



Getting in Touch with Numbers: Intuition and Mathematical Platonism

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Mathematics is about numbers, sets, functions, etc. and, according to one prominent view, these are abstract entities lacking causal powers and spatio-temporal location. If this is so, then it is a puzzle how we come to have knowledge of such remote entities. One suggestion is intuition. But 'intuition' covers a range of notions. This paper identifies and examines those varieties of intuition which are most likely to play a role in the acquisition of our mathematical knowledge, and argues that none of them, singly or in combination, can plausibly account for knowledge of abstract entities.

I. Mathematical platonism

Mathematics is apparently about numbers, sets, functions, and the like. According to the doctrine which has become known as mathematical platonism, such mathematical entities actually exist, but exist as abstract entities. Platonists claim that these entities lack causal powers and spatio-temporal location. They also claim that they exist objectively and independently of our thoughts, concepts, and language. They are realists about entities which exist in a *platonic* realm outside of space and time, and the causal nexus. If this is so, then it is a puzzle how we come to have knowledge of such remote entities. It is particularly puzzling for those of us who believe that human beings are physical beings wholly located in space and time. But the question remains even for those who disagree with this physicalistic view of humankind. If the ontology of pure mathematics consists of such entities, and we do have mathematical knowledge, then what is the process by which we acquire such knowledge?

We acquire our knowledge of the existence and nature of concrete objects by causally interacting with them, however indirectly. If abstract objects lack causal powers, then it would seem that we cannot come to know of their existence and nature by a similar process.¹ Some platonists argue that they need

¹ The *locus classicus* for this puzzle is Paul Benacerraf, "Mathematical Truth," reprinted in P. Benacerraf and H. Putnam (eds.), *Philosophy of Mathematics*, 2nd edition (Cambridge: Cambridge University Press, 1983), pp. 403–20.

8

not concern themselves with epistemological problems. There are, according to them, good reasons for believing that mathematics is about platonic objects and there are good reasons for believing that we have mathematical knowledge, therefore, any lack of an account of how we come to have that knowledge is not a threat to platonism. In other words, they appeal to an argument like the following:

1. Mathematics is about platonic objects
2. We have mathematical knowledge
- ∴ 3. We have knowledge about platonic objects.

Call that Argument A.

The burden of proof, such platonists claim, is on anti-platonists to prove that we cannot have such knowledge. I do not agree with this view. I believe that on this issue the burden of proof rests on the platonists, but I shall not argue for that here. Argument A does not address the issue of providing an account of the process by which we acquire knowledge of platonic entities. My concern is with those platonists who do offer such an account, in particular, with those who claim that we acquire such knowledge by a process or faculty of intuition.

Some platonists appear to argue, as a corollary to Argument A, that since we do not acquire mathematical knowledge by causal interaction via sense perception, we must acquire it by intuition. They argue thus:

4. We do not acquire mathematical knowledge via sense perception
- ∴ 5. We acquire knowledge about platonic objects by intuition.

This argument begs a number of questions and clearly avoids the issue I wish to address. I seek an account of how a process of intuition yields knowledge of platonic objects.

There are two other prominent accounts advanced by contemporary platonists. One is the claim, advanced by Quine and Putnam, that we know of platonic objects by a process of postulation and scientific confirmation.² The other, advanced by Crispin Wright, is that platonic knowledge is conceptual knowledge.³ I shall not be concerned with these accounts, except where it is necessary to distinguish an intuitionist account from one or other of them.

Some philosophers claim that abstract entities do, in some sense, have causal powers. Some of these philosophers call themselves platonists. I think

this is inappropriate. Be that as it may, in this paper, by 'platonic objects' I shall mean acausal objects which, if they exist, exist objectively and mind-independently, and by 'platonic knowledge' I shall mean knowledge of the existence and nature of platonic objects.

II. Varieties of intuition

Intuition has been suggested as the means by which we gain platonic knowledge. An immediate problem is that the term 'intuition' is employed with a wide range of meanings.⁴ If these uses of the term have anything in common, it is that intuition is related to the acquisition of belief by a process which is apparently immediate and non-inferential. I shall identify those uses which are most often associated with the purported acquisition of platonic knowledge and show that none of them refers to a process which both exists and could perform as claimed. It is important to examine each notion separately. There is the danger that evidence for one intuitive process may be adduced as evidence for another, somewhat different, process. One process may genuinely yield some sort of knowledge, but be incapable of yielding *platonic* knowledge. The other, *if it existed*, might yield platonic knowledge, but any evidence for the former would, of course, be irrelevant to the existence of the latter.

