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COSMOS

(In this paper our aim is to show how, in the first place, the emergence of philosophy paved the way for the development of the various sciences and, secondly, how the continued subordination of the sciences to philosophy, although largely beneficial so far as the exact sciences were concerned, gravely hampered the development of the empirical sciences. To bolster our second argument, we propose to consider in detail mathematics and medicine as examples of the exact and empirical sciences respectively. In stressing the influence of philosophy upon the emergent sciences, we are by no means discounting influences in the reverse direction. But that is another story.)

In the sixth century B.C. there came about a striking development in the history of Greek thought when, in the Ionian city of Miletus, all mythopoeic forms of thought were abandoned by certain thinkers who sought to explain the world about them in terms of its visible constituents. Here, for the first time, was displayed an attitude of mind unhampered by any religious belief in divine, arbitrary intervention. Natural explanations were introduced which took the place of supernatural and mystical ones. Various circumstances seem to have contributed to the development of this outlook which brought about the transition from mythological conjecture to rational explanation, namely, the material prosperity of commercial Miletus, the special opportunities for contact with other cultures and, most important of all, the absence of a priestly caste jealous to guard its religious traditions. The Milesians were not subject to the constraint of any religious degree.

Like their mythopoeic predecessors, these Ionian thinkers firmly believed that there was an orderliness inherent in the world around. Again like their predecessors, they attempted to explain the world by showing how it had come to be what it is.¹ But, instead of invoking the agency of supernatural powers, they sought for a unifying hypothesis to account for this order and, to a greater or lesser extent, proceeded to deduce their natural explanations of the various phenomena from it.

When we claim that there arose in the sixth century B.C. an attitude of mind unhampered by any religious belief in divine intervention, we are not seeking to maintain that all religious elements were completely eradicated from the minds of the Presocratic philosophers and that they sought to explain the world about them in the cold light of pure reason. This is patently not the case. For philosophy was turned by the Pythagoreans into a religious way of life and the strong religious elements in their thought exercised a considerable influence upon the development of the exact sciences in particular. In Empedocles' thought, too, his religious beliefs exercised a considerable influence upon his physical theories.² However, it is important to notice that even where religion and philosophy are most closely intertwined nowhere at this period do we find any recourse to a supernatural agency to account for the origin and continued operation of the world.³

Two elements, then, characterise early Greek philosophy, the search for natural as opposed to supernatural and mystical explanations, and secondly, the search for a unifying hypothesis. Both of these elements, as we hope to show proved influential in paving the way for the development of the sciences. When we turn to the Pythagoreans, however, we find that, although they satisfy the second of our criteria for a definition of philosophy at this date, they clearly do not seek for natural causes in the manner of their Ionian predecessors. It might, therefore, be objected that their outlook should not be regarded, strictly speaking, as philosophical. This objection, we believe, cannot be upheld. For the unifying hypothesis upon which the Pythagoreans rely itself presupposes the Ionian search for natural explanations and could not have been propounded before these thinkers had introduced such explanations to take the place of supernatural and mystical ones. Like the Ionians, the Pythagoreans were not hampered in outlook by any belief in divine intervention. Aristotle, we may notice, recognises that the Pythagoreans followed closely in the footsteps of the Ionians. In the *Metaphysics*,⁴ after commenting upon this point of difference between them - "the so-called Pythagoreans employ stranger principles and elements than the physicists, the reason being that they took them from non-sensible things" - he adds - "they still concern themselves wholly with nature; they generate the universe and watch what happens to its various parts and affections and activi-

THE SUN AND THE PLANETS

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ties; and they use up their first principles and causes upon these things, as if they agreed with the other physicists that Becoming is just so much as is sensible and is embraced within what they call the universe. And yet, as I said, they maintain causes and first principles that are adequate to lead up to the higher kinds of reality - that are indeed better fitted to them than to discussions about nature!"

The late Professor John Austin has drawn a simile which well depicts the state of affairs in the Presocratic period. He writes:⁵

"In the history of human enquiry, philosophy has the place of the initial central sun, seminal and tumultuous: from time to time it throws off some portion of itself to take station as a science, a planet, cool and well regulated, progressing steadily to a distant and final state".

This simile is extremely apposite for in the Presocratic period the sciences first began to be thrown off from the central sun, philosophy, and to evolve slowly towards a separate and independent, or, to keep the simile, a "cool and well regulated existence". Mathematics came into being as a theoretical science as a direct result of the attempt of the Greeks to explain the world about them. The Pythagoreans by putting forward as their unifying hypothesis the theory that "all things are number" brought into being a theoretical study of mathematics divorced from practical considerations. The evidence of both Proclus and John Stobaeus supports this interpretation for the former in his commentary on Euclid's Elements tells us on the authority of Eudemus that "Pythagoras changed the study of geometry giving it the form of a liberal discipline, seeking its first principles in ultimate ideas and investigating its theorems abstractly and in a purely intellectual way" (In Eucl. 65, 11 Fr. DK14.6a), while the latter informs us that Aristoxenus in his treatise on arithmetic said that Pythagoras was the first to carry that study beyond the needs of commerce (DK58B2). The development of Greek mathematics proved rapid and this study reached its impressive climax in the third century B.C.

