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IN: PLATO AND PARMENIDES

INTRODUCTION

CHAPTER I

THE EARLIEST PYTHAGOREAN COSMOGONY

THE best evidence for the date of Parmenides' life is furnished by Plato's dialogue. This contains an imaginary conversation of Socrates with Parmenides and his pupil Zeno when they were visiting Athens for the Great Panathenaea. Socrates was then 'quite young', perhaps eighteen to twenty; Parmenides is about sixty-five, Zeno about forty. Socrates' age fixes the date for the meeting at about 450 B.C. That would place Parmenides' birth somewhere about 515 B.C. In his poem he makes the goddess address him as a young man. If we suppose him to have been thirty, the poem would be written about 485 B.C. This date would be consistent with the fact that Heraclitus' fragments contain no reference to Parmenides, whom he would certainly have denounced even more vigorously than the other philosophers whom he names, including Xenophanes. And, on the other hand, some have seen in Parmenides a denunciation of Heraclitus as the arch-offender against reason.

The remarkable features of Parmenides' system will become intelligible only when we see his poem as a protest against the fundamental assumptions of the earlier systems which he is concerned to criticise and reject. Some of his expressions indicate that he knew the Milesian cosmogony of Anaximander and Anaximenes. But his work belongs, not to the Ionian, but to the Italian tradition. There is evidence that he had broken away from the Pythagorean school, which alone was established in Southern Italy, and he would therefore be likely to define his own position mainly in contrast with theirs. If Pythagoras settled at Croton about 530 B.C., and if Parmenides was born about 515 B.C., his teachers must have been the immediate pupils of the master. We must, accordingly, examine such traces as remain of the primitive Pythagorean cosmogony.

The peculiar difficulty here confronting us, as it confronted Aristotle and Theophrastus, is the absence of early documents. The fragments attributed to Philolaus (end of the fifth century)

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are all under suspicion of forgery. Plato, though familiar (at least after his first visit to Sicily in 388/7) with Pythagorean philosophy, never attributes any doctrine to an individual, with the exception of Philolaus. Aristotle ascribes various and conflicting views to 'the Pythagoreans', or 'some Pythagoreans'; and of his books on the Pythagorean philosophy only a few fragments remain. This state of affairs is due to the tradition of the school, not to claim discoveries as the achievements of individual members, but to ascribe them to the founder. Later authorities repeat this attribution uncritically, assigning to Pythagoras himself much that must belong to later times. Modern doxographers can do no more than collect the testimonies down to and including those of Aristotle and his pupils, and set them down under the heading of 'the Pythagorean School'. The lack of documents, however, does not leave us altogether without witness. We have some unquestioned information about Pythagoras. The philosophers of the fifth century, notably Empedocles and the Eleatics, were influenced by Pythagoreanism or reacted against it. Finally, common sense may tell us that some elements which persist throughout the later Pythagorean literature are obviously primitive and archaic.

We shall not be concerned here with Pythagoras as the founder of a religious community, but only with such traces as remain of his rationalised cosmology. There is no serious ground for doubting his claim to eminence among the founders of mathematical science. For his intellectual attainments we have the evidence of his contemporary Heraclitus, a hostile witness, as well as that of Empedocles and Herodotus, Aristotle and Aristoxenus.¹ It was universally believed by the ancients, whose testimony modern scholars are not in a position to disprove, that Pythagoras was the author of the doctrine that numbers are the real nature of things. It is probable, moreover, that this intuition was prompted and confirmed by his discovery that the perfect consonances which formed the framework of all musical scales (*harmoniai*) were expressible in terms of ratios between the numbers 1, 2, 3, 4: the octave being 2:1, the fifth 3:2, the fourth 4:3. These four numbers are the *tetractys* of the decad: $1 + 2 + 3 + 4 = 10$. The decad 'contains the whole nature of number'² (since all nations count up to 10 and then begin again) as well as 'all the consonances'. The *tetractys* was a symbol of great significance and, like other such symbols, capable of many interpretations. The source of the doctrine in the field of music explains the conclusion, as stated by Aristotle, that 'the whole Heaven or visible universe is a musical

¹ See the passages referred to by Burnet, *E.G.P.*, 97-99.

² Ar., *Met.* 986a, 8; *Phys.* 206b, 32; *Met.* 1084a, 10 (Plato).

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scale or number'. From first to last, the fundamental distinction between the two main traditions, Ionian and Italian, is that whereas the Ionian sought the nature of things in some kind of matter, the Italian laid stress on the principle of limit or form, which first appears as geometrical shape and number.¹

The cosmogony, then, which we seek to reconstruct takes numbers as the ultimately real things in nature. The evidence is provided, in the first place, by certain statements in Aristotle about the earliest Pythagorean doctrine known to him, going back to at least the middle of the fifth century. Secondly, these statements are confirmed by the first document which gives a connected account of Pythagoreanism. Diogenes Laertius has preserved an extract from the *Successions of Philosophers* by Alexander Polyhistor, who, writing in the first century B.C., professed to reproduce what he had found in Pythagorean treatises. Independent studies by Wellmann and Delatte led to the conclusion that Alexander's source was probably a contemporary of Plato in the fourth century.² No later writer could have escaped the influence of Plato himself and in particular of the *Timaeus*.³ The first paragraph of Alexander's summary runs as follows:

'The first principle of all things is the One. From the One came an Indefinite Two, as matter for the One, which is cause. From the One and the Indefinite Two came numbers; and from numbers, points; from points, lines; from lines, plane figures; from plane figures, solid figures; from solid figures, sensible bodies. The elements of these are four: fire, water, earth, air; these change and are wholly transformed, and out of them comes to be a *cosmos*, animate, intelligent, spherical, embracing the central earth, which is itself spherical and inhabited round about.'

The opening sentences are in substantial agreement with Aristotle, who begins his historical account of the Pythagoreans with a brief

¹ Ar., *Met.* 1028b, 15: δοκεῖ δὲ τοῖς τὰ τοῦ σώματος πέριχα, ὅσον ἐπιφάνεια καὶ γραμμὴ καὶ στίγμη καὶ μονάς, εἶναι οὐαίαι, καὶ μᾶλλον ἢ τὸ σῶμα καὶ τὸ στερεόν. *Met.* 1090b, 5.

² Diels-Kranz, *Vors.*, 58 [45], B 1a. Diog. L. viii, 24-33. Wellmann, *Hermes* 54 (1919), 225. Delatte, *Vie de Pythagore* (1922).

³ The fact that Alexander uses a few phrases (e.g. 'the Indefinite Dyad' for the Unlimited) which became current in Plato's school is no evidence against the pre-Platonic content of the doctrine. In every history of early philosophy, ancient or modern, the writer inevitably uses some language which is familiar to his contemporaries and to some degree anachronistic, however carefully he may try not to falsify the thought he is conveying. M. Robin (*Théorie plat. des Idées*, p. 650) holds that Theophrastus' attribution of 'the Indefinite Dyad' to Pythagoreans as well as to Plato can be defended on the supposition that Pythagorean contemporaries of Plato are meant.

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statement of the doctrines held by the school in the latter part of the fifth century (the time of the Atomists, Leucippus and Democritus) and earlier.

'Bred in the study of mathematics, which they were the first to advance, they thought that the principles of mathematics are the first principles of all things. Of these principles numbers are by nature the first; and in numbers, rather than in fire or earth or water, they found many resemblances to things that exist and come into being. . . . Further, they saw that the properties and ratios of musical scales were expressible in numbers. Since, then, all other things seemed in their whole nature to be modelled after numbers, and numbers seemed to be the first things in Nature as a whole, they supposed that the elements of numbers are the elements of all things, and that the whole Heaven is a musical scale or number' (*Met. A*, v, 985b, 23).

