

1637 PAB
ZHMUD, L.
1989

CS
Numbers

'All is number'?

ZHMUD' Leonid Ja., *All is number? « Basic doctrine » of Pythagoreanism*

reconsidered: Phronesis XXXIV 1989 270-292. | La doctrine des nombres n'était pas essentielle dans le pythagorisme puisqu'elle n'apparaît qu'avec Philolaos, c'est-à-dire un siècle après la fondation de l'école, qui était d'abord une association (ἐταρεία) politique dont les membres ne partageaient pas nécessairement les mêmes opinions philosophiques; en effet, les ἀρχαί des premiers pythagoriciens sont très diverses. Aristote est le principal responsable de la confusion entre pythagoriciens et ceux qui fondaient tout sur les rapports de nombres. [4488

To find consensus omnium in scientific literature devoted to early Pythagoreanism is not an easy task. Absence of unambiguous solutions for what seem to be the most important problems, to say nothing of an abundance of contradictory interpretations of more particular points, is a typical feature of the "Pythagorean question".

So, for example, one of the most prominent modern students of Pythagoreanism, W. Burkert, in the wake of E. Frank,¹ tends to show that in fact, early Pythagoreanism did not contain any philosophy or science.² All subsequent interpretations of Pythagoreanism have been seriously influenced by his book. But it has convinced those who were dealing with the problem first-hand least of all. J. Philip and C. de Vogel have shown Burkert's denial of early Pythagorean philosophy to be unsubstantiated,³ while K. von Fritz and B.L. van der Waerden reject his other thesis – about the absence of scientific progress in the early Pythagorean school.⁴ On this issue, however, J. Philip believes that scientific studies of Pythagoreans begin as late as the end of the V-th century, and B.L. van der Waerden touches upon the philosophy of Pythagoreans most sparingly, so that it is not yet possible to speak of any unanimity having been established.

Nevertheless, on one important problem the standpoints of nearly all the participants of this controversy, which has lasted almost a century and a half, come very close together. The question is about number as the main principle of Pythagorean philosophy. Both those who take it seriously and

¹ Frank, E., *Plato und die sogenannten Pythagoreer*, Halle, 1922.

² Burkert, W., *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaos und Platon*, Nürnberg, 1962. I have used a revised English translation of this book: *Lore and Science in Ancient Pythagoreanism*, Cambridge (Mass.), 1972.

³ Philip, J., *Pythagoras and Early Pythagoreanism*, Toronto, 1966; Vogel, C. de., *Pythagoras and Early Pythagoreanism*, Assen, 1966.

⁴ Fritz, K. von, *Grundprobleme der Geschichte der antiken Wissenschaft*, Berlin/New York, 1971; idem, "Pythagoras", *RE*, 47, 1963; Waerden, B.L. van der, *Die Pythagoreer: Religiöse Bruderschaft und Schule der Wissenschaft*, Zürich/München, 1979.

those who think that until Philolaos the Pythagoreans did not exceed the limits of quasiphilosophical arithmological speculations, proceed from the belief that the well-known maxim "all is number" gives a correct interpretation of the main doctrine of early Pythagoreanism. Thus, the archē of Pythagoras and, accordingly, of early Pythagoreans who are claimed to have never departed from the teaching of the head of their school, is number.

Let me specify at once: I am not going to undertake one more effort at interpreting this idea. I am interested in other things. The Pythagorean number is manifestly struggling out of the rank of pre-Socratic archai, all of which, without exception, were thought of as corporeal and extensive. In what way did it find itself side by side with water, air, fire and seeds? If it was also thought of as corporeal, then what is the world which consists of corporeal numbers or units?

Of course, the originality of the idea itself should not provoke in us any suspicions of its authenticity. After all, τὸ ἀπειρον of Anaximander also looks out of place between the water of Thales and the air of Anaximenes. But such suspicions will inevitably arise if we try to clear up the following: from what sources is it known that Pythagoras and after him Pythagoreans had deduced all the world from numbers?

It is known that reconstruction of the pre-Socratics' theories should primarily be supported by genuine fragments of their writings and only then can we use the evidence of later authors, for example, Aristotle or Theophrastus. Unfortunately, it is impossible to follow this principle in the case of Pythagoras as he himself did not write anything. There exist, however, quite a few other ways of verification, even though not absolutely reliable. First of all, Pythagoras had numerous disciples and followers who, if the later tradition is believable, had infinite faith in the authority of their Teacher. May be they left some books which set forth his philosophical doctrines? – Alas, it is of no use looking for such writings: the ancient sources do not contain any credible evidence for their existence. In such a case we should probably turn to what is said about Pythagoras by his contemporaries Xenophanes and Heraclitus and at the same time enlist the evidence of other authors of the V-th century: Empedocles, Herodotus, Democritus, Ion of Chios, Glaucus of Rhegium and the anonymous author of *Dissol. logoi*. The early tradition about Pythagoras, as we can see, is considerable, and attempts to find in it some doxographic information are by no means fruitless: Xenophanes, for example, did mention the teaching of Thales.⁵ But despite the fact that all these pieces of evidence have been

⁵ Lebedev, A.V., "Thales and Xenophanes", *Ancient Philosophy as Interpreted by Bourgeois Philosophers* (Moscow, 1981), 1-16.

8

“All is number”?

“Basic doctrine” of Pythagoreanism reconsidered

LEONID JA. ZHMUD'

To find consensus omnium in scientific literature devoted to early Pythagoreanism is not an easy task. Absence of unambiguous solutions for what seem to be the most important problems, to say nothing of an abundance of contradictory interpretations of more particular points, is a typical feature of the “Pythagorean question”.

So, for example, one of the most prominent modern students of Pythagoreanism, W. Burkert, in the wake of E. Frank,¹ tends to show that in fact, early Pythagoreanism did not contain any philosophy or science.² All subsequent interpretations of Pythagoreanism have been seriously influenced by his book. But it has convinced those who were dealing with the problem first-hand least of all. J. Philip and C. de Vogel have shown Burkert's denial of early Pythagorean philosophy to be unsubstantiated,³ while K. von Fritz and B.L. van der Waerden reject his other thesis – about the absence of scientific progress in the early Pythagorean school.⁴ On this issue, however, J. Philip believes that scientific studies of Pythagoreans begin as late as the end of the V-th century, and B.L. van der Waerden touches upon the philosophy of Pythagoreans most sparingly, so that it is not yet possible to speak of any unanimity having been established.

Nevertheless, on one important problem the standpoints of nearly all the participants of this controversy, which has lasted almost a century and a half, come very close together. The question is about number as the main principle of Pythagorean philosophy. Both those who take it seriously and

¹ Frank, E., *Plato und die sogenannten Pythagoreer*, Halle, 1922.

² Burkert, W., *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaus und Platon*, Nürnberg, 1962. I have used a revised English translation of this book: *Lore and Science in Ancient Pythagoreanism*, Cambridge (Mass.), 1972.

³ Philip, J., *Pythagoras and Early Pythagoreanism*, Toronto, 1966; Vogel, C. de., *Pythagoras and Early Pythagoreanism*, Assen, 1966.

⁴ Fritz, K. von, *Grundprobleme der Geschichte der antiken Wissenschaft*, Berlin/New York, 1971; idem, “Pythagoras”, *RE*, 47, 1963; Waerden, B.L. van der, *Die Pythagoreer: Religiöse Bruderschaft und Schule der Wissenschaft*, Zürich/München, 1979.

those who think that until Philolaus the Pythagoreans did not exceed the limits of quasiphilosophical arithmological speculations, proceed from the belief that the well-known maxim “all is number” gives a correct interpretation of the main doctrine of early Pythagoreanism. Thus, the **archē** of Pythagoras and, accordingly, of early Pythagoreans who are claimed to have never departed from the teaching of the head of their school, is number.