Perhaps the commonest non-technical notion of intuition is that of intuition as apparently unjustified belief which seems immediate (not a result of inference) yet is accompanied by a feeling of conviction. We might call this *intuition-as-hunch*. A detective may claim that she has solved a crime by intuition. Having considered a bewildering array of clues and suspects, she has an inexplicable hunch that the butler did it. Following up on the hunch, she eventually pieces together a convincing case against the guilty party. Similarly, mathematicians often claim that they discover mathematical truths by intuition, discoveries which are later confirmed by formal proof. We can have such hunches about anything and many of our hunches (perhaps a surprising number) turn out to be true. So this notion does not apply to any particular kind of belief-content and the existence of such intuitions cannot be denied.

Now it may be that no special faculty is at work here, at least, not one which yields knowledge. Successful hunches may be no more than lucky guesses. Perhaps detectives, mathematicians, and the rest of us tend to re-

² W. V. O. Quine, *From a Logical Point of View*, 2nd edition (New York: Harper and Row, 1961), ch. 1, *The Ways of Paradox* (New York: Random House, 1966), ch. 20, and elsewhere. Hilary Putnam, "Philosophy of Logic," in his *Mathematics, Matter and Method*, 2nd edition (Cambridge: Cambridge University Press, 1979).

³ Crispin Wright, *Frege's Conception of Numbers as Objects* (Aberdeen: Aberdeen University Press, 1983).

⁴ A useful historical survey of the varying use of the term in philosophy, theology, and psychology is given in N. Noddings and P. Shore, *Awakening the Inner Eye: Intuition in Education* (New York: Teachers College Press, 1984), ch. 1 and 2, although they do not make much progress in disentangling the various concepts. T. Bastick identifies twenty properties associated with the general use of the term in his *Intuition: How We Think and Act* (Chichester: J. Wiley and Sons, 1982).

member the successful hunches and to forget those which lead nowhere.³ If so, successful hunches give no support to the claim that intuition is a source of platonic knowledge. Suppose, on the other hand, that there is such a faculty, and that it is a reliable source of true beliefs. Such a faculty could involve unconscious (and very rapid) inference. Or it could involve a direct apprehension of the state of affairs which comes to be known. So, the notion of intuition-as-hunch suggests two possible processes for knowledge by intuition.

A somewhat different process of intuition as direct apprehension is invoked by the claim that we have a special faculty for intuiting platonic entities or platonic states of affairs, rather than any state of affairs. Again, a more technical notion of intuition is invoked by the claim that intuition is part of the process of ordinary sensory perception. Finally, there is the claim that the truth of certain propositions may be intuited when we contemplate them.

We have five different processes which involve some notion of intuition and which are relevant to the claim that we can obtain platonic knowledge by intuition:

- (1) intuition as unconscious inference (inferential intuition),
- (2) intuition as direct apprehension of any state of affairs (ESP intuition),
- (3) intuition as part of the process of ordinary sensory perception (perceptual intuition),
- (4) intuition of the truth of certain propositions (cognitive intuition),
- (5) intuition as direct apprehension of platonic entities or platonic states of affairs (direct platonic intuition).

I claim that these five processes exhaust the possible ways for intuition to yield propositional knowledge of an external reality, and hence to yield platonic knowledge. I now examine each in turn.

III. Inferential intuition

When mathematicians claim that they discover mathematical truths by intuition, they are usually talking about a sudden realization accompanied by a firm conviction that a certain mathematical proposition is true. This mode of discovery contrasts with processes whereby mathematicians come to believe a mathematical proposition as the result of following a piece of reasoning or a

proof (be it formal or informal). Discovery by conscious inference involves becoming convinced as the result of understanding why something is the case. Intuition, on the other hand, seems to involve conviction without such an understanding.

