

LINEAR STRUCTURAL PATTERNS BASED ON AN ARRAY OF NUMBERS DERIVED FROM THE DOUBLE-ENTRY MULTIPLICATION TABLE [of Pythagoras]

Juan Luis Diaz* with Merry MacMasters**

1.

I became interested in the double-entry multiplication table (Fig. 1, left) in 1972 when a student in my sculpture workshop at the Academia de San Carlos (Mexico City) showed me a single-digit form of it (Fig. 1, right) with the numbers to be multiplied limited to integers in the range 1 to 9. Except for the 3×3 array in the upper left-hand corner, it does not look like the ordinary double-entry multiplication table because products, when composed of two digits (i.e. from 10 to 81), have been replaced by a one-digit number. Two-digit products are reduced to one-digit numbers by the following procedure: for the multiplication $4 \times 6 = 24$, one adds 2 and 4 to get the digit 6; for $4 \times 7 = 28$, one adds 2 and 8 to get 10, and then one adds 1 and 0 to get 1.

The first part of my investigation of designs coincides with a study carried out by a group of English investigators [1]. My inquiries, however, have taken a course that allows me to find more design possibilities and then to find projections of these in three dimensions. This last step leads to two procedures: one is discussed below; the other, in which I convert the square into a cube and proceed to locate numbers within it to produce a spatial geometry, will be discussed in a forthcoming article.

right) to be the mirror image of the third one (lower left) and the second one (upper left) to be the mirror image of the fourth (lower right). Similarly, each diagonal divides the 8×8 array into two triangles in which the numbers are mirror images.

Also I found that if one adds the numbers in each column or in each row of the array and reduces the result to a single digit, as described above, the number 9 is obtained in each case. Thus, for the first row the sum is 36, which reduces to 9. Also, if in any row or column in the 8×8 array the first number is added to the last, the second to the second but last (seventh), the third to the third but last (sixth), and the fourth to the fifth, each sum is 9.

The last observation led me to the idea of calling these couples *complementary numbers*. Thus, 1 is the complementary number of 8; 2 of 7; 3 of 6; 4 of 5; and 9 of 0. I employ the term 'complementary' because I note a parallelism with its use in the case of complementary colors as presented by Alfred Hicethier in his system of color classification [2]. In the method of design that I conceived on the basis of the above observations and that I shall now discuss, I employ the 8×8 array of numbers.

2.

I became intrigued by the patterns of the numbers in the square array (Fig. 1, right). I noted that if the column of nines (right) and the row of nines (bottom) are eliminated, the 8×8 array of numbers, when divided into four quadrants of 16 numbers, shows the first quadrant (upper

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In Fig. 2(a) are shown points for the locations of the number 1 in the 8×8 array (Fig. 1, right) and lines connecting each point to the others. In Fig. 2(b) the pattern of points and lines is shown for 8, the complementary number of 1. Fig. 2(c) shows Figs. 2(a) and 2(b) superimposed. Fig. 2(d) shows the superimposition of these points, but in this case each point is connected by lines only to points of an unlike number. Patterns of the points and lines for 3 and 6 and combinations of their patterns are shown in Figs. 2(e) to 2(h). The patterns for 2 and 7, 4 and 5, and 9 can readily be constructed.

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1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	2	4	6	8	1	3	5	7	9
3	6	9	12	15	18	21	24	27	3	6	9	3	6	9	3	6	9
4	8	12	16	20	24	28	32	36	4	8	3	7	2	6	1	5	9
5	10	15	20	25	30	35	40	45	5	1	6	2	7	3	8	4	9
6	12	18	24	30	36	42	48	54	6	3	9	6	3	9	6	3	9
7	14	21	28	35	42	49	56	63	7	5	3	1	8	6	4	2	9
8	16	24	32	40	48	56	64	72	8	7	6	5	4	3	2	1	9
9	18	27	36	45	54	63	72	81	9	9	9	9	9	9	9	9	9

Fig. 1. Left: The double-entry multiplication table. Right: The double-entry multiplication table presented in single-digit form.

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2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
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3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
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Fig. 1. Left: The double-entry multiplication table. Right: The double-entry multiplication table presented in single-digit form.