Let me specify at once: I am not going to undertake one more effort at interpreting this idea. I am interested in other things. The Pythagorean number is manifestly struggling out of the rank of pre-Socratic **archai**, all of which, without exception, were thought of as corporeal and extensive. In what way did it find itself side by side with water, air, fire and seeds? If it was also thought of as corporeal, then what is the world which consists of corporeal numbers or units?

Of course, the originality of the idea itself should not provoke in us any suspicions of its authenticity. After all, τὸ ἄπειρον of Anaximander also looks out of place between the water of Thales and the air of Anaximenes. But such suspicions will inevitably arise if we try to clear up the following: from what sources is it known that Pythagoras and after him Pythagoreans had deduced all the world from numbers?

It is known that reconstruction of the pre-Socratics’ theories should primarily be supported by genuine fragments of their writings and only then can we use the evidence of later authors, for example, Aristotle or Theophrastus. Unfortunately, it is impossible to follow this principle in the case of Pythagoras as he himself did not write anything. There exist, however, quite a few other ways of verification, even though not absolutely reliable. First of all, Pythagoras had numerous disciples and followers who, if the later tradition is believable, had infinite faith in the authority of their Teacher. May be they left some books which set forth his philosophical doctrines? – Alas, it is of no use looking for such writings: the ancient sources do not contain any credible evidence for their existence. In such a case we should probably turn to what is said about Pythagoras by his contemporaries Xenophanes and Heraclitus and at the same time enlist the evidence of other authors of the V-th century: Empedocles, Herodotus, Democritus, Ion of Chios, Glaucus of Rhegium and the anonymous author of **Dissoi logoi**. The early tradition about Pythagoras, as we can see, is considerable, and attempts to find in it some doxographic information are by no means fruitless: Xenophanes, for example, did mention the teaching of Thales.⁵ But despite the fact that all these pieces of evidence have been

⁵ Lebedev, A.V., “Thales and Xenophanes”, *Ancient Philosophy as Interpreted by Bourgeois Philosophers* (Moscow, 1981), 1-16.

frequently examined both together and separately,⁶ no one has succeeded in discovering in them even a slight hint of the philosophic teaching about number.

Nevertheless, even here our possibilities are far from being exhausted. Let us turn to the early Pythagoreans – they could not have passed over in silence the basic doctrine of the founder of the school! More than that: if we start from the standard description of the Pythagorean school as blindly repeating all that αὐτὸς ἔφα, this doctrine should also form the basis of the philosophy of Pythagoras' followers. Before discussing what the early Pythagoreans say about number, however, let us dwell on two other interrelated problems.

There is a widespread belief that Philolaus' book, which appeared in the last quarter of the V-th century, is the first written record of Pythagorean doctrines. From this it is inferred that earlier this teaching was spread orally and that none of the Pythagoreans wrote books. How is this to be reconciled with the evidence for the existence of writings by Hippasus, Alcmaeon, Menestor, Hippon and some other Pythagoreans – since all of them did live before Philolaus?⁷ Possibly, our sources are wrong, and these are not true Pythagoreans? Who, then, are the true ones, and by what criteria are they to be singled out?

In the overwhelming majority of works on Pythagoreanism this problem is not raised openly, and a doctrinal criterion is implicitly used as the main working method. A Pythagorean is one who speaks about Number. Here we are faced with an obvious **petitio principii**: that which itself is in need of being proved is taken as a starting premise. Irrespective of this error, however, the doctrinal criterion is neither universal, nor the most convincing one. Regarded as Platonists and Peripatetics are not only and not so much those who shared the faith in Ideas, a prime mover, or four types of causes, but those who are named disciples (followers) of Plato and Aristotle in our sources. It seems indisputable that the question of who were Pythagoreans is also to be settled on the basis of reliable data of ancient tradition and not proceeding from adherence to the number philosophy.

⁶ For bibliography on the problem see: Zhmud', L.Ya., "Pythagoras in the Early Tradition", *Vestnik drevnei istorii*, 1985, 121-142.

⁷ Although not one fragment of Hippasus' book has remained, it should be believed that it really did exist. The oral tradition could hardly have preserved the sufficiently detailed information about his scientific achievements. Cf. Burnet, J., *Greek Philosophy. 1. Thales to Plato* (London, 1914), p.70.

It was just this criterion which H. Diels used for selecting representatives of the Pythagorean school in his edition of the fragments of the pre-Socratics. The main source (but not the only one) he had relied on was the well-known catalogue of Pythagoreans found in Iamblichus (Vit.Pyth.267). Diels believed that this catalogue went back to the Peripatetic Aristoxenus⁸ who learned from the last Pythagoreans and possessed most valuable information concerning this school. This idea of Diels was supported by M. Timpanaro Cardini,⁹ while Burkert gave new arguments in favour of Aristoxenus.¹⁰ With a few other considerations added to this, the argumentation may be presented as follows:

- 1) It is evident that the catalogue was not compiled by Iamblichus: his writings contain 18 more names of Pythagoreans which are absent from the catalogue.
- 2) The catalogue is independent of pseudo-Pythagorean literature; it does not include the names of about 20 "Pythagorean" writers which are given in the collection of H. Thesleff.¹¹
- 3) All the persons in the catalogue whose chronology can be established belong to the period before Aristoxenus, i.e. – from the VI-th to the middle of the IV-th century.
- 4) The origin of a number of Pythagoreans indicated in the catalogue differs from the data of other sources, but coincides with those given by Aristoxenus.
- 5) The origin of the greatest number of names (43) is given as Tarentum, the homeland of Aristoxenus, while from two other centres of Pythagoreanism – Croton and Metapontum – there are given 29 and 38 names respectively.

The total number of names (218) and the very principle of constructing the catalogue (according to the place of origin) indicate that Aristoxenus, besides oral tradition, had found support in some other documentary sources. This is also testified to by the fact that about three quarters of the catalogue names are to be encountered only in the catalogue itself. Of course, it cannot be claimed that only these 218 people are "true Pythagoreans" and that none of the representatives of this school has been left

⁸ Diels, H., *Antike Technik* (Leipzig, 1924), p.23. For the first time this idea was expressed by E. Rohde, "Die Quellen des Iamblichus in seiner Biographie des Pythagoras", *Kleine Schriften* (Tübingen, 1901), v.II, p.171.

⁹ Timpanaro Cardini, M., *I. Pitagorici, Testimonianze e frammenti* (Firenze, 1964), fasc.3, p.38-39.

¹⁰ Burkert, *op.cit.*, p.105, n.40.

¹¹ Thesleff, H., *Pythagorean Writings of the Hellenistic Period*, Åbo, 1965.

outside the catalogue. As was noted by M. Timpanaro Cardini, the catalogue is valuable evidence about the people included in it, but not sufficient to disprove the Pythagoreanism of those on whose names it is silent, if there is other (let me add – reliable) evidence on that score.¹² Not included in the catalogue, for example, are the names of the Pythagoreans Democedes, Calliphon, Ameinias, Cercops; there is no Amyclas, but there is his friend Cleinias, although Aristoxenus mentions them both (fr.131 Wehrli). There are no disciples of Philolaus, such as Simmias and Cebes who figure in Plato's "Phaedo"; there is Ecphantus, but absent is his countryman and contemporary Hicetas mentioned by Theophrastus (50 A 1). Neither is Democritus mentioned who, according to his contemporary Glaucus of Rhegium, was taught by Pythagoreans (D.L. IX, 38); but at the same time, the names of Parmenides, Empedocles and even Melissus are mentioned here.