Following hard upon the heels of mathematics, astronomy began to develop as a separate science. The first steps towards the development of scientific astronomy might be traced back as far as Anaximander, traditionally the second of the Milesian philosophers, who, by developing the assumption that the world was orderly, gave to the heavenly bodies a proportionate arrangement and thereby took an important step towards the mathematisation of the cosmos. (Briefly Anaximander's astronomical theories are as follows: he held that the earth was cylindrical in form and that it remained in position at the centre of the universe because it was equidistant from all the extremes. The stars he held to be wheels of fire enclosed in mist. At points on these wheels were breathing-holes, certain pipe-like passages, through which the fire shone. Eclipses of the sun and moon were caused when these openings were blocked and the waxing and waning of the moon by their opening and closing. Primitive though these ideas are, they none the less mark an enormous advance on the allegories and mythological fancies which had gone before. No sun god sails his bark daily across Anaximander's skies ever ready to do battle against a lurking and voracious serpent whose victory in the fight was considered to be the cause of eclipse. Explanations were sought by him in terms of purely natural causes. In describing how the fire was able to shine through the holes in the mist enshrouded wheels, Anaximander makes use of a mechanical analogy in a manner which can be paralleled in scientific explanations at the present time and his theory that the earth remains in position because it is equidistant from all the extremes bears a startling resemblance to the modern Principle of Sufficient Reason).

However, it was the Pythagoreans who contributed most to the development of scientific astronomy for it was their influence which brought about its close and fruitful marriage with mathematics. The great contribution of the Greeks to astronomy was their formulation of geometrical systems to represent the motions of the heavenly bodies. The most important developments of this idea took place in the Academy and later, but the idea itself that the motions of the heavenly bodies could be given geometrical expression had its origin in the Presocratic period.

The influence of philosophy, then, upon the exact sciences was, for the most part, extremely beneficial. However, when we turn to the natural sciences, there is a different tale to tell, for, apart from the first beneficial effect derived from an outlook unhampered by any belief in supernatural intervention by a personal god, the continued subordination of the natural sciences to philosophy gravely hampered their development. A study of Presocratic biological thought, especially, shows clearly how harmful was the attempt of the philosophers to apply their general unifying hypotheses in this sphere and to deduce their explanations of the particular biological phenomena from them. In the last few centuries an hypothesis (unless in the realm of some highly abstract science such as mathematics) has come to be regarded as not genuinely scientific unless it is constantly tested by rigorous appeals to sense experience and kept, modified or abandoned according to the support that the phenomena give it. The early philosophers, however, felt no necessity to check their "philosophic" hypothesis in the light of further empirical evidence and failed to recognise as a possibility that further evidence could invalidate their theory.⁶

Two examples from fifth century biological thought may be cited to illustrate how harmful and unscientific was the attempt to deduce particular biological explanations from a general philosophical hypothesis. From our surviving evidence of Empedocles' thought it is clearly apparent that his general assumptions about the world at large were made to account for individual details within it. The consistency with which he seeks to deduce his particular explanations from his philosophical hypothesis is impressive, but hardly conducive to the development of biology as a science. As we shall see, even when he makes observations which are clearly contradictory to his general hypothesis his faith in that theory is not by any means shaken.

All living organisms, according to Empedocles, are composed of the four elements, earth, air, fire and water mingled in different proportions. We learn that upon this basis Empedocles sought to explain the separation of animals into their various species: those that have the most fire in their make-up fly up into the air, those with a preponderance of water become aquatic, while those that are heavier spend their life on earth.⁷ This attraction of "like to like" is an important axiom of Empedoclean physics. The double growth of plants, upwards and downwards, is also explained upon this basis. The earth in the roots is the cause of the downward growth and the fire in the shoots that of the upward.⁸ Generally, the fiery part of an animal or plant tends upwards and the earthy part downwards. There are, however, exceptions to this rule which Empedocles himself notices. He points out that in the case of such creatures as shellfish and turtles it is the earthy part that is uppermost.⁹ Here can be clearly seen the harmful nature of the deductive approach to the development of biology as a science. Empirical evidence obviously contrary to an a priori theory is not held to invalidate that theory which is regarded as inviolate.¹⁰

A similar illustration may be drawn from the theories of Diogenes of Apollonia, a philosopher who wrote somewhat later than Empedocles, and a man whose influence is out of all proportion to his intellect.¹¹ Diogenes, reviving the philosophical theory of Anaximenes, maintained that air was the source and substrate of all things. Our evidence shows how he endeavoured to base his biology upon his philosophical postulate with extremely harmful results to the former. One particular theory among many may be cited to illustrate this. It is recorded that Diogenes believed that other creatures were inferior in intellect to man because they breathed air which derived from the earth and was therefore moister. It, then, apparently occurred to Diogenes that according to this theory birds would be the most intelligent of all creatures since they breathe the purest air. This thought, however, did not shake his faith in the theory and his reaction was to introduce a saving clause and maintain that this was not really the case since the flesh of birds was of a particularly solid nature and it prevented the air from permeating¹² completely throughout their bodies by checking it in the vicinity of the stomach. Trivial though these examples doubtless are, they nevertheless clearly illustrate the harmful effect on the natural sciences of their continued subordination to philosophy.

Apposite though Professor Austin's simile is, we must be careful that it is not overpressed or employed too rigidly. It is true that medicine, like biology, benefited greatly from the new rational outlook derived from natural philosophy. Yet we must bear in mind that medicine is a practical art almost as old as man himself. It thus had an existence independent of the unified knowledge of the Presocratic philosophers. But, as we shall argue, it did not become truly scientific until it had felt the influence of that attitude of mind which the early Greek philosophers were the first to apply to the world about them. In the middle of the fifth century B.C. medicine felt for a second time the influence of philosophy. This time the influence was more direct and much less beneficial.

