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The Mathematical Contributions of Francesco Maurolico to the Theory of Music of the 16th Century (The Problems of a Manuscript)

TITO M. TONIETTI*

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Part I

1.1 Introduction

During the course of his long lifetime, the 16th-century mathematician Francesco Maurolico (1494–1575)¹ also took a constant interest in music. He even continued his musical studies after the age of 70 years. Three extant documents testify to this activity:

- (1) the *Manuscript latin Par. Lat. 7462* in the Bibliothèque Nationale in Paris, the edition of which we recently terminated (Maurolico 2000)
- (2) the *Musicae traditiones carptim collectae* contained in the *Opuscula Mathematica*, published posthumously at Venice in 1575 (Maurolico 1575a)
- (3) the manuscript of Christophorus Clavius, *F. M. Boetianae musicae compendium* in the Biblioteca Gregoriana in Rome (Clavius 200?).²

An occasional note can also be found in the *San Pantaleo* collection 115/32 fol. 51r, in the Biblioteca Nazionale in Rome (Maurolico 2001).

When Maurolico divided philosophy into various parts, in his *Grammaticorum rudimentorum libelli sex* of 1528, he wrote:

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Part I

1.1 Introduction

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Mathematica idest disciplinabilis comprehendit arithmetica, geometria, musica, et astronomiam ... Musica de vocum rationibus et qualitativibus. (Macri 1901, pp. 257–258)³

Also, in his division of sciences (in the preface to *De sphaera liber unus* of his *Opuscula Mathematica*), he again placed music at times under physics and at times under arithmetic, depending on the kind of classification accepted (Maurolico 1575a, p. 3).

Maurolico thus followed the tradition that had seen several of the leading figures of antiquity taking an interest in music: Plato, Aristoxenus, Euclid, and Claudius Ptolemaeus (Aristoxenus 1954; Euclid 1557; Ptolemaeus 1682; Platone 1994; Platone 1999).

By means of his table 4 of *Musica* (M. III, 4)⁴ (here, Figure 1), repeated several times, with variations, in his writings, he linked the sun and the planets with the musical notes and the Greek modes. In this, he referred to Cicero (M. III, 2 and 6), who had described the music of the heavenly spheres in the *Somnium Scipionis* at the end of his *Repubblica* (Cicero, 1992). However, his main sources, to which he repeatedly turned, were Severinus Boethius (M. IV, 24; V, 2; VI, 1–7; VII, 10 and 12; IX, 15; XII, 20; Boezio 1867), Guido D'Arezzo (M. IV, 13; VIII, 1, 7, 17, 26, 28, 35, 44; IX, 10; Guido D'Arezzo 1963), and Faber Stapulensis (M. IV, 24; VII, 11–12; Faber Stapulensis 1496a).

Like them, Maurolico followed the Pythagorean tradition, which had fixed the musical notes by means of the ratios created by the first few whole numbers, 1:2, 2:3, 3:4. From these intervals, called *diapason*, *diapente*, and *diatessaron* by the Greeks and subsequently the octave (*do – do*), the fifth (*do – sol*), and the fourth (*do – fa*) in modern Europe, the other intervals could be calculated, thus creating all the notes in the scale. The various tables of *Musica* were variants, in their numbers, their names, or some other particular, of this general approach, which, in other words, presents the notes by means of a series of whole numbers, or fractions, in a geometrical sequence.

The theory of music chosen by Maurolico, however, was not the only one possible. Other scholars, such as Aristoxenus, had adopted a completely different approach to the problem. However, Maurolico put all these aside, as the Pythagoreans traditionally did. We shall see, however, that the controversy between the two schools re-emerged because it was specifically connected with a mathematical problem of which there is evidence in his folios.

Then, there were also the moderate innovators who began to appear on the scene of the 16th century: while following the general mathematical approach based on ratios, they introduced other numbers and consequently other intervals. By now, the most famous of these was Gioseffo Zarlino, who also used the ratios 4:5, 5:6, and 3:5 for musical intervals. Thus, the classification of the intervals gained the major third, which took the place of the Pythagorean ditone, the minor third, the sixth, and so on. Maurolico seems to also ignore Zarlino deliberately, despite the books by the latter were already in circulation before the dates present in the manuscript (Zarlino 1558). While he did not, therefore, use the new numerical ratios, he did, however, at times write about major and minor thirds. Indeed, in table 11 (M. XI), we present a page where he broke down all the old and new intervals, major and minor, into apotome and diesis, seeing that the Pythagoreans had been in the

						<i>supremum coelum</i>
$3\frac{5}{9} \dots$	$\sharp$ $\flat$	tonus				
4	a	tonus	*	<i>Antehypate</i>		
$4\frac{1}{2} \dots$	7^{us} g	tonus	$\natural$	<i>Nete</i>	<i>Inferior in tritono</i>	<i>Mixolydius</i>
$5\frac{1}{16} \dots$	5^{us} f	diesis	$\natural$	<i>Paranete</i>	<i>Superne diesim altam</i>	<i>Lydius</i>
$5\frac{1}{3} \dots$	3^{us} e	tonus	♂	<i>Paramese</i>	<i>Infra diesim altam</i>	<i>Phrygius</i>
[6] modi	8^{us} p ^{us} D	tonus	$\odot$	<i>Mese</i>	<i>Media in ditono</i>	<i>Dorius Hypomixolydius</i>
$6\frac{3}{4} \dots$	6^{us} c	diesis	♀	<i>Lichanos</i>	<i>Superne diesim imam</i>	<i>Hypolydius</i>
$7\frac{1}{9} \dots$	4^{us} $\sharp$ $\flat$	tonus	♀	<i>Parhypate</i>	<i>Infra diesim imam</i>	<i>Hypophrygius</i>
8 ...	2^{us} a	tonus	☾	<i>Hypate</i>	<i>Superior in tritono</i>	<i>Hypodorius</i>
9	g	tonus	•	<i>Proslambanomenos</i>		
$10\frac{1}{8} \dots$	f					
						<i>centrum</i>

Fig. 1. From table 4 in *Musica*, the correspondence between numbers, notes, and stars.

habit of dividing the tone 8:9 into two unequal semitones, called major (apotome) and minor (diesis). But he did not stop here, and by substituting the apotome for the diesis, he created new ‘increased’ intervals, and vice versa, by substituting the diesis for the apotome, he created new ‘diminished’ intervals. He appeared to be interested in calculating all the possible combinations, and he even reduced the musical scale to a series of such semitones. His taste for combination and mathematical symbolism also appeared in table 1. Marin Mersenne and the young Gottfried Wilhelm Leibniz were subsequently to conceive music as the art of calculating the combinations between notes (Mersenne 1636; Leibniz 1666). From Johann Sebastian Bach to Arnold Schönberg, the musical scale was to be considered as broken down into the various semitones, even if, obviously, these were subsequently considered as well tempered, that is to say, all equal, at least approximately.

