

Source: World & I
Date: 05/01/2001
Document ID: AA20010514020006441
Subject(s): Mathematicians; Mathematics; Personal profiles

Citation Information: ISSN: 0887-9346; Vol. 16 No. 5; p.
140-147
Author(s): Michael R Williams

He lived with numbers

Known to students around the world for his famous theorem, Pythagoras led a school of number mystics whose collective efforts helped define much of what we know today as mathematics. Schoolchildren who venture beyond elementary arithmetic soon encounter the Pythagorean theorem ($a^2 + b^2 = c^2$), an expression relating the two sides of a right triangle to its hypotenuse. And adults who have long since forgotten most of their math may still remember the name Pythagoras even if they don't fully appreciate his theorem's place in mathematics.

Despite his achievements, the man is as much myth as reality. Pythagoras was one of the earliest Greek philosopher/mathematicians, and few contemporary records survive from his day. Most references to him come from the writings of Aristotle (384-322 B.C.), who lived a few hundred years after Pythagoras' time. It is rather like trying to reconstruct the life of George Washington from your great-grandfather's vague memories of things he heard as a child. One difference is that the ancient Greeks maintained a largely oral tradition, so their unwritten stories no doubt spanned centuries better than unwritten stories would today. Some facts about Pythagoras are clear, while others seem to have been made up to explain how this remarkable man became one of the founding fathers of mathematics.

The merchant's son

Pythagoras' father, Mnesarchos, was a merchant trader in the eastern Mediterranean. He settled on Samos after he was made a citizen of the island in appreciation for his having delivered a shipload of grain during a famine. Mnesarchos married a local woman, Pythais, and Pythagoras was born on Samos between the 50th and 52d Olympiads (580-568 B.C.; one Olympiad equals four years).

Mnesarchos continued his trading business, and Pythagoras probably traveled with his father for at least part of his youth. He was well educated for a young man of his day. By the time he was a teen he had mastered the arts of poetry. He could recite the great Greek epics about the battles for Troy and could play the lute, a talent he would later use in an attempt to heal the sick.

At the time there would have been almost no formal education in anything resembling science or mathematics. Pythagoras' travels brought him into contact with many different people, however. He evidently picked up some appreciation of Babylonian technology and philosophy through contact with scholars at Tyre and might even have been exposed to Egyptian culture, if not directly, then certainly through contact with other merchant traders. While stories abound of his early travels to such faraway places as India,

1631
WILLIAMS, M.R.
2001

A1631

(C)

these must be considered legends rather than facts.

Pythagoras was born into a situation that suited him and his inquiring mind. Although great progress had been made in areas such as architecture and government, civilizations were still asking why rather than how. For some reason, the sixth century B.C. saw the simultaneous beginnings of the rise of reason in many parts of the world. The Greeks began their tradition of philosopher mathematicians, Buddha (c. 563-483 B.C.) began to ask philosophical questions in India, and, in China, Confucius (c. 551-479 B.C.) was proposing his own system. It was certainly an age when new ideas were being explored and abstract generalizations began to replace the more concrete realities of life.

The only real formative incident we know of occurred when Pythagoras was in his late teens. The philosopher Thales (c. 625-546 B.C.) and his student Anaximander (c. 610-547 B.C.) lived in Miletus, a town located on the mainland of Asia Minor not far from Samos. It is known that Pythagoras visited these two and the school they had established. Although Thales was by then an elderly man, and so might not have had any direct influence on Pythagoras, we do know that he attended lectures given by Anaximander. Thales was one of the first Greeks to become interested in mathematics. He had visited Egypt and had incorporated its astronomical, mathematical, and technical knowledge into a Greek context. While Thales' philosophy was rather primitive by today's standards (he thought, for example, that the whole world was made up entirely of water in various states), he was one of the first to generalize from concrete problems in subjects such as architecture to more abstract concepts such as lines and angles. Anaximander's lectures likely inspired Pythagoras to consider the role of numbers in the structure of the universe and in human affairs.

Early Greek mathematics

While the Greeks are best known for their discoveries in geometry, the mathematics of Euclid was 200 years in the future. At the time of Pythagoras the Greeks were divided into various "tribal" groups, usually based in individual cities. Each city group had its own culture and way of representing things like numbers. The various number systems became more or less popular in Greek culture as the city-states waxed and waned in importance. The Greeks didn't use the zero as we do today and were unfamiliar with the concept of negative numbers.

As one example, many Greeks used an alphabetic additive number system somewhat akin to the more familiar Roman numerals. The Greeks simply assigned the values of 1 to 9 to the first nine letters of their alphabet. They didn't see any particular reason to stop at this point, so they kept on assigning numerical values (keeping a couple of older forms of letters from an earlier alphabet). The Greek numerals thus were much more cumbersome to use than even the later Roman numerals.

Numbers were represented by writing down these letters so that the sum of the values would add up to the required value. For example, 34 would be AA and 556 would be phiNzeta. When we do a simple arithmetic problem such as $3 + 5 = 8$, it is immediately apparent that it has a relationship with $30 + 50 = 80$. The equivalent Greek formulas were $\gamma + E = H$ and $\lambda + N = \pi$, which show no such relationship. Thus, even simple arithmetic in such a system required the memorization of a huge number of rules and procedures.