From the facts cited it follows first, that the catalogue is far from being complete (which is, of course, not only Aristoxenus' fault), and secondly, that it was compiled not at all on a doctrinal basis, otherwise it would not have included the names of such original philosophers as Parmenides and Empedocles. Most probably, the catalogue presents people connected as teachers and disciples in succession, which implies perception and development of ideas advanced by early Pythagoreans, along with adherence to the way of life initiated by Pythagoras, but does not make all this compulsory.

If we proceed from the catalogue and absence of data contrary to it, then Hippasus, Alcmaeon, Menestor and Hippon who are mentioned in it should be admitted to be Pythagoreans in no lesser degree than Philolaus or Archytas.¹³ What then, do the early Pythagoreans say about number as the substance of the world? Strange though it is, they say nothing. Their **archai** are natural qualities, as in Alcmaeon or Menestor, or natural elements, as in Hippasus or Hippon. Moreover, the philosophy of number is altogether absent in them.

This leads us to the obvious question: was the teaching about number basic for the philosophy of Pythagoras at all? If it was, then why is it that, during a whole century – from the end of the VI-th to the end of the V-th century – it not only did not evoke any direct response beyond the school,

¹² Timpanaro Cardini, *op.cit.*, p.39.

¹³ We are concerned here only with those early Pythagoreans about whose philosophic doctrines some evidence has been left. On Paron and Petron (not mentioned in the catalogue) see: Burkert, *op.cit.*, p.114, 170.

but its traces are also lacking in the Pythagoreans themselves? If we do not wish to think that the central dogma of Pythagorean philosophy was secret,¹⁴ then it would be quite reasonable to suppose: either this dogma was not central, or it was not a dogma at all.

Only very few of those who write about Pythagorean philosophy arrive at such a paradoxical conclusion and, as far as my knowledge goes, there is no one who would attempt to develop it. J. Burnet, for example, believed that “Pythagoras himself left no developed doctrine on the subject (i.e. about the interrelationship of numbers and things – L.Zh.), while the Pythagoreans of the fifth century did not care to add anything of the sort to the tradition.”¹⁵ It is possible to agree with this, but nevertheless: was number an ontological principle in Pythagoras’ philosophy, as it was asserted in hundreds of books before Burnet and after him? “The teaching about number certainly does not belong to ancient Pythagoreanism. Until the second half of the V-th century we do not possess any reliable evidence of it” – this is how the situation was estimated by O. Gigon.¹⁶ In such a case it should be made clear just what occupied the “vacant” place of Pythagoras’ fundamental principle, from whom did the number philosophy arise, and how was it transferred to the founder of the school and his direct followers? To do this, it is necessary to overstep the limits of early Pythagoreanism.

Judging by the remaining evidence, Philolaus was the first of the Pythagoreans to have regarded number from a philosophic viewpoint, although he too only partially justifies our expectations. The cosmos of Philolaus has arisen and consists not of numbers or corporeal units, but of things unlimited (boundless) and limiting – τὰ ἄπειρα καὶ τὰ περαίνοντα (44 B 1-2). It is these two kinds of things that Philolaus terms φύσις and ἀρχή of all (44 B 1,6); he has no other beginnings. And number in Philolaus appears in an epistemological but not ontological context. (It is important to note here that out of Philolaus’ fragments only 1-7, 13 and 17 are now recognized as genuine.)¹⁷

“All that is cognizable, certainly has number. For it is impossible for us to think or perceive something without this” (44 B 4). Does it follow from this

¹⁴ Concerning the “secrecy” of early Pythagoreanism see: Zhmud’, L.Ya., “Scientific Studies in the Early Pythagorean School (according to the Sources of the V-IV-th cc.B.C.)”, *Problemy antichnogo istochnikovedeniya* (Moscow/Leningrad, 1986), 153-175.

¹⁵ Burnet, J., *Early Greek Philosophy* (London, 1920), p.107.

¹⁶ Gigon, O., *Der Ursprung der griechischen Philosophie: Von Hesiod bis Parmenides* (Basel, 1945), p.142.

¹⁷ Burkert, op.cit., p.238-277; Kirk, G.S., Raven, J.E., Schofield, M., *The Pre-Socratic Philosophers* (Cambridge, 1983), 2nd ed., p.324.

that *things* are composed of numbers or arise from them? Such a conclusion is not even implied here, for we already know *what* the world consists of in Philolaus.¹⁸ Just how number is connected with the cognizable can be understood with the help of another fragment: "If all things are unlimited, there will be nothing cognizable at all" (44 B 3). Thus, what is unlimited, be it by number or size, cannot be apprehended and recognized.¹⁹ At the same time, the existence of things which impose limits, bring definiteness to this world, makes it possible to count and measure something, to find its number – that is to cognize.²⁰ Let us note here that this definiteness is introduced not by number – number itself is the result of activity of a limiting principle.

An example of such cognition is given by Philolaus himself when he outlines the foundations of Pythagorean musical theory. What is an octave as understood by Philolaus? It is the ratio 1:2, the fifth - 2:3, the fourth - 3:4 etc. (44 B 6). Having established these numerical relations, we have thereby *cognized* the harmonic intervals.

Despite the fact that in Philolaus we find what may be called mathematical theology, for instance, consecration of an angle of the triangle to various gods,²¹ in no other of his authentic fragments shall we find a more thorough treatment of his epistemological principle. Moreover, it would be in vain to search for identification of numbers and things²² in his fragments,

¹⁸ Burns, A., "The Fragments of Philolaus and Aristotle's Account of Pythagorean Theories in Metaphysics A", *Classica et Medievalia*, 25, 1964, p.107.

¹⁹ Schofield remarks: "He (Philolaus) probably means to claim that if things are *not countable*, we cannot think them, nor be acquainted with them" (Kirk, Raven, Schofield, op.cit., p.327).

²⁰ Nussbaum, M.C., "Eleatic Conventionalism and Philolaus on the Conditions of Thought", *Harvard Studies in Classical Philology*, 83, 1979, 92-93. In her commentary of the fr. B 4 Nussbaum points out: "In this fragment it is usually assumed that we meet some rather peculiar Pythagorean theory about the magical powers of number, conceived of as separate entities in their own right. I would like to argue that we should translate and interpret this fragment in the most straightforward and ordinary way . . . it makes sense . . . on this purely ordinary level, without the introduction of any extra doctrinal apparatus" (ibid, p.88).

²¹ Hübner, W., "Die geometrische Theologie des Philolaus", *Philologus*, 124, 1980, N 2.

²² In fr. B 7 Philolaus calls Hestia (Central fire) that which had first arisen and was in the very centre of the (celestial) sphere. At the same time τὸ πρῶτον ἀρμυσθέν = τὸ ἓν. All the context indicates that by τὸ ἓν is meant not a numerical unit, but the One, i.e. what had come after fitting together (ἀρμυσθέν) of τὰ ἄπειρα καὶ τὰ περαίνοντα. Such meaning of fr. B 7 is also reflected in the Wortindex of W. Kranz. On the contrary, in fr. B 8 (which is considered not genuine) Iamblichus reinterprets Philolaus' words already in his own sense: "According to Philolaus the unit (μονάς) is the beginning of all: does

or for assertions that “all is number”. Strictly speaking, there are no such words in any one of Pythagoreans. They appear for the first time only in Aristotle.

Yet the main thesis of Pythagorean philosophy could not have been entirely invented by Aristotle; he must have relied upon something! – Undoubtedly, he must, and he did rely – upon Philolaus himself in particular. Let us indicate a few other possible sources.

