accordingly each in itself a whole, a One with an individual quality.

In many dialogues he discusses the original and typical problems of apparently reluctant opposites which he obviously named with the quality of the Two. The Two is described as the principle of "differentiation".¹⁹ It defines the polarity between which life and development take place and thus makes up the certainty of being. The human being lives in this world of polarities and is the only living being in a position to recognise this and to appreciate it consciously. In his cave parable in particular, he describes the various stages in the awareness of human beings. Plato considers that only those who recognise the original image-like ideas behind the appearances, which he describes as shadow images and who act according to these principles are on the way to wisdom.

In later years Plato loved to link the ideas to numbers using Pythagorean lines of thought. It is largely unknown in what detail he directly allocated the ideas to the numbers in his philosophy beyond the numbers One and Two. There is no mention of this to be found in his personal written estate. We find more on this question in the "indirect" records, which are the indirect or doxographic records of witnesses of verbal, non-literary statements of Plato; these are reports first written down by Plato's pupils and then produced independently of his published literary works. The texts of these indirect records are far more numerous than his own writings. Plato himself attached much more importance to passing on knowledge verbally, probably for the same fundamental reasons which caused the

Pythagoreans for example not to describe the number Five. He explicitly emphasises that he would never entrust the innermost core of his teaching to writing and therefore to misunderstanding or even disfavour.²⁰ On this he says, *"there is no writing from me and there will never be; for it cannot be pronounced like other things which one can learn but it ... occurs suddenly as though ignited by a leaping spark, a light in the soul which keeps itself alight from now on."*²¹

The texts and interpretations of the doxographic written records are dubious and unexplained and several schools have built up around them. The reports centre on the discussion of the principle of the One and the Two, to which the principles of the numbers are allocated. According to the indirect records, Plato tried to attribute the ideas to higher and simpler principles, the so-called ideal numbers. These were said to be located above the ideas and to exist only in a strictly limited number, in contrast to the ideas which are as numerous as the notions. Aristotle writes in his chapters on Plato and his theories on the relationship between numbers and ideas, *"But if the ideas are not numbers, their existence becomes quite impossible. For what should the principles be from which they could arise?"*²² One assumed that Plato only used the numbers One to Nine, because all the other numbers arise from them. The numbers above Nine correspond to the details of the world, but are not new independent principles but a combination of the nine archetypes. They form the infinite complexity of life.

Most other interpretations of these verbal records are more or less uncertain and are usually of a speculative character which is why they are not given any further consideration here. It is undisputed, however, that Plato studied the quality of the numbers all his life and saw in them far more than their counting function in mathematics. In his dialogues it becomes clear that discussion of the content of the numbers was nothing unusual in ancient philosophy. His pupil Cephalus recounts the following from a discussion between Plato and Permenides, *"If therefore One is, so necessarily is also the number. And if the number is, then many is also and an infinite quantity of being. Or does the number not become infinite in quantity and in having being in itself? If now each number has being in itself, so must each individual part of the number have it in itself."*²³ Such dialogues are to be found frequently in the writings of Plato. They show the importance of the theory of number for the thinkers of that time. The examination of the knowledge of the quality of the numbers seems to have been a sort of basis for understanding all areas of knowledge. Nobody studying Plato's philosophy today hears this knowledge unless he dares to approach the original sources on his own, free of prejudice. It is largely unknown that Plato himself raises this subject repeatedly parallel to his generally known theory of ideas and even, according to the indirect records, describes the numbers as the original ideas. Some scientists of the 20th century suspect this significance once again, thus also the well known physicist and cofounder of quantum mechanics, Werner Heisenberg, who once said about Plato's theories, *"Plato certainly came nearest to the truth, if admittedly in an old fashioned way: the last to which human research has access may well be a type of mathematical order"*.²⁴

IV. Aristotle

Aristotle was a lifelong member of the Platonic academy and is considered to be his master's most important pupil. He counts as the most influential of all philosophers because he founded the real scientific philosophy in that he was the first to divide it up into different individual scientific disciplines, which form the foundation of all later work in this science right up to the present day.²⁵