The Greeks split their ideas about arithmetic into two divisions. The first, which they called logistic, was the actual manipulation of numbers in arithmetic processes; what they called arithmetic was much more akin to what we would term number mysticism or even number theory. Elite Greek citizens would have had little need to do "logistic" math, because they kept slaves who were trained in elementary accounting systems to do such mundane tasks. Likely, the difficulty of dealing with such a number system caused Pythagoras to think more about arithmetic than logistic. Indeed, many historians of mathematics have implied that it was just this

difficulty with numbers that caused the Greeks to develop their more abstract considerations of geometry. Pythagoras eventually developed his consideration of numbers into a complete philosophy of numerical meaning. Most of his teachings were in terms of the meanings of individual numbers and how they related to the physical world and even to social relationships.

Pervasive numbers

The meaning of each number was derived from a mixture of geometric considerations and the philosophy of how numbers influenced the world. Numbers were considered to be more than mere counts of things: They were the actual fabric of the physical universe, much as atoms are considered the building blocks of our physical reality today. The number one (1), for example, was not considered a true number but rather the origin of the entire system, because all the rest were made up of multiple units of this progenitor.

All the other integers were divided into various classifications, with the odd numbers being male and even numbers female. Numbers that could be represented by a triangular or square pattern of dots were considered "triangular" (e.g., 3, 6, 10, ...) or "square" (4, 9, 16, ...) numbers.

Four was considered to represent justice because it was both the sum and product of the first true number (2), while 5 represented marriage because it was the union of the first female (2) and male (3) numbers. Ten was thought of as the best number because it was the union of the first four integers ($1 + 2 + 3 + 4 = 10$), which formed a perfect triangle when represented as rows of dots:

Various properties of numbers were considered good or bad. For example, the number 6 was thought of as "perfect" because the only smaller numbers that would divide into it evenly (the so-called aliquot parts) were 1, 2, and 3. Six was, itself, a perfect sum of its aliquot parts. Some numbers, such as 12, whose aliquot parts (1, 2, 3, 4, 6) added up to more than 12, were called "abundant," while other numbers, such as 8 (aliquot parts 1, 2, 4), were called "deficient" because they were greater than the sum of their aliquot parts. These concepts were expanded into "friendly numbers" (such as 220 and 284, the aliquot parts of each number adding up to the other) and various aliquot sequences where the divisors of each term add up to the next in sequence.

The study of these properties of numbers remains an interesting area of mathematical research. These number-theoretic problems are nontrivial, and it is only now that some of the problems first posed by Pythagoras and his followers are being solved. The theoretical results are so complicated that even a mathematician may find it impossible to enumerate the thousands of special cases that must be considered in a proof. Instead, mathematicians have had to use sophisticated computer systems to solve these ancient problems.

To illustrate the seemingly simple, yet difficult, problems that fascinated Pythagoras, it is only necessary to point out that a large number of perfect numbers are known (6, 28, and 496 are the smallest) but that none of them are odd. There is no known mathematical reason why there should not be an odd perfect number, but none has ever been discovered despite extensive searching with powerful computers.

One of Pythagoras' first recorded discoveries, likely helped by his knowledge of music, was the relationship between harmonious notes given off by a vibrating string. If the string is shortened to half its original length, then the note it produces is one octave higher. Similarly, a string of two-thirds the original length would produce the note we usually term the fifth. Another early discovery concerned the very real problem of filling space with regular figures—today usually termed "tiling the plane." It was common knowledge among the masons who created mosaic floors that squares and triangles could be used to fill up an area. Nonetheless, it was Pythagoras who first systematized the tiling options to show that

six equal triangles, four squares, or six hexagons could fit around a point in a plane and that these facts were related.

The Croton school

After Pythagoras began teaching these ideas in Samos, he drew a number of students into a "school" where they developed a communal system of living. Around 518 B.C. Pythagoras left Samos and moved to the Greek outpost of Croton (sometimes called Crotona and now known as Crotone) in the toe of the Italian peninsula. His reasons for leaving Samos are not clear. Some say it was because he felt his teachings were not being properly appreciated there, while others indicate that he fled for his life after arguments with Polycrates, the local tyrant.

Croton was a wealthy seaport, and it was to these wealthy families that Pythagoras presented his teachings. The "school" he set up was more akin to a mystical commune than to anything we would apply that name to today. Its members (some say as many as 300) held no personal possessions and had to follow a strict vegetarian diet. The vegetarian rule was because Pythagoras believed in the transmigration of souls and was reluctant to kill another living creature. The reasons behind some of the other rules are more difficult to determine: Members, for example, were forbidden to eat beans, stir a fire with an iron implement, touch things that had fallen down, or look into a mirror beside a light.

Combined with their many rules was the tradition that all discoveries were to be kept secret from outsiders. In keeping with the society's communal nature, all discoveries were attributed to Pythagoras, regardless of whether he was the actual discoverer. This attribution of discoveries continued long after the master's death.