Apart from this, Maurolico appeared to be a conservative in music, just as it was clear from the arrangement of the planets in his tables that he accepted the geocentric system. A few years later, Giovan Battista Benedetti (1530–1590) expressed ideas that were different from Maurolico’s. He criticised the Pythagorean music of the spheres, for physical reasons, and referred, in his letters to his musician friend, Cipriano de Rore, to the new ratios proposed by Zarlino. Indeed, as these ratios created particular problems for the tuning of instruments like organs and harpsichords, he offered some practical solutions. Last, Benedetti never failed to call on the ear, to judge harmonies, a thing that Maurolico never wrote (Benedetti 1585, pp. 190–191; 277–283).

Table 1. Comparison between *Musica* (A6), *Musicae traditiones* (S7) and *Clavius'* manuscript (C13).

Ω	A6	S7	C13
Boetianae musicae compendium	No	Yes	Yes
(I) De Musicae subiecto. De sono, voce et modulatu De primis vocum intervallis et proportionibus De tono, diesi et apotome et eorum proportionibus	Yes	No	No
(II) Theoria musices proportiones 30	Only 1–20	1–32	o
(III) Septem planetae table 1	Yes	No	No
Repastinatio	No	No	Yes
(IV) Speculatio super consonantiis Comma esse diesis et apotomes differentiam Tonum esse minorem quam 9 commata maiorem quam 8 Diesim esse minorem 4 maiorem vero 3 commatibus Apotomen minorem 5 maiorem 4 commatibus	Yes	No	Yes, only in part
(V) Ex calculo Boetii	Yes	No	No
(VI) Calculus Boetii Scholium super calculum Boetii unde constabit diesim excedere 3 commata et dimidium. Apotomen autem maiorem 4 ^{or} commatibus et dimidio	Yes Yes	Yes No	No No
(VII) Medietas	Yes	No	No
(VIII) De Icosichordo Guidonis Hoc est Icosichordum Octochordis Lyra cum suo preambulo Icosichordum Guidonis cum sua expositione Mercurii Lyra Hexachordum Guidonis Ordo Compendii	Yes Yes Yes Yes	Yes, table with a short comment No No No	Yes, table with short introduction No No No
(IX) De octo modis modulatum, quos vocant tonos De genere diatonico, chromatico et harmonico Modorum proprietates	Yes	Yes	Yes, only effects
Tropi	No	No	Yes
(X) Regulae contexendi Symphonias Cantus praeceptiones	Yes	Yes	No
(XI) Instrumentorum auctores Instrumenta secundum sonorum proportiones construi debere De organis, tibiis, monochordis, harpichordis instrumentis. De cithara. Compendium praxis musicae	Yes, only scanty mention	Yes	No
(XII) Systematum calculus Regula Compositionis	Yes	Yes, only results	No
(XIII) Notularum proportio	Yes, only figure 38	No	No
(XIV) Species ... irrationalium	Yes	No	No

We are not interested now, however, in dealing exhaustively with these problems. Here, we prefer to pose the question whether our scholar from Messina, although dealing mainly with musical matters, was led to engage in calculations and demonstrations of some mathematical interest. We intend to show that this is what emerges from our edition of *Musica*. On the contrary, the personal contributions of Maurolico were left out of the *Musicae traditiones*, published posthumously in 1575. This appears to be the main difference between the two texts, which makes the edition of the manuscript *Par. Lat. 7462* particularly interesting. At the end, we discuss by whom, Maurolico himself or others, and why, the original version was changed, together with the history of the manuscript.

1.2 How Many Commas? A New Demonstration

Boethius had dominated the scene for a thousand years. Now, Maurolico had found a mistake in his famous *De Institutione Musica*. The result was correct all the same, but the mathematical procedure used to arrive at it was decidedly wrong. Let us now examine his reasoning. From the tables and calculations found scattered throughout the manuscript, for example on fol. 32r (figure 4) or on fol. 11r (figure 32), Maurolico concluded that the tone was equal to 8:9 and consequently the ratio of the ditone was 64:81. From these, he obtained a ratio of 243:256 for the diesis, and the subtraction of this from the tone left the ratio of the apotome equal to 2,048:2,187, fol. 32v (figure 5) (M. IV, 19). The apotome and the diesis did not divide the tone into equal parts and were in any case represented by increasingly large numbers. The initial simplicity of Pythagoras' mathematical model for music was being lost, leaving an irksome, ugly asymmetry, which seemed to be inherent and impossible to eliminate; however, much of the intervals were reduced. At the foot of fol. 32v, Maurolico wrote:

Differentia vero diesis et apotomes dicitur comma quod elicitur per subtractionem unius proportionis ab alia, ut infra patet (M. IV, 21).⁵

Thus, the famous or infamous comma was calculated to be 524,288:531,441 on fol. 33r (figure 8). What greater desire could there be (for a Pythagorean) than to produce at least a kind of basic minimal interval, a kind of atom for music, that can be used to break down all the others and consequently to obtain them? To break them down and to obtain them, of course, by means of operations that are accepted as valid. But the experienced mathematician might start to suspect that this was not possible. Maurolico continued:

Constabit etiam quod diesis maior est, quam tria commata minor autem quam quatuor.

Apotome autem maior, quam .4^{or}. commata, minor quam .5.

Unde et tonus excedet .8. commata et minor quam novem commata nascitur quae omnia ex longo et multarum figurarum calculo constari possunt lege Boetium et Fabrum in musicis elementis (M. IV, 22–24).⁶

They thus had to be satisfied with more or less fractional approximations and inequalities, which could only be obtained by means of long calculations involving several figures. Very true, seeing that the comma itself required six.