Like Plato, he devoted himself to the detailed search for the origin of all knowledge and struggled unceasingly with the theory of a finally explainable original image, an archetype. He did not favour his master's solution, who believed he had found the answer to the question of the original in the theory of ideas. On the contrary, Aristotle became an opponent of Plato's theory of ideas. While Plato only allowed that general ideas have true reality and saw the individual things only as imperfect copies derived from the ideas (similar to the relationship of the ideal circle to the ellipse that really exists), Aristotle confronts this with the entities in the individual things. For him generality is not an ideal original image belonging to another world. If generalities are stated, they can only ever basically be about individual things existing in time and space. Therefore, according to Aristotle, a generality cannot represent anything unchanging or an original image. Now however, Aristotle sees, just as Plato saw, that the numerous "trees" come and go, while the "tree" as a generality continues to exist, unaffected by changes in appearance. He does not describe this superordinate symbol as a "notion",

as Plato did, but as a "form" expressed in the individual things. Aristotle calls the completely unformed and indefinite from which the forms appear "cloth" or "matter". However, the matter, taken on its own regardless of any form, has no reality. The fact that Aristotle tries to allocate the multitude of entities to different forms, proves that in the end he too, like Plato, follows the principle of abstraction. Here it becomes difficult to make the difference between his "forms" and Plato's "ideas" clear. Later in the history of philosophy these statements of Aristotle repeatedly lead to misunderstandings.²⁶

Aristotle also sought after an "original measurement" on which the multitude of details mentioned above could be based and arranged in a hierarchy. Like his teacher, he finally turned to mathematics, the science of calculation and numbers, and discovered the significance of the numbers as the basic measurement: *"When one seeks a measure one looks everywhere for something possessing the characteristics of unity and wholeness and this then counts as simple, be it with regard to quality or to quantity. Where it is felt to be impermissible that anything should be added or detracted, we are dealing with an exact measure; that is why the measurement of the number is the most exact."*²⁷

Gripped by the theories of Pythagoras who many years before him had described the numbers as the "measure of all things", he also came across the symbolic side of the numbers, quite apart from their "counting" aspect in mathematics. Like his predecessor in philosophy, Aristotle was quite clear that if there were to be any such thing as finally explainable entities, these would necessarily demonstrate the principle of undividedness. The assumption of an original unit²⁸, from which the multitude of individual things develops, lead him inevitably in turn to the subject of the numbers. For what expresses the principle of unity better than the One itself? The infinite variety of individual things is described by nothing better than by the unlimited series of numbers developing from their origin, namely from the number One. Finally Aristotle was unable to evade the fascination of the numbers and writes in "Metaphysics" on the "mathematical objects" (numbers): *"It is therefore the One which is the measure for everything, because we recognise what the being consists of, by taking it to pieces, either with regard to quantity or to its nature. The One is therefore undivided, because everywhere the original is undivided.(...)"*

*The number derives from the One and the uncertain Two; these are addressed as the principles and basic elements of the number.*²⁹

In his writings on metaphysics Aristotle finally also discusses in detail the subject of the numbers in isolation and in connection with the Platonic ideas in several chapters: 1. Mathematical Objects, 2. The Ideas, 3. Ideal Numbers. In his essay on the mathematical objects he discusses the numbers very fundamentally and in particular the theory of the Pythagoreans and also examines them. Here he comes up against several apparent contradictions which lead him to doubt the entire theory: he takes the guiding principle of the Pythagoreans: "Everything is number" very literally and applies it directly to everything material. If everything is number, then every material must be a number. Every tree must be a number. According to his theory, however, matter can only consist of the material. As the numbers are not matter in themselves, the tree also cannot be a number. Consequently the statement "Everything is number" does not apply to the tree. This thought game makes Aristotle sceptical and he discusses the importance of the numbers for the Pythagorean school using other examples: According to Aristotle, the Pythagoreans allocate a certain number to the soul.³⁰ Here again he believes that he has found a flaw in the theory: *"Again with the statement that the soul is no number, one has to divide the numbers into odds and evens; if one now finds that the soul is neither odd nor even, it is clear that it is not a number."*³¹ He gives his reasons in his own way, which is not clear beyond a single last doubt, that after all not everything can consist of numbers.