Suppose that this sort of intuition is no more than unconscious (and very rapid) inference. If so, then it must be the same process which allows me, when reading a detective novel, to have an intuition that the butler did it. The point is that *intuition-as-unconscious-inference*, if it works, works just as well in fictional and hypothetical realms as in any other. So we can agree that knowledge can be obtained by this process, without agreeing that it can yield existential knowledge. Such intuition allows us to make connections which can, in principle, be made by conscious inference. It is simply achieving by unconscious inference what can also be achieved by conscious inference. If the claim is that intuitive knowledge is no more than a special kind of inferential knowledge, then we need an account of how inference can yield platonic knowledge. It is inference, not intuition, which would be doing the important work in such an account. The question of whether inference alone can provide such knowledge is outside the scope of this paper.⁴

IV. ESP intuition

Suppose that knowledge by intuition involves a direct apprehension of the state of affairs involved. This assumes some sort of clairvoyant or telepathic faculty. A platonist could argue as follows. There is evidence that we can directly intuit states of affairs by some sort of extra-sensory perception. For example, psychics foresee the future, perceive the location of distant objects, and read minds. There is no evident causal link with the states of affairs in such cases. Therefore, it must be a non-causal process. If we can make contact with concrete states of affairs by a non-causal process, then it should be possible for us to make contact with states of affairs which are causally inert by means of the same, or a similar, process.

There is little or no good evidence for such a process or faculty. An epistemology which relies on the results of paranormal research is on very shaky ground.⁷ Besides, any evidence we have for the existence of telepathy or clairvoyance should most plausibly be regarded as evidence for a causal process the details of which are as yet undiscovered. How else are we to explain the matching of concrete (non-platonic) states of affairs with true beliefs

⁶ The Quine/Putnam account may be seen as an account of how platonic knowledge is acquired by inference.

⁷ An extensive, critical examination of such results is given in Paul Kurtz (ed.), *The Skeptic's Handbook of Parapsychology* (Buffalo: Prometheus Books, 1985). Andrew M. Colman offers a brief but devastating refutation of the "best" evidence for the existence of telepathic powers, in his *Facts, Fallacies, and Frauds in Psychology* (London: Unwin Hyman, 1987), ch. 7.

⁵ "And such is the way of all superstition, whether in astrology, dreams, omens, divine judgements, or the like; wherein men, having delight in such vanities, mark the events where they are fulfilled, but where they fail, though this happen oftener, neglect and pass them by." Francis Bacon, *Novum Organum* (1620), in F. H. Anderson (ed.), *The New Organum and Related Writings* (New York: Liberal Arts Press, 1960), p. 51.

about those states, unless we suppose that that it is the states of affairs which give rise to the beliefs? The evidence for ESP is meagre enough without its being expected to give support for a mysterious and unknown non-causal process. The claim that platonic knowledge can be acquired by a non-causal process is not supported by the fact that we have successful hunches about a wide range of non-platonic states of affairs.

V. Perceptual intuition

The third notion of intuition to be considered sees intuition as part of the ordinary process of sensory perception. One argument for the involvement in perception of a process worthy of the title *intuition* goes as follows. When I see that there is a tree in front of me or see that the tree in front of me is an oak, then I acquire beliefs that are immediate upon seeing the tree and do not seem to arise from any rational or inferential process. But seeing a tree is one thing, seeing that it is a tree or that it is an oak is another. The former is a case of perception-*of*, the latter are cases of perception-*that*. Perceptual intuition supposedly closes the gap between perception-*of* and perception-*that*.

Perceptual intuition, whatever it may be, seems to be a causal process. Light reflected from the tree enters the eye, causing activity in the visual cortex. That activity causes certain sensations, which in turn (no doubt mediated by other occurrent mental states) give rise, via the process of perceptual intuition, to perceptual belief. So, even if we accept that intuition plays a role in sensory perception, it is puzzling how this could result in knowledge of platonic objects. Light does not bounce off such objects.

It may be argued that the perceptual process sometimes, perhaps always, gives knowledge of entities which have no causal role in the process. In the example of the tree, the causal interaction is with the front surface of a time-slice of the tree not with the tree itself, yet I come to believe and know that there is a tree in front of me, not just a slice of a tree. Intuition is posited as the process which closes the gap between the spatio-temporal slice which is causally interacted with and the object which is known but not interacted with. But the known object is an object with causal powers. It has causal powers in virtue of the fact that its spatio-temporal parts have causal powers. A platonic object lacks parts with causal powers and therefore could not be known by such a process. Indeed, according to the usual conception of platonic objects, they have no spatio-temporal parts, since they exist outside of space and time. Even if it is accepted that we can have perceptual knowledge of an object without causally interacting with that object, but with only part of it, we still do not have an account of how we can have knowledge of platonic objects.