'The first principle of all things is the One.' Alexander's summary represents the second principle, which he calls the Indefinite Two, as derived from the One. Eudorus¹ (first century B.C.) also declares that the Monad is the first principle of all things and 'the supreme god', whereas the two 'secondary principles of the nature of elements, the opposites (Limited and Unlimited) under which they ranged their two columns', are not strictly principles but posterior to the Monad. It has been doubted whether this doctrine was a feature of the original system, and in what sense this 'One' or Monad is to be understood. As a religious philosophy, Pythagoreanism unquestionably attached central importance to the idea of unity, in particular the unity of all life, divine, human, and animal, implied in the scheme of transmigration. The Table of Opposites, in which a column of goods and an answering column of evils are ranged under Limit and Unlimited, shows clearly how the whole view of the world was coloured by conceptions of value, foreign to the Ionian tradition. Nor is there any ground for rejecting the testimony that the principle of Unity, in some form, was regarded as divine.² We should expect, moreover, something analogous to the one God of Xenophanes, the One Being of Parmenides, the Sphere of Empedocles. A system of the Italian type, seeking the reality of things in form rather than matter, will not take for its

¹ Simplic., *Phys.* 181, 7 ff. (R.P. § 70).

² Hippol., *Ref.* 1, 2, μονάδα μὲν εἶναι ἀπεφώνητο τὸν θεόν. Aet. 1, 7, 18, Πυθαγόρας τῶν ἀρχῶν τὴν μονάδα θεόν καὶ τάγαθόν. O. Gilbert (*Arch. Gesch. Phil.* xxii (1909), 155) defends these statements against Zeller; but he thinks that Unlimited matter (ἄπειρον) must have been equally eternal with the One, the divine Unity which informs it (p. 165).

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starting-point an unlimited and indiscriminate mass like that 'Boundless' which Anaximander called 'divine', but which is the ancestor of the 'all things together' of Anaxagoras. As Aristotle observes, 'the Pythagoreans suppose that supreme beauty and goodness are not present in the beginning; for, although the beginnings of plants and animals are causes, beauty and perfection are rather in their outcome' (*Met.* 1072b, 30). The world itself is a living creature. The element that makes it 'divine' will be the principle of beauty and goodness which is manifest in the perfection of its completed order (κόσμος). It is possible that this principle was from the first called Unity or 'the One', and regarded with religious reverence as the object of human aspiration. It must certainly be distinguished from the first unit of number, which provides, as we shall see, the starting-point for cosmogony.

Some obscurity in our sources is due to the confusion of these two senses of 'the One' (τὸ ἓν or ἡ μονάς). This expression is sometimes synonymous with the Limit (πέρας), which figures as the good member of the primary pair of Opposites, while the Unlimited or the Dyad is the bad.¹ So where Aristotle speaks of Limit and Unlimited, Alexander has 'the One and the Indefinite Two'. Again Theophrastus writes:

'Plato and the Pythagoreans make the distance between the real and the things of nature a great one, but hold that all things wish to imitate the real; yet since they make a sort of opposition between the One and the indefinite dyad, on which essentially depends what is indefinite and disordered and, so to speak, all shapelessness, it is absolutely impossible that for them the nature of the whole should exist without the indefinite dyad; they say it has an equal share in things with, or even predominates over, the other principle; whereby they make even the first principles contrary to one another. Hence those who ascribe causation to God hold that even God cannot guide everything to what is best' (*Metaph.* 33, trans. Ross).

Here, of course, Theophrastus is thinking mainly of the *Timaeus*; but the passage illustrates the use of 'the One', not for an all-embracing whole, but for the good principle within that whole, which, as good, is in dualistic conflict with the principle of disorder and shapelessness, the Unlimited.

On the other hand, 'the One' sometimes means the unit of arithmetic, 1, standing at the beginning of the series of numbers. Number being defined as a plurality of units (πληθος μονάδων), the

¹ Eudorus, *loc. cit.*, ἄλλο μὲν ἐστὶν ἐν ἡ ἀρχὴ τῶν πάντων, ἄλλο δὲ ἐν τῷ τῇ δυάδι ἀντικείμενον.

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first unit is not a number, but the 'beginning of number'. As we shall see, it is the product of the two opposites, Limit and Unlimited, which are combined in its nature.

We can now understand Aristotle's statement that, although numbers are 'first' among mathematical objects, they are themselves derived from ulterior elements, which he proceeds to describe (*Met.* 986a, 15):

'Evidently these philosophers also consider number to be a principle, both as the matter of things and as their modifications and states.¹ And as elements of number they have the Even and the Odd; and of these the Odd is limited, the Even unlimited.'

The limited Odd and unlimited Even correspond to Alexander's One and Indefinite Two. The fundamental pair of opposites are Limit and Unlimited: Odd and Even are only the exemplification of these universal principles in the sphere of number.² This appears from the Table of ten Opposites ranged in two columns, which Aristotle in the same context attributes to 'other' Pythagoreans. Here Limit and Unlimited head the list, followed by Odd and Even, Unity (*ἕν*) and Plurality (*πληθος*). The complete list, as given here,³ is as follows:

Limit	Unlimited
Odd	Even
Unity	Plurality
Right	Left
Male	Female
Resting	Moving
Straight	Crooked
Light	Darkness
Good	Bad
Square	Oblong

Aristotle evidently regards this list as primitive, since he doubts whether the medical theory of Alcmaeon derived the notion of pairs

¹ This remark has reference to Aristotle's attempts to equate the principles of the earlier philosophers with one or another of his own four causes. It means that the Pythagoreans treat numbers as, in some sense, both material and formal causes of things.

² So Ross, *ad loc.*, and O. Gilbert, *Arch. Gesch. Phil.* xxii (1909), 29. Since 2 is the first even number, and the even falls under the unlimited, the phrase 'indefinite dyad', though Plato gave it a peculiar sense with reference to his Great-and-Small, is not inappropriate to earlier Pythagorean conceptions.

³ Ross gives the references to other forms of the list, in which some of the items and the number of items vary.

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of opposites from these Pythagoreans or the Pythagoreans from him; and Alcmaeon is described as a younger contemporary of Pythagoras.¹ After the first three, the items seem to be arranged in no logical order. There is nothing about any one of them to suggest a later date. It seems obvious that the ten pairs stand for ten different manifestations of the two primary opposites in various spheres; in each there is a good and an answering evil. At *Philebus* 16c, Plato speaks of a gift from heaven to mankind sent down through the agency of some Prometheus, together with a most illuminating fire; 'and the ancients, who were superior to us and dwelt nearer to the gods, have handed down a tradition that all things that are said to exist consist of a One and a Many and contain in themselves the connate principles of Limit and Unlimitedness'. The Prometheus of this revelation can hardly be other than the divine man, Pythagoras.² Proclus is echoing the *Philebus* when, in considering the principles of all mathematics, he speaks of Limit and Unlimited as coming first after the One and 'pervading all things that are and generating all things from themselves' (*Eucl.* I, p. 5).

The first thing that they generate is the arithmetical unit, 1. After the mention of the limited Odd and unlimited Even, Aristotle proceeds:

'And the unit (*τὸ ἕν*) consists of both these, for it is both even and odd; and from the unit (proceeds) number'.

That 'the one' here means the unit of arithmetic is clear from its being both even and odd and from the reason given for so regarding it: 'the unit partakes of the nature of both, since when added to an even number it makes it odd, and when added to an odd number it makes it even; hence the unit is called "even-odd"'.³ So Limit and Unlimited combine to produce the first unit; and 'from the unit (proceeds) number'. Numbers, which are pluralities of units, can be most simply obtained by adding one unit to another; (not by division, for the unit is always held to be indivisible).⁴ The process will be further considered presently. Here let us observe that the plurality of numbers is not original, but derived. The system does not start, like Atomism, with an unlimited plurality of units.

¹ This description is not in all the manuscripts, but, as Ross says, is 'likely enough to be true'.

² So O. Gilbert, *op. cit.*, p. 38. Plato could not so describe a doctrine which had taken shape in his own life-time.

³ Ar., frag. 199R, ap. Theon. 1, 5, p. 22, Hiller.