Disappointed for the time being by the mathematical objects and the theories of the old philosophers, he turned again in the following chapters to Platonic ideas in order to clarify their relationship to the numbers. However, Aristotle saw very quickly that the theory of ideas alone did not end his search for simple original images. The ideas which describe as a generality the notions on which the individual things are based present in turn only another indefinite quantity of superordinate entities which are neither unlimited nor ordered in a hierarchy. Aristotle said of this, *"The disciples of the theory of ideas are wrong in that they describe as an idea what in a multitude of objects is the uniform feature. The reason for their error lies in the fact that they cannot say which substances will last indefinitely alongside the individual things which can be perceived with the senses. They therefore represent them as having a form identical to that of the ephemeral things with which we are familiar; they describe*

them as human beings *per se* and horses *per se* by simply adding the words "*per se*" to the name of the things we perceive."³² Aristotle sees that the multitude of ideas corresponds far more to the infinite quantity of details perceived by the senses than to the entities which cannot be further reduced and which are what is really being sought.

In the last chapter "The Ideal Numbers", he finally deals with what the "number" and the "idea" have in common, in the hope of finding a solution to his problem here. Aristotle discusses in the following the question of to what extent the numbers relate to the ideas and which is to be seen as the higher principle. However, as is described above, neither the numbers nor the ideas were able to fulfil his expectations of last explainable entities, he finally throws out the hypothesis of the existence of ideal numbers and concludes the discussion of this question as follows:

*" But if the ideas are not numbers, then their very existence becomes impossible. From which principles should they have arisen? (...) But to put the ideas a priori in front of the numbers is just as impossible as to place them behind them. (...) The result of all that is that the number cannot be the cause of things in the sense of being the originating cause, neither the number itself nor anything comprised of oneness; nor can it be (the cause) either as matter or as the notion and form of things. Nor is it that in the sense of being the cause."*³³

At this point Aristotle ends the discussion about the numbers as independent original image-like beings. Nor is there any more to be found on this subject in his other writings. Aristotle rejects the numbers themselves, because he cannot find them in matter in reality and therefore they cannot be real. The Aristotelian view contradicts the Pythagorean because it argues objectively and makes matter and form the starting condition. In this sense he is also the founder of the materialistic sciences which finally declare the spirit to be the product of matter and do not consider matter to be an expression of the spirit. The Pythagoreans, like the theory of ideas to which they gave rise, go the opposite way. They are guided by the existing order of natural numbers in which the Trias – the principle of the spirit – precedes the Tetras – the principle of matter.³⁴

V. In Conclusion

The above paper shows that, in ancient Greek philosophy, knowledge of the qualitative principles of the numbers played a fundamental role. The irresistible logic of the Pythagorean theories had a considerable effect on all early Greek thinkers. Socrates, Plato and Aristotle were very significantly influenced by these theories and examined this fascinating subject very intensively as has been described above. While in essence the theories of those early thinkers form the foundations of our modern sciences, neither modern natural scientists nor philosophers are aware of the importance of the Greek view of numbers. How was it possible that such an fundamental element of the original philosophy of the West could almost have been forgotten? There is an obvious explanation for this:

The philosophers of the ancient world did not differentiate between the natural sciences and the humanities as contradictory ways of looking at the world – on the contrary. Unity was their supreme principle, and so they regarded spirit and matter as a whole. Accordingly the early philosophers were always also natural scientists who saw their task in finding universal basic principles in both fields of knowledge. They found this harmony, this bonding of spirit and matter again in the double nature of the numbers. The numbers show both a counting, calculating side and also a qualitative, content side. With the historical division between the natural sciences and the humanities it seems, however, that the numbers lost their fundamental bonding role. The possibilities of predicting and forecasting opened up by the new natural sciences developed far greater powers of attraction than the theories of wisdom. Also, the progress in the natural sciences after the division was felt for a long time as a liberation. The discussion of the qualitative values was left to religion. The religions withdrew for reasons of supposed self-preservation from the findings of the natural sciences. They showed no interest in such findings nor were they willing to integrate them into their understanding of the world. With the withdrawal of the religions, the so-called objective, "pure" natural sciences blossomed, but in turn they excluded the question of meaning.

