

Minor, Syria and Egypt. Since a polytheistic religion cannot deny the valid claims of foreign deities for recognition, it was impossible for Roman religion to expand into a religion of the Empire or to resist the steady infiltration of cults and philosophic theories that were alien to the Roman heart and mind. Confused within and buffeted from without by these conflicting concepts, it lost its determinate hold on the religious beliefs of the Roman populace. Yet, when it finally succumbed to the spiritual teachings of Christianity, it asserted its true nature by imposing on these a wealth of traditionally Roman cult and ceremony.

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Euclid and Pythagoras

There is divinity in odd numbers.—*Sir John Falstaff*
"There is luck in odd numbers." This saying, considered as superstition by some and as religious dogma by others, has had a long history in many different nations. It was current among the Greeks; Vergil quotes it in his *Eclogues* (8.76): *numero deus impari gaudet*. Our own language offers many examples, like this engaging quatrain from the Irish novelist, Samuel Lover:

"Now, Rory, leave off, sir; you'll hug me no more;
That's eight times to-day that you've kissed me before."
"Then here goes another," says he, "to make sure,
For there's luck in odd numbers," says Rory O'More.

Light is thrown on the explanation of this belief by a study of the relation between Euclid and Pythagoras. The contents of Euclid's *Elements* were determined to a great extent by the mathematical discoveries of Pythagoras and his school. Proclus goes so far as to say that Euclid's whole thirteen books were devoted to explaining the five "divine solids of Plato," first investigated by Pythagoras. But this is a one-sided view of the relation between the two men. The *Elements* contain a great deal beside solid geometry. Books VII, VIII and IX deal with the theory of numbers considered as falling into two classes, the prime numbers, 2, 3, 5, 7, 11 . . . which are not the product of other numbers, and the composite numbers, 4, 6, 8, 9, 10 . . . each of which is divisible by one or more primes. The question arises whether the list of prime numbers ever comes to an end. Are there infinitely many primes or is one of them bigger than all the others? Theorem 20, Book IX, proves that no matter how far we go along the list of primes, there still remain others. There is no biggest prime. For if there were, the absurd result would follow that we could find a composite number not divisible by any prime! To find such a number we would need only to multiply together all the primes up to and including our biggest prime and add to this result the number one. The large number so obtained would be composite, since larger than the biggest prime, and not divisible by

any prime since, whenever we tried to divide it by a prime, we would get a remainder of one. This important theorem has always been held in honor by mathematicians. Let us for our part try to see its connections with the religious doctrines of the Pythagoreans.

One of the most noticeable features of these doctrines is the emphasis laid upon the fact that every number is either even or odd (see, e.g., Aristotle, *Metaph.* 1.5.986a 17; τοῦ δὲ ἀριθμοῦ στοιχεῖα τό τε ἄρτιον καὶ τὸ περιττόν). Pythagoras asserted, moreover, that the odd numbers are superior to the even ones (e.g. Ps. Plut. v. Hom. 145: τῶν ἐφείητε ἀριθμῶν τὸν μὲν ἄρτιον ἐνδεὲς καὶ ἀτελὲς, τὸν δὲ περιττὸν πληρὴν τε καὶ τέλειον) and he explained this assertion by saying that odd numbers do not allow themselves to be divided into two equal parts (ibid: δαιρέσειν οὐκ ἐπιδέχεται). It is clear that his belief in the superiority of odd numbers must have its roots deep in human nature, for it has been held by different races over the face of the globe. And we shall see that he has given it the right explanation.

If odd numbers owe their superior qualities to the fact that they cannot be divided into two equal parts then the prime numbers must enjoy a very great superiority, since they do not allow division of any kind. Pythagoras used to represent a number with a row of dots and then experiment with the ways in which these dots could be rearranged to form a rectangle. He was much impressed by the distinctive character of any row which represented a prime number; it could not be formed into a rectangle at all. In a usually misunderstood passage of the *Metaphysics*, Aristotle mentions the importance which Pythagoras attached to the fact that every number except the primes could be formed as the product of two numbers. The odd numbers are better than the even numbers because they are not divisible by two; the prime numbers are the best or "oddest" of all, because they are not divisible by any number.