Our mathematician from Messina added a date here: 'Monday 30 December 1566', the earliest date in the manuscript, as if he desired to fix an important fact in time. Perhaps, he himself had no desire to engage in calculating and hoped to find the calculations already performed in books. But then, he found that the calculations presented by Boethius in his third book were not correct. He copied them out on fol. 12r (figure 10) and found that his illustrious predecessor had made a (venial?) mistake: he had subtracted the terms of the ratio for the comma, obtaining 7,153, then he had added the number to itself nine times, obtaining 64,377, and he had compared this with the tone, for which, following the same method, he had obtained 59,049. This had proved to be greater than 57,224, that is to say, 8 multiplied by 7,153.

Sed in hoc errat, quod in divisione proportionis utitur differentiis aequalibus. Cum debeat uti proportionalibus: sic enim multiplicantur proportiones non per aequales differentias. Verum si vitavit laborem multiplicandi; et tamen conclusit verum; excusatur (M. VI, 3).⁷

In other words, seeing that the notes are fixed in the Pythagorean model by numbers arranged in a geometrical sequence and *not* in an arithmetic sequence, the addition or the removal of an interval means multiplying or dividing the terms (as is the case) and not adding or subtracting them. Clearly, multiplying a six-figure number by nine is much easier than multiplying it nine times by itself. Seeing that the result was correct, Maurolico seems to forgive Boethius. Again, he inserted the date: 'last day of January 1567'. The 1575 editors of *Musicae Traditiones* placed this page as a final seal, although they changed the wording.

Et tamen, sicut nos proportionaliter calculando, experti sumus, Boetius veritatis scopum attingit. (Maurolico 1575a, p. 160)⁸

The *Musicae Traditiones* had been made to begin with the *Boetianae Musicae Epitome*, and the editors would have found difficulty in finishing the work with a criticism, which put Boethius in too bad a light. However, not all the folios of Maurolico were published in the posthumous edition of 1575. The differences, not at all negligible, between our *Musica* and the *Musicae Traditiones* are discussed in greater detail below, together with the reasons behind them.

In fact, our mathematician from Messina had by no means re-established precision *calculando*. On fol. 12v, we found a *scholium* on Boethius, with his personal solution to the problem of the number of commas in the Pythagorean tone.

Disponantur hi numeri . 64 . 65 . 66 . 67 . 68 . 69 . 70 . 71 . 72 . Cumque 72–64 sit ratio toni, patet quod ratio 72–71 est minor, quam octava pars toni.

Sed 531441–524288 quae est ratio commatis est sicut 72–71 $\frac{16425}{531441}$ minor scilicet, quam 72–71. Ergo tanto fortius minor, quam octava pars toni (M. VI, 4–5).⁹

Therefore, instead of carrying out long and laborious calculations, Maurolico gave an elegant demonstration in a few lines. Nowadays, although, we find it a bit too concise because the customs of calculation and demonstration have changed with the passing of time. Let us therefore try to investigate that ‘patet quod ratio ...’, that is to say, ‘it is evident that ...’.

We must divide the tone into eight parts and compare the part obtained a_8 with the comma. The tone was equal to 8:9, or also equivalently, to 64:72. Therefore, to obtain a_8 , we must calculate the following proportional means:

$$64 : a_2 = a_2 : 72 \quad a_2 : a_4 = a_4 : 72 \quad a_4 : a_8 = a_8 : 72$$

It had long been known¹⁰ that the arithmetic mean between two numbers is always greater than the geometrical mean, and consequently, the ratio between 72 and a_8 is greater than that between 72 and arithmetic mean. Now, at last, it is clear why Maurolico had chosen that arithmetic sequence of numbers from 64 to 72: to calculate, at a glance, the arithmetic means between 64 and 72, that is 68, then between 68 and 72, that is 70, and last, between 70 and 72, that is 71. Thus, ‘it is evident that the ratio 72–71 is less than the eighth part of the tone’. Then, he took the ratio of the comma 531,441–524,288 and rewrote it as $72-71 \frac{16,425}{531,441}$; thus, the comma proved in turn to be less than 72–71. Therefore, the comma proved to be *a fortiori* less than the eighth part of the tone. ‘...for this reason, eight commas are less than a tone’. Q. E. D.

When they did not make mistakes, like Boethius, arithmeticians and theoreticians of music had to engage in intricate extractions of roots or mind-boggling multiplications involving several dozens of figures to obtain their results. Compare the calculations of Faber in *Elementa Musicalia* or those of Benedetti for the much easier syntonic (or Didymus) comma 81:80, which includes numbers composed of 18 or 20 figures (Faber Stapulensis 1496a II, p. 35; Benedetti 1585, p. 280).¹¹ Maurolico cannot have loved calculations, and at times, he even got them wrong. Some of his mistakes are found at the beginning of fol. 20v (M. VIII, 51) or on fol. 10r (M. Marg. 6). Thus, he preferred a rapid, elegant demonstration. I believe that it is original because I do not know of any others that are similar in the treatises of the period. The number of commas contained in the tone had been a recurrent topic in books since antiquity, but the demonstration is generally missing. This demonstration of Maurolico is probably the first one that is correct and is without doubt the first one that is also brief and elegant.

Anyway, in the evolution of the mathematical sciences, the tendency to substitute calculations with reasoning was one of the important stimuli for invention. This impulse emerged from his documents dealing with music, but the editors of the 1575 edition seemed to be more interested in leaving the tradition of Boethius intact than in understanding the originality shown in this case by Maurolico.

Our mathematician from Messina also improved, in a simple manner, the inequalities that defined the diesis and the apotome by means of a suitable number of commas, reducing them both by half a comma. He may well have been satisfied with his new results, seeing

that he again wrote the date in the margin: '6 February 1567' (M. VI, 7). Apart from other considerations dictated by the original projects and by clarity, we have preferred to arrange the folios of the manuscript in the order of our edition, to underline the results achieved by Maurolico. This aspect was completely lost in the Venetian edition of 1575.