⁴ Rep. 525D: Mathematicians deride any attempt *αὐτὸ τὸ ἕν τέμνειν . . . μή ποτε φανῇ τὸ ἕν μὴ ἔν ἀλλὰ πολλὰ μέρη*. Is this what is referred to at *Μενο*, 77A: *παῦσαι πολλὰ ποιῶν ἐκ τοῦ ἑνός, ὅπερ φασὶ τοὺς συντρίβοντας τι ἐκάστοτε οἱ ἀκώπτοντες*?

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Aristotle's next words, 'and numbers are, as we said, the whole Heaven' (physical world), seem to us to take a considerable leap. But Aristotle is not here outlining the Pythagorean process of cosmogony; he merely re-states the point he is making at the moment: that numbers in this system are the material and formal causes of things, replacing the material water or air of Milesian physics. Alexander's summary here helps to fill the gap (as it seems to us) between numbers and visible and tangible bodies. When numbers have been derived from the One and the Indefinite Two,

'from numbers came points; from points, lines; from lines, plane figures; from plane figures, solid figures; from solid figures, sensible bodies.'

This stage takes us from arithmetic to geometry, from numbers to the solid body in three dimensions.

The transition was facilitated by the ancient practice of representing numbers by arranging units in geometrical patterns. Nicomachus,¹ where he turns to discuss the various kinds of 'linear', 'plane', and 'solid' numbers, remarks that the use of numerical symbols like ι for 10, κ for 20, is a mere human convention: 'the natural, unsophisticated, and simplest way of representing them is to set out the units in each number side by side, thus:

α for 1
 $\alpha\alpha$ for 2
 $\alpha\alpha\alpha$ for 3, etc.'

Iamblichus (*Nicom.* 57) adds that this is the older method. Nicomachus goes on to say that the unit holds the position of a point ($\sigma\eta\mu\epsilon\iota\omicron\nu$) and is the starting-point of intervals and numbers, but not as yet an interval or number, just as the point is the starting-point of line and dimension, but not as yet a line or dimension. The unit is without interval or dimension ($\acute{\alpha}\delta\iota\delta\omicron\sigma\tau\alpha\tau\omicron\varsigma$); the first interval appears in 2, the next in 3, and so on, interval being that which is between two terms.² The first dimension is called 'line',

¹ *Arithm.* pp. 82 ff.

² Cf. Plato, *Rep.* 546B, $\alpha\upsilon\tilde{\xi}\eta\sigma\epsilon\iota\varsigma \dots \tau\rho\epsilon\iota\varsigma \acute{\alpha}\pi\omicron\sigma\tau\acute{\alpha}\sigma\epsilon\iota\varsigma \tau\acute{\epsilon}\tau\tau\alpha\rho\alpha\varsigma \delta\acute{\epsilon} \delta\rho\omicron\upsilon\varsigma \lambda\alpha\beta\omicron\upsilon\sigma\sigma\alpha\iota$, and *Parm.* 149A ff. n contacts involve $n + 1$ terms ($\delta\rho\omicron\iota$). After tracing the development in the meaning of $\delta\rho\omicron\varsigma$ from its early use for boundary-stones and boundary lines, Mrs. Markwick concludes: 'As the idea of a series of numbers became more familiar, though each number was still "a collection of units" ($\mu\omicron\nu\acute{\alpha}\delta\omega\nu \sigma\upsilon\sigma\tau\eta\mu\alpha$, *Nic.*, *I.A.* vii, 1), the word $\delta\rho\omicron\varsigma$ was transferred from a unit of one number to a unit of a series of numbers, and $\acute{\epsilon}\kappa\tau\iota\theta\acute{\epsilon}\nu\alpha\iota$ was used to denote the setting-out of the terms in the series (Theon, p. 22, 17). . . . Thus $\delta\rho\omicron\varsigma$ when used of numbers always implied something discon-

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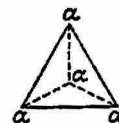
for line is that which is extended in one direction; two dimensions make surface; three, the solid. So in numbers, the unit is the starting-point of all number, which proceeds in one dimension, unit by unit; then linear number is the starting-point of plane number, which is broadened out as a surface in a second dimension; and the plane number is the starting-point of solid number, which acquires depth in the third dimension.

Plane numbers, Nicomachus continues, start from three as their ultimate root, the triangle being the most primitive and elementary plane figure. So, by adding to the original unit the natural numbers (2, 3, 4, 5 . . .) successively, we obtain the series:

$$\begin{array}{ccccccc} & & & & \alpha & & \\ & & & & \alpha & \alpha & \\ & & \alpha & & \alpha & \alpha & \\ \alpha & & \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha \end{array}$$

$$1 + 2 = 3, \quad 3 + 3 = 6, \quad 6 + 4 = 10, \quad 10 + 5 = 15 \dots$$

The first solid number is the pyramid with triangular faces, represented by four units:



Further on (p. 108) Nicomachus dwells on the distinction between square and oblong numbers. He tells us that the ancients, Pythagoras and his successors, found 'the other or otherness' in 'two', 'the same or sameness' in 'one'. They regarded 'one' and 'two' as the two principles of all things. These differ only by 1; accordingly 'the other' is originally that which is other by 1 unit, not by any other number; and the word 'other' is properly used only of two things (one and the other), not of a greater number. Moreover, 1 is the formative principle of all uneven number, 2 of all even number; hence it is reasonable to say that odd number partakes of sameness, even number of otherness. This is illustrated by the formation of square and oblong numbers. If we start with 1 unit, and add the successive odd numbers in the form of gnomons,¹

tinuous. It was first taken from the boundary-stones, then applied to units in a figured number, then to terms in a series.' (Unpublished dissertation on the Concept of Continuity, chap. i.)

¹ The gnomon is defined by Aristotle as the figure which, when added to a square, increases its size but does not alter its form. For this and other uses of the word, see Heath, *Thirteen Bks. of Euclid*, i, 370.

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the resulting figure will always be *the same*, a square :

$$\begin{array}{cccc}
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 1 + 3 + 5 + 7 + \dots
 \end{array}$$

If we start with two units, and dispose the successive even numbers round them in the same way, we shall obtain a series of oblongs, constantly *differing* in shape :

$$\begin{array}{cccc}
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 \alpha & \alpha & \alpha & \alpha \\
 \hline
 2 + 4 + 6 + 8 + \dots
 \end{array}$$

These are the oblong numbers strictly so called, forming a figure of which one side always exceeds the other by 1 unit only.

That this distinction of square and oblong numbers was significant to the earliest Pythagoreans is evident, since square and oblong appear in the list of ten Opposites given above. It is mentioned again by Aristotle where he compares the Pythagorean Unlimited with Plato's Dyad, the Great and Small :

'The Pythagoreans identify the Unlimited with the Even. For this, they say, when it is enclosed and limited by the Odd provides things with the element of unlimitedness. An indication of this is what happens in numbers : if gnomons are placed round the unit and apart (from the unit ?),¹ in the latter case the resulting figure is always *other* (ἄλλο), in the former it is always *one* (ἓν)' (*Phys.* 203a, 10).

The use here of 'one' for 'the same' and 'other' for 'different', together with Nicomachus' remarks about the proper use of the terms 'one' and 'other' may possibly confirm the statement attributed to Aristotle, that 'Pythagoras called matter "other" (ἄλλο) as being in flux and a thing that is always becoming other'.²

¹ The oblong figures obtained by putting gnomons round 2 must be meant, however the words *καὶ χωρὶς* be interpreted.

² Ar., frag. 207R (Damasc., *Princ.* ii, 172, Ruelle). 'Ἀριστοτέλης δὲ ἐν τοῖς Ἀρχαίοις ἰστορεῖ καὶ Πυθαγόραν ἄλλο τὴν ὕλην καλεῖν ὡς ρευστὴν καὶ αἰεὶ ἄλλο καὶ ἄλλο γιγνόμενον. Delatte (*Vie de Pythagore*, 236) and Rostagni (*Il verbo*

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At *Met.* 1087b, 26 Aristotle mentions philosophers who set 'the different or the other' (τὸ ἕτερον καὶ τὸ ἄλλο) or plurality in opposition to the one (τὸ ἓν). Ps.-Alexander refers this view to 'other Pythagoreans'. Perhaps there is another trace of this expression at *Met.* 1080b, 6, where Aristotle, discussing Platonists and Pythagoreans together, speaks of 'those who say that the One is the first principle and substance and element of all things, and that number consists of the one and an other' (ἄλλου τινός). However this may be, we shall encounter this term 'other' where Plato generates numbers in the *Parmenides* (143B).