Today we are standing at a turning point. The discoveries of the natural sciences and the facts they create have long ceased to be only liberating. Quite apart from the risks and dangers they hold,

human beings see themselves confronted with a ever increasing flood of data and, due to the neglected theories of wisdom, have less and less criteria for action. The modern human being is searching for knowledge and is drowning in information. In such a situation, it is becoming increasingly clear that the natural sciences, despite constant refinement of their tools, can always only observe and describe the relationships of life but, without calling on another spiritual world (e.g. the Platonic world), cannot interpret it. On its own science cannot explain satisfactorily why matter occurs and what moves it.³⁵ The natural scientist and philosopher Carl Friedrich v. Weizsäcker has pointed out, *"Physics does not explain the secrets of nature, it leads us back to deeper secrets."*³⁶

The message of the natural scientists >coincidence, not meaningfulness, rules all existence< is heard increasingly frequently and arouses dissatisfaction among human beings. The question of a comprehensive and recognisable order on which all areas of knowledge are based is increasingly being placed once again. It is reasonable enough to recall the origins of Western philosophy when the Greek philosophers still felt the world to be a unity and thus also the inner order of things. They went out into the world with this certainty. Their aim was to integrate the human beings into the order of the world. The idea of the Pythagoreans of recognising the numbers as the basic and order-giving principles seems to us today to have been an inspired presentiment of those early thinkers. Today, two and a half millennia later, their irresistible logic cannot be dismissed. Their strongest argument is still the proof that harmony is based on whole numbers and is supported by the constant experience that numbers reveal and found order.

A more exact examination of history shows that the knowledge of the qualitative significance of the numbers did not die out with the Pythagoreans but stands in an unbroken tradition as one of the oldest themes of philosophy. All the great religions go back to the relationship of the numbers, processing and interpreting them in their mythologies. In this respect let us remember St. Augustine, who, in his >De doctrina christiana< recalled the necessity of interpreting numbers for a proper explanation and interpretation of the Holy Scriptures.

The thoughts of the Pythagoreans have been reflected in all eras until the present day. Plato, Plotinus, St. Augustine, Kepler, Spinoza, Leibniz, Spengler and others have kept the awareness of them alive and they meet with a resonance from several natural scientists of our own era. The mathematician Kronecker has revitalised the question of whether the order of numbers was discovered or invented in his field of specialisation with his statement: *"God made the integers and everything else is the work of human beings"*. Gottfried Wilhelm Leibniz spoke earlier as a mathematician and philosopher of the *"preestablished harmony"* of the numbers as a matter of course. His unfulfilled dream in life was to invent a universal number language. Today however, his name is only connected with the discovery of infinitesimal calculation.

If we look more closely we find the significance of the natural numbers again in today's natural science. The order of the natural chemical elements follows the order of whole numbers in its structure which facilitated the compiling of the PSE (the periodic system of the elements). The contemporary natural scientist John D. Barrow³⁷ suspects a high priority importance of the numbers for the modern times and writes:

"Philosophy is a steadily evolving subject; but that part of it that deals with the nature of mathematics is stuck in a timewarp. The philosophy of mathematics has hardly progressed at all when compared with the development of philosophy or of mathematics. Now is the time to rejuvenate a study of the meaning of mathematics amongst philosophers. (...)"