The members of the society were divided into two groups. Those who lived in the school and ascribed to the rigorous lifestyle regulations were known as mathematikoi. A second group lived in the town. They did not have to follow the strict regulations but nonetheless attended the lectures given by Pythagoras and his mathematikoi. This second group, known as "the listeners" or akousmatics, was composed mostly of people with an interest in the subjects being discussed, but some were also serving a probation to prove their worthiness to enter the society proper.

Those attending the school formed a close-knit group that jealously protected its teaching and methods. Much like some modern lodges (the models for which can be traced back to the Pythagorean school), the members were only initiated in the "secret knowledge" after they became trusted associates. Despite this secretiveness and the fact that the culture was transmitted primarily through the oral tradition rather than the written word, we do know that Pythagoras married Theano, the daughter of the ruler of Croton, who evidently wrote a biography of her husband. Unfortunately, no trace of this document exists today.

Expanding the reach of mathematics The investigations at the Croton school expanded into areas that had received little previous study Pythagoras divided mathematics into two realms: the discrete and the continuous. The discrete was further divided into the absolute (arithmetic) and the relative (music), while the continuous was split into the stable (geometry) and the moving (astronomy). These divisions had lasting value. In fact, the medieval academic curriculum (known as the quadrivium) was based on these four subjects.

In geometry the Pythagorians certainly established the foundation on which the work of Euclid was based. They systematized the elementary concepts of tiling the plane and extended these to provide the definitions of elementary concepts of geometry. The Pythagoreans were the first to define the basic elements of geometry and provide the fundamentals for mathematical proofs. Pythagoras was, for example, the first person to define a point as "unity having position." He also discovered that certain geometrical quantities were irrational (could not be measured in terms of other associated quantities). The simplest of these irrational items was

that the diagonal in a square could not be a simple multiple of any of its sides, or even a multiple of some simple proportion of the length of the side.

Certainly the famous theorem that bears his name would have led eventually to this result, but the discovery of irrational measures in simple geometrical figures came as a surprise to the people of the day. This is a much greater leap than is apparent on the surface: The evidence shows that Pythagoras likely didn't think about the relationship the same way we do today but rather considered the actual areas of the squares erected on the sides of the triangles. Although the relations in the Pythagorean theorem were well known to other early societies (particularly the Babylonians and Chinese), we owe the first formal proof of the concept to the master. In providing this proof, he showed the way for mathematicians for the next 2,500 years.

The Pythagoreans also made contributions in the field of astronomy (more properly, cosmology). These were not based on any observational evidence but were rather a philosophical system of the world developed from their belief that numbers were the fundamental origin of all things. While there is no doubt that the school in Croton proposed many of the speculations attributed to Pythagoras, the only hard evidence we have comes from one of the mathematikoi, Philolaus, who wrote down some of the "secret knowledge" to be revealed upon his death.

Pythagoras seems to have been the first to propose that Earth was really a moving sphere, but this likely was because a sphere was a more perfect geometric shape than any other. The Pythagorean school proposed that there were actually 10 heavenly bodies orbiting around a central fire (not the Sun, which was simply one of the 10). Recall that 10 is one of the best numbers, and anything as important as the entire cosmos must be built up of elementary items that conformed to the ideal Pythagorean numerical system. The facts that only 8 bodies were known and that no one had ever seen this central fire were dealt with simply. The Pythagoreans proposed a dark "counter-earth" positioned between Earth and the fire, thus blocking our view of both.

The last act

About 508 B.C. the residents of Croton became angry at the political power held by Pythagoras and his followers. The exact reasons are surrounded in the usual myths and are not at all clear. The final outcome was that the school was abandoned and the mathematikoi were dispersed to other parts of Greece, taking their knowledge and methods with them. Thus, the discoveries were preserved in a hundred different locales and formed the basis for the advances made by the many great Greek philosopher/mathematicians in the next few hundred years. Pythagoras escaped to another Greek outpost (reports vary, but Tarentum and Metapontium are often cited) and died there about 500 B.C. His wife and two daughters apparently attempted to resurrect the glory of the original school but were unsuccessful.

On the Internet

"PYTHAGORAS" AT UNIVERSITY OF TENNESSEE AT MARTIN

http://www.utm.edu/research/iep/pl_pythagor.htm "PYTHAGORAS OF SAMOS" AT

MOUNTAIN MAN GRAPHICS, AUSTRALIA <http://www.magna.com.au/>

-prfbrown/pythagor.html "PYTHAGORAS OF SAMOS" AT UNIVERSITY OF EVANSVILLE

<http://plato.evansville.edu/public/burnet/ch2a.htm>

Michael R. Williams teaches computer science at the University of Calgary, Alberta, Canada.

Copyright Washington Times Corporation May 2001

©2001 Bell & Howell Information & Learning Services; All Rights Reserved.

Only fair use, as provided by the United States copyright law, is permitted. Bell & Howell Learning & Information Services makes no warranty regarding the accuracy, completeness or timelines of the Publications or the records they contain, or any warranty, express or implied, including

any warranty of merchantability or fitness for a particular purpose, and shall not be liable for damages of any kind or lost profits or other claims related to them or their use.