A large part of Pythagorean arithmetic (the theory of the nature and properties of numbers, as distinct from the art of calculation) consisted of a study of the various series resulting from arranging units in geometrical patterns. There is, therefore, nothing strange in the statement that Pythagoras specially studied 'the arithmetical form of geometry'.¹ The two sciences were not yet distinguished ; for at the earliest stage the unit of arithmetic appears to have been simply identified with the geometrical point 'having position', and lines, surfaces, and solids were built up of adjacent points. This method of building solids is already attested by Speusippus, following Philolaus.² Enlarging on the properties of the *tetractys* of the decad, he tells us that

- 1 is the point 
- 2 is the line 
- 3 is the triangle 
- 4 is the pyramid 

di *Pitagora*, 43) accept this, citing Ar., *Met.* 1087b 26. Ross (on *Met.*, *loc. cit.*) regards it as 'most improbable, since Aristotle in his preserved works never refers to the views of Pythagoras. But he may well have ascribed the view to certain Pythagoreans, and there may easily have been late Pythagoreans, influenced by Platonism, who adopted such a view.' Cf. Robin, *Théorie plat. des Idées*, 660. This is no doubt true ; but it is equally possible (though Zeller, I, i, p. 470³, denies this) that ἄλλο was applied to the second element before Plato ; after him we might expect to find ἕτερον rather than ἄλλο. O. Gilbert (*Arch. Gesch. Phil.* xxii (1909), 149) infers from κινούμενον being ranked under ἀπειρον in the Table of Opposites that Pythagoras regarded unlimited matter as in perpetual movement. See further below, p. 152.

¹ Diog. L., viii, 12, τὸ ἀριθμητικὸν εἶδος αὐτῆς.

² Speusippus, frag. 4, Lang. Diels-Kranz, *Vors.* 5, 44 [32] Philolaos, A 13. Burnet, *E.G.P.* 3, 290.

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Sextus remarks that the construction of lines, planes, and solids by simply adding one unit-point to another was (as we should expect) earlier than the method of representing a single point as 'flowing' into a line, the line into a surface, the surface into a solid.¹ We may add that the fluxion method yields a geometrical progression in place of the series 1, 2, 3, 4.

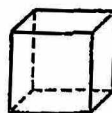
The point flows into a line



the line flows into a square



the square flows into a cube



On this view the minimum solid will be, not the pyramid, but the cube. Now Plato takes the pyramid and the cube as the figures of his two extreme elements, fire and earth, and has two geometrical progressions 1, 2, 4, 8 and 1, 3, 9, 27 (representing the even and the odd numbers respectively) as the basis for the *harmonia* of the world-soul. Each progression stops at the cube, as the first solid number (*Timaeus*, 35B).

Like Sextus, Proclus also contrasts with this fluxion view the 'more Pythagorean' account of point, line, surface, solid as analogous to the numbers 1, 2, 3, 4 (*Eucl.* I, p. 96). It is easy to conjecture why the earlier view was abandoned. According to it a line is a row of unit-points set side by side; the surface is a row of such lines; the solid, a row of surfaces. Some of Zeno's arguments against the Pythagoreans turned on this conception of magnitudes as consisting of discrete points or units juxtaposed. In the later method the 'flowing' of a single point into a line secures the continuity and infinite divisibility of magnitudes, and provides also for irrational quantities represented by incommensurable lines. The discovery of the irrational $\sqrt{2}$ and of the incommensurability of the diagonal of the square must have been made at a very early stage in geometry. It would follow upon the discovery of 'the Pythagorean theorem' (*Euclid*, i, 47) which may be due to Pythagoras himself, though the evidence is not conclusive. There can be little

¹ *Ar., de Animi*, 409a, 4, ἐπεὶ φασὶ κινηθεῖσαν γραμμὴν ἐπίπεδον ποιεῖν, στιγμὴν δὲ γραμμὴν, καὶ αἱ τῶν μονάδων κινήσεις γραμμαὶ ἔσονται. ἡ γὰρ στιγμή μόνος ἐστὶ θέσιν ἔχουσα.

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doubt that the earliest Pythagoreans, before these difficulties arose, simply built all geometrical magnitudes by adding unit-points.

Following Alexander's summary, we have now reached the geometrical solid, by a continuous evolution proceeding from the elements of number to the units of number, which have now been identified with the points making up lines, surfaces, and solids. The summary continues without a break:

'from solid figures (come) sensible bodies. The elements of these are four: fire, water, earth, air; these change and are wholly transformed, and out of them comes to be a cosmos . . .'

The description of fire and the rest as 'elements' must, as Wellmann remarks, come from Empedocles; and their complete transformation is Heraclitean.¹ These features cannot, therefore, belong to the primitive system, in which Air is not one of four elements on the same level, but still identified with the Unlimited void. But the preceding statement 'from solid figures, sensible bodies' can be accepted in the light of Aristotle's testimony. We have seen that the geometrical solid was held actually to consist of the unit-points composing its lines and surfaces. In this way the solid can be said to be a number (plurality of units). Now, where Aristotle contrasts the Pythagorean view of number with the Platonic, he tells us that the Pythagorean numbers had no existence apart from sensible bodies, but sensible things actually consist of the numbers present in them.² The units in these numbers, moreover, have spatial magnitude (1080b, 19, 32): they are the indivisible magnitudes (ἄτομα μεγέθη, 1083b, 13) or atoms composing the physical body. It thus appears that the transition, 'from solid figures, sensible bodies', can hardly be called a transition at all; numbers, as the real nature of sensible things, occupy physical space.

To Aristotle, who denied the existence of indivisible magnitudes, and held that mathematical entities are abstractions incapable of motion (the essential characteristic of all physical objects), such a view appeared crude and impossible; but his criticism ascribes it to the Pythagoreans:

¹ D.L., viii, 35, ἀ μεταβάλλειν καὶ τρέπεσθαι δι' ἄλλων. Empedocles' elements are not transformed into one another, and Plato's are not completely transformed, earth being excluded.

² The references are collected by Ross in his note on *Met.* 986a, 16. He adds: 'Aristotle insists that the Pythagorean theory of numbers as the substance of things was no mere symbolism, but a literal account of the nature of the physical world (989b, 33, N 1091a, 18).'

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'They employ less ordinary principles or elements than the physical philosophers, the reason being that they took them from non-sensible things (for the objects of mathematics, except those of astronomy, are without motion); yet all their discussions and investigations are concerned with Nature. They describe the generation of the Heaven, observing what takes place in its parts, their attributes and behaviour, and they use up their causes and principles upon this task, which implies that they agree with the physicists that the real is just all that is perceptible and contained in what they call "the Heaven". But, as we said, the causes and principles they assign are adequate for the ascent to the higher orders of reality, and indeed appropriate to these rather than to the study of Nature. How there is to be motion, if nothing is presupposed save Limit and Unlimited, Odd and Even, they altogether fail to explain; or how, without motion and change, there can be coming into being and perishing, or the behaviour of the bodies that move in the heavens. Further, even if we granted that spatial magnitude consists of these elements, or if this were proved, how could some bodies be light, others heavy? ¹ For, to judge by what they assume and maintain, what they say applies to sensible bodies just as much as to mathematical; hence they have said nothing about fire or earth or the other bodies of that sort, I suppose because they have nothing to say which applies *peculiarly* to perceptible things' (*Met.* 989b, 29 ff.).

Aristotle is objecting precisely to the identification of the geometrical solid with the sensible body. The Pythagoreans did not confine their evolution to the world of mathematical abstractions. Within that world you might, by a logical process, start from the One and arrive at the figures of geometry. Such a process would be the reverse of a logical analysis, taking the geometrical solid and analysing it into its surfaces, the surfaces into lines, the lines into points, and so on. But the synthesis, having arrived again at the solid figure, ought not to cross the boundary into the physical world, without explaining how the solid can acquire motion in space and perceptible properties like weight.