1.3 *The Music of the Spheres*

Our *Musica* contains several tables, which are all similar. According to the ancient Pythagorean tradition, the stars generated notes by their movements in the sky: the so-called music of the heavenly spheres. Thus, every star corresponded to a particular note and a particular musical mode. The modes produced effects on people's souls, leading them to adopt the kinds of behaviour illustrated by the doctrine of *ethos* in music. The names of the planets also recalled the days of the week. To take a simple example, the planet Mars 'corresponded' to Tuesday and to the Phrygian mode, which rendered people bellicose (M. I, 24; IX, 18–25). Thus, the links that held the cosmos together, and regulated it, were revealed in music. Consequently, the tables present a synthesis of the structure of the cosmos, which is expressed in numbers. But what might these numbers be? At times, the tables differ from one another in their choices. Almost all of them, however, attributed the number 6 to the sun, to the Doric mode (the first), and to the note *mese*, that is to say, the mean. Almost all of them explicitly contain the tone ratio 8:9 and also the number 4 at the top (M. tables 2–10). In this way, the sun and the mean note could be presented as the proportional mean 6 between 9, the earth, and 4, the firmament. However, there must have been something unsatisfactory in the model, seeing that Maurolico rewrote the tables so many times, changing the numbers as well, proportionally, of course. Thus, in table 8, they were increased in the ratio 4:3, in table 5 in 16:9, 4:1, 12:1, etc. This numerical choice was, on the one hand, arbitrary and, on the other, unsatisfactory due to the presence of fractional ratios (and also because it was asymmetrical with respect to the octave, as we shall shortly see more clearly). The recurrent problem can be seen above all in table 6, which is the object of particular re-elaboration in the manuscript, where the numbers of the notes, which had increased to 20, that is to describe Guido D'Arezzo's eicosichord, were multiplied by 2 and 6. Here, in fact, Maurolico wrote, on fol. 6v:

Quod si quis velit has fractiones redigere ad numeros integros, multiplicet hos terminos singulos per 384, nam hic numerus continet omnes terminorum fractiones (M. VIII, 5).¹²

At last, in figure 16 of fol. 19r, all the numbers became whole numbers (a bit too big, alas!). The three tables chosen for the 1575 edition (Maurolico 1975a, pp. 148, 154, 156) completely ignore this question, which, on the contrary, is present in the manuscript.

Unlike the problem found for the commas, the performance of multiplications does not show particular intelligence. However, it does allow us to understand Maurolico's position better in the scientific culture, which based the world above all on proportions. Among other points, Maurolico followed the assumption of those who (like Cicero) 'felt' that

the planets that were furthest away emitted the highest notes. His justification might have been connected with their greater speed of movement. But was not their distance from the centre important, then? What about the size of Jupiter and Saturn, compared with the small dimensions of the Moon, Mercury, and Venus? It was a controversial question; for Boethius, it was rather the distant planets that emitted the lower notes. This was even written in Newton's *Scholium* on the eighth proposition in the third book of *Principia* (Tonietti 2000). We should not forget, either, that in the same book of *Harmonices Mundi Libri Quinque* that contained the famous third law, Kepler also made the planets sing (Kepler 1619). The music of the spheres remained extremely varied, just as the sciences did, as well.

In the tables, the numerical values decrease as the height moves towards the shriller notes, and consequently, they are inversely proportional to it. The numbers can thus be imagined as the length of the strings that generate the notes. The fact that the height was inversely proportional to the length of the relative string had been an ancient discovery, attributed by tradition to the Pythagoreans. Like them, Maurolico also seemed to believe that the height of the notes increased with the tension of the strings, but always in a linear manner, as he wrote on fol. 34r and fol. 29v:

Et nervus remissus gravior: intensus acutius (M. I, 4).¹³

Corpus magis densum tremit velocius, sicut chorda aenea nervo et intentus remisso (M. II, 2).¹⁴

The idea, on the contrary, that the height of the sound was proportional only to the square root of the tension was to be brought to maturity by Vincenzo Galilei and Marin Mersenne (Cohen 1984; Gozza 1989; Bailhache 1993).

1.4 Theory of 'Ictus' and Roots

The manuscript of *Musica* was filled up by Maurolico also with extractions of roots. Clearly, calculations with proportions could not be reduced to multiplications and divisions. But why should the determination of proportions for music, in accordance with the tradition of Boethius and Pythagoras, have involved the extraction of roots and venturing onto the dangerous ground of incommensurable quantities? In fact, no trace of this was left in the pages published in the 1575 edition. In general, music was connected with arithmetic, which dealt with discrete quantities, such as whole numbers, and not with continuous quantities, which were reserved for geometry. No roots are found in Boethius or Faber, either in the musical part or in the arithmetical part of their treatises (pending further verification). They argue that incommensurable ratios cannot generate consonances and should be excluded for this reason. Maurolico follows this idea too, but he adds a 'physical' explanation, which was to enjoy high consideration for a couple of centuries, until Euler (Euler 1739, chap. 2; cf. Bailhache 1995, p. 6): the so-called theory of ictus [beats]. On fol. 29v, we read:

Consonantias consistere in proportionibus commensurabilibus nam incommensurabiles sonos impossibile est concordare. sicut impossibile est correspondere tremores incommensurabilium velocitatum, quandoquidem concordantia sive consonantia fit ex ictuum correspondentia (M. II, 12).¹⁵

The explanation is repeated also on fol. 31r:

Non solum in proportionibus commensurabilibus, sed etiam in praecipuis numeris consistunt vocum musicarum consonantiae. Quoniam incommensurabiles proportiones (quoniam irrationales et ignotae sunt) semper faciunt dissonantiam: quoniam voces in tali proportionem constitutae, propter incommensurabilitatem, non per ordinatos ictus sed semper diversos (quae diversitas parit discordantiam) invicem sibi respondet (M. IV, 1–2).¹⁶

The idea of justifying consonances by means of the more frequent coincidence of beats created by the air vibrating in the ear is thus already found here in Maurolico (also repeated in Maurolico 1575a, proposition 11, pp. 150–151). This was also the conviction of Benedetti (Benedetti 1585, p. 283). We do not know how much Isaac Beeckman, who offers a similar explanation, may have read of his two predecessors; therefore, we cannot exclude the possibility that his idea on this subject was independent of them. However, H. F. Cohen fails to also consider Maurolico, when he attributes the origin of the idea alternately to Benedetti, ‘the first propounder of what we have termed the coincidence theory of consonance’ (Cohen 1984, p. 78), to Beeckman, ‘there can be no doubt that the priority concerning the discovery of the coincidence theory of consonance (...) belonged to Beeckman’ (Cohen 1984, p. 196), and again to Benedetti, ‘... the first to formulate it, however primitive and provisional, was Benedetti ...’ (Cohen 1984, p. 199).