The outstanding mathematician Archytas who could be expected at least to take interest in number philosophy, if not glorifying number, did in reality prefer to busy himself with other things. He has nothing that could be of interest to us. On the other hand, two of his younger contemporaries, Eurytus and Ecphantus, who did not show their worth in mathematics (in the same way as Philolaus), display obvious interest in this subject.

Ecphantus is an example of an eclectic, so typical of late pre-Socratics. In accord with atomists he taught that the world consists of atoms and void, but is governed by reason and not by necessity as Anaxagoras believed (51 A 1,4). According to Aetius, Ecphantus was *the first* to believe that the Pythagorean monads (units) are corporeal (51 A 2). Apparently, he terms units those minute bodies the world is composed of. What follows from it? If Ecphantus was really the first to come to the idea of number atomism, it can in no way be projected to the early school and ascribed to Pythagoras. Those who will disagree with Aetius will have to find the traces of earlier existence of this doctrine, which, so far, nobody has been able to do.

Number atomism which, beginning with P. Tannery and Fr. Cornford, was ascribed to early Pythagoreans, in reality has proved to be only an academic construction.²³ The very fact of mathematical atomism existing before physical atomism (i.e. before the second half of the V-th century) is highly doubtful. As to attempts at interpreting the puzzles of Zeno as a reaction to the number atomism of the early Pythagoreans, they have been repeatedly rejected and now this idea has no active proponents.²⁴

The corporeal monads of Ecphantus are mentioned very briefly, in one sentence. If this idea be developed, such teaching, very likely, would be called number atomism. It is only strange that it appeared nearly one hundred years after the time it ought to have disappeared. Indeed, it was once a commonplace in almost every work devoted to Pythagorean philosophy that the discovery of irrationality made by Hippasus struck a heavy blow to number atomism. Number (**arithmos**) for Greek mathematicians is

not he say that ἐν - ἀρχὰ πάντων”?

²³ Furley, D.J., *Two Studies in Greek Atomists* (Princeton, 1967), p.44-56.

²⁴ Burkert, *op.cit.*, p.285-288; Kirk, Raven, Schofield, *op.cit.*, p.277-278.

always a multitude composed of units (Eucl.Elem.VII, def.2); and the diagonal of a square, being incommensurable with its sides, cannot be expressed either by a whole, or by a fractional number. How, then, can things consist of numbers? Hippiasus was a younger contemporary of Pythagoras. His discovery should have stopped at once the development of number philosophy, at its very beginning. But in reality we see that at the turn of the V-th century Ecphantus, not at all embarrassed by the problem of irrationality, comes to advance an idea which would naturally be expected from Pythagoreans before Hippiasus!²⁵

If Pythagorean number atomism really began with Ecphantus, so, most likely, it also ended with him. His contemporary, Philolaus' disciple Eurytus developed similar ideas, but along slightly different lines. As far back as early Pythagoreans, when they were creating the theory of the so-called figured numbers, they represented various geometrical figures by counting stones, **psephoi**, such as a triangle, a square etc. Starting from these operations, which were of purely mathematical significance, Eurytus began to lay out figures of a man, or for example, a horse of **psephoi**. Having thus drawn a silhouette of man, he took a certain number of **psephoi**, say 250, and arranged them in such a way that they became as it were, the limits of his figure. In such a way he "determined" the number of the man (45 A 2,3). Aristotle tells us about it in all seriousness, but it is hard to believe that Eurytus attached deep meaning to his activity. However, if he really was going to accomplish a revolution in cognition in this way, he obviously met with no sympathy from his contemporaries.²⁶ No information has remained about any other of his ideas, and we do not even know just what numbers he had arrived at; those given in ps.-Alexander's commentary on the "Metaphysics" evidently have been taken at random. Yet, even taking it seriously, it is impossible to deduce identification of man or horse with number from Eurytus' idea, for he did not say that they *consisted* of numbers or corporeal units.²⁷

We have exhausted all the examples worthy of attention and we have nowhere come across either the main thesis of Pythagorean philosophy, or

not he say that ἔν - ἀρχὰ πάντων"?

²⁵ The attempts to date Hippiasus to the middle of the Vth century or even later are unsatisfactory. He is mentioned several times along with the authors of the turn of the VI- to V-th century, for example, with Heraclitus (14 A 7, A 1 a), Lasus of Hermione (14 A 13) and is depicted as participant of a political revolt against Pythagoras of the end of the VI-th century (14 A 5).

²⁶ Theophrastus speaks of him manifestly ironically (45 A 2).

²⁷ Kirk, G.S., Raven, J.E., The Pre-Socratic Philosophers (Cambridge, 1961), p.313-317.

any serious development of it. Of course, the views of Philolaus, Ecphantus and Eurytus could be put together under the name of number philosophy. But were they really continuations of an early Pythagorean doctrine that has not reached us? Each of them regarded number from his *own* standpoint, and as these standpoints are well understood in the context of philosophy of the Eleatics and atomists as well as Pythagorean science, there is no necessity of connecting them with an antecedent Pythagorean philosophy of number. Without going into the details of appraisal of these three later Pythagoreans, it is possible to say that they were on the periphery of philosophic thought of that time. The end of the V-th to the first half of the IV-th century is the period of the highest peak of Pythagorean science and flourishing of Platonic philosophy. Against this background the belated polemics of Philolaus with Eleatics and the modification of atomistic doctrine by Ecphantus (to say nothing of a “philosophy” of Eurytus) cannot be regarded otherwise than symptoms of decline of Pythagorean philosophy as well as, in fact, of pre-Socratic philosophy as a whole.²⁸

Does this mean that we are witnessing here the end of number philosophy among the Pythagoreans, which died off not being able to develop? In a sense – yes. But a more interesting fate was destined for it by an unpredictable motion of human thought: scarcely having died away it was revitalized again. To put it more exactly, it was just at this time that the doctrine of number as the essence of all things came into existence, being cast into those forms which were later to be taken over by the generations to come. But it was not Pythagoreans who were responsible for it, and not even Plato. The “Pythagorean” thesis “Things are numbers” owes its birth to the disciples of Plato, and in the first place to Aristotle.²⁹

Aristotle gives us more information about Pythagoreans than all his contemporaries taken together. A great number of works of scholarship are devoted to interpreting this information,³⁰ but despite a marked progress in this field, the main problems facing the student have not yet been satisfactorily solved. Up to now, for instance, it remains obscure what were the

²⁸ See: Nussbaum, *op.cit.*, p.82ff.

²⁹ On Speusippus and Xenocrates see: Burkert, *op.cit.*, p.53ff.

³⁰ See, for example, Zeller, E., *Die Philosophie der Griechen*, 4 Aufl. (Leipzig, 1887), V.I., p.343ff.; Gilbert, O., “Aristoteles Urteil über die Pythagoreische Lehre”, *Archiv für Geschichte der Philosophie*, 22, 1909; Frank, *op.cit.*; Cherniss, H., *Aristotle's Criticism of Pre-Socratic Philosophers*, Baltimore, 1955; Burns, *op.cit.*; Philip, *op.cit.*, *passim*; Burkert, *op.cit.*, p.28ff.

sources Aristotle mainly relied on and who is meant when he says "Pythagoreans" or "so-called Pythagoreans".

How far the current state of the problem is from finally being solved can be judged if only by the diversity of answers to the latter question, and this also implies a partial solution of the former. In Aristotelian Pythagoreans Frank saw Archytas and his disciples, Burkert-Philolaus, Philip-Pythagoras himself, while the majority of other scholars prefer to see the Pythagoreans "in general", ignoring quite a few obvious discords or trying to single out these or other groups, generations etc. Meanwhile, without having solved these problems it is impossible to make progress in what is most important and to appraise how adequate the Aristotelian interpretation of Pythagorean philosophy has been.