Here we may digress for a moment to observe that we find a similar transition in Plato. At *Laws* 893E the Athenian distinguishes generation or coming into existence (*γένεσις*) from other processes of motion and change. The generation of all things occurs

¹ Cf. *Ar., de caelo*, 300a, 15: Certain Pythagoreans construct nature and the universe out of numbers; but natural bodies have weight and lightness, whereas their units, when put together, cannot make a body or have weight.

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'when a starting-point (*ἀρχή*) receives increase and reaches the second stage, and from that the third, and so by three stages *acquires perceptibility for percipients*'. Here the starting-point is the indivisible line (Plato's substitute for the point, which he condemned as a geometrical fiction); the second stage, the indivisible surface; the next, the indivisible solid; and the last is the solid body perceived by the senses.¹ In Plato's case, the *Timaean* furnishes a link between the geometrical solid and the sensible body in the theory which assigns to each of the four primary bodies the structure of a regular solid.² It is interesting to observe that the whole course of Euclid's *Elements*, itself based on earlier handbooks of geometry compiled at the Academy, is covered by this sentence in the *Laws*. Euclid starts with the definition of the point and ends with the construction and inscription in the sphere of the regular solids which were known as the 'Platonic' or 'cosmic figures'. Hence Proclus remarks that 'with respect to subject matter, Euclid's entire discourse is concerned with the cosmic figures: he begins with their simple constituents and ends with the complexity of their construction, their inscription in the sphere and their mutual proportions. Hence some have thought that the scope of the several books is to be referred to the cosmos and their utility is to be explained with reference to the contemplation of the universe' (*Eucl.* I, p. 70). Proclus himself points out that Book I, for example, is concerned with the most primitive rectilinear figures, the triangle and the parallelogram; these genera include the principles of the elements, the isosceles and the scalene and the figures composed of these, namely the equilateral triangle and the square, which yield the construction of the figures of the four elements, fire, air, water, earth. Thus the scope of Book I fits into the scheme of the whole treatise and contributes towards the study of the elements of the cosmos (*ibid.* p. 82). Even so, however, the cosmic figures provide only the element of limit or form which is the intelligible factor in sensible things. Plato has also to recognise the pairs of opposite qualities, like hot and cold, which cause our sensations. These are represented as 'motions and powers', imagined as existing in a disorderly chaos apart from the element of geometrical form and number added by the Demiurge. Thus Plato has advanced far beyond the early Pythagoreans' simple assumption

¹ So A. T. Nicol, *Indivisible Lines*, C.Q. xxx (1936), p. 125. This passage in the *Laws* will be further discussed later, p. 198.

² It is not impossible that the shapes of the regular solids may have been associated with the elements before Plato. *Act.*, ii, 6, 5, attributes this to Pythagoras. The theoretical construction of the figures, completed by Theaetetus, is an entirely different matter.

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that solid and sensible body were the same thing. Alexander's summary completely ignores the difficulty of equipping a geometrical figure with sensible qualities.

The first Pythagoreans thus appear as unaware that their cosmogony really consists of two chapters: the first mathematical, terminating in the geometrical solid, and the second physical, beginning with the first sensible body. As this distinction gradually came to be realised, the two parts of the system were differently affected by criticism and external influences. The physical part, which originally, as we shall see, had some kinship with the philosophy of Anaximenes, was modified in various ways to accommodate features borrowed from the later Ionian systems. Hence, at this point in Alexander's summary we find the intrusion of Empedocles' four 'elements' and Heraclitus' doctrine of complete transformation. On the other hand, the mathematical chapter is not affected by changes in opinion about the constitution of matter and the causes of physical change. It is only open to criticism on mathematical grounds, such as Zeno's arguments turning on the dilemmas of discrete or continuous quantity. The consequent modifications, traceable, for instance, in Plato's scheme, do not alter its main outline. This reappears in Sextus' account of Pythagorean doctrine, underneath the superficial changes chiefly due to Plato.

The Pythagorean physicists quoted by Sextus argued that the first principles must be not only imperceptible, like the atoms of Epicurus, but also incorporeal. But not all the incorporeal things that are prior to bodies are ultimate elements. Thus the solid figures, though prior to bodies in conception, are themselves reducible to planes, and these again to lines, and lines to numbers; for, as drawn from point to point, a single line involves the number 2. Finally 'all numbers fall under the One, since 2 is a *single* 2 and 3 is *one* particular thing, and 10 is a *single* compendium of number'. Moved by these considerations Pythagoras declared that the Monad is the first principle of things, by participation in which each several thing is called one. This principle is conceived, in its self-identity, as a Monad; but when added to itself, in respect of its otherness (*καθ' ἐτερότητα*) it creates the Indefinite Dyad. These, then, are the two principles of things (*adv. phys.* ii, 255-262). After some illustrations of the various forms in which the contrariety of these two principles is manifested, Sextus states the synthesis balancing the above analysis. 'Thus, as the highest principles of all things have emerged the primary Monad and the Indefinite Dyad; and from these, they say, arise the unit of number and the number 2. The unit comes from the primary Monad; the number 2, from the Monad and the Indefinite Dyad; for 2 is twice 1, and

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before 2 existed 'twice' did not exist but was taken from the Indefinite Dyad. Thus 2 came from both principles. And in the same way the remaining numbers were produced from them, the unit always acting as limit while the Indefinite Dyad generates 2 and extends the numbers to infinite plurality. In the same way they construct the cosmos and all that it contains. The point is ranked under the head of the unit; for both are indivisible, and just as the unit is a starting-point in numbers, so is the point in lines. The line is regarded as corresponding to 2, since both are conceived by way of transition, and again, the length without breadth conceived as lying between two points is a line. The plane corresponds to 3, for the plane is not regarded as mere length, like 2, but has taken to itself, in the third place, the dimension of breadth. When three points are set down, two at an interval opposite to each other and the third over against the middle of the line formed by the two, but in another dimension, the result is a plane. And the solid figure or body, such as the pyramid, is ranked under 4; for when another point is placed above the three set down as above described, the result is a pyramidal figure of solid body' (*ibid.* 276-280). After noting that this method of building up the solid by adding one unit-point to another is earlier than the conception of the single point 'flowing' into one dimension after another, Sextus concludes: 'In this way, with numbers taking the lead the solid bodies are produced; and from these finally sensible bodies also: earth, water, air, fire, and in general the cosmos. This, they say, is ordered according to a musical scale. Here, once more, they hold fast to the numbers which contain the ratios of the consonances composing the complete scale: the fourth, $\frac{4}{3}$, the fifth, $\frac{3}{2}$, and the octave, $\frac{2}{1}$ ' (*ibid.* 283).

Whatever may be the date of Sextus' immediate authorities, it is clear that the scheme of the mathematical chapter remains substantially unchanged. But by Sextus' time it was realised that the evolution was a logical process, not one that ever took place in time. The first Pythagoreans, on the other hand, certainly conceived of the development from the first unit of number to a plurality of units as a physical process in actual space. We know this from certain brief references in Aristotle to a Pythagorean cosmogony which cannot be much later than the end of the sixth century. At *Met.* 1091a, 12, Aristotle complains of Pythagoreans and Platonists for representing numbers as 'generated'; there can be no generation of eternal things. 'As for the Pythagoreans, there can be no doubt that they do represent them as generated; for they openly say that when the unit had been constructed, whether out of planes or surface (*χρoιας*) or out of seed or from things they are

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at a loss to describe, immediately the nearest parts of the Unlimited began to be drawn and limited by the Limit.' That this is not merely a generation of numbers but also a physical process of cosmogony is plain from Aristotle's next words, which declare that 'they are describing the making of a cosmos and mean what they say in a physical sense'. Accordingly he dismisses the subject as belonging to physics, not to metaphysics. As we have seen, if sensible bodies simply are numbers, pluralities of units which are themselves atomic magnitudes, then the generation of numbers from a single unit is the same thing as the generation of sensible bodies from a single atom.