A somewhat different account of the involvement of intuition in sensory perception makes a more direct claim to knowledge of abstract objects. When

I see a shirt, I become acquainted not only with the shirt but with many of its properties. Suppose it is a red shirt. Then I am acquainted with its redness, and, perhaps, not only with its redness, but with the universal redness. Now universals are abstract entities which lack causal powers, so I do not see the redness of the shirt. Rather, the story goes, I see the red shirt but intuit its redness. Mathematical structuralists, who believe that mathematics is about structures or patterns rather than individual objects, often employ such an account.⁸ They claim that when we see some dots on a page, we may intuit a pattern which, unlike the dots, is an acausal entity existing outside space and time.

This is just mystery-mongering. There are three possibilities. Either we (unconsciously) infer from what we perceive that platonic entities exist, in which case we need an account of how such inferences are possible. I have already discussed and dismissed the possibility of such inferential intuition. Or we directly apprehend platonic entities. Such a process is either the ESP intuition I have discussed or direct platonic intuition to be discussed later. Or some other process is being invoked, about which we are told nothing. Unless we are told more, the question of how perceptual intuition yields knowledge of platonic objects remains unaddressed. Besides, we may doubt the existence of an epistemic gap between perceiving an object and acquiring knowledge of its properties, a gap which the mysterious intuitive process supposedly closes. Is my acquaintance with the redness of the shirt any more than my coming to know that the shirt is red? And can I not come to know that the shirt is red as the result of my causal interaction with the red shirt? On the other hand, if entities like properties and patterns must be invoked, why not suppose that they have causal powers? If redness is to be part of an account of my perception of a red shirt, then it seems plausible to say that it is the redness, shape, and other properties of the shirt which give rise to my perception of the shirt.

A recent intuitionist account of mathematical knowledge, more detailed than most, is that of Charles Parsons.⁹ The basis of his account is the claim that we can intuit a type when we perceive (or imagine) an object as a token of that type.¹⁰ In other words, it is a variation on perceptual intuition. As such, I claim that it depends either on a causal link between the type and the perceiver, or on a mysterious *ad hoc* process to bridge the epistemic gap. Parsons seeks to close the gap by labelling the abstract types as "quasi-concrete" objects.¹¹ This is reminiscent of, and no more convincing than, Descartes'

⁸ For example, Charles Parsons, "Mathematical Intuition," *Proceedings of the Aristotelian Society* 80 (1979-80): 145-68 and "The Structuralist View of Mathematical Objects," *Synthese* 84 (1990): 303-46.

⁹ Parsons (1979-80) and (1990).

¹⁰ Parsons (1979-80), pp. 153-55.

¹¹ Parsons (1990), p. 304.

story of the pineal gland which can be moved both by the immaterial soul and by the subtle animal spirits of the material body.¹² Either these quasi-concrete objects have causal powers and there is an epistemic gap between them and what Parsons calls "pure" abstract objects, or they have no causal powers and, therefore, cannot be given in ordinary sense perception.

VI. Cognitive intuition

The fourth notion of intuition to be examined is what I have called cognitive intuition. The idea is that we can intuit the truth of certain propositions when we contemplate them. I do not wish to suggest that active or conscious contemplation is a necessary condition for acquiring knowledge in this way, although it does play an important role in some accounts. What is important is that it is only certain kinds of truths which can be intuitively known in this way. This distinguishes cognitive intuition from intuition-as-hunch where there is no such restriction.