Aristotle, certainly, understood that he was dealing with a school which by that time had existed for almost two hundred years (Met. 985 b 24, 1078 b 21), but in his account he makes no attempt to show the development of its ideas and to present its separate stages. He characterizes the Pythagoreans "in general" and states their views as a whole. At the same time, in a number of places he notes the discrepancies between certain groups of Pythagoreans (Met. 986 a 25; De coelo 300 a 14; De an. 404 a 16); in other cases this is clear from his account itself. Still more different from the "general Pythagorean" system of views are the opinions of individual Pythagoreans listed by him.

Now we are approaching the most interesting point. There is no doubt that Aristotle was aware of the teachings of Alcmaeon, Hippasus and Hippon, for he mentions them in his treatises. Most probably, he also knew about Menestor who is repeatedly referred to by Theophrastus (DK 32). In the "Physics" (216 b 26) there is talk of a certain Xuthus who is named a Pythagorean in a later commentary (DK 33). Aristotle was no less well-informed about the views of later Pythagoreans, such as Philolaus, Eurytus, Archytas. Two of his special writings were devoted to Alcmaeon and Archytas (D.L.V. 25). It would seem that the views of these thinkers ought to be the subject of his analysis, but we observe quite a different picture. The records of separate Pythagoreans in Aristotelian treatises are few and much less informative than the presentation of "general Pythagorean" philosophy. Still more surprising is the fact that when mentioning these people by their names, in *no* place does he call them Pythagoreans, and speaking about Pythagorean philosophy on the whole, he *never* gives any name.

Thus we have, as it were, two non-intersecting lines: the views of individual Pythagoreans are treated quite separately from the number philosophy

belonging to no person and referred to as Pythagorean by Aristotle. Who, then, was its author? Aristotle, in his extant works, mentions Pythagoras only twice (Met. 986 a 30; Rhet. 1398 b 15), on the first occasion in connection with a so-called table of ten opposites.³¹ But in both cases nothing is said about number. Philolaus whose book Burkert considered to be the main, if not the only source of Aristotle on Pythagoreanism,³² is mentioned only once, in a rather unimportant passage (E.E. 1225 a 30) and without any connection with number. And again Philolaus is not called a Pythagorean! As a matter of fact, *nobody* is called a Pythagorean by Aristotle.³³ How can this be explained?

It should be remembered here that Aristotle is interested in stating the views of pre-Socratics or Platonists not for themselves, but as a starting point for critical analysis in the course of developing his own theories.³⁴ The task thus set by him required, in its turn, systematization of the earlier doctrines on the basis of his own principles. No wonder that in the course of this work Aristotle repeatedly resorted to far-fetched or merely wrong interpretations. Recognition of this obvious fact has nothing in common with the current opinion that he systematically distorted or simply did not understand the teachings of pre-Socratics. One should not, however, run into another extreme trying to prove that Aristotle is always in accordance with the truth. H. Cherniss has convincingly shown that even to his teacher Aristotle sometimes ascribed views which are not only absent from Plato's dialogues, but are exactly contrary to them.³⁵

It is possible to believe that in the case of Pythagoreans we are also faced with a similar tendency. This is all the more probable as here Aristotle was confronted with a much more complicated problem. Setting out to give an analysis of philosophic doctrines of Pythagoreans, he inevitably found himself before a choice: either the teaching of each one should be set forth separately – in which case it would be clear that all of them are different, or all of them should be presented as a single whole – then a certain common denominator is necessary which would characterize the school and at the

³¹ As was long ago shown by I. Wachtler (De Alcmaeone Crotoniata, Leipzig, 1896, p. 1 sqq.), there are no reasons at all to see an interpolation in mentioning Pythagoras in A of "Metaphysics".

³² Burkert, op.cit., p.235-238.

³³ Cherniss, op.cit., p.385. The misunderstanding with the "Pythagorean Paron" was explained away by Burkert (op.cit., p.170).

³⁴ This is not denied even by Guthrie who tried to moderate the criticism toward Aristotle that increased after the works of H. Cherniss (Guthrie, W.K.C., "Aristotle as a Historian of Philosophy", Journal of Hellenic Studies, 77, 1957).

³⁵ Cherniss, H., The Riddle of the Early Academy (Berkeley, 1945), 7ff., 72ff.

same time would distinguish it from philosophers of other trends. Why was it number that was chosen as the relevant characteristic? Evidently, this was because some later Pythagoreans really did say something about number, while the Milesians, Eleatics or atomists said nothing at all. Most probably, Aristotle was not able to find any other common feature, for it is indeed very difficult to do so.

Modern history of philosophy not only left unsolved, but has not even raised the question: what is there in common between the theories of Pythagoras, Hippasus, Alcmaeon, Hippon, Menestor, Philolaus, Archytas, Eurytus, Ecphantus and other representatives of Pythagoreanism? The cause of such irresoluteness is clear: to those familiar with their doctrines there seems to be very little hope of finding a common ground in them. But if one does attempt to reveal the “all-Pythagorean doctrine”, then it is to be searched for in the teachings of concrete Pythagoreans and not in number philosophy belonging to no one. It is difficult to imagine that along with the thinkers named above, there existed some other unknown Pythagoreans who, in anonymous treatises that have vanished without any trace, developed number philosophy as stated in Aristotle. Or should we suppose the existence of an oral but carefully guarded teaching which Aristotle alone had unexpectedly got access to?

Search along these lines will hardly give any results. There is too much in favour of the idea that the number philosophy presented in Aristotle was created not by obscure Pythagoreans, but by Aristotle himself. One of the causes of its having appeared was that Aristotle regarded the Pythagoreans as predecessors of the mathematically coloured philosophy of Plato (*Met.* 987 a 30, b22ff). He was not alone in this view. Speusippus and Xenocrates also appear to have transferred their own interpretation of Plato’s doctrines to Pythagoreans. In any case, it is to them that certain ideas go back which, beginning with the III-century B.C., were called specifically Pythagorean.³⁶ As Burkert showed, the well-known identification of a unit with a point, of two with a line, of three with a plane (the simplest plane is a triangle) and of four with a solid (pyramid) stemmed from Speusippus and Xenocrates’ interpretation of Plato’s “*Timaeus*”.³⁷ This idea is presented, in particular, in the fragment from Speusippus’ treatise “On Pythagorean numbers” (fr.4 Lang=44 A 13).

³⁶ In fact, already Theophrastus attributes to the Pythagoreans the doctrine of Monad and Indefinite Dyad (58 B 14) created in the Early Academy (Burkert, *op.cit.*, p.57f., 62ff.).

³⁷ Burkert, *op.cit.*, p.23f., 55f., 66-69.

Burkert believes that Aristotle made a clear distinction between Pythagoreanism and Platonism and, unlike the Platonists, did not ascribe any ideas of his own to the Pythagoreans. It is true that the number philosophy of Pythagoreans outlined by Aristotle differs from that taught by the Platonists. But this is due to the fact that we are faced with an *Aristotelian interpretation* of Pythagoreanism that could not but differ from the views of Speusippus and Xenocrates, as it was an attempt to squeeze into notions and schemes of his own philosophy something that belonged to quite a different way of thinking. Besides that, one has an impression that the mathematical philosophy of Platonists was not so much based on the *philosophic* views of Pythagoreans as on their *mathematics*, while Aristotle was prone to see in the Pythagoreans just the philosophic predecessors of Platonism.