An examination of Presocratic thought reveals how the philosophers began increasingly to widen the scope of their enquiries and extend their views of the macrocosm (the world at large) to the microcosm (man). Since man was part of the furniture of the world, they held that the same materials and rules which functioned in the world at large were also applicable to man himself. The implications of this idea were most fully drawn out in the fifth century B.C. and as a corollary to this, the attempt was made to found medical theory, too, upon the general unifying hypothesis. The dangers inherent in this procedure, however, did not go unnoticed and we find the author of the medical treatise Ancient Medicine delivering a spirited polemic against those who sought to introduce a philosophical postulate as a basis for medical theory. He argues vigorously that medicine has no need of "new-fangled" hypotheses which are incapable of verification; but it has long had its own methods by which many excellent discoveries have been made in the past and by which many more will be made in the future.¹³ Yet in spite of this warning there are several works in the Hippocratic Corpus which clearly show the adverse influence of philosophy.

Let us now turn to our special pleading where we proposed to use mathematics and medicine as examples of the exact and empirical sciences respectively. Of all the branches of science which engaged the attention of the Greeks, it was in pure mathematics and in geometry especially that they achieved their greatest success. It is here that we find the most fruitful influence of philosophy upon the development of science.

The Greeks themselves claim to have derived their mathematics from Egypt. Aristotle, for example, records his belief that mathematics evolved in a highly theoretical way as the invention of a leisured class of Egyptian priests.¹⁴

"Hence when all such inventions were already established, the sciences which do not aim at giving pleasure or at the necessities of life were discovered, and first in the places where men began to have leisure. That is why the mathematical arts were founded in Egypt; for there the priestly caste was allowed to be at leisure."

Our earliest source, Herodotus, also believes that mathematics had their origin in Egypt. His reasons for doing so, however, are quite dissimilar. He believes that geometry originated in practice, i.e. that it owed its origin to the recurrent necessity for the re-measurement of the land periodically flooded by the Nile.

"And any man who was robbed by the river of part of his land would come to Sesostris and declare what had befallen him; then the king would send men to look into it and measure the space by which the land was diminished, so that thereafter it should pay the appointed tax in proportion to the loss."¹⁵ From this, to my thinking, the Greeks learnt the art of measuring land...

An extract from the history of mathematics, written as part of the systematisation of knowledge which went on in the Lyceum, and preserved in the writings of the Neo-Platonist, Proclus, records Eudemus's views on the origin of geometry. It is interesting to note that in spite of his being a Peripatetic his sympathies lie in this matter with Herodotus rather than Aristotle and he assigns to geometry a practical origin, finding it like the former in the land-measurement necessitated by the Nile floods.

"But since we must speak of the origin of the arts and sciences with reference to the present world-cycle, it was, we say, among the Egyptians that geometry is generally held to have been discovered. It owed its discovery to the practice of land-measurement. For the Egyptians had to perform such measurements because the overflow of the Nile would cause the boundary of each person's land to disappear."¹⁶

Eudemus tells us that it was "Thales who, after a visit to Egypt, first brought this study to Greece" and adds "not only did he make numerous discoveries himself, but he laid the foundations for many other discoveries on the part of his successors, attacking some problems with greater generality and others more empirically". Proclus has preserved for us some of the discoveries which Eudemus ascribed to Thales, namely that the circle is bisected by its diameter;¹⁷ that the base angles of an isosceles triangle are equal¹⁸ and that vertically opposed angles are equal.¹⁹ In addition he informs us that the theorem that two triangles are equal in every respect if they have two angles and one side respectively equal was referred by Eudemus to Thales with the comment that the latter's method of measuring²⁰ the distance of ships out at sea necessarily involves the use of this theorem.

From the above it can be seen that Eudemus credited Thales with full knowledge of the theory behind his discoveries. As we saw, Thales is held to have introduced geometry into Greece from Egypt. However, our surviving sources of information about the nature of Egyptian mathematics - the most important of which is the Rhind Papyrus (this papyrus which is now in the British Museum is named after its purchaser, A.H. Rhind, who bought it in Luxor; it was written after 1800 B.C. but, as its writer assures us, it derives from a prototype written during the Middle Kingdom 2000-1800 B.C.) - give us no evidence to suggest that Egyptian geometry had advanced beyond certain "rule of thumb" techniques of practical mensuration. Nowhere do we find any attempt to discover why these techniques worked, nor anything resembling a general and theoretical mathematics. It seems most unlikely, then, pace Aristotle, that the Greeks derived their mathematics from the Egyptians. But could Thales nevertheless have been the founder of theoretical mathematics in Greece as Eudemus claims? Here again the answer must be negative. The first of the three discoveries attributed to him by the Peripatetic most probably represent "just the neatest abstract solution of particular problems associated with Thales".²¹ Sir Thomas Heath points out that the first of these propositions is not even proved in Euclid.²² As for the last of them, Thales, who has acquired quite a reputation for practical savoir-faire, could very easily have made use of a primitive angle-measurer and solved the problem in one of several ways without necessarily formulating an explicit theory about the principles involved.

Van der Waerden, on the other hand, believes that Thales did develop a logical structure for geometry and introduced into that study the idea of proof.²³ He also seeks to derive Greek mathematics from Babylon. This is a very doubtful standpoint. Babylonian mathematics with its sexagesimal place-value system had certainly developed beyond the primitive level reached by the Egyptians. Nevertheless, a feature common to them both is that neither made any attempt at proof. Our evidence suggests that the Greeks were influenced by Babylonian mathematics, but that this influence occurred at a date considerably later than the sixth century B.C. If the Greeks had derived their mathematics from Babylonian sources, one would have expected them to have adopted the much more highly developed place-value system. Moreover, the Greeks themselves, who are extremely, indeed over-generous in acknowledging their scientific debts to other peoples, give no hint of a Babylonian source for their mathematics.