The idea that we can know that certain propositions are true simply by contemplating them is a familiar one. It usually applies to the so-called analytic and/or logical truths, such as 'All bachelors are male' and 'If p implies q , and p , then q '. Explanations as to why this is possible vary, but appeal is usually made to the fact that the propositions are true in virtue of the meanings of the terms used, or in virtue of the concepts employed, or by convention. These examples of analytic truths do not have existential import and I know of no serious claim that we can have cognitive intuitive knowledge of the existence of non-platonic objects. If it is claimed that we can have cognitive intuitive knowledge of the existence of platonic objects, then we need an explanation as to why it is not possible in the case of non-platonic objects. Either the platonic knowledge must be explained as some sort of conceptual knowledge or a special kind of cognitive intuition must be invoked which applies especially (and only) to platonic knowledge. The claim that platonic knowledge is conceptual knowledge is an alternative to the claim that platonic knowledge is intuitive knowledge, a discussion of which is beyond the scope of this paper. An account of platonic knowledge as conceptual knowledge might include some process of intuition, but so long as that intuition was no different than that involved in acquiring non-platonic conceptual knowledge, then it is conceptual knowledge which is doing the important work in such an account. The notion of a special platonic cognitive intuition is simply *ad hoc* and explains nothing. It won't do to claim that we have a special faculty for contemplating platonic objects, as a result of which we can cognitively intuit certain truths about them. That is just a version of direct

platonic intuition (discussed below), a version which incorporates perceptual intuition.

VII. Direct platonic intuition

The final notion of intuition which is invoked to account for platonic knowledge is direct apprehension of platonic entities. This is usually described in terms of a faculty analogous to sense perception. Kurt Gödel (in a frequently quoted passage) expresses this notion as follows:

But, despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception.¹³

It is clear that the claim that intuition is a faculty analogous to sense perception is quite distinct from the claim that intuition is part of the process of sense perception. Unfortunately, because the same term is used for both notions there is a danger of conflation. If your account of sense perception includes perceptual intuition and you also claim that we have direct platonic intuition which is a faculty analogous to sense perception, then it may be that your direct platonic intuition will include a process analogous to perceptual intuition. In fact, you may wish to claim that it is the very same process which is involved in both faculties. However, any evidence or argument for the existence of perceptual intuition will be irrelevant to the claim that we have direct platonic intuition.

Before going further, a distinction already alluded to should be noted. In the case of perception, we distinguish perception *of* objects from perception *that* something is the case. I may see the tree (perception-of) and I may see that the tree is swaying (perception-that). I may hear the bells and I may hear that the bells are ringing. The relation between these two kinds of perceiving is controversial. The usual view is that perception-of gives rise to perception-that, and that it is possible to have the former without the latter. I can see the trees without seeing that the trees are swaying or without seeing (and thus knowing) anything about the trees. However, on some accounts, perception-that is seen as more fundamental. On such accounts, only if I perceive that something is the case can I perceive objects. Whatever is the case, it is fairly clear that we cannot have perceptual knowledge without both perception-of and perception-that.

If there really is a process of intuition analogous to sense perception, we should expect this distinction to apply to it as well. We should expect that

¹² René Descartes, *The Passions of the Soul* (1649), in M. D. Wilson (ed.), *The Essential Descartes* (New York: New American Library, 1969), pp. 362–63.

¹³ Kurt Gödel, "What is Cantor's continuum problem?" in P. Benacerraf and H. Putnam (eds.), *Philosophy of Mathematics*, 2nd edition (Cambridge: Cambridge University Press, 1983), pp. 483–84.

we can have both intuitions of platonic objects and intuitions that certain mathematical claims are true. It is not clear whether Gödel is conflating or observing the distinction when he suggests that "we do have something like a perception...of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true." But exegetical concerns need not detain us. Some philosophers have claimed that we can have intuition of platonic objects, while others have claimed that we only have intuition-that.¹⁴ Many others have not been clear on the issue. The acquisition of platonic knowledge by intuition-that alone is a variety of cognitive intuition that I discussed and dismissed in the previous section. It remains for me to argue that the knowledge of abstract objects which is required by platonism cannot be based on, or involve, a special platonic intuition-of.

There is one other possibility that should be mentioned. Some philosophers believe that propositions are intensional objects and that intuitive knowledge consists of having an intuition of such objects. For example, to know by intuition that five is a prime number is to be related in some way to the proposition that five is a prime number. On such accounts, intuition-that is subsumed under intuition-of. If this is so, then my argument that platonic knowledge cannot be based on intuition-of will eliminate this possibility.