Aristotle certainly used the doctrines of some later Pythagoreans, but relying on them he created a number philosophy such as had never existed in Pythagoreanism. And as Aristotle was unable to substantiate it in any way, he turns to an original solution: the thesis about number he ascribes to the school as a whole and to nobody in particular, while in discussing the views of individual Pythagoreans he never speaks about their belonging to the school. Probably, this was done to smooth away the striking contradictions between the number doctrine and the views of early Pythagoreans reported by Aristotle himself. In the “Metaphysics”, for example, it is repeatedly asserted that none of the Pythagoreans had said anything about sensible principles (Met. 989 b 30ff; 990 a 17), while Hippasus’ *archē* was fire (Met. 984 a 7) and Hippon’s water (Met. 984 a 4). According to Aristotle, Pythagoreans gave explanations of all things by means of quantitative characteristics, yet in Alcmaeon and Menestor we find only qualitative opposites, principally warm and cold (Met. 986 a 27f.; 32 A 5). The Pythagoreans believed that the soul is harmony (De an. 407 b 30; Pol. 1340 b 18), and Hippon wrote that the soul consisted of water (De an. 405 b 2). Aristotle gives four (!) quite different Pythagorean views on the soul, giving no explanation whatever of this strange fact.

The teachings of early Pythagoreans are strongly resistant to being connected with the number doctrine. In fact, Aristotle himself confirms once more that the sensible, corporeal beginnings and the qualities connected with them were at the basis of natural philosophy of the Pythagoreans known to him – and in this way they did not differ from other pre-Socratics. If Pythagoras really asserted that “all is number”, his followers do not at all look like adherents persistently repeating what “he himself said”. Incidentally, the very expression αὐτὸς ἔφα appears for the first time only in

Cicero (De nat. deor. I.5,10) and it may well go back to the pseudo-Pythagorean treatises written in Doric dialect. But even supposing they are an intermediate source and this expression had arisen in the acousmatic tradition, it is clear that it is connected with religious doctrines and not with scientific or philosophic theories. This could have been said by some adherent of metempsychosis, while Pythagorean scientists and philosophers did not by any means suffer from lack of originality or independence.

Yet, with due regard for these qualities of theirs, it is difficult to understand how the influence of the central idea of the Master could have been avoided by them. But should not we try to find continuity in their views instead of artificially contrasting Pythagoras' number against the qualitative principles of the early Pythagoreans? In essence, only one thing is necessary for this: to reject the idea that the number doctrine formed the corner stone of Pythagoras' philosophy. After what has been said above of the teachings of Pythagoreans of the V-th century, it will not be very difficult to do.³⁸

We begin our reasoning with an indirect proof: let us suppose that the number philosophy contained in Aristotle belongs to Pythagoras himself. What is it in itself? Aristotle gives three different and mutually contradictory variants of this doctrine, and that is a true indication that we are dealing with an interpretation of his own.³⁹ In the first place, things *are* numbers, in the sense that numbers serve as the material basis of the world: τὸν ἀριθμὸν . . . ἀρχὴν εἶναι καὶ ὡς ὕλην τοῖς οὐσι καὶ πάθη τε καὶ ἔξεις (Met. 986 a 16); ἀριθμοῦς εἶναι αὐτὰ τὰ πράγματα (Met. 987 b 28); τὰ σώματα ἐξ ἀριθμῶν εἶναι συγκείμενα (Met. 1083 b 11). Secondly, the Pythagoreans *liken* things to numbers: ἐν δὲ τοῖς ἀριθμοῖς ἔδοκουν θεωρεῖν ὁμοιώματα πολλὰ τοῖς οὐσι καὶ γιγνομένοις (Met. 985 b 27), μμήσει τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν (Met. 987 b 11). Thirdly, the *elements* of numbers are simultaneously the elements of things: τὰ τῶν ἀριθμῶν στοιχεῖα τῶν ὄντων στοιχεῖα πάντων ὑπέλαβον εἶναι (Met. 986 a 2), τοῦ δὲ ἀριθμοῦ στοιχεῖα τό τε ἄρτιον καὶ τὸ περιττόν, τούτων δὲ τὸ μὲν ἄπειρον, τὸ δὲ πεπερασμένον (Met. 986 a 18).

It is obvious that the second thesis is contradictory to the first one: likened to number can be only what is not number. Meanwhile, Aristotle persistently repeats that Pythagorean number is precisely a material first principle, although not giving any concrete example to support it. Thus, we

³⁸ Kirk, Raven, op.cit., p.261.

³⁹ Cherniss, Aristotle's Criticism, p.386ff.

still cannot say just *what things* or *elements* were likened to numbers by Pythagoreans. Yet on the other hand it can be safely asserted that they did not say anything about an independent existence of number *outside the physical world* (Met. 987 b 27, 1080 b 17; 1086 b 16; Phys. 203 a 6). To put it more exactly, number had never been an independent entity for them, but always the number of something.⁴⁰ But this feature is typical of all the pre-Socratics; none of them had spoken about an independent existence of abstractions simply because the division of the world into the sensible and the ideal had not yet taken shape.⁴¹ Corporeal were not only τὸ ἄπειρον of Anaximander or τὸ ἕόν of Parmenides, but also Φιλία of Empedocles, Νοῦς of Anaxagoras etc.⁴²

Stressing the corporeality of the Pythagorean understanding of number, Aristotle obviously wanted to separate this school from Plato and his disciples, who for the first time raised the problem of the ontological status of abstractions including the mathematical ones.⁴³ So far as the Pythagoreans did not say, like Plato, that number belongs to the world of Ideas and did not consider it to be an abstraction as Aristotle himself did, it means that their number is material – probably in such a way the logic of Aristotelian thought may be restored. But not considering number to be ideal is not at all the same as considering it to be material. It is not known if the Pythagoreans had a philosophic definition of number at all; it is quite possible that they were satisfied by a purely mathematical definition: number is a multitude composed of units (Aristox.fr. 23 Wehrli).⁴⁴ At all events, Aristotle himself writes that the number of the Pythagoreans is a mathematical number, and they know of no other (Met. 1080 b 16, 1083 b 13). Aristotle's attribution to the Pythagoreans of the doctrine that things consist of material numbers has long been disputed; attempts to refute it have been made even by those who do not deny the number philosophy of Pythagoreans itself.⁴⁵ Neither does the third thesis of Aristotle look any better: it is also at variance with the first two and is not at all built in the spirit of pre-Socratic philosophy. According to Aristotle, the Pythagoreans believed that the στοιχεῖα of things are πέρας and ἄπειρον, and of numbers περιττόν and ἄρτιον,

⁴⁰ Nussbaum, op.cit., p.89-92.

⁴¹ Guthrie, W.K.C., A History of Greek Philosophy (Cambridge, 1963), v.I, p.64.

⁴² Sambursky, S., The Physical World of the Greeks (London, 1960), p.18-19, 82; Krafft, Fr., Geschichte der Naturwissenschaft I (Freiburg, 1971), p.237, 257.

⁴³ Cherniss, Aristotle's Criticism, p.36f.

⁴⁴ The same definition is also given in Euclid (Eucl. El.VII, def.2).

⁴⁵ See for example: Zeller, op.cit., p.349; Gilbert, op.cit., p.40f.; Burns, op.cit., p.112-115; Kirk, Raven, Schofield, op.cit., p.333.

identifying the limit with odd and the unlimited with even. Both these pairs certainly occupy the first place in the table of opposites which, according to Aristotle, belonged to one group of Pythagoreans (Met. 986 a 22). It was long customary to see in this table the result of a later systematization,⁴⁶ although its main ideas may well go back to early Pythagoreanism. It is quite probable too, that some Pythagoreans of the IV-th century B.C. *did connect* even with unlimited and odd with limit (in what way we are not going to find out now),⁴⁷ but where are the indications that they had identified them? They are not to be found even in Philolaus who repeatedly returns to discuss τὰ ἄπειρα καὶ τὰ περαίνοντα. The other eight pairs of opposites are also connected with each other to a certain degree, which is far from pointing to their identity.