The difficulty we feel in identifying the two processes becomes considerably less if we think of the evolution as having happened once for all and produced a cosmos which is everlasting and will never be destroyed and rebuilt. This was not so in the Ionian systems. For them cosmogony was a purely physical process of change; a world would at some moment begin to evolve, last for a time, and then perish to be replaced by another. In the Italian tradition there is not this succession of worlds. The world is one and everlasting.¹ So we have not to think of the generation of numbers as occurring again and again. The cosmogony is like Plato's in the *Timaeus*, where there is the same possibility of doubt whether (as most authorities hold) the cosmos never had a beginning at all, or order was created out of disorder once for all 'at the beginning'.

In Aristotle's description of the cosmogonical process there are two stages: (1) the formation of the first unit; (2) the subsequent 'drawing in' of the Unlimited, which is progressively limited by the Limit, so as to produce more and more units. Other notices, presently to be quoted, leave no doubt that the Unlimited in this system is the 'boundless breath' which is also called 'the void'. This extends outside the limited world, which, as a living creature, breathes it in.² It unmistakably corresponds to the boundless Air of Anaximenes, that breath or air which encompasses the whole cosmos and is compared to the human soul, which is also air.³

(1) Aristotle complains that the Pythagoreans 'seem at a loss to

¹ Ar., frag. 201R, τὸν μὲν οὐρανὸν εἶναι ἕνα. Burnet's suggestion that Pythagoras probably believed in a plurality of co-existent worlds (*E.G.P.*, 109) is baseless and contradicts such evidence as we have. Zeller (17, 550) regards it as certain that the Pythagorean cosmos was never destroyed.

² Aet., ii, 9, 1, οἱ μὲν ἀπὸ Πυθαγόρου ἐκτὸς εἶναι τοῦ κόσμου τὸ κενόν, εἰς δ' ἀναπνεῖ ὁ κόσμος καὶ ἐξ οὗ. Ar., *Phys.* 203a, 6, οἱ μὲν Πυθαγόρειοι ἐν τοῖς αἰσθητοῖς (οὐ γὰρ χωριστὸν ποιοῦσι τὸν ἀριθμὸν) καὶ εἶναι τὸ ἕξω τοῦ οὐρανοῦ τὸ ἄπειρον. Πλάτων δέ...

³ Anaximenes, frag. 2.

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describe how the first unit was constructed so as to have magnitude'.¹ Obviously this is the first unit of number conceived as for the first time (if the process is temporal) formed or composed so as to have magnitude and occupy a position in space. Aristotle found a difficulty in understanding how and of what elements such a unit could be formed. He offers two suggestions, which should not be brushed aside as baseless conjectures. They must have been prompted by known features of the system.

The first is that this unit was composed of planes or surface (*γοιὰ* is the Pythagorean word for *ἐπιφάνεια*, the visible coloured superficies or boundary, *πέρας*, of a physical body, *de sensu* 439a, 33). A unit composed of planes is a solid; and, as we have seen, the minimum solid is the pyramid, consisting of four unit-points and having four equilateral triangular surfaces.

The second suggestion is that the constituents of this first unit with magnitude might be 'seed' (*σπέρμα*). This biological conception fits the notion of the world as a living and breathing creature, which, like other living things, would grow from a seed to its full form. It also fits in with the position of the male principle under Limit, the female under Unlimited, in the Table of Opposites, and the statement that 'the beginning of numbers is the first unit, which is male and like a father begets all the other numbers; while the number 2 is female, also called the even'.² This imagery survives even in the *Timaeus* (50D), where the Form is compared to a father, the Recipient (space) to a mother, and the nature that arises between them to their offspring. We find it also in one interpretation of the *tetractys*: 'The sixth *tetractys* is of things that grow (*τῶν φνομένων*): the seed is analogous to the unit and point, growth in length to 2 and the line; growth in breadth to 3 and the surface; growth in thickness to 4 and the solid' (Theon, p. 97). Aristotle himself seems to refer to the identification of unit and seed, where he asks 'Does number come from its elements as from seed?' and objects that 'nothing can come from that which is indivisible'.³ Indivisibility is the essential characteristic of the arithmetical unit. This view could be combined with the previous suggestion. The four units composing the pyramid might be regarded as 'seed', if the living world is to grow from this first body into all three dimensions.

It is still, perhaps, necessary to insist that too little attention is paid by historians of early philosophy to traditional images like this

¹ Met. 1080b, 20, ὅπως δὲ τὸ πρῶτον ἐν συνέστη ἔχον μέγεθος ἀπορεῖν εἰκόσκειν.

² Hippol., Ref. I, 2, 6 (*Dox.* 556).

³ Met. 1092a, 32, ἀλλ' ὡς ἀπὸ σπέρματος; ἀλλ' οὐχ οὐδὲν τε τοῦ ἀδιαίρετου τι ἀπελθεῖν.

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of the father, the mother, and the seed. They are preserved in poetry long after philosophers of the more prosaic sort have discarded them and grammarians have come to treat them as mere arbitrary 'metaphors'. They are in fact survivals from a time when they were the only language available for speculation and were much more literally meant than we imagine. It is a commonplace of analytical psychology, confirmed by daily experience, that they still remain as the language of dreams. That is why they are charged with emotion in the poetry which preserves them, and also why the modern poets who renounce them and rack their brains to invent images never used before fail to produce the proper effect of poetry. The Greek poets thought otherwise. There is much light to be gained from a study of their traditional store of so-called metaphors, on the assumption that they embalm the philosophy of pre-scientific ages.

The next point is that there is ground for connecting the first unit with fire. Ross (on *Met.* 1091a, 15) illustrates Aristotle's phrase 'the first unit constructed so as to have magnitude' from Philolaus, frag. 7: 'the first thing formed, the unit (τὸ πρῶτον ὁμοιοῦν, τὸ εἶν) in the midst of the Sphere is called Hestia', and frag. 17: 'the cosmos is one and it came into being from the centre'. In the astronomical system attributed to Philolaus this central Hearth of the universe has become an independent body round which all the other heavenly bodies, including the Earth, revolve. In the earlier Pythagoreanism and in Alexander Polyhistor's summary the Earth was still in the centre.¹ But it has been argued by Hilda Richardson² that 'the earliest generations of the Pythagorean school conceived of fire as existing at the heart of their central, spherical earth. It was only the separation of this fire from the earth and the conversion of the earth into a planet that was late'. She adduces Simplicius' statement that, as opposed to the Philolaic system, the 'more genuine' Pythagorean doctrine was that of a fire in the midst of the earth, endowing it with life and heat.³ Hestia and Earth are already identified in Sophocles (frag. 615, Pearson) and Euripides (frag. 944, N²), and 'it may be considered at least probable that this identification, whoever was responsible for it, was partly due to the conception of the earth as containing fires within itself'—a notion to which volcanoes and hot springs would

¹ Cf. Burnet, *E.G.P.*², 111 and 297 ff.

² *C.Q.* xx (1926), p. 119.

³ Simplicius, *de caelo*, 512, 9. She points out that this is not to be regarded as a later modification of the Philolaic system on Zeller's ground that it implies a rotation of the earth (Zeller, ², 1, p. 420). There is no such implication.

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naturally give rise. She also cites Anatolius *On the Decad*,¹ according to whom the Pythagoreans held that 'a certain unitary fiery cube' is situated in the midst of the elements, and Parmenides, Empedocles, and others followed them in 'placing this monadic nature, like a hearth, in the centre'. Since both Parmenides' and Empedocles' systems were geocentric, this fiery unit could only be at the core of the Earth. In view of the identification of the Unlimited with Air or darkness, 'it seems certain', as Burnet remarks, 'that Pythagoras identified the Limit with fire'.² Miss Richardson, accordingly, concluded that the first unit with magnitude in our cosmogony was this fiery unit at the centre, round which the boundless mist or darkness has 'condensed to form the hard solidity of earth'.