Scientists believe there to be one Universe with a single universal legislation from which all the diverse subdivisions of science ultimately receive their marching orders. In recent years, the search for this single 'Theory of Everything' has become the new Grail of fundamental science. If found, its content will be a piece of logically consistent mathematics. But what is mathematics and why do we entrust it with the secret of the Universe? Why do we look to mathematics for answers to ultimate questions about the nature of physical reality? What is the foundation upon which this magical mathematics rests? Indeed, what is mathematics and why does it work? If we cannot answer these questions our scientific explanations of the Universe are based ultimately upon things we do not understand, upon the intangible mysteries that lie behind the impregnable battlements of a castle in the air. (...)"

Why do things keep following the path mapped out by a sequence of numbers that issue from an equation on a piece of paper? Is there some secret connection between them; is it just a coincidence; or is there just no other way that things could be?"³⁸

There are similar approaches in the vanguard of physics. Penrose³⁹, who became well known with Hawking for his description of black holes, writes in his book:

"We shall find ourselves driven towards a Platonic viewpoint of things. ... Plato's world is an ideal world of perfect forms, distinct from the physical world, but in terms of which the physical world must be understood. It also lies beyond our imperfect mental constructions; yet, our minds do have some indirect access to this Platonic realm through an "awareness" of mathematical forms, and our ability to reason about them. (...)

To me the world of perfect forms is primary (as was Plato's own belief) – its existence being almost a logical necessity. (...)

*I hope that I have persuaded the reader of the close and genuine relationship – still deeply mysterious – between the Platonic mathematical world and the world of the physical objects. I hope also that the very presence of this extraordinary relationship will help the Platonic sceptics to take that world a little more seriously as a "world " than they may have been prepared to do previously. Indeed, some might well go further than I have been prepared to go in this discussion."*⁴⁰

There are many approaches of this type in current scientific papers. Some speak of so-called "finite elements", others of the necessity of a new science of general principles. Ian Stewart calls them morphomatics in his book "Die Zahlen der Natur" published by Spektrum Verlag, Heidelberg in 1998. Behind all these thoughts there is the suspicion that the numbers are separate entities and that, apart from their formal mathematical significance, they also have a deeper, content aspect. The old wisdom of the Pythagoreans seems to reassume a high degree of current significance. At this point two important questions are raised:

- 1.) Is there a continuous wisdom of numbers leaving its tracks time and again in the various theories of wisdom around the world. Can it be proved that the special numbers hold the same or similar content – over and beyond the individual cultures?
- 2.) If this were the case; do we rediscover these archetypal characters in modern science? Do the results of scientific discoveries confirm these age-old wisdom of numbers?

The question has to be asked once again, why are the natural numbers "natural". Do the numbers today follow the example of the Pythagorean and Platonic philosophies and have a universal effect, acting as a link between the natural sciences and the humanities?

The reconciliation of science and wisdom – taken for granted in the ancient world – is one of the greatest challenges of our time.

Bibliography:

- Apelt, Otto: Platon – Sämtliche Dialoge. Felix Meiner Verlag, Hamburg 1988.
- Barrow, John D.: Warum die Welt mathematisch ist. Campus Edition Pandora, Frankfurt 1993.
- Barrow, John D.: Ein Himmel voller Zahlen; Akademischer Verlag – Spektrum der Wissenschaften.
- Bindel, Ernst: Pythagoras. Freies Geistesleben.
- Bor, Jan und Petersma, Errit: Illustrierte Geschichte der Philosophie. Scherz-Verlag 1997.
- Eigler, Gunther: Platon – Werke in acht Bänden. Wissenschaftl. Buchgesellschaft Darmstadt 1990.
- Fischer, Ernst Peter: Aristoteles, Einstein & Co. Piper, München 1995.