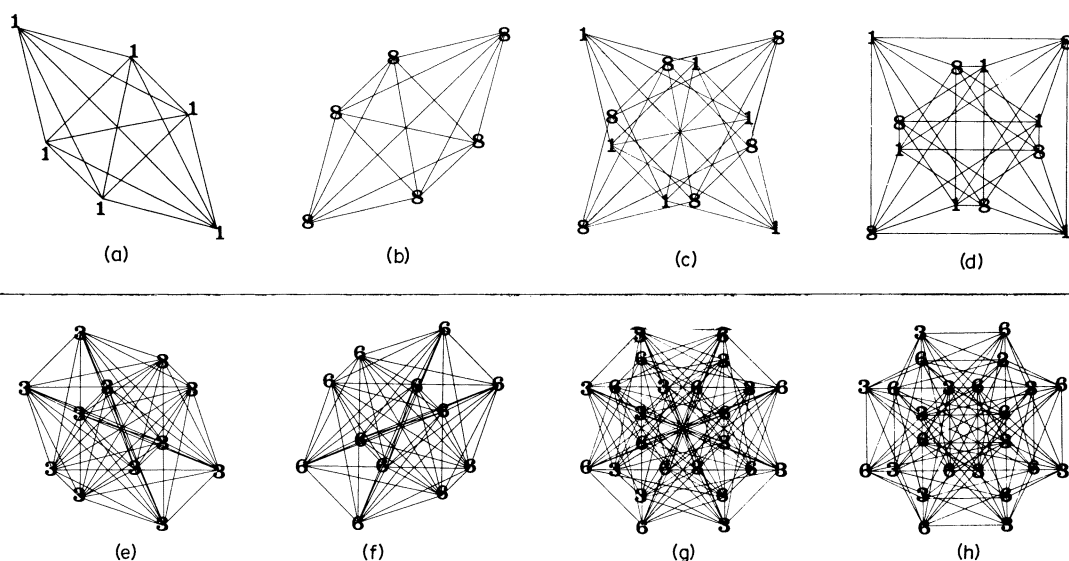


Fig. 2. Some linear patterns obtained by connecting points representing like or complementary numbers in the 8×8 array in Fig. 1 (right).

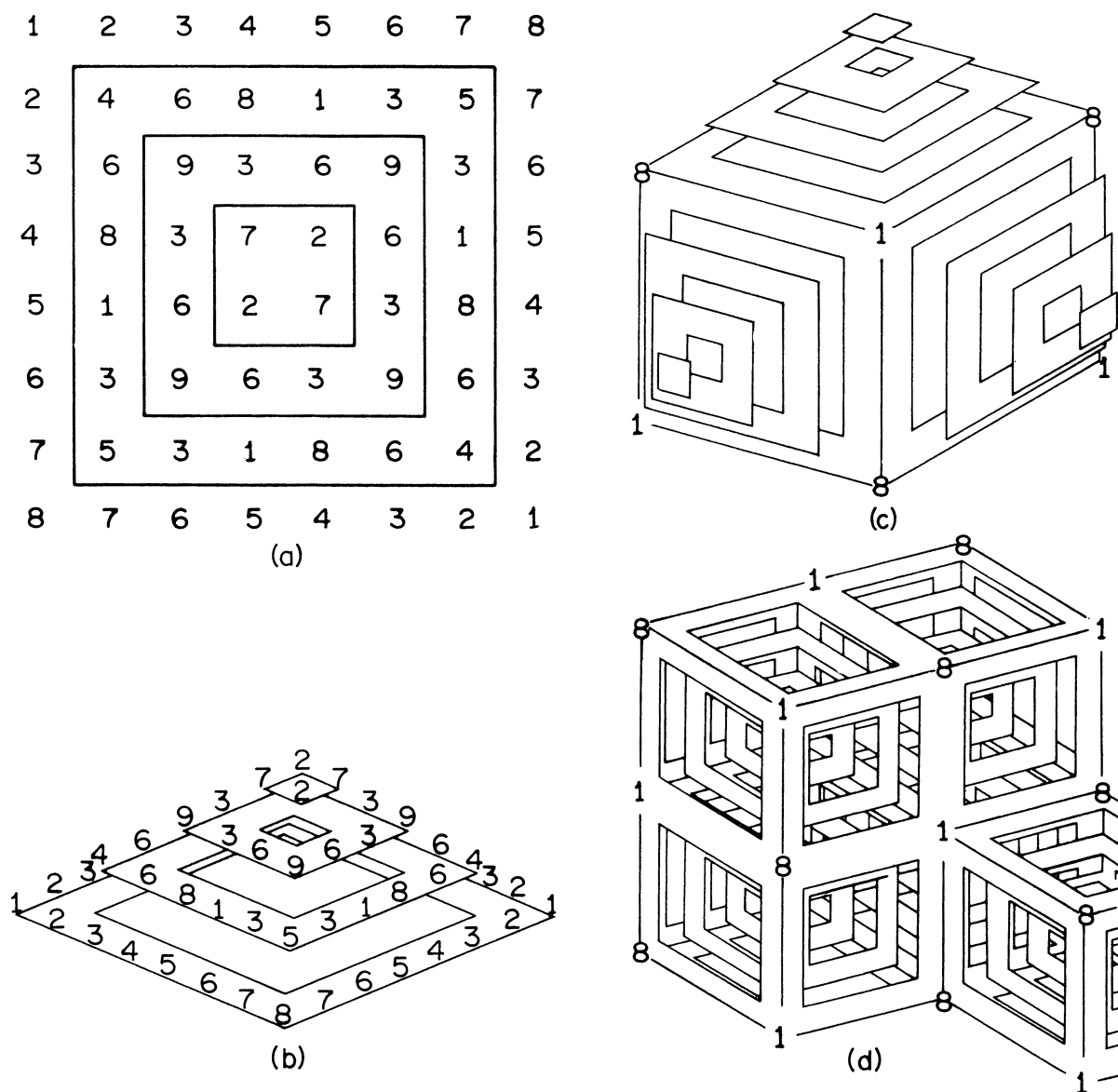


Fig. 3. (a) The 8×8 array divided into concentric frames. (b) The concentric frames expanded into a pyramidal form. (c) A 3-dimensional structure utilizing the convex pyramidal form. (d) A 3-dimensional structure utilizing the concave pyramidal form.