It is not necessary to prove in detail that the concept στοιχεῖον does not at all go back to the Pythagorean school. The Pythagorean mathematicians divided numbers into the odd and even ones, but the doctrine that περιττόν and ἄρτιον are the “elements” of numbers belongs not to mathematics, but to philosophy; and it is not otherwise attested in early Pythagoreanism. By and large, the division of the world into two kinds of entities (things and numbers) with their being subsequently joined together through identification of their first principles could have appeared only after Plato. Thus, the third formulation of the number doctrine, as well as the first one, is the interpretation of Aristotle himself.⁴⁸

There remains only the thesis that Pythagoreans likened things to numbers. Here for the first time we gain a more or less firm footing. First of all, this is mentioned not only by Aristotle, but also by Aristoxenus: Pythagoras made great advance in arithmetic, πάντα τὰ πράγματα ἀπεικάζων τοῖς ἀριθμοῖς (fr.23 Wehrli).⁴⁹ Secondly, Aristotle has several examples of similar comparisons (Met. 985 b 29-30, 1078 b 22; E.N. 1132 b 21) which are also known from other sources.

However, looking more intently at just what is being likened, these examples will be rather disappointing to those searching for Pythagoras' number philosophy. As I already said, cases in which sensibly perceived

⁴⁶ Burkert, op.cit., p.51ff.; Kahn, Ch., “Pythagorean Philosophy before Plato”, The Pre-Socratics, ed. A.P.D. Mourelatos (New York, 1974), p.170.

⁴⁷ See for example: Taylor, A.E., “Two Pythagorean Philosophers”, Classical Review, 40, 1926. A natural drawing together of the paired concepts πέρας/περιττόν and ἄπειρον/ἄρτιον can also be supposed here.

⁴⁸ Cherniss, Aristotle's Criticism, p.17, 44-45, 390.

⁴⁹ Similarity with Aristotle does not entail that Aristoxenus could not have independent Pythagorean sources.

things are likened to numbers are unknown before Eurytus, and it is very likely that Eurytus was the first to take this path (45 A 2). The tradition preserved by Aristotle is more ancient than Eurytus, but it leads in quite a different direction. It does not speak of things, but of abstract, mostly ethical concepts: “justice” corresponds with 4, “marriage” with 5, “opportunity” with 7 etc. Were there enough of such sayings for Aristotle to exclaim: “Why, all is number with them!”? The question is not easy to answer, but it is much more difficult to find more serious confirmations of this “doctrine” besides the ideas of Philolaus, Ecphantus and Eurytus mentioned above and a few acousmata of the sort τί τὸ σοφώτατον; ἀριθμός.

In those cases where Aristotle tries to do it himself, his tendentiousness is evident. He says when analysing the astronomical system of Philolaus that the Pythagoreans are so much attached to the number 10 that they had specially thought out a tenth celestial body – the “antichthon” (Met. 986 a 10-12). However, from his other passage it appears that the “antichthon” was introduced for explaining a greater frequency of lunar eclipses than solar ones (De coelo 293 b 21).⁵⁰ It is known that Philip of Opus had spoken against the explanation of lunar eclipses by the “antichthon” (58 B 36), hence he knew that it had been introduced exactly for this purpose.

Thus, from what Aristotle says about number, we can attribute to Pythagoras only the idea of similarity and correspondence of some notions with numbers, as mentioned in the pseudo-Aristotelian “Magna Moralia” (1182 a 11). These correspondences are sometimes not devoid of wit (justice is equal times equal, or 2×2), but no deep philosophical sense is to be elicited from them. Their author himself had hardly aimed at putting it into these sayings. The acousmatic tradition shows that some Pythagoreans believed in the magic power of number, but even if this tendency does go back to Pythagoras (cf. Heracl. Pont. fr. 44 Wehrli), there had not been formed any distinct philosophical doctrine on its basis.

It becomes obvious that we do not succeed in reconstructing Pythagoras’ philosophy on the “number basis”. At the same time, the task is not at all hopeless. Scattered about in the work of Aristotle, there are quite a few ideas most likely going back to Pythagoras. First of all, this is true of the Pythagorean cosmogony some records of which have survived (Met. 1091 a 13f.; Phys. 203a5, 213b22; fr. 201 Rose). According to this cosmogo-

⁵⁰ Burnet, *Early Greek Philosophy*, p.303; Dreyer, J.L.E., *A History of Astronomy from Thales to Kepler* (New York, 1953), 2nd ed., p.47.

ny, the world is formed as a result of interaction of two principles, πέρας and ἄπειρον, the latter being thought of as a boundless air (πνεῦμα) surrounding our world, and at the same time as a void (κένον). The nearest part of this unlimited pneuma is breathed in the world and is bounded by a limit. Further, this part of the pneuma separates natural things from one another marking thereby the beginning of their existence.⁵¹

Despite the vagueness of many details in the Pythagorean cosmogony, it is possible to say with certainty that numbers do not participate in the emergence of the world. Τὸ ἓν mentioned in Met. 1091a16 is not at all a unit, but the One, all the world as a whole.⁵² Aristotle's attempts to connect the cosmogonic process with number (Phys. 213 b 26-27) show that he had no support in the Pythagorean tradition.

The antiquity of the Pythagorean cosmogony is seen from its archaic character and identification of air with void disproved by Empedocles and Anaxagoras. The criticism of this cosmogony by Xenophanes who claimed that the deity which he had identified with the whole world does not breathe, also shows it to belong to the VI-th century.⁵³ "It was just at this time", wrote C. de Vogel, "immediately after the Milesians and as a reaction to them, such dualistic explanation of cosmos was to arise. It was here that, in contrast to ἄπειρον of Anaximander, the principle of limitation named πέρας was postulated by the creative mind of a true philosopher".⁵⁴

To what extent did the teaching of Pythagoras, in particular his **archai** πέρας and ἄπειρον, influence the philosophic views of his followers? It should be noted first of all that there did not exist any general Pythagorean philosophy; each of the Pythagoreans developed his own system of views. In some features this may have been similar to the ideas of other Pythagoreans and Pythagoras himself, but *there is not one philosophic doctrine that would be shared by all the Pythagoreans.*

For Hippasus the beginning was fire. He closely resembles Heraclitus in this (they are often mentioned together), but is at variance with Hippon whose beginning was water. Hippon, in his turn, is quite unlike his younger contemporary Philolaus: in choosing **archē** he followed Thales, although accounting for his choice by other considerations.

⁵¹ A curious peculiarity, probably derived from Aristotle, is reported by Ioannes Philoponus: "The Pythagoreans say that void and air breathed in by heaven separate animals from plants" (De gen. anim. comm., p.107.14).

⁵² Cherniss, Aristotle's Criticism, p.39.

⁵³ Kahn, op.cit., p.183-184.

⁵⁴ Vogel, C. de, Philosophia. Part 1. Studies in Greek Philosophy (Assen, 1970), p.85.

Dualism, which comes from Pythagoras' positing of opposite **archai**, is evident in Alcmaeon, Menestor and Philolaus, but it has nothing in common with Hippasus' doctrine.⁵⁵ Both Hippasus and Hippon touched upon cosmogonic questions, but these problems were quite alien to Alcmaeon, Menestor and Archytas. Philolaus was influenced by Parmenides' philosophy, Menestor and Hippon adopted some ideas of Empedocles, Ecphantus followed the atomists and Anaxagoras. Some Pythagoreans chose as principles natural elements (fire, water), others metaphysical entities (πέρας and ἄπειρον), while Alcmaeon and Menestor spoke not about elements at all, but about qualities (cold-hot, moist-dry etc.). Some Pythagoreans thought our world to be unique (51 A 3; Arist.fr. 201 Rose), others believed in plurality of worlds (Heracl.Pont.fr.113 Wehrli).