It was the Greeks who developed an abstract and theoretical mathematics with its ideal of a rigorously deductive proof. Why should this have been the specific contribution of the Greeks? The answer lies in their outlook upon the world about them. Behind the multiplicity of the phenomena, as we have seen, the Greeks were convinced that there was to be found an underlying order. Their attempts to account for this order led them to put forward a unifying hypothesis from which they proceeded to deduce their explanations of the particular phenomena. It was this outlook²⁴ which gave rise to theoretical mathematics when, in Southern Italy, there was put forward, as such a unifying hypothesis, the theory that "all things are number". Upon this very simple foundation arose the towering edifice of theoretical mathematics and here can be seen the enormous debt of mathematics to philosophy.

In view of the practice of later Pythagoreans of referring to Pythagoras himself their own particular discoveries, both out of respect and in order to invest them with the authority of his illustrious name, we must obviously exercise great caution in attributing any theory to the Master himself. Nevertheless, there seems to be no reason to doubt the tradition that Pythagoras himself made the discovery that the chief musical sounds were expressible by simple numerical ratios. If musical sounds could be reduced to numbers, then why not everything? Accordingly, in the manner familiar among the Presocratics, the theory that "all things are number" was put forward to account for the underlying order in the world.

It was this effort, then, to find in number the unity behind the diversity of the phenomena which gave birth to theoretical mathematics in the Greek world. Our tradition supports this interpretation. Proclus in his Commentary tells us,²⁵ on the authority of Eudemus, that Pythagoras "changed the study of geometry, giving it the form of a liberal discipline, seeking its first principles in ultimate ideas, and investigating its theorems abstractly and in a purely intellectual way". Even though Eudemus believes that theoretical mathematics began in Greece with Thales, he is nevertheless aware of the contribution made by the Pythagoreans in freeing mathematics from practical applications. Furthermore we learn from Stobaeus²⁶ that Aristoxenus in his treatise on arithmetic said that Pythagoras was the first to carry that study beyond the needs of commerce. Finally, and most important, Aristotle tells us in the Metaphysics:²⁷

"Contemporaneously with these philosophers (i.e. Leucippus and Democritus) and before them, the Pythagoreans, as they were called, devoted themselves to mathematics; they were the first to advance this study, and having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being - more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity - and similarly almost all other things being numerically expressible); since, again, they saw that the attributes and the ratios of the musical scales were expressible in numbers; since, then, all other things seemed in their whole nature to be modelled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and number".

It seems safe to accept, then, that in the latter part of the fifth century Greek mathematics had attained considerable success, especially with the theory of numbers and geometry, which the Pythagoreans were credited with promoting.

It can readily be seen how this Pythagorean belief would lead to the development of arithmetic, i.e. the theory of number as distinct from logistics, the art of calculating and solving particular problems. One contribution of the Pythagoreans, for example, was their classification of numbers by exhibiting them as dots in the form of geometrical figures. Although we no longer speak of "triangular" or "oblong" numbers, we still to this day employ the term "square" number.

However, it was in geometry that the Greeks achieved their greatest success. It is true that the Pythagorean method of representing numbers by arrangements of dots does provide some link between arithmetic and geometry, but this in itself is insufficient to explain the strong geometrical turn taken by early Greek mathematics. Is it possible to account for this trend? We believe that it is and that we must turn again to philosophy for the answer. The geometrical nature of Greek mathematics, it appears, is a direct result of the discovery that no pairs of integers could express the ratio between the diagonal and side of a square: no matter what positive integer is assigned to the side of a square, it is impossible to represent the diagonal by a corresponding integer. No square number can be divided into two equal square numbers and the diagonal and side are therefore incommensurable.

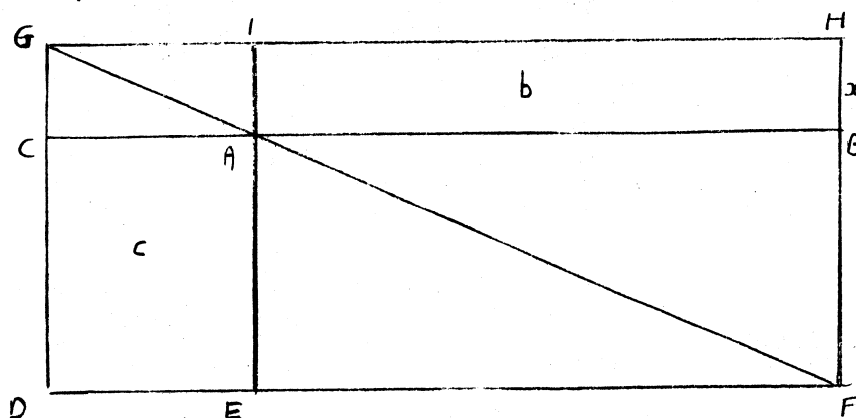
It is possible to gain some idea of the great consternation which attended this discovery. Our tradition records the dismay which greeted it. Iamblichus preserves the tale that the man who first revealed this incommensurability of the diagonal and side of a square was drowned at sea.²⁸ This story is no doubt apocryphal; but it nevertheless gives some idea of the shock caused by this discovery.

It is extremely important to realise that the discovery of incommensurability represented not only a crisis in mathematics but also in philosophy. A serious flaw had been discovered in the whole Pythagorean outlook upon the world about them. How could they express the order which they were convinced was inherent in the world by the claim that "all things are number", if it could actually be proved that the diagonal and side of a square are incommensurable?