In my design method which I call the *transmutations system*, I utilize patterns such as those shown in Fig. 2. This system is, however, not limited to the series of superimposed pairs of patterns of points and lines for complementary numbers, that is, those that add to 9. It includes combinations such as the superimposed patterns for the pairs 1 and 4, and 2 and 8. Among these, one can distinguish families of patterns; for example, the family of 1 comprising the pair 2 and 8 (whose digits add up to 10 but reduce to 1); the pair 3 and 7; the triplet 5, 2 and 3; the triplet 6, 1 and 3, etc. Families may also be complementary according to my terminology, such that the family of 1 is complementary to the family of 8.

4.

The patterns of lines produced as described above can be used as the basis of 2-dimensional work such as paintings and drawings. To provide a 3-dimensional basis for the design of sculpture and architectural forms, it was necessary for me to find a way to arrange the points in space. I did this by dividing the 8×8 array of numbers into concentric frames (Fig. 3(a)) and by separating the frames by equal amounts to form a pyramid (Fig. 3(b)) or, when viewed from underneath, a concave pyramidal form. Illustrations of applications of these convex and concave forms are given in Figs. 3(c) and 3(d).

Two 3-dimensional models based on the application of my system are shown in Fig. 4. They were made using the pattern for the complementary numbers 3 and 6 (Fig. 2(h)) as the starting point, where the points of unlike numbers are interconnected. The upper model is the structure standing on one edge to show its top appearance which permits comparison with Fig. 2(h). In the lower model, the structure, with its convex surface up, is viewed downward at about 45° . It is a roof-type structure and differs from the upper model only in that certain connecting struts, deemed to be structurally superfluous, have been eliminated.

5.

Using my design system, I have constructed a number of models of roofs (such as the lower model in Fig. 4), bridges and towers, and I have made some sculptures and 2-dimensional paintings. My work has been subsidized by Ingenieros Civiles Asociados (I.C.A.) of Mexico City. I have been able to consult with their engineers and have received help in making the models. Also, I have had access to their computer. With the use of a computer program for my system, I have been able to obtain plotter drawings of structures showing plan, elevation and perspective. Many of my designs depicted in this way are

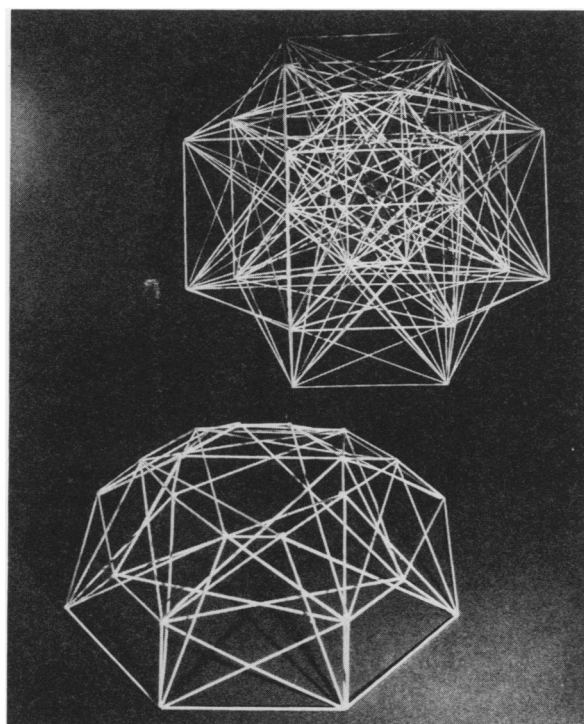


Fig. 4. Models utilizing the convex pyramidal form and Fig. 2(h). (Photo: J. Shulman, Los Angeles, Calif., U.S.A.)

being studied for possible architectural and engineering applications.

Although the double-entry multiplication table has led me into a domain that I had not envisaged, I feel certain that in the past it has served in a similar way, and I should not be surprised if it were found to be of importance in the design of architectural and engineering structures, in visual art and even in music. Through my urging, Felipe Hartasanchez in Mexico City has begun studying the application of my system to music. Recently, Barbara Hero derived a musical scale from the double-entry Pythagorean table to serve as a basis for the composition of her paintings [3]. The Pythagorean table presents the products of integers (1,2,3, . . .) and the reciprocals of integers ($1/1$, $1/2$, $1/3$, . . .).

References

1. K. Albarn, J. M. Smith, S. Steele and D. Walker, *The Languages of Pattern* (London: Thames & Hudson, 1974).
2. A. Hickethier, *El Cubo de los Colores* (Paris: Editorial Bouret, n.d.).
3. B. Hero, Paintings Based on Relative Pitch in Music, *Leonardo* 8, 13 (1975).