- Franz, Marie-Luise von: Zahl und Zeit. Klett-Cotta 1990.
- Gadamer, Hans-Georg und Schädewaldt, Wolfgang: Idee und Zahl. Studien zur Platonischen Philosophie, Heidelberg 1968.
- Hawking, Stephen: "A brief history of time: from the big bang to black holes". Bantam Press 1988
- Hegel, Georg Wilhelm Friedrich: Werke in zwanzig Bänden. Auf der Grundlage der Werke von 1832-1845. Redaktion Eva Moldenhauer und Karl Markus Michel, Frankfurt/M, Suhrkamp 1979
- Ifrah, Georges: Universalgeschichte der Zahlen. Campus.
- Karlson, Paul: Vom Zauber der Zahlen. Ullstein Verlag, Berlin 1954.
- Mansfeld, Jaap: Die Vorsokratiker Griechisch/Deutsch, Auswahl der Fragmente, Übersetzung und Erläuterung. Philipp Reclam Stuttgart.
- P.M. Perspektive: Die Welt der Zahlen.
- Penrose, Roger: Shadows of the Mind, A Search for the Missing Science of Consciousness. Oxford University Press 1994.
- Shanks, Daniel: Solved and Unsolved Problems in Number Theory. Spartan Books, 1962.
- Stenzel J: Zahl und Gestalt bei Platon und Aristoteles. Leipzig und Berlin 1933.
- Störig, Hans Joachim: Weltgeschichte der Philosophie. Verlag W. Kohlhammer, Stuttgart 1985.
- Weizsäcker, Carl Friedrich von: Ein Blick auf Platon. Reclam 1981.
- Wells, David: Das Lexikon der Zahlen. Fischer Taschenbuch Verlag, Frankfurt am Main 1990.
- <http://www.pyrrhon.de/platon/xenokrat.htm>

1. Stephen Hawking: " A brief history of time"

2. Edward Teller, born 1908 in Budapest, Professor for Physics in Washington, Chicago and Berkeley is a Senior Fellow of the Hoover Institution at Stanford University. He worked on the Manhattan Project in Los Alamos and was an early protagonist of developing the hydrogen bomb. For his research he was awarded the Enrico Fermi Medal and the Albert Einstein Prize.

3. See "Das Studium an der HSG"/University St. Gallen 1996 by Edward Teller

4. Carl Friedrich v. Weizsäcker: "Ein Blick auf Platon" Reclam 1981

5. Marie-Luise von Franz: Zahl und Zeit, Klett-Cotta 1990, Preface

6. C. G. Jung, W. Pauli: Naturerklärung und Psyche, Page 44

7. Marie-Luise von Franz: Zahl und Zeit, Klett-Cotta 1990

8. Stobaios aus Aristoxenos' Schrift >Über die Arithmetik< ; Übersetzung aus: Jaap Mansfeld: Die Vorsokratiker Griechisch/Deutsch)

9. Oktave 2:1; Quinte 3:2; Quarte 4:3

10. "Darin, daß er [Platon] das Eine als Substanz faßte und das Eine und die Zahlen nicht bloß von anderem ausgesagt werden ließ, näherte er sich der Ausdrucksweise der Pythagoreer, und ebenso darin, daß er die Zahlen als die Ursachen für die Wesenheit des Übrigen ansah." ("...in that he (Plato) understood the One as a substance and did not allow the One and the numbers just to be pronounced by others, he approached the way of expression of the Pythagoreans and also in that he saw the numbers as the cause for the being of everything else"). [Aristoteles: Metaphysik, S. 43. Digitale Bibliothek Band 2: Philosophie, S. 4121 (vgl. Arist.-Metaph., S. 21-22)]

11. Goethe: "Faust", Zeile 1112

12. Von Jamblichos überliefert. Aus Paul Karlson: Vom Zauber der Zahlen. Ullstein Verlag, Berlin 1954, S. 103f.

13. In der Schrift des Philalaos (Pythagoreer) aus Jaap Mansfeld: Die Vorsokratiker Griechisch/Deutsch

14. Explanation: Classical mathematics based on Euclidian geometry is based on ideal prerequisites such as the circle, the triangle, the square etc. In nature such ideals do not occur. In them the "idea of a circle", for example, is represented as a multitude of ellipses. This prerequisite of the ideal is, however, no longer taken for granted by every scientist today. Fractal geometry tries today to overcome this contradiction between ideal and reality by no longer seeing the ideal, but taking complex patterns as the basis for understanding reality.