The Pythagoreans could not bring themselves to abandon, even in the face of a proof to the contrary, an outlook invested with the authority of the Master himself. One of their reactions to this discovery was as ingenious as it was infertile. They devised a system for the formation of the succession of what they called "side" and diagonal numbers which gave closer and closer approximations to the value of $\sqrt{2}$ and thereby enabled them to retain their old theory of proportion. This system of numbers is described by Theon of Smyrna (pp.42.10-44.17 Hiller). The side and diagonal were described by a series of paired numbers such that the square on the diagonal was always equal to twice the square on the side plus or minus one, i.e. $d_n^2 = 2a_n^2 \pm 1$. Plato seems to be referring to some such system of approximation in the *Republic* (546c), where he contrasts the "rational diameter of five" with the "irrational" (diameter). If the square of side five is taken, the diagonal is equal $\sqrt{2 \cdot 5^2}$ or $\sqrt{50}$. This is the Pythagorean "irrational diameter" of five and the "rational diameter" is the approximation $\sqrt{50}-1$ i.e. 7.³⁰

Another and much more fertile reaction of the Pythagoreans was to restrict the scope of their theory of proportion. Although they found it impossible on their old theory of numbers to solve such equations as $x^2=2$; $x^2=3$; etc. they could quite easily construct geometrically lines describable as $\sqrt{2}$, $\sqrt{3}$ etc. For this reason they turned to geometry and by the use of geometrical methods they were able to perform the equivalents of algebraical operations. A simple example may serve to show how the Pythagoreans were able to solve geometrically problems which we would consider to belong to algebra. The algebraic expression $bx=c$, where b and c are known, was interpreted by the Pythagoreans as the need to construct a rectangle with given side b and area c and solved by their method of application of areas. Their procedure was as follows:

The line AB was drawn equivalent in length to b . This line was then produced to C and on the extension AC the rectangle ACDE was drawn of area c . DE was produced to F so that EF was equal in length to AB. FB was then joined. In the rectangle ABEF the diagonal AF was joined and produced through A to meet the corresponding extension of DC at G. GH was then drawn parallel to DF and of equal length. FB was produced to meet GH at H and correspondingly EA was produced to meet GH at I. In the resulting figure:



$CG=AI=BH$ =the length sought (x) and the rectangle $ABHI=bx$.

From the figure $\triangle FGD = \triangle FGH$ (two angles and corresponding side).

Similarly $\triangle CGA = \triangle AGI$ (" " " " ").

and $\triangle AEF = \triangle ABF$ (" " " " ").

But $\triangle FDG = \triangle CGA + \triangle AEF$ + the rectangle $ACDE$ (c).

and $\triangle FGH = \triangle AGI + \triangle ABF$ + the rectangle $ABHI$ (bx).

Therefore the rectangle $ACDE$ (c) = the rectangle $ABHI$ (bx)
and BH is the required straight line, x . 31

By this geometrical algebra the Pythagoreans were able to perform operations equivalent to such processes as addition, subtraction, multiplication, division, the extraction of the square root and even the solution of certain types of quadratic equations.

The conviction of the Greeks, then, that there was an order inherent in the world about them and the Pythagorean attempt to account for this order upon the basis of number led to the development of theoretical mathematics in the Presocratic period. Here is to be found the greatest and most fruitful influence of philosophy upon the development of science. The finest flowering of Greek mathematics occurred in the third century B.C. when we encounter such great names as Euclid, Archimedes and Apollonius. The seeds of this development, as we have sought to show, were sown in the Presocratic period.

Let us turn now to the example we have chosen of the empirical sciences, medicine. The influence of philosophy upon this study we described as two-fold in nature. Although at first beneficial, subsequently it was harmful and gravely hampered the development of this science.

Some records of early medicine in Babylon, Assyria and Egypt have survived. Although our evidence is in an incomplete and fragmentary state, it is still possible to gain some over-all impressions from it. We find no indication that anything of the real nature of disease was discovered by the ancient physician. Diseases were regarded as being marks of the displeasure of the gods or were held to be caused by the intrusion of a demon. The prime purpose of the physician was to appease the god or drive out the demon which had possessed the sick man's body. In order to do so he employed prayers, supplications, sacrifices, spells and incantations. The surviving Egyptian medical papyri consist, for the most part, of prescriptions of drugs and are interspersed with magical spells which were believed to impart efficacy to the prescriptions which follow. Many of the remedies prescribed contain noxious or offensive ingredients. The intention was, presumably, to make them as unpalatable as possible to the possessing spirit and so give it no inducement to linger in the patient's body.³²

Magic and superstition are also found in Greek medicine. The practice of "incubation" in the temples and belief in faith-healing affords a good illustration of this.³³ Both magic and superstition, however, are rare in the Hippocratic Corpus and the majority of the works contained in the Corpus are completely free from both mythological conjecture and magical intervention. The emancipation of medicine from superstition and its subsequent development as a science was the outcome of precisely the same attitude of mind which the Ionian philosophers were the first to apply to the world about them. Their attempt to explain the world in terms of its visible constituents, as we have seen, brought about the transition from mythological conjecture to rational explanation. The clearest evidence of this relationship between philosophy and medicine is the fact that the medical literature of the fifth and fourth centuries B.C. is in Ionic. Although the dialect of the island of Cos, the home of the Hippocratic School, was Doric its members wrote in Ionic.