On the basis of the above sentences, Maurolico becomes as good a candidate as Benedetti, for those who enjoy disputes about priority, even if we cannot by any means exclude the possibility of simultaneity. As a result, in Cohen’s figure 64 (Cohen 1984, p. 199), which lists the dates and the names of Benedetti, Beeckman, Mersenne, Descartes, and Galileo, the name of Maurolico should without doubt be added, with at least his manuscript of 1566 (with parts that were probably prior to this date), which was partly published in 1575.¹⁷

There was another very ancient problem, which had always presented the risk of evoking irrational numbers in such a way as to link music with mathematics more closely than the simple use of numerical proportions could. Because one of the most famous controversies of ancient culture, perhaps *the* controversy par excellence, was now directly represented in music. There is no need to provide further details about this subject because everybody, whether mathematician, physicist, philosopher, historian, or musician, has his own idea about it, and these ideas often diverge. Let us immediately see how this emerges from music.

Is it possible to divide the tone into two equal parts? As its ratio was equal to 9:8, dividing the tone in this way meant calculating the proportional mean between 9 and 8. Maurolico could easily introduce the proportional mean in his tables because between 4 and 9, the proportional mean was equal to a nice whole number, 6. However, as the octave of 9 is 4.5, this does not fit in with the module of the octave. But the proportional

mean between 9 and 8 is equal to $\sqrt{72}$, a number that, to the dismay of the Pythagoreans, contains $\sqrt{2}$. Therefore, all the treatises of the Pythagorean and Platonic tradition state, for the well-known reasons, that it is impossible to divide the tone into two equal semitones. This can be found, for example, in Euclid's *Sectio canonis* (Euclid 1557). Those who, on the contrary, admitted this possibility included Aristoxenus and his followers. One of them, in the generation after that of Maurolico, was the bearer of a famous name: Vincenzo Galilei (Galilei 1581).

Which side was Maurolico on? The former one, without doubt. Yet, we repeat, all the same his papers were full of extractions of roots. Was not he completely sure? Did he want to verify it directly and find his own personal reasons for it? The precise statement, which goes back to Boethius, is quoted in *Boetianae Musicae Epitome*, which is at the beginning of the *Musicae Traditiones* of 1575.

Tonum non posse dividi per aequalia: quandoquidem toni ratio sesquioctava non est, quae quadrati ad quadratum numerum: et perinde medium proportionalem numerum, qui proportionem per aequalia secet, non suscipit. Sic non datur locus Aristoxeno tonum per aequalia secari debere, asserenti. (Maurolico 1975a, p. 148)¹⁸

The subject is also mentioned in the numbered list, which started from the definition of sound. However, it is only at number 31, towards the end of the list, in the part which is not extant in our manuscript. It says:

Neque igitur Aristoxenus, qui tonum per aequalia: neque Philolaus, qui aliter divisit, audiendus est. (Maurolico 1975a, p. 153)¹⁹

If Maurolico had been satisfied with statements like this, he would not have carried out the calculations that are found in the manuscript. But how did he arrive at the decision to trust Boethius? Thus, on fol. 10r, he obtained the tone 9:8, and by subtracting the diesis 256:243, he obtained the apotome 2,187:2,048 (figure 29). The tone 2,187:1,944 was thus divided by the number 2,048 into two parts.

Et quoniam numerus medius proportionalis inter .2187. et .1944. est maior quam 2061. Propterea proportio .2048. ad .1944. minoris semitonii minor est quam dimidium toni (M. XII, 15–17).²⁰

The same folio contains the relative calculations. First, Maurolico made a mistake in multiplying 2,187 by 1,944 because he thought that the answer was 3,018,528; he extracted the square root of this and obtained 1,737, which was clearly wrong. Then, he corrected the result to 4,008,528 and recalculated the square root. He had again made a mistake and obtained 2,002, which was not correct, as he verified. Then, he started again from the number that he believed to be correct, 4,018,528, and extracted the square root again thrice. He now obtained 2,004. This was the first number that he had considered in his text for the proportional mean, which was subsequently corrected to 2,061. At last, he realised that he had made mistakes at various points in his multiplication and corrected it again. As a result, the folio had become so messy that we preferred to transcribe it several times to point out every mistake separately. In the end, he obtained the right answer, 4,251,528, the

square root of which, extracted on fol. 9v, was now equal to (approximately) 2,061 (Marg. 6), which is the number found in the text.

As regards, for the technique used to calculate square roots, see the treatises of the period. They contain the procedure with the characteristic diagrams and the numbers with a stroke, which have remained in the manuscript. In his *Summa De Arithmetica*, Luca Pacioli had given a similar procedure (Pacioli 1494). See also the doctoral dissertation of Jean-Pierre Sutto (Sutto 1997).

On fol. 27r, Maurolico went even further. He obtained the ratio for the comma and at least started to extract the square root for the terms 531,441 and 52,4288 and also for their difference, 7,153. Then, he even appears to have sought the proportional mean between those terms for the comma, as if he wanted to divide it, in turn, into two equal parts, that is to calculate the schysma. Here, surprisingly, even if this multiplication was much longer than the other one, he only made one mistake, which he realised immediately. He extracted the square root of the result thrice, also on fol. 28r. On fol. 27v, he tried to find the proportional mean between 2,187 and 2,048, that is to say, he divided (using approximations) the apotome into two equal parts. When, on the contrary, he calculated the proportional mean between 2,304 and 2,048, it was the tone that he was now dividing into two equal parts. He represented this operation even more clearly, by writing '9 . r72 . 8', where 'r' was the symbol used in that period for the square root.