15. Heraclitus: "Panta rhei" ("Alles fließt") oder "Keiner steigt zweimal in den gleichen Fluss"
16. Aus Platon: Der Staat 7. Buch (Platon im Dialog mit Glaukon)
17. For example, the Platonic bodies named after him.
18. Platon: Philebos
19. Platon: Phaidon
20. Platon, Briefe VII, 344 C [vgl. Störig: Weltgeschichte der Philosophie S. 156]
21. Platon, Briefe VII, 341 CD [vgl. Störig: Weltgeschichte der Philosophie S. 156]
22. Aristoteles: Metaphysik: Zweite Abteilung – Die angefügten Stücke: IV. Die Frage der unsinnlichen, unbeweglichen Substanzen.
23. Kephalos aus Platon: Parmenides
24. Aus Helmut Kuhn: "Idee und Zahl – Studien zur platonischen Philosophie" (Heidelberg 1968)
25. Division of Philosophy into the disciplines: Logic, Epistemology, Natural Philosophy, Metaphysics and Ethics

26. Hans Joachim Störig: "We cannot avoid one very important objection, however, namely that that Aristotele, having first banned with vehemence the general existence of Plato's ideas from his system, then allowed these to re-enter through a backdoor, for his forms look very similar to the Platonic ideas and could be mistaken for them. (Weltgeschichte der Philosophie, W. Kohlhammer Verlag, Stuttgart 1985)

27. Aristoteles: Metaphysik: Zweite Abteilung – Die angefügten Stücke: IV. Die Frage der unsinnlichen, unbeweglichen Substanzen
28. Plato expressed the principle of oneness as the idea of the good, the beautiful, the One.
29. Aristoteles: Metaphysik: Zweite Abteilung – Die angefügten Stücke: Einheit Verschiedenheit Gegensatz
30. In the sources of the Pythagorean writings handed down to us there is no allocation of the soul to a certain number to be found. Such a direct allocation would contradict the content of the Pythagorean abstraction of numbers because the notion soul does not remain strictly in the abstract of the numbers, but is linked far more with the actual, the individual. If Aristotele still claims this, we must on the one hand take into account that during his lifetime there could have been several other written records. On the other hand we must also consider the possibility of a misunderstanding, whereby originally one was concerned with the (individual) notion soul and thus typically allocated a principle, but did not mean generally the soul principle – similar to the allocation of a soul to an astrological original type for the purpose of characterisation.

31. Aristoteles: Organon; Die Topik; Buch 3, Kapitel 6.
32. Aristoteles: Metaphysik: Erste Abteilung. Die Hauptstücke; IV. Das begriffliche Wesen; 5. Ergebnisse für den Begriff des Wesens
33. Aristoteles: Metaphysik: Zweite Abteilung – Die angefügten Stücke: IV. Widerlegung des Dualismus
34. Note: That the Pythagorean allocation of the numbers Three and Four to spirit and matter is not arbitrary is shown by the preceding description of the numbers (see chapter 1 Pythagoras and the Pythagoreans)
35. Consider here Gödel's Theorem. The mathematician Hilbert wanted to prove the logical freedom of contradiction, the consistency of mathematics of itself. Kurt Gödel, a Viennese mathematician put an end to this. He proved that the mathematical truths are more than constructions of axioms and rules. But it was clear to Gödel that one must leave mathematics in order to understand them completely. Otherwise there always remain undecidable statements.

36. PM 12/98
37. Professor for Astronomy at the University of Sussex in Brighton
38. John D. Barrow: Ein Himmel voller Zahlen; Akademischer Verlag – Spektrum der Wissenschaften
39. Rouse Ball Professor for Mathematics at the University of Oxford, Member of the Royal Society
40. Roger Penrose: "Shadows of the Mind: A Search for the Missing Science of Consciousness", Oxford University Press 1994