Why are the doctrinal differences so great in Pythagoreanism? First of all, because it had not arisen as a philosophic school, and belonging to it had never been determined by following the sum of certain doctrines.⁵⁶ It is just for this reason that such original thinkers as Parmenides and Empedocles are called Pythagoreans in our sources. Originally the name "Pythagoreans" was given to the people who belonged to a Pythagorean political society, ἑταιρεία,⁵⁷ and later usually to those who had Pythagoreans as teachers and/or did scientific research in one of the fields where Pythagoras' first followers had distinguished themselves: in mathematics, astronomy, acoustics, as well as in medicine and related sciences (anatomy, physiology, botany etc.). The Pythagoreans were much closer to each other in science than in philosophy.

Let us once more return to the role of number ideas in early Greek thought. Is every attempt to find support in a countable regularity to be understood as number philosophy? It is one thing to assert that sensibly perceived things consist of units, another to believe that all in the world is arranged in accordance with the number principle, and a third to look for concrete number regularities in nature. We are faced not with different stages of number philosophy, but with heterogeneous trends, and if the first two of them can really be termed "philosophy of number", the scientific hypothe-

⁵⁵ The question is, of course, in contrasting two equally corporeal principles, and in no way in dualism of form and matter.

⁵⁶ Gruppe O., *Über die Fragmente des Archytas und der älteren Pythagoreer* (Berlin, 1840), p.60.

⁵⁷ Already Zeller remarked the political origin of the term Πυθαγόρειοι (op.cit., p.314, n.1). See also: Minar, E., *The Early Pythagorean Politics in Practice and Theory* (Baltimore, 1942), p.15ff.; Burkert, op.cit., p.30, n.8.

sis was moving along the lines of the third. It was just here that it became possible not only to advance some ideas, but also to prove them, which led Pythagoras to the discovery of the numerical relations of harmonic intervals. This discovery laid the foundation for the Pythagorean acoustics which relied both on mathematics and on physical experiment; on the other hand, it was also to contribute to the development of arithmological speculations which tried to express by number what is inexpressible by it. However, arithmological speculations existed in Greek culture before and besides Pythagoras,⁵⁸ while for the Pythagoreans they were a by-product of the development of mathematical studies, but not the essence of their natural philosophy.

It often happens that to the “number mysticism” are ascribed things which in reality have the character of scientific hypotheses, for example, the idea of celestial harmony.⁵⁹ Contrary to the notions coming down from Iamblichus and Porphyrius that celestial harmony is a certain mystical doctrine, the testimony of Aristotle shows that we are in effect faced here with a physical teaching (De coelo 290 b 12ff; fr. 13 Ross). Pythagoreans who were predecessors of Archytas asserted that sound is initiated by motion (18 A 13) and that there cannot be sound without motion (47 B 1). It follows that each motion should be accompanied by sound – this conclusion did suggest itself. And if the celestial bodies are in constant motion, they must produce sounds, even though we do not hear them. According to the Pythagorean model, the bodies which are nearer to the centre move more slowly, while with the increase of the distance from the centre the velocity of motion and the pitch of the sound will increase. The correspondence of distances between the celestial bodies to the numbers of harmonic intervals is of a secondary nature in this theory, being in a way a mathematical interpretation of an originally physical hypothesis. Yet this idea, although proved to be wrong, did point to the right path toward the answer: there really exists a mathematically formulated regularity discovered by Kepler between the velocity of planet motion and their distances to the centre.

How little the idea of celestial harmony is connected with the supposed Pythagorean number philosophy is shown by the fact that already Anaximander arranged his celestial “rings” according to the number principle.

⁵⁸ See for example: Usener, H., “Dreiheit”, *Rheinisches Museum*, 58, 1903; Roscher, W.H., *Die Sieben-und Neunzahl im Kultus und Mythos der Griechen*, Leipzig, 1904.

⁵⁹ It is generally called “the harmony of the spheres”, yet this is a later and therefore misleading name. The planetary spheres appeared as late as the IV-th century, in Eudoxus, while Aristotle speaks merely about celestial harmony.

The *distances* between the heavenly bodies (as distinct from the number of the bodies and their mutual disposition) given by him are no better and no worse than those supposed by the Pythagoreans. What in that case prevents us from pronouncing Anaximander to be the founder of number philosophy?⁶⁰

In many pre-Socratics we find an inclination to these or other numbers, which is not necessarily explained by the influence of Pythagoras' ideas. Empedocles taught that the world consists of four elements. Why just of four, and not of two, as in Parmenides? Probably, because Empedocles considered such a quantity to be necessary and sufficient and not because of his special predisposition to the number four. His younger contemporary, Ion of Chios, on the contrary, proved that all exists by three and there are three beginnings of the world: fire, air and earth (36 A 6, B 1). In this case one has the impression that Ion's choice was dictated by a conscious polemic with Empedocles and Parmenides, and not only by his predisposition to the number three. Reasoning by analogy, a reproach for following the Pythagoreans can be addressed to Aristotle too: in his theory the sublunary would also consist of four elements, no less and no more!

Clearly, it is not that everybody who tries to count something should be suspected of sympathy for the Pythagoreans. Nevertheless, there are examples where it is possible to speak with some degree of certainty about the influence of their ideas. What is true and important is that in these cases the question is not so much about number mysticism, or even of number philosophy, as about the impact of the mathematical discoveries of the Pythagoreans on the natural science. Here is an example of how the Pythagorean ideas about proportion were reflected in the Hippocratic treatise "On Regimen": "If it were really possible to find for the nature of each man a correct proportion of food to exercise, without inaccuracy of excess or of defect, this would be a right way to his health" (De victu I.2). The Pythagorean physicians were also occupied with searching for numerical relations of food, drink and physical exercise (58 D 1).

A no less interesting example is to be found in Empedocles who believed that different parts of human body consist of four elements which are in a certain proportion to each other (31 B 96-98). The bones, for example, consist of two parts of water, two parts of earth, and four parts of fire, the nerves consist of one part of fire, one of earth and two of water etc. In these

⁶⁰ Krafft, for example, stresses that the relative distances between the planets given by the Pythagoreans do not mark a new step as compared to Anaximander and that Pythagoras should not be considered the founder of this kind of number speculations (Krafft, op.cit., p.220, 222).

reasonings, at the first sight naive, many would be able to see if not a discovery, then at least an insight into the idea of a chemical formula.⁶¹ It would be possible to give some more examples, but it is already clear without them that mathematical ideas served as one of the valuable instruments of human thought, which was mastering the vast and diverse material that is the subject of natural science. If many steps in this direction were far from being fruitful, even so, each of them shows a lively interest in the cognition of the surrounding world, an interest altogether different from the fruitless numerological contrivances of Neopythagoreans and Neoplatonists. Those who are looking for concealed mysteries hidden in the Pythagorean Number should apply to Iamblichus, but not to Pythagoras.⁶²

Institute of the History of Science and Technology, Leningrad

⁶¹ See for example, Gomperz, Th., *Griechische Denker* (Leipzig, 1922), v.1, p.190-193. On the influence of the Pythagorean teaching about proportions on Heraclitus see: Fränkel, H., "A Thought Pattern in Heraclitus", *American Journal of Philology*, 53, 1938.

⁶² I read Carl Huffman's article "The Role of Number in Philolaus' Philosophy", *Phronesis*, 33, 1988, 1-30, just after finishing my paper. I have not added references to it, but I am glad to acknowledge that his reasoning is very close to mine on some important points.