(2) Cosmogony would thus begin with the formation of the first solid, probably a pyramid, the fiery seed from which the world is to grow. The next point is the nature of the process whereby the unit is multiplied. We have here three statements from Aristotle. The first has already been quoted:

'When the first unit had been constructed . . . at once the nearest parts of the Unlimited began to be drawn and limited by the Limit' (1091a, 15).

'The Heaven is one, and from the Unlimited it draws in upon itself time and breath or the Void, which keeps the places of individual things always distinct' (frag. 201R).

'The Pythagoreans also asserted the existence of the Void, and that there comes into the Heaven, from the unlimited breath which the Heaven breathes, the Void also, which keeps things distinct, the Void being regarded as a sort of separation or division between things that are next to one another; and this occurs first in numbers, for the Void keeps their natures distinct' (*Phys.* 213b, 22).

The last statement, about numbers, is intelligible if we remember that numbers are composed of atomic units, which are kept distinct

¹ Anatol., p. 30, Heiberg = *Vors.*², 28 [18], Parmenides, A, 44, περὶ τὸ μέσον τῶν τεσσάρων στοιχείων κείσθαι τινα ἑναδικὸν διάπυρον κύβον . . . τὴν μοναδικὴν φύσιν ἐστίας τρόπον ἐν μέσῳ ἰδρῦσθαι.

² *E.G.P.*², 109. There is a curious survival of this association in Ar., *de gen. et corr.* 335a, 15 ff.: Food is akin to matter; what is fed is the form taken with the matter. Hence fire, as the ancients say, is the only element that is 'fed'; for fire, alone or pre-eminently, is akin to form, because its natural tendency is to move to the boundary (ὅρον), in which the form or shape of things consists; and everything tends naturally to its own place.

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by intervals of vacancy (or air), as bodies which touch one another are kept from coalescing into one body. We have already seen how Aristotle describes the method by which the Unlimited is limited in the case of numbers—by adding units arranged in gnomons round 1 or 2. These patterns give a picture of the unit-points separated by blank intervals. The expansion of the pattern illustrates the multiplication of the units as more and more of the Void is drawn in.

The 'drawing in' of the unlimited breath has a close analogy in the medical theory of Philolaus,¹ who taught that our bodies are constructed from the hot and have no part in the cold. The seed, which constitutes the living creature, is hot, and so also is the womb, the place (τόπος) in which the seed is deposited. After birth the creature draws in the breath from outside, and this is cold. It is needed in order that the heat of the body may be cooled 'by the drawing-in of this imported breath'.² The analogy is so close that Frank³ attributes the cosmogony referred to by Aristotle to Philolaus himself; but against this there is the identification of the unlimited air with the 'void'—a feature which belongs to the beginning rather than to the end of the fifth century. Air was established by Empedocles as one elementary body on a level with the other three. As such it figures in Alexander Polyhistor's summary: 'from solid figures, sensible bodies; the elements of which are four: fire, water, earth, air'. In contrast with this, the earliest cosmogony has only two primitive factors: Fire or Light, associated with limit, and the dark Air, identified with unlimited void, the 'Night' of pre-scientific cosmogonies.

Alexander's summary, in spite of its mention of the four elements, retains traces of the original opposition of Fire and Air.

'The air about the earth is stagnant and unwholesome, and everything in it is mortal; but the uppermost air is always in motion, pure and healthy, and everything in it is immortal and so divine.⁴ Sun, moon, and stars are gods; for in them preponderates the Hot, which is the cause of life . . . Men have

¹ Anon. Lond. 18, 8 (= Vors.⁵, 44 [32], A, 27).

² τῇ ἐπεισάκτῳ τοῦ πνεύματος ὀκλή. Cf. Ar., Phys. 213b, 23, ἐπεισέναι αὐτὸ τῷ οὐρανῷ ἐκ τοῦ ἀπείρου πνεύματος ὡς ἀναπνέοντι καὶ τὸ κενόν. frag. 201, ἐπεισάγεσθαι δ' ἐκ τοῦ ἀπείρου χρόνον τε καὶ πνοήν καὶ τὸ κενόν. Mel. 1091a, 17, εὐθὺς τὸ ἔγγιστα τοῦ ἀπείρου εἴλετο καὶ ἐπεραίνετο ὑπὸ τοῦ πέρας.

³ Plato u. d. sog. Pyth. 326 ff.

⁴ Wellmann (Hermes, 1919, 244) notes the reminiscence of this at Phaedo 111 A, B, where the lower air in which we live is contrasted with the aether on the 'true surface' of the earth, where the climate is so tempered ὥστε ἐκείνους ἀνθρώπους εἶναι καὶ χρόνον τε ζῆν πολὺ πλείω τῶν ἐνθάδε.

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kinship with the gods because man partakes of the Hot; hence God takes thought for us . . . A ray from the sun penetrates through the 'cold aether' (as they call the air) and the 'dense aether' (as they call the sea and moisture). This ray descends even to the depths and thereby quickens all things. All things live, which partake of the Hot—that is why plants also are living creatures—but not all have soul. Soul is a detached part of both the hot and the cold aether, for it partakes also of the cold aether. Soul is distinct from life, and it is immortal because that from which it is detached is immortal' (D. L., viii, 26–28).

But if the unlimited Air, considered as the breath of the living world, corresponds to the Air of Anaximenes, it is important to note that its status is radically changed. Anaximenes made Air the ultimate 'nature' or substance of all things; but in the Pythagorean view the nature of things lies in the opposite principle of number, the units composing and bounding sensible bodies; the unlimited is rather the empty space not occupied by body but separating bodies and their parts. In the Atomism of Leucippus and Democritus, body is 'being', the void is 'not-being'. By that time the void had become sheer vacancy, no longer confused with air. But the idea that empty space is 'not-being' seems to appear already in Parmenides; and it follows that the identification of air and void must belong to the earliest Pythagoreanism which Parmenides was criticising.

Now, if the atoms can be identified with fire, and air is not a second element in their constitution but rather the vacancy which keeps them apart, this cosmogony agrees with a doctrine discussed by Aristotle in the *de caelo* iii, 5. That chapter criticises all who hold that there is only one primary body, and the latter part deals with those who make this body fire. Among these some simply regard fire as composed of the finest and smallest particles. Others give the fire-particles the shape of the pyramid because the pyramid is the sharpest of figures, as fire is the sharpest and most penetrating of bodies, and because all bodies are composed of the finest body and all solid figures of pyramids. Simplicius (621, 6), who has earlier mentioned Hippasus and Heraclitus, questions to what school this last theory should be assigned. Heraclitus did not say that fire was pyramidal; and 'the Pythagoreans, who do say that fire consists of pyramids, do not say that fire is the element of the rest, if (or since, εἴτε) they say that fire itself comes from water and air, as water and air come from fire'. This last feature, however, is not part of the theory as stated by Aristotle. It seems to be inferred by Simplicius from Aristotle's subsequent criticism.

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But this criticism is directed not solely against the pyramidal view, but generally against all who take fire as the one primary body. Aristotle says (304a, 21) that both types of theory are open to the same objections. If the primary body is indivisible (*ἄτομον*), the mathematical reasons previously urged against any atomic magnitudes will apply. Then follows an objection from the physical point of view (*φυσικῶς*). This appears to assume, as an ascertained fact, that fire, air, and water turn into one another, and that these changes involve increase or decrease in volume, which cannot be explained by the hypothesis of a void, because Aristotle has disproved the existence of void. These arguments may dispose of the theory, if all Aristotle's own assumptions are granted; but there is no reason to suppose that the Pythagoreans would have granted them: they certainly believed in atomic magnitudes and in the void. If we ignore the Aristotelian assumptions, the doctrine as stated will fit the primitive cosmogony we have been reconstructing. The pyramid is the minimum solid and the fiery atom. The generation of numbers and of a plurality of bodies will be the multiplication of the first fire atom; all bodies will be aggregates of such atoms, and fire will be the only elementary body. Air or void merely keeps the units distinct. Water and Earth could be obtained by packing the atoms more closely with less void between them—a conception resembling Anaximenes' rarefaction and condensation, and actually attributed to Hippasus and Heraclitus by Theophrastus.¹ The attribution to Heraclitus is, no doubt, mistaken; but the device is appropriate to an atomic theory.² Even Plato, who goes as near as possible towards eliminating the void, invokes larger or smaller interstices to explain differences of weight and density.

