

Piano Pages - harmonic theory by Brian Capleton

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Music, mathematics and philosophy

Some history

Music is not considered one of the sciences today, but from the Middle Ages the study of music as a science (even if called a 'Liberal Art') was integral to the learned man's understanding of the world. Boethius helped establish it as one of the four disciplines of the *quadrivium*, in which music was studied together with arithmetic, geometry and astronomy. This was not, however, the kind of academic subject 'music' is today, but rather, was very much concerned with 'harmonics' - the study of the mathematical roots of harmony - in the context of Ptolemaic astronomy, which was itself a part of the quantitative *harmony of the spheres* 'tradition'. The universe (the motions of the planets and stars) was considered to be built on 'musical' harmonic principles - the same principles of harmony found in practical music.

The origin of this great 'tradition' is attributed to Pythagoras (c. 582 - 497 BC). One of its most important proponents was Plato, who was revered as a source of ancient wisdom, and whose *Timeaus*, which contains enigmatic references to the Pythagorean ideas, was known and studied before the renaissance.

By the 17th century and the rise of the 'scientific age' music was still inseparable from science, and even in the middle of the 18th century William Tans'ur described a 'Master of Musick' in *A New Musical Grammar*, 1746, as someone who

'must not only be a grammarian, but also a Master of Letters and Languages, in order to unfold what is locked up in the Closets of the Learned - He must be an Arithmetitian (sic) and able to explain Numbers, and even the Misteries (sic) of Algebra; and also a Geometrician, to evince a great variety, the Original of Intervals, Consonant and Dissonant...'²

Mathematics and music - the esoteric connection

For over two thousand years then, something called 'music' was studied in what could be called a 'scientific' way. There is however something missing from this historical picture of musical science: *context*. What would be the point of all this 'learned' scientific understanding of 'music' in relation to music as it is experienced? What, if anything, has it got to do with the *experience* of music?

1873
CAPLETON, B.
2003

The idea of the 'harmony of the spheres' (*harmonia mundi*), or 'music of the spheres' (*musica mundana*), was largely received as the science of 'harmonics' - the study of the relationships between whole number 'harmonic ratios', musical intervals, and the orbital speeds and distances of the planets. The authority for this 'science' was referred back through its major proponents like Boethius or Ptolemy, to the 'ancient wisdom' of Plato or to its supposed originator, Pythagoras. This referral of authority continued at least until the eighteenth century, despite the fact that this 'science' always had its opponents - probably the most notable being Plato's pupil Aristotle. The question is (and this would apply to any of the opponents), did Aristotle have any real appreciation of what it was he set out to discredit? As far as Aristotle was concerned, and as far as concerns most of the other commentators on the 'tradition', whether they are its opponents or propagators, the 'tradition' of the *harmonia of the spheres* was an assertion about planetary orbital speeds and distances, and an assertion that they produced sound from their motion. In fact, there is no extant writing by any Pythagorean that explicitly states this. There are only attestations and insinuations. The problem here, is that the situation is rather like early attestors of, say, Christianity, stating "He (Christ) said the Kingdom of Heaven is within you", and then later attestors asserting that Christianity holds Heaven to be the inside of the human body.

From the point of view of the history of science, Pythagoras is of course considered an 'early scientist', responsible for the discovery of the harmonic ratios associated with the octave, 'perfect' fifth, and 'perfect' fourth. He is well known as the discoverer of the *Pythagoras' Theorem*, that in any right-angled triangle 'the square on the hypotenuse is equal to the sum of the squares on the other two sides'. But like other 'scientists' in the past, including Kepler (who deeply respected Plato and the notion of the Platonic Solids) and Newton (who was immersed in alchemy, and ultimately concluded that the explanation of gravity was that it was the manifestation of God's Will), Pythagoras was apparently deeply involved in thought that would never be considered 'scientific'. The reasons the modern world considers him to be 'important' now, as a figure in the history of science, may be merely peripheral to what was important to Pythagoras himself.