The pages presented here lead us to the delicate general question of whether and in what sense Maurolico admitted the numerical representation of irrational proportional means. Traditional considerations, which he also quoted, that incommensurable ratios were unable to generate consonances tend to throw a negative light on this possibility. Maurolico had first taken holy orders, then he had become a Benedictine abbot at Castelbuono, at the Abbazia del Parto (Macrì 1901, p. 57). For an 'official' representative of religion like him, therefore, the statement contained in *Boetianae musicae epitome* of 1575 must undoubtedly have been valid:

Solus enim Deus infinitus. (Maurolico 1575a, p. 147)²¹

When, in the pages about music, the problem cropped up again, that irrational numbers could not be considered or used like other numbers, it found its definitive justification for him. By contrast, Simon Stevin wrote: *Que racine quelconque est nombre*, as the third of his *Thèses mathématiques* (Stevin 1585, p. 738).

Consequently, no ideal obstacle was to stop the Dutchman from calculating, and supporting, the division of the octave into 12 equal semitones (Stevin 1966; Tonietti 2003).

In view of what has been said, the papers about *Species quantitatum rationalium et irrationalium* are not out of place in an edition that deals mainly with music, even if, among Maurolico's works, they are closer to *Arithmeticon libri duo* (Maurolico 1575b).

1.5 A New Symbolism

Here, in *Musica*, Maurolico also showed a peculiar inclination towards the use of abstract and general symbolism. At the edge of fol. 34v, he defined the intervals by means of couple of letters, instead of numbers (Table 1). He even betrayed a taste for the symmetric circular combination $ab\ bc\ ca$ to which we have become accustomed, after centuries of historical evolution, from Leibniz to quaternions.

Last but not least, Maurolico left aside calculations with numbers to give more general rules, which could be used for operations on proportions that were useful for music. He defined a *Regula compositionis* and a *Regula subtractionis* by means of letters arranged like the diagrams on fol. 9r, reproduced in section XII of *Musica* in figures 23 and 24 (Figure 2).

He cannot have been too sure of himself at the beginning. He made a mistake in the first diagram and initially placed e in a wrong position with respect to f . Then, he was forced to correct the mistake in the text. A proportion is defined by an ordered couple of terms. Given two proportions, they may ‘continue’, obtaining three more terms from the two couples of terms. These three contain both the starting proportions, and the new one sought, which describe their composition. The rule makes it possible to obtain the proportion for the addition of two musical intervals. It is also possible to ‘subtract’ two proportions by means of the second diagram, which allows us to obtain the proportion for the difference of musical intervals.

In the project of 1569, the *Regulae* are called ‘*Systematum calculus*’ (M. VIII, 53). Visualised by means of these rules, the musical proportions can be obtained rapidly and in a particularly clear manner. These diagrams used by Maurolico are not found in the treatises of arithmetic of the period and are to be considered as his own original creations. Usually, only those for multiplication between couples and division are shown, with their characteristic cross.

The *Regulae* were excluded from the 1575 edition by the editors. They only published the result at the end (Maurolico 1575a, p. 160). Thus, Maurolico was again reduced to the level of a simple expounder of tradition, without any particular flash of inspiration. On the contrary, we have shown that the folios of the manuscript edited by us are much richer and contain elements of interest also for historians of mathematical sciences.

One curious detail. In performing the divisions here, Maurolico wrote the divisor on the left and the dividend on the right. We do not know whether this was relatively common at the time or a habit of our mathematician from Sicily.

1.6 The Greek Language

Finally, we would like to draw attention to two more small details that throw a little light on an important question, which, however, has remained obscure and controversial. Did Maurolico know Greek?

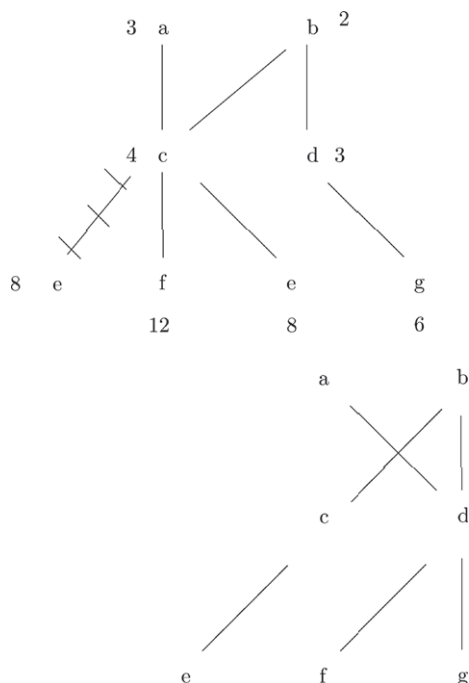


Fig. 2. From figures 23 and 24 in *Musica*, diagrams to compose proportions.

He used the Greek names of the strings to indicate the musical notes, for example:

Appellantur etiam hi .8. primi nervi Proslambanomenos, Hypate, Parhypate, Lichanos, Mese, Paramese, Paranete, Nete (M. I, 25; table 4).²²

But the fact that he gave their meaning also in Latin, as the added string, main string, for the index finger, middle and high (table 10) did not mean very much because the translations could easily be found in treatises. Yet, he wrote on fol. 29r:

Lichanos quae consistit in indice digito subsequenti.
Etymologia caeterarum chordarum per se patet (M. X, 12).²³

and on fol. 35v:

Quorum vocabulorum etymologia facile patet (M. I, 25).²⁴

Would he have written that the etymology was ‘patently’ clear and ‘readily’ apparent if he had not known Greek? To this, we could also add the irony against Boethius.

Hinc multum sudat Boëthius in vocabulis nervorum Grecis ... (see section 3.1).²⁵

His nephew, Francesco Marolì Baron della Foresta, stated that Francesco Maurolico had been taught Greek by his father (Baron della Foresta 1613). Macrì later wrote that Maurolico was ‘dottissimo nella greca favella’ (Macrì 1901, p. 144).

He even said that the Villacanense manuscript contained some translations from Greek (Macri 1901, p. 115). However, we have no definite confirmation of this so far. Even if their significance is limited, these little words in our *Musica* have a certain weight and should be included among the elements supporting the above hypothesis.