Beyond comparison, the most "sacred" of all the prime numbers is seven. To what does this number owe its peculiar position? What religious experience led the Hebrew people to the feeling that it is good to rest one day in seven rather than one in eight or six? An explanation for the Pythagorean worship of the number seven is given by Philolaus, a contemporary of Plato. His answer is of interest to us since our week still has seven days; there can be no doubt that it gives an accurate and complete explanation. He says that seven is the most sacred number because it is the largest prime number less than ten (see, e.g., Joannes Lydus, *De Mens.* ii.11, p. 72). It is natural to take ten as a limit since, having ten fingers, we count up to ten and then begin over again (cf. *ἑνδεκα, δώδεκα* etc.), while the importance of having a prime can be seen in the following way. Suppose that our week had not seven days but six. Then the impressiveness to our minds of the first

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or holy day would be lessened by its having a rival, so to speak, in the fourth day, seeing that each of them would stand at the beginning of a half week of three days. The same trait of the human mind can be observed in music. In three-quarter time the first of the three notes in a bar is the only one to receive an accent; but in six-eight time the fourth note also receives some accent because of an inevitable though perhaps subconscious realization on the part of both performer and listener that it stands at the beginning of the second half-bar. In a week of six days the unavoidable singling out of the fourth day would give it a psychological importance which would detract in an undesirable way from the commanding position of the first or holy day. The number of minutes in an hour is sixty, a number chosen simply because it is far from being prime, seeing that a half-hour, a quarter-hour, etc. all contain an exact number of minutes. But if the beginning of a new hour had the same religious significance as the beginning of a new week, its importance on the face of the clock would be detracted from by the presence of certain other periods which naturally stand out above the rest, namely the half-hour and the quarter-hours.

It was because Pythagoras was interested in questions like these that the Greek mathematicians invented those theorems about the divisibility of numbers which make up Books VII, VIII and IX of Euclid's Elements. In the theorem quoted above, about the infinitude of the number of primes, mathematicians have always recognized a cornerstone of the theory of numbers, a stone on which they have erected a magnificent edifice devoted to the theory of distribution of primes. But from our point of view, as students of Greek religion, we may also honor this theorem as the one in which Euclid proves that God has created an inexhaustible supply of the best, the most divine, the "oddest" of all numbers, the primes.

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The Sanctity of the Cabbage

The fragment of Hipponax 37D=40B is generally considered obscure and difficult. Athenaeus (9.370) quotes our passage as well as the fg. Ananiaios 3D=4B as evidence for the holiness of the plant, of which no doubt seems possible. In the same place Athenaeus states that cabbage was used (the expression *παρεσκευάζετο* means 'was cooked') to protect lying-in women against witchcraft. In the Hipponax passage the sanctity appears to be reinforced by the seven leaves of the plant, for which, however, I can find no botanical evidence (cf. W. Roscher, *SAWA* 1906, 41). The difficulty of the fragment lies in lines 2 and 3: *ἢ θύεσκε Πανδώρα* (Schwenn RGVI XV 36.5 reads

Πανδώρα without explanation, perhaps because he identified Pandora with Anesidora the earth goddess) *θαργηλίσσιν ἐγχυτον πρὸ φαρμακοῦ*. The question is whether Pandora sacrifices 'enchyton,' a decoction or soft cake: the word seems to be used only as an adjective; it could here be an adverbial accusative or it may be used as a noun. Thus the dative *ῖ* would be instrumental. But in that case it is hard to understand why the escaping culprit (?) or lover (?) should seek refuge with or implore the plant itself. It seems better, then, to take *ῖ* as the indirect object of *θύεσκε* (note the iterative), so that Pandora (a *hetaira*?) The name is not listed in RE 8 s.v. *Hetairai*.) offers a libation to the plant itself. Have we here a survival of the tree and plant cult? That Ananiaios swears by the cabbage might support this view, were it not for the fact that we know of the euphemistic substitution of animate and inanimate objects for the name of a god.

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The Anabasis and Katabasis in Eleusis

The Eleusinian festivals, celebrated in the days of Athenian greatness in honor of Demeter and Persephone, were held in Eleusis, a town of Attica not far from Athens. These rites, according to different authors in classical times and in the Christian era, had great influence on the religion of the ancient world.

Clement of Alexandria, the most authoritative critic of this cult, called the Eleusinian mysteries a mystical drama. Some hostile writers have suggested that the mysteries were a dramatic representation of the experiences of Persephone and Demeter, showing their passage through Hades and the coming of a new birth and a new order. The Homeric Hymn to Demeter is the ancient evidence of this belief and this gives only a part of the story, and that part elaborated and adorned with poetic ornament. We must supplement the Hymn from other sources and then separate the essential parts of the myth, which are likely to be just those in which we can detect the symbolic language.

The celebration did not take place entirely in Eleusis, but Athens was the starting-point. Every year there were two celebrations, the Greater Eleusinia, the Katabasis, celebrated in the autumn, and the Lesser Eleusinia, the Anabasis, celebrated in the spring at Agrae on the banks of the Ilissus.

The Lesser Mysteries and the Greater had many points of similarity, even though the Lesser Mysteries were considered only as a prelude to the Greater (Scholiast on Aristophanes, *Plut.* 849: *ἔστι τὰ μικρὰ ὡς περ προκαθαρσις καὶ προάγγελος τῶν μεγάλων*). The Lesser Mysteries were presented on a much smaller scale, but initiation in the Lesser was generally required before the candidate could present himself for initiation into