A general objection to knowledge by intuition is that intuition, whatever its exact nature, is fallible. People's intuitions concerning the same matter often differ from person to person, or vary from occasion to occasion. Intuitions are subjective and therefore cannot provide a reliable source of knowledge. Gödel responds to this criticism by saying that he does not "see any reason why we should have less confidence...in mathematical intuition than in sense perception." He also suggests an analogy between the deceptions of the senses and the set-theoretical paradoxes.¹⁵ If the analogy between sense perception and intuition is to be taken seriously, then this is a reasonable response to the objection. If the fallibility of sense perception can be incorporated into our theory of knowledge (as it must if that theory is to be plausible), then the fallibility of intuition does not provide grounds for barring it from our theory of knowledge. Of course, if we insist on the infallibility of mathematical knowledge, then mathematical intuition would have to be in some sense an infallible process. But such a doctrine is highly implausible and finds little favour with contemporary epistemologists. Moreover, to insist on the infallibility of mathematical intuition would threaten the grip of the analogy with perception. In what way could infallible intuition be analogous with fallible sense perception?

Any subjective element of intuition should also not be a bar to incorporating it into our epistemology. Perception also has a subjective element. How we feel and what we believe and desire can affect what we perceive. If careful observers can be objective, then why not careful intuiters (such as trained mathematicians)? On the other hand, if subjectivity is an ineliminable aspect of perception, then either perceptual knowledge of an objective reality is impossible, or it is possible in spite of the subjectivity. The former is implausible and would lead to radical scepticism. To maintain the possibility of objective intuitive knowledge in the face of such scepticism would loosen the grip of the analogy with perception. If objective perceptual knowledge is possible in spite of a subjective element, then it will be because, as well as everything else that is going on, the right sort of relationship exists between the perceiver and an objective external reality. Gödel makes a similar claim for intuition:

It by no means follows, however, that [intuitions], because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective.... Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality.¹⁶

Unfortunately, it is the positing of this "other kind of relationship" which threatens to undermine the case for intuitive knowledge. What is the nature of this relationship? All we have to go on is the analogy with perception. But a distinguishing feature of the relationship between the knower and reality in the case of perception is causation. Perception is a causal process. It is puzzling how a process could be analogous to perception and yet not involve causation. Furthermore, no independent evidence is offered for our having a faculty of intuition, or even for recognizing when we are exercising it. Talk of this mysterious faculty of intuition is tainted with an air of occult mysticism. The notion is *ad hoc* and the analogy "broken-backed" or "flabby."¹⁷

These criticisms may be unfair if Gödel's analogy is intended in only a restricted sense. One suggestion is that the common feature between sense perception and mathematical intuition that Gödel is drawing our attention to is that in the case of sense perception we have no choice as to what seems to be the case, and the same is true when we contemplate the axioms of set theory.¹⁸ As Berkeley puts it:

¹⁶ Gödel, p. 484.

¹⁷ Bob Hale, *Abstract Objects* (Oxford: Blackwell, 1987), p. 79 and Charles Chihara, "A Gödelian Thesis Regarding Mathematical Objects: Do they Exist? And can we perceive them?" *Philosophical Review* 91 (1982), p. 217.

¹⁸ John Bigelow, in conversation.

¹⁴ Charles Parsons argues for intuition-of, in his (1979-80), p. 151, as does Richard Tieszen in his *Mathematical Intuition: Phenomenology and Mathematical Knowledge* (Dordrecht: Kluwer, 1989), p. 5. Mark Steiner claims that there is only intuition-that, in his *Mathematical Knowledge* (Ithaca: Cornell University Press, 1975), p. 131.

¹⁵ Gödel, p. 484.

When in broad daylight I open my eyes, it is not in my power to choose whether I see or no, or to determine what particular objects shall be present to my view: and so likewise as to the hearing and other senses; the ideas imprinted on them are not creatures of my will.¹⁹

Is Gödel simply pointing out that we have the same lack of choice with respect to the axioms? If so, then we are no closer to an account of *how* platonic knowledge is acquired by intuition. Or perhaps his comments should be seen as offering evidence for the existence of another process for acquiring knowledge of objective, external reality. But if it is evidence for anything, it is evidence for a *causal* process, especially in the light of his metaphor of the axioms "forcing" themselves upon us as being true.