Part II

2.1 The Formation of the Original Corpus

We have seen that the differences between the version printed at Venice in 1575 and what remained in the Paris manuscript are numerous and highly significant. Now we shall try to understand the reasons for this, and to do so, we shall attempt to reconstruct the history of these folios about music, left by Maurolico, whose circumstances are complex, incomplete, and at times, even contradictory. Let us start by relating how the musical *corpus* was composed, and let us therefore repeat, for the sake of convenience, here which texts, mainly dealing with musical topics, are now extant. The abbreviations refer to the edition of *Opera Mathematica* in www.maurolico.unipi.it

- (1) *Par. Lat.* 7462, at the Bibliothèque Nationale in Paris, referred to in this edition by the name of *Musica A6*
- (2) *Musicae traditiones carptim collectae* contained in *Opuscula Mathematica*, published posthumously at Venice in 1575 S7
- (3) fol. 51r of *San Pantaleo* 115/32, at the Biblioteca Nazionale in Rome A19
- (4) the manuscript of Christophorus Clavius, *F. M. Boetianae musicae compendium*, APUG Fondo Curia 2052, at the Biblioteca Gregoriana in Rome C13.

Let us now reconstruct everything that Maurolico may have written about music or may have done in connection with music in his whole life. This will serve to examine better what elements of this *corpus* (indicated as Ω) passed into our sources, and why, and what, on the contrary, was lost. Furthermore, we shall also understand the relationships between the various witnesses. For this purpose, it is a great help that Maurolico often left, among his writings, a list of the studies that he had carried out up to that moment, which he often called *index lucubrationum*. The results of our research are the following, in which we indicate, together with the year (when it is known, or hypothesised) the text from which the information is taken.

- 1528: *Grammaticorum rudimentorum libelli sex*: ‘... Boethi arithmetica et musica elementa ... nullo praeunte praeceptore per memet ipsum intellexi’ S1 (Maurolico 1528, p. 7).²⁶
- 1540: dedication to Pietro Bembo, in *Cosmographia* 1543: ‘aliquot meas lucubrationiculas ... In secunda sectione. ... Boetianae musicae compendium. Musicae speculativae et practicae compendium ex Guidone, aliisque authoribus: in quo vocum consonantium ac dissonantium ratio plene discutitur’ S2 (Maurolico 1543).²⁷

- 1550: Abbate del Parto: 'Quivi [Abbadia del Parto, Castelbuono] dimorava il buon Pastore con la sua diletta greggia, salmeggiando con essi loro in Choro, ed attendendo in camera alla speculazione Mathematica', 'nella quale [chiesa di San Giovanni Battista dei Cavalieri Gerosolimitani a Messina, dove poi sarebbe stato sepolto] udivasi assiso in Choro cantar sovente con festevole sembiante' (Baron della Foresta 1613).²⁸
- Villacanense: *Index lucubrationum*: 'De musica ... compendium musicae Boetii cum quibusdam scholiis ad intervallorum proportionem facientibus' (Macrì 1901, p. xvii).²⁹
- Villacanense: among the autograph writings contained in the codex: 1. Synoptic table, among the applications of mathematics to bodies, 'musica' '17. Boetii arithmetica (pag. 208 a 212). Le più utili teoriche di Boezio ridotte in un compendio che termina con le parole: Catanae die 28 januarii 1554. Lector vale; caetera in quibus Boetius speculatur, plus habent fastidii quam jucunditatis; ideoque negligenda duximus. 18. Musica Boetii (pag. 213 a 218). Opuscolo del tutto simile al precedente' (Macrì 1901, pp. xxiii–xxv).³⁰
- 1556: letter to Juan de Vega: '... Et quamvis Euclides a continuis, Boetius autem noster a discretis exordium capientes discrepent' (Macrì 1901, p. li; Moscheo 1998, p. 289), 'De quibus [numeri lineari, piani, solidi] Euclides, Boetius, ac Jordanus' (Macrì 1901, p. lii; Moscheo 1998, p. 290), 'Item tam discreta quam continua quantitas aliis aut aliis applicata rebus, aliam atque aliam generat scientiam, quae arithmeticae, ut calculus, rhythmica, musicae: aut geometriae subiacet, ut/ astronomia, geographia, chorographia perspectiva, de quibus postea latius loquemur' (Macrì 1901, p. liii; Moscheo 1998, p. 291), 'In Jordani autem Arithmetice atque Boetii ...' (Macrì 1901, p. lvi; Moscheo 1998, p. 292), "... sine quibus ['*Spherica elementa Theodosii*'] nemo astronomica primordia satis perpendere queat, sicut neque sine arithmetice musicam, cum pure mathematica materialibus scientiis anteponenda sint.' (Macrì 1901, p. lvi; Moscheo 1998, p. 293), 'Quid enim ego, per immortalem deum rogo, nocerem, si elementorum Euclidis, sphaericorum Theodosii ac Menelai, conicorum Apollonii, cylindricorum Sereni, operum Archimedis, arithmetice Jordani, musicae, perspective, astronomiae ac mechanicarum inventionum definitiones, conceptus, postulata, problemata et theoremata in unum congregarem?' (Macrì 1901, p. lxxiii; Moscheo 1998, pp. 303–304), 'O felix seculum, non solum sculptores machinatores, fabri, pictores, tibiicines, musicique praestantissimi, publice conducuntur, sed etiam rethorum ac philosophorum ad commune commodum opera exquiruntur' (Macrì 1901, p. lxxv; Moscheo 1998, p. 305).³¹
- 1558: *Index lucubrationum*, in *Sphaerica Theodosii*: 'De musica' S4 (Maurolico 1558).
- 6 May 1567: *Par. Lat. 7471*: 'In Sextum libellum. De Musicae subiecto. De sono, voce, et modulatu. De primis vocum intervallis et proportionibus. De tono, diesi et apotome, et eorum proportionibus. De Icosichordo Guidonis. Comma esse diesi et apotomes differentiam. Tonum esse minorem, quam .9. commata, maiorem quam .8. Diesim esse minorem .4. maiorem vero .3. commatibus. Apotomen minorem .5. maiorem .4. commatibus. Scholium super calculum Boetii, unde constabit diesim excedere .3. commata et dimidium. Apotomen autem maiorem .4. or commatibus et

dimidio. Speculatio super consonantiis. De octo modis modulatum, quos vocant tonos. De genere diatonico, chromatico et harmonico. Regulae contextendi Symphonias. Instrumenta secundum sonorum proportionem construi debere. De organiis, tibiis, monochordis, harpichordis instrumentis. De cithara. Compendium Praxis musicae' (Moscheo 1988, p. 540).³²