Here there can be seen the first and most advantageous influence of philosophy upon medicine. In two treatises in the Corpus, Airs Waters Places and The Sacred Disease this rational attitude is especially marked. In these works it is emphasised that all diseases arise through natural causes. Both of them lay particular stress upon the moistening of the brain as a cause of disease. The former work attempts to expound the effects of climatic and topographical considerations upon health. In the latter treatise the uniformity of nature is stressed in the sense that no disease can be any more "divine" than any other. It is argued that all diseases arise through natural causes and that men think some are of divine origin only because of their inexperience and their wonder at the peculiar symptoms which characterise them. We quote the following passages from this work

to illustrate the rational outlook derived by medicine from philosophy.

"But this disease (epilepsy) is in my opinion no more divine than any other; it has the same nature as other diseases, and the (same) cause that gives rise to individual diseases. It is also curable, no less than other diseases, unless by long lapse of time it be so ingrained as to be more powerful than the remedies that are applied. Its origin, like that of other diseases, lies in heredity..... Another strong proof that this disease is no more "divine" than any other is that it affects the naturally phlegmatic, but does not attack the bilious. Yet, if it were more divine than others, this disease ought to have attacked all equally, without making any difference between bilious and phlegmatic."

(Chapter 5).

"This disease styled sacred comes from the same causes as others, from the things that come to and go from the body, from cold, sun, and from the changing restlessness of the wind. These things are divine. So that there is no need to put the disease in a special class and to consider it more divine than the others; they are all divine and all human. Each has a nature and power of its own; none is hopeless or incapable of treatment."

(Chapter 21).³⁴

In spite of this first beneficial influence of philosophy upon medicine, its later influence was far from advantageous and the subordination of the latter study to philosophy gravely hampered its development. In the middle of the fifth century, as we observed earlier, the philosophers sought to extend their views about the world at large to man himself and, as a corollary to this, began to base their medical theory upon their philosophical postulate. (The idea that man and the outside world are made of similar materials and behave according to similar rules, already to some extent implicit in Anaximenes' thought, can be clearly discerned in Heraclitus, who believes that man's very life is bound up with his surroundings. But it was not until the middle of the fifth century that the implications of this idea were fully drawn out.)

Empedocles affords the best illustration of the manner in which the attempt was made to base medicine upon such a postulate. Our evidence clearly reveals how he extended his views from the world at large to man himself. We find that the same four components, earth, air, fire and water, which he believed to make up the world, also give rise by their mixture to man's flesh, blood, bones etc.

Another philosopher whose theories exercised a strong adverse influence upon medicine is Diogenes of Apollonia. Theophrastus has preserved for us a short account of his theory of health. Brief though our information is, it is nevertheless apparent that he, too, based his theory upon his philosophical postulate. According to Diogenes, health was the result when a large amount of air in a normal condition mingled with the blood and lightened it, penetrating throughout the whole of the body. Whenever the condition of the air was not normal and failed to mix with the blood, the latter coagulated, became weaker and denser and sickness ensued.

The attempt to apply philosophical postulates to medicine in this manner is vigorously attacked by the author of the Hippocratic treatise Ancient Medicine. In this remarkable little work we find for the first time some recognition of the distinction between science and philosophy. The author is clearly conscious of the opposition between the dogmatic method of the natural philosopher and the more empirical method of the physician. This is especially apparent in the first two chapters of the work, where he carefully points out that

"(medicine) has no need of a "new-fangled" postulate, as do insoluble mysteries, which necessarily require the use of a postulate, if an attempt be made to discuss them, for instance the mysteries of heaven and of the regions below. If anyone were to express his opinion about the condition of these, it would not be plain either to the speaker himself or to the audience whether the statements were true or not. For there is no test the application of which would bring certain knowledge. But medicine has long had everything to hand, with a principle and method already discovered, by which many excellent discoveries have been made over a long period; while what remains will be discovered, if the enquirer be competent and familiar with discoveries already made, conducting his researches with these as his starting point. But whoever

spurns and rejects all these, attempting to conduct research after a different method and fashion, and then declares that he has made some discovery, deceives and is deceived. He attempts the impossible."³⁵

Again, in Chapter 12, we find him declaring proudly that medicine has been able to rise by reasoning from deep ignorance to approximate exactness and therefore its discoveries should be admired as being the result of excellent and correct research, not of chance.

But notwithstanding this vigorous polemic against the attempt to base medical theory upon a philosophical postulate philosophy continued to exert its baneful influence upon medicine. This adverse influence of philosophy is clearly revealed in many of the works in the Hippocratic Corpus. In The Nature of Man, for example, Empedocles' influence is apparent. In this treatise the number of the humours which were believed to be active in the human body is limited to four, viz. blood, phlegm, yellow bile and black bile. This view is the counterpart of the four element theory and it seems to have been the influence of the latter which led to the number of humours being limited in this manner. Like the elements themselves these four humours are characterised by the qualities hot, cold, moist and dry.

Two other treatises which reveal strong philosophical influence have already been mentioned by us earlier, viz. Breaths and The Sacred Disease. Both these works are much influenced by the theories of Diogenes of Apollonia. The former work is, perhaps, more of a sophistic essay than a serious medical treatise. But it apparently became known as a Hippocratic work at quite an early date, for it is referred to in Meno's Iatrica (Chapters V & VI) and is in the list of Erotian. Its author is even more guilty of making dogmatic assertions and failing to verify or reject them by experiment than the philosophers themselves. Like Diogenes, he believes that air, which is of fundamental importance in the world generally, is instrumental in causing disease. In addition to this major influence, there are one or two minor reminiscences of Diogenes in the work. In Chapter III, for example, the theory is put forward that the sea has air in it which is breathed by fish and other aquatic creatures. This theory is apparently derived from the philosopher (cf. Aristotle de Respiratione 2,471a3 DK64A31). At the very end of the same chapter we find the poetic description of air as the *γῆς ὀξυγμία*. This expression is also found in Euripides' Troads 884. If the author of Breaths is not simply imitating Euripides, they could both be copying some famous utterance of Diogenes. We know that the latter's thought was well known in Athens at this period for his views are parodied by Aristophanes in the Clouds (225ff.).