Besides, it is not at all clear that the cases are similar. Perhaps there are axioms which we feel bound to accept. One such may be the axiom of extensionality which states that two sets are equal if and only if every element of one is an element of the other, and vice versa. But other axioms of set theory such as the axioms of choice, infinity, and reducibility have been hotly debated. Also, recall Cantorian set theory with the Continuum Hypothesis (CH) as an axiom, and non-Cantorian set theory with its negation ($\neg$ CH) as an axiom. Neither CH nor $\neg$ CH *forces* itself on us as being true.²⁰ Finally, and crucially, axioms with existential import do not have this feature, especially when they are given a platonistic construal. I, for one, do not feel bound to accept that the empty set exists independently as an acausal object.

James Brown argues that the mysteriousness of the purported process of intuition should not be a bar to our accepting Gödel's explanation. He suggests that sense perception is equally mysterious.

In the case of ordinary visual perception of, say, a teacup, we believe that photons come from the physical teacup in front of us, enter our eye, interact with the retinal receptors and a chain of neural connections through the visual pathway to the visual cortex. After that we know virtually nothing about how beliefs are formed. The connection between mind and brain is the great problem of the philosophy of mind. Of course, there are some sketchy conjectures, but it would be completely misleading to suggest that this is in anyway 'understood'. Part of the process of cognition is well understood; but there remain elements which are just as mysterious as anything the platonist has to offer.²¹

But the objection to intuition is that it is *wholly* mysterious. Brown himself supplies sufficient detail about perception to show that that process is not wholly mysterious. There may be gaps in our understanding, but there are

parts of the process that we do understand. Continuing scientific investigation is gradually filling in those gaps and giving us no reason to suppose that our overall picture is radically mistaken. The mystery of direct platonic intuition goes deeper than this. When investigating an unknown causal process, scientists have a fairly clear idea of what sort of hypotheses may appropriately be tested. Whatever the outcome of their tests, further hypotheses will be suggested. But in the case of investigating a non-causal process, it is not even clear what, if any, hypotheses are appropriate. An analogy between the mysteriousness of perception and the mysteriousness of intuition cannot be sustained.

Brown tries another line of attack. He suggests that the correctness of our theory of perception is irrelevant to whether or not we can have perceptual knowledge. He points out that our ancestors' beliefs about perception were quite wrong and that it is possible that our beliefs about it are equally wrong.

Nevertheless, whether we are right or wrong in our account of perception, we are (and our ancestors were) rightly convinced that there are material objects such as trees and tables which exist independently of us and that they somehow or other are responsible for our knowledge of them. Whether we have the right theory, the wrong theory or no theory at all, it is reasonable to reject Berkeley's idealism.²²

Brown's argument seems to be that the perception/intuition analogy amounts to no more than the claim that *if* we have faculties of perception and intuition, then we can use those faculties to obtain knowledge of the external world, irrespective of what theories we have about those faculties. I shall not dispute this conditional claim. The same claim is true of faculties of precognition, clairvoyance, and religious mysticism, *if* we have such faculties. But the claim is worthless as a response to the request that platonists explain how we acquire knowledge of the existence of platonic objects. It is a variation on Argument A and its corollary.

Before leaving direct platonic intuition, I turn to two bold, but contrasting, responses to the objection that the analogy between perception and intuition is quite unhelpful because one is a causal process while the other is not. Penelope Maddy offers the bold conjecture that the analogy holds good because both are causal processes.²³ She claims that sets of physical objects are located in space and time, and that we can perceive them just as we can perceive physical objects, for example, by looking at them and seeing them. Maddy is concerned only with sets because she believes that all of mathematics can be reduced to set theory. So, according to Maddy, sets are entities with causal powers. There are a number of problems with Maddy's position but I shall not discuss them here. Her brand of *physicalistic* platonism does not as-

¹⁹ George Berkeley, *The Principles of Human Knowledge* (1710), in *Berkeley's Philosophical Writings*, edited by D. M. Armstrong (New York: Collier Books, 1965), p. 72. Of course, Berkeley concludes that the source of the ideas is the will of God, rather than material objects, but the relationship is with an objective reality for all that.

²⁰ Gödel concedes as much when he talks of verifying an axiom by other means, such as by examining what can be proved with its help and what cannot, pp. 477, 485.