Pythagoras was supposed to have attributed great importance to Number, which apparently led to so-called 'important' mathematical discoveries on the one hand, and the so-called 'nonsense' of numerology on the other. The Pythagoreans have also been ridiculed because they are supposed to have been forbidden to eat beans. But we only 'know' any of this as a result of attestations, because nothing written by Pythagoras exists, and the Pythagorean 'sect' was in any case steeped in secrecy. The attestations also tell us something else. Whatever Pythagoras actually taught, he is supposed to have 'healed' his disciples through the *practical* use of music - by playing music to them. This seems not to have much to do with the assumed 'science' of Number, and is certainly not such a ridiculous assertion. The psychological or even physical 'healing' power of music has long been claimed, and is even to some degree acknowledged now in the modern field, in music therapy. If there is a connection between the *experience* of music (and its effects on the auditor) and 'mathematics'

or Number, then Aristotle is not the place to look for clues. Clues to this connection are to be found in the ideas surrounding Pythagoras, and the prime source for this is Plato.

The difficulty here is that contemporary philosophy is poised 'at the ready' to dictate how Plato should be read, and what he meant. Contemporary philosophy, 'analytical' or not, is primarily analytical in its approach, using its established, pre-existing *modus operandi* of questioning and arguing from within the thinking box of the human mind, in the condition it already happens to find itself. It cannot change its state of mind, because there is no way of 'working out' how to do this from within the box. Even if it did, it would be considered to have left the academic discipline of 'philosophy', and moved into say, Divinity, Religion, or Mysticism.

And yet even the very word 'philosophy' has its original meaning as 'lover of wisdom', which can hardly be divorced from Divinity, Religion, or Mysticism. Its original and proper meaning connects with love, wisdom, and the love of wisdom. Plato's reverence of Socrates is obvious in his writings, and Socrates was declared to be the wisest man in Greece - whilst Socrates himself said he 'knew nothing'. The only way to read Plato is to know nothing to begin with, which means to forget everything his thinking, analysing interpreters have said about what he means, and to realise that in an environment where rational discursive argument is the norm, what appears as rational discursive argument may be *used* to teach something that transcends it.

Mathematics and music - the exoteric connection

In its physical, acoustical manifestation, music is indeed imbued with 'mathematics'. Really, this is true of all physical phenomena. Consistency can only be possible in all physical phenomena, because there is always a 'level' at which the *relationships* between the objects of physical phenomena, are *consistent*. What is possible, and what is impossible in these relationships, is determined, it has been argued, by abstract mathematical 'logic'. Some of this 'logic' appears in music in a most direct way, however, especially if we are using strings or pipes. The primary intervals in Western music are already 'contained' within the tone of a single string or 'pipe' (any wind instrument), in the 'Chord of Nature' or 'harmonic series'.

Intrinsically connected with these intervals are the harmonic ratios:

Unison.....	1:1	cents
Semitone.....	16:15	111.731
Minor tone.....	10:9	182.404
Major tone.....	9:8	203.910
Minor third.....	6:5	315.641
Major third.....	5:4	386.314
Perfect fourth.....	4:3	498.045
Augmented fourth.....	45:32	590.224
Diminished fifth.....	64:45	609.777
Perfect fifth.....	3:2	701.955
Minor sixth.....	8:5	813.687
Major sixth.....	5:3	884.359
Harmonic minor seventh.....	7:4	968.826
Grave minor seventh.....	16:9	996.091
Minor seventh.....	9:5	1,017.597
Major seventh.....	15:8	1,088.269
Octave.....	2:1	1,200.000

Some of the mathematical relationships between these numbers exhibit interesting, if not remarkable properties, and the study of these relationships is what the old science of *harmonics* was all about. It is still the basis of the Theory of Temperament.

This is the beginning (but certainly not the end) of the existential connection of mathematics and music.