The Sacred Disease is a work written expressly to explain on rational grounds the nature and causes of epilepsy and at the same time to combat the superstitious beliefs held about this disease. The author is strongly influenced by philosophy and attempts to apply a philosophical hypothesis to medicine in a manner which would have brought down upon his head the wrath of the writer of Ancient Medicine. Like many other medical writers of this period the author of The Sacred Disease is an eclectic and the psychological theory contained in his work reflects in some respects the influence of Alcmaeon of Croton as well as that of Diogenes. The latter, however, is undeniably eclectic himself, and the influence of the former on the medical work may have been at second hand. Alcmaeon's physiological researches had led him to the belief that the brain was the essential organ of intelligence and perception. This belief Diogenes may have derived from Alcmaeon since Theophrastus informs us (de Sens. 39ff.) that the former held that the sense-organs were connected to the brain. Diogenes had revived the monistic theory of Anaximenes and adopted air as his first principle. He conceived of it as psychical in nature so that for him it was not only the primary substance but the principle of intelligence as well. (The immediate influence of Anaxagoras may be found here.) Upon the basis of these two theories, that the brain is the seat of the intelligence and that air is the source and principle of intelligence in the organism, is elaborated a comprehensive explanation of disease. Epilepsy, the most important of these for the medical author's present purpose, is caused by the stoppage of air in the veins by a flow of phlegm from the brain.

Later in the work (Chapter 16 Littré) the doctrine is advanced that moisture is harmful to thought. This theory, too, seems to have been derived from Diogenes, who appears himself to be indebted to Heraclitus for it.³⁶ Theophrastus

tells us (*de Sens.* 44) that Diogenes believed that moisture (*ἰκμῶς*) hampers the intellect. This word appears to have been a technical term used by Diogenes himself. The word is rare in Attic and occurs a further three times in the tradition relating to Diogenes; viz. in Alexander (*Quaest. Nat.* II, 23 DK64A18), the scholium on Apollonius Rhodius IV, 269 (DK64A18) and in Aristophanes' parody of Diogenes' theories at *Clouds* 225 (DK64CI).

The influence of Heraclitus is also marked in the *Corpus*. In the treatise *Nutrimet*, a later Heraclitean, skilfully copying his master's aphoristic style, applies a theory of perpetual change to the assimilation of food by a living organism. *Regimen* I is also written in the Heraclitean style and reminiscences of Anaxagoras and possibly Empedocles and Archelaus can be found in the work.³⁷ Finally the influence of Pythagorean numerology may be traced in the work *Sevens* (cf. especially Chapter 5) and perhaps in the importance assigned to "critical days" generally, since Celsus actually refers to them as "Pythagorici numeri".³⁸

Although, then, the influence of philosophy proved, for the most part, extremely beneficial so far as the exact sciences were concerned and impressive heights were reached within the short space of some three centuries, its initial effect upon the natural sciences was gravely to hamper their development. The continued subordination of the natural sciences to philosophy led to the application of general unifying hypotheses in this sphere also and the attempt to deduce explanations of the particular phenomena from them was far from conducive to the furtherance of empirical science. We may conclude then with the paradox: that although philosophy performed the historical task of bringing a scientific attitude into the study of nature, at the same time it was one of the factors that hindered the most advantageous development of this study.

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Notes:

1. Euripides, we may note, speaks of the philosopher who studied "the ageless *νόστος* of undying *φύσις*", whence it was composed and in what way" (Frag. 910 DK59A30).
2. There has been considerable controversy over the relationship between Empedocles' two poems, *Purifications* and *On Nature*. While some scholars have claimed that the latter held simultaneously beliefs which were quite incompatible, others have argued that these works belong to different stages of the poet's life (it is a topic of further controversy which work was the first to be written). Cornford, however, has convincingly argued that the two works are not incompatible at all but that Empedocles has attempted to reconcile Pythagorean views on the nature and destiny of the soul with Ionian physical science (cf. *From Religion to Philosophy* (London, 1912, p. 224)).
3. Parmenides in Fragment B12 speaks of the "daimon" (Aphrodite, presumably) who "directs the course of all things". But even if he had in mind a personal god which is extremely doubtful, it must be remembered that he is here recording the *θεοῦ δόξας* where *θεὸς ἐν πλείστοις ἀνθρώποις*. Empedocles in his *Purifications* (Fragment B112) describes himself modestly as an "immortal god, Honoured among all as is fitting". In the poem *On Nature* (Fragment 23.11) he exhorts his hearer to accept his account because he hears it from a god. Unfortunately the meaning of this line is unclear and there may be a reference to the Muse mentioned earlier in Fragments 3.3 and 4.2 rather than to Empedocles himself. In Fragment 111 he lists the powers which he claims that he will impart to his hearer, viz. the power to repel and still the winds, to cause drought after rain and rain after drought, and to bring back the dead from Hades. Although Empedocles makes extravagant claims to be able to influence the natural course of events, his account of the origin and continued working of the world, though based upon the authority of a god, makes no appeal to divine intervention. Like the Ionians he seeks to explain the world about him in terms of purely natural causes. By appeal to divine authority, he gives an account of the origin of the world which cuts out divine intervention.