²¹ J. R. Brown, "π in the Sky," in A. D. Irvine (ed.), *Physicalism in Mathematics* (Dordrecht: Kluwer Academic Publishers, 1990), p. 108.

²² Brown, p. 109.

²³ Penelope Maddy, "Perception and Mathematical Intuition," *Philosophical Review* 89 (1980): 163–96 and *Realism in Mathematics* (Oxford: Clarendon Press, 1990).

sert that we can have knowledge of the existence of acausal objects, so it is outside the scope of this paper.

The mirror image of Maddy's position is Richard Tieszen's claim that Gödel's analogy is appropriate because neither mathematical intuition nor sense perception are causal processes.²⁴ His claim is based on Husserl's phenomenological accounts of the two processes. Husserl's account of sense perception is mostly concerned with an analysis of what he calls perceptual intuition. He gives a similar, but less developed, analysis of mathematical perception. Tieszen first develops an account of mathematical intuition along phenomenological lines and then argues for the analogy between the two kinds of intuition.

According to the phenomenological view, perceptual intuition is thought of "in terms of sequences of indexical partial perceptions of objects and not in terms of some sort of special relation, causal or otherwise, and independent of this process, which is supposed to put us in touch with real objects."²⁵ The emphasis is on intentional cognitive acts and the contents of those acts. The objects towards which those acts are directed are "bracketed" (as the phenomenological jargon has it), and play no part in the analysis. The reason for this seems to be that the object may or may not exist and "[w]hether the object exists or not depends on whether we have evidence for its existence, and such evidence would be given in further acts carried out through time."²⁶ That this view of perception entails idealism seems inescapable. There is no clear distinction between what is perceived and the perception of it or between actual existence and evidence for existence. If the existence of an object *depends* on our having certain evidence, and that evidence, in turn, depends on our performing certain acts of perception, then it is difficult to see in what sense such an object could be mind-independent.

Tieszen provides further evidence for the idealism implicit in this view when he states that:

The point of phenomenological reduction is just that we are to investigate the structure of acts and sequences of acts, and to do so does not require considerations about causality. [W]e are not committed to providing an analysis of the condition that the state of affairs referred to by *S* causes *M* to believe that *S*.... On our view there is an analogy between mathematical and perceptual intuition and an analysis of any condition about causes is not involved in either case. It should be noted that we are not saying that ordinary perceptual objects are not given as objects to which we are causally related, or as objects that may be causally related to one another. On the contrary, they would certainly be recognized as so given, unlike mathematical objects.²⁷

No doubt, the pink rats that the inebriate hallucinates are *given* as objects to which he and other objects are causally related. But they are not actually causally related because they do not actually exist. This is one reason why the inebriate does not *know* that there are pink rats in front of him. It may be that the Husserl/Tieszen analysis provides a splendid account of what goes on when someone has a perceptual or mathematical *intuition*, and it may be that those two processes are analogous. But the analysis does not provide an account of processes by which we acquire knowledge of external reality. The analysis of perceptual intuition is insufficient as an account of how we acquire perceptual knowledge since it lacks the crucial link of causation, and this insufficiency makes it useless as an analogy for a process by which we acquire knowledge of platonic objects.

VIII. Conclusion

I conclude that although there may be actual faculties or processes to which the term 'intuition' may be appropriately applied, none of them can plausibly account for platonic knowledge. To posit another sort of intuition especially for platonic knowledge would be *ad hoc* and, therefore, cannot help to explain how it is that we can have knowledge of the existence and nature of platonic objects. More than a century later, Frege's warning should still be heeded. "We are all too ready to invoke inner intuition, whenever we cannot produce any other ground of knowledge."^{28, 29}

²⁴ Tieszen, *Mathematical Intuition*.

²⁵ Tieszen, pp. 63-64.

²⁶ Tieszen, p. 23.

²⁷ Tieszen, pp. 179-80.

²⁸ Gottlob Frege, *The Foundations of Arithmetic*, translated by J. L. Austin, 2nd edition (Oxford: Basil Blackwell, 1968), p. 19.

²⁹ My thanks to Alan Musgrave, Charles Pigden, John Bigelow, and Pavel Tichý for their helpful comments on earlier drafts of this paper.