We have now traced the whole process leading to the existence in actual space of a plurality of sensible bodies. We are given no further details that can with equal probability be assigned to the earliest form of the system. Theophrastus has a general complaint against the Pythagorean-Platonist tradition that it confines attention to the ruling principles.

¹ Theophr., *Phys. Op.* 1 (Dox. 475), 'Ἰππασος . . . καὶ Ἡράκλειτος . . . πῦρ ἐποίησαν τὴν ἀρχὴν καὶ ἐκ πυρὸς ποιοῦσι τὰ ὄντα πύκνυνται καὶ μανύσονται καὶ διαλύονται πάλιν εἰς πῦρ ὡς ταύτης μιᾶς οὐσίας φύσεως τῆς ὑποκειμένης.

² As Aristotle observes (*Met.* 988b, 34), the most elementary thing would be the primary thing from which others are produced by combination (*συγκρίσει*); and this property would belong to the finest and subtlest of bodies; hence this account of how other things are produced would best fit the doctrine that fire is the first principle. *πύκνυνται* is regarded as reducible to *σύγκρισις*, *Phys.* 260b, 11.

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'Starting from this first principle or principles, one might demand that they should at once go on to give an account of the successive derivatives, and not proceed to a certain point and then stop; for this is the part of a competent and sensible man, to do what Archytas once said Eurytus did when he arranged certain pebbles and explained that this is the number of man, this of horse, and this of something else. But, as it is (*ῥῶν*), most of them go to a certain point and then stop, as those do who set up the One and the Indefinite Dyad: after generating numbers and planes and bodies they leave out practically everything else, merely touching on other things and explaining no more than that some things proceed from the Indefinite Dyad, e.g. place, the void, the unlimited, others from numbers and the One e.g. soul and some other things; and they generate time and the heavens simultaneously and several other things, but of the heavens and the remaining things they make no further mention' (*Met.* 6a, 15).

This passage may refer specially to the Pythagoreans, since Speusippus, Xenocrates, and Plato himself are separately mentioned later.¹ Could anyone who had read the last fifty pages of the *Timaeus* accuse Plato of 'making no further mention of the remaining things'? But the general impression left by Aristotle's notices is that the earliest Pythagoreans were not concerned with a detailed study of nature, or the meteorology of the Ionians. 'They have said nothing about fire or earth or the other bodies of that sort, I suppose because they have nothing to say which applies peculiarly to perceptible things' (*Met.* 990a, 16). They were interested in 'the many resemblances they seemed to see in numbers, rather than in fire or earth or water, to the things that are and come to be (such and such a property of numbers being justice, another soul or reason, another opportunity and so on with practically everything else); and moreover they saw that the properties and ratios of the musical scales are expressible in numbers. Since, then, all other things seemed in their nature to be modelled after numbers, and numbers to be the first things in the whole of nature, they supposed the elements of number to be the elements of all things, and the whole Heaven to be a musical scale or number. And all the properties of numbers and scales which they could show to agree with the

¹ 'Place, void, the unlimited' is a description which suits the Pythagorean Unlimited better than Plato's Space, which was not void. 'Time and the heavens simultaneously' is true of Plato; but Aristotle speaks of time as entering the Heaven with 'breath and void' from the Pythagorean Unlimited (*frag.* 201).

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attributes and parts and the whole arrangement of the Heaven, they collected and fitted into their scheme' (*Met.* 985b, 27).

Here it may be noted that these 'resemblances' (*ὁμοιωματα*) between things like Justice and the properties of numbers explain why Aristotle sometimes says that things represent (*μυεῖσθαι*) numbers, rather than simply *are* numbers. A sensible body, as we have seen, can be said to *be* the unit-atoms composing it; but if a man says that 'Justice is the square number' he cannot mean that Justice is a plane figure composed of four unit-points; obviously he means that the square figure is a symbol which represents or embodies the nature of fairness, just as when an honest man was called 'four-square without reproach', no one imagined that his figure really had four corners. The two modes of describing the relation of things to numbers are perfectly compatible, being respectively appropriate to different orders of 'things'.

Shortly after this passage comes the statement about the elements of number and the generation of numbers from the unit, ending 'and numbers, as we said, are the whole Heaven'. The world-order, *cosmos*, in which cosmogony terminates was not conceived, as by the Ionians, as the arrangement of the four great concentric masses of earth, water, air, and fire. The Pythagorean sciences are arithmetic, geometry, astronomy ('sphaeric'), and music, the sciences which discover the element of number, measure, proportion, in the cosmos and are studied in order to bring the soul into harmony with the objects of its contemplation. Accordingly for them the visible world is not Anaximander's battlefield in which the warring opposites perpetually encroach on one another's provinces and pay the penalty of their injustice. Rather it is the harmonious disposition of earth and the heavenly bodies according to the intervals of the musical scale. The same sciences in Plato's scheme of higher education lead to the same end, the assimilation of the soul to principles of symmetry and concord. As Socrates says earlier in the *Republic* (500b): 'One whose thought is set on reality will not have leisure to look downwards upon the field of human interests, to enter into the strife of men and catch the infection of their jealousies and feuds. His eyes are fixed upon an unchanging order; the things he contemplates neither inflict injustice nor suffer wrong, but observe due proportion and order; and of these he studies to reproduce the likeness in himself as best he can. A man cannot fail to imitate that with which he holds converse with wonder and delight. So the philosopher, holding converse with the divine and orderly, becomes, so far as man may, both orderly and divine.'¹

¹ The original sense of *cosmos* was social and political: the 'right order' of a state, army, or other group (cf. W. Jaeger, *Paideia* i, 108, E.T.). This

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The Ionian 'inquiry into the nature of things' had no bearing on conduct and no point of contact with politics. But Pythagoras, as Plato remarks in the only passage where he mentions him by name, was pre-eminently valued for his private converse with his disciples, to whom he bequeathed a 'way of life' which marked them out from the rest of mankind (*Rep.* 600b). This way of life was characterised by Aristoxenus: 'Every distinction they lay down as to what should be done or not done aims at converse with the divine. This is their first principle, and their whole life is ordered with a view to following God!' (ap. Iambl., *V. P.* 137). The 'following' or 'imitation' of the divine has been variously construed in different religious systems. It is probable that the Pythagorean construction is faithfully reproduced in the *Timaeus* (90b):

'If a man is engrossed in appetites and ambitions and spends all his pains upon these, all his thoughts must needs be mortal and, so far as that is possible, he cannot fall short of becoming mortal altogether, since he has nourished the growth of his mortality. But if his heart has been set on the love of learning and true wisdom and he has exercised that part of himself above all, he is surely bound to have thoughts immortal and divine, if he shall lay hold upon truth, nor can he fail to possess immortality in the fullest measure that human nature admits; and because he is always devoutly cherishing the divine part and maintaining the guardian genius (*daemon*) that dwells with him in good estate, he must needs be happy (*eudaimon*) above all. Now there is but one way of caring for anything, namely to give it the nourishment and motions proper to it. The motions akin to the divine part in us are the thoughts and revolutions of the universe; these, therefore, every man should follow, and . . . by learning to know the harmonies and revolutions of the world, he should bring the intelligent part, according to its pristine nature, into the likeness of that which intelligence discerns, and thereby win the fulfilment of the best life set by the gods before mankind both for this present time and for the time to come.'

In this passage Plato shows how the life of religious and moral aspiration was identified with the pursuit of truth about the order of the world. Philosophy is the achievement of immortality. The goal is attained by purifying the soul of lower desires and worldly ambitions, so as to set free the divine part to apprehend the harmony of the *cosmos*, and reproduce it in the harmony of the microcosm.

conception was first projected into external Nature, and then rediscovered there and set up as a pattern to be reproduced in socialised humanity.