4. A8, 989b29 (DK58B22).
5. "Ifs and Cans" Proceedings of the British Academy Vol.XLII (1956) p.131.
6. It is, perhaps, worth observing here that even Aristotle himself, whose thought clearly reveals his recognition of the value of carefully collected observational data, does not measure up to the ideal of modern empirical science in that he failed to recognise the need to corroborate his hypotheses by constantly testing the conclusions which can be drawn from them under circumstances intentionally called into being for the purpose.
7. Aëtius V,19,5 (DK31A72). Aristotle adopts this theory cf. de Resp.477a30.
8. Aristotle, de Anima B4.415b28 (DK31A70).
9. Plutarch, Quaest.Conv. I,2,5 p.618B (DK31B76).
10. Cf.my earlier article "Empedocles's Fiery Fish" Journal of the Warburg and Courtauld Institutes Vol. XXVIII (1965) p.314.
11. Cf. Theophrastus' comment that Diogenes, in attempting to deduce all his explanations from his hypothesis "strays in many places from good sense" de Sens 48 (DK64A19).
12. Ibid. 44.
13. Chapter I (W.H.S.Jones' translation with slight modification in "Philosophy and Medicine in Ancient Greece" Bulletin of the History of Medicine, Suppl.8 1946.
14. Metaphysics 981b20-25.
15. Book II,109.
16. Proclus Commentary on Euclid's Elements I p.64.16 (Friedlein).
17. Ibid.,p.157,10 (DK11A20).
18. Ibid.,p.250.20.
19. Ibid.,p.299.1.
20. Ibid.,p.352.14.
21. G.S.Kirk in Kirk & Raven The Presocratic Philosophers (Cambridge,1957) p.84.
22. Greek Mathematics (Oxford,1921) Vol.I p.131.
23. Science Awakening (trans.by A.Dresden Gröningen,1954) p.89.
24. Aëtius tells us that Pythagoras was the first to apply the term κόσμος to the world because of the order in it (II.1.1 DK.14.21 and cf.D.L.VIII,48 DK28A44). In spite of Kirk's arguments to the contrary (cf.Heraclitus pp.312-314 and Kirk & Raven p.159), it may well be correct that Pythagoras or, at any rate an early Pythagorean used the word in this sense (cf.E.R.Dodd's discussion in Plato Gorgias A Revised Text with Introduction and Commentary (Oxford,1959)Note on 508a3 p.338). Kirk himself in his later consideration admits that the word has the sense of "world" in Empedocles Fragment 134.5 and it should be noted that in Parmenides Fragment 4.3 κόσμος must surely have the meaning of "world". The text can hardly mean "scattered in order" as Kirk maintains (Heraclitus, p.313). C.H.Kahn, strangely neglecting Aëtius' evidence altogether, has recently argued for a Milesian origin of the term (Anaximander and the Origins of Greek Cosmology (New York,1960) Appendix 1,p.219).
25. In Euclid. 65,11F. DK14A6.
26. I.1 pr.6 (p.20,1W.) DK58B2.
27. 985b23 DK58B4. (Ross' Translation).

28. V.P.246-7, p.132 Deub.Cf., too, Schol. in Eucl. Elem. X p.417 Heib.
29. We know from a passage in Aristotle what the traditional proof was. In the Prior Analytics (I,23 41a23-27) he declares that the diagonal and side of a square are incommensurable because otherwise an even number would have to be equal to an odd number. This argument is decidedly Pythagorean in character and the proof found in Euclid's Elements X, Appendix xxvii (Heiberg) turns on this very point and may also be Pythagorean. This proof may be paraphrased briefly as follows:
- Let ABCD be a square with diagonal AC and side AB
 If AC is commensurable with AB, the ratio AC:AB may be represented by $m:n$, where m and n are integers prime to each other and $m > n$ since $AC > AB$.
 Then $AC:AB = m:n$
 and $AC^2:AB^2 = m^2:n^2$.
 But $AC^2 = 2AB^2$.
 Therefore $m^2 = 2n^2$, whence m is even and n is odd.
 Since m is even, let $m = 2p$
 $(2p)^2 = 2n^2$
 $4p^2 = 2n^2$
 $2p^2 = n^2$
 Therefore n is even.
 Since the assumption that AC is commensurable with AB leads to the impossible conclusion that the same number (n) is both odd and even, the assumption must be false.
 Therefore AC is not commensurable with AB.
30. Cf. Proclus' Commentary on Plato's Republic II,27,11f. (Kroll): "The Pythagoreans set forth this elegant theorem about the diameters and sides, that the diameter when added to the side of which it is the diameter becomes a side, and the side when doubled and added to its own diameter becomes a diameter."
31. We have no less an authority than Eudemos that this theory of application of areas is Pythagorean in origin cf. Proclus' Commentary on Euclid's Elements I pp.419.15ff.: "These things, says Eudemos, are ancient, being discoveries of the Muse of the Pythagoreans, I mean the application of areas, their exceeding and their falling short...."
32. A good example of this is to be found in the Hearst Papyrus (85): "O ghost, male or female, who dwellest in this my flesh, in these my limbs. Lo I brought thee excrements to devour! Beware, hidden one, be on your guard, concealed one, escape!"
33. Cf. Aristophanes' Plutus (665ff.) where he burlesques these temple rites and gives a farcical account of the proceedings in the temple of Asclepius.
34. W.H.S.Jones' translation.
35. W.H.S.Jones' translation (with slight modification).
36. Cf. Stobaeus Flor. V.8 (DK22B118) and Ibid. V.7 (DK22A117).
37. Chapter 4ff.
38. On Medicine Book III,4,15.