- 1569: lecturer, University of Messina: '... legere in predictis studiis publicis istius civitatis ... predictam lectionem mathematicae geometrie arithmetice speculative astrologie musicae speculative prospective et omnium aliarum rerum et instrumentorum quae ad hanc scienciam mathematicam spectant ...' (Macrì 1901, pp. lxxix–lxxx; Moscheo 1998, p. 334).³³
- *Par. Lat. 7466: Index lucubrationum Maurolyci*: 'De musica. ... Compendium musicae Boetii cum quibusdam scholiis ad intervallorum proportionem facientibus... Musica ex Boetio, ex Graecis authoribus, ex Fabro. His additur compendium nostrum, theoriam vocum et consonantiarum [modorum modulationum] omnem paucis comprehendens ... Boetii Arithmetica, et Musica, cum compendio Jacobi Fabri et tractatu nostro brevissimo ... 17 sep. 1570' A10 (Clagett 1974, pp. 180, 183, 188, 189; Moscheo 1998, p. 347).³⁴
- 1575: *Index lucubrationum, Arithmeticorum libri duo*: 'De musica ... Compendium boetianae musicae, cum optimis speculationibus et calculo, ac modulatum ratione et systematum proportione' S8 (Maurolico 1575b).³⁵
- 1593: Antonio Possevino quoted Maurolico 'among the authors of practical and speculative music' (Moscheo 1998, p. 223).
- 1613: *Index lucubrationum, Vita dell'Abbate del Parto*: 'De musica ... Compendium boetianae musicae, cum optimis speculationibus et calculo' (Baron della Foresta 1613).³⁶
- List of manuscripts and books *apud haeredes Sylvestri Maurolyci*: '[33] Nonnulla ad Musicae Theoricam spectantia' (Moscheo 1988, p. 418).³⁷ 'In the mid-17th century, almost all the manuscripts of Maurolico, including his studies on the works of Archimedes, were still in the hands of his heirs' (Moscheo 1988, p. 113).

Therefore, by 1528, our man from Messina had studied and learnt by himself what Boethius had written about arithmetic and music. We often find the arithmetic of Boethius followed immediately by his music because the one was considered an introduction to the other, thus creating a traditional couple. In 1540, Maurolico wrote that he had compiled a compendium of the music of Boethius and a theoretical and practical compendium of music taken from Guido D'Arezzo and other authors (Faber?). In these works, he probably began to add some of his own ideas, discussing '*plene*', the '*ratio*', of consonances and dissonances. By this date, therefore, the composition of Ω had already started.

However, he did not seem to be interested only in the theory of music. His nephew, Baron della Foresta, described him as joyfully, taking part in the church singing, both at his abbey at Castelbuono and in Messina. This should not come as a surprise because music

has always been a part of the training of priests and members of other Catholic religious orders. Moscheo also hypothesises the participation of Maurolico in the musical activities at the court of Pietro Barresi (Moscheo 1998, pp. 168–169).

From this point on, music appeared in all the lists of studies performed by Maurolico. From the list added to the Villacanense manuscript, which is dispersed, but again was studied by Macrì, we learn that the compendium of Boethius still existed, together with some comments about the proportions for musical intervals. Indeed, as Macrì states, the compendium of music from Boethius must have been a part of the Villacanense manuscript, together with the arithmetic of Boethius. It is significant that Maurolico left some criticisms of Boethius, the scholar of arithmetic, here as well because they can be compared with those that he made of his music. We do not know whether the date of 1554 for the arithmetic can also be transferred to the music. Probably, the compendium on music does not bear any date because it had already been compiled previously. In any case, it cannot have contained much more than the old compendium from Boethius because it composed of only six pages. However, around 1554, then, Ω was an integral part of the Villacanense manuscript, and it was not yet very long: six pages.

Also, in his famous letter of 1556 to Juan de Vega, Maurolico wrote about music here and there, but not too much, compared with the other disciplines discussed. It reappeared when there was a mention of Boethius, who was compared with Euclid because he had considered discrete quantities, whereas the Greek geometrician had dealt with continuous quantities. The science of discrete quantities was arithmetic, under which came music, just as astronomy and perspective came under geometry. Without arithmetic, it would not have been possible to discuss music, just as it would not have been possible to discuss astronomy without the Sphericals of Theodosius. The pure mathematical sciences were to be placed before the material ones. In the end, music was incorporated, together with all the other works of Euclid, Menelaus, Apollonius, Archimedes, etc., to create a single whole. Last, music was not only theory for Maurolico, and he put ‘musicians and flautists’ among the various artists and philosophers who had made that century happy.

Ω returned as ‘*De musica*’ in 1558, but it cannot have changed much. On the contrary, in the *Index lucubrationum* added to *Par. Lat.* 7466 A10, we can notice an expansion. Together with the compendium from Boethius, Faber, and other authors, Maurolico said that he had added a ‘*compendium nostrum*’, which included all the theory of notes and consonances in a few lines, with a ‘*tractatu nostro brevissimo*’.

At this point, we can follow the dates found in our edition of *Musica*, to see how Ω grew between 1566 and 1569. So the comments and the decisive criticisms of Boethius appeared between the end of 1566 and the early months of 1567 (fol. 12r and fol. 12v). In May 1567, we have the list of subjects for music, some already discussed and others, perhaps, to be discussed subsequently (Moscheo 1988, p. 540). In June 1567, Pietro Barresi, prince of Pietraperzia, asked Maurolico to add and to consider other musical subjects, including means (fol. 7v). In November 1567, our mathematician from Messina was working on Guido D’Arezzo’s eicosichord (fol. 6v–7r). Finally, in March 1569, he made a further

note, under the heading ‘*Ordo Compendii*’, on the state of the situation, and what was still lacking to give a satisfactory order to the musical material (fol. 20v). The years passed and Maurolico was already more than 70 years old. He would have to accelerate if he wanted to complete his work, filling in the gaps and bringing it together ‘*in unum*’. He received a further stimulus in 1569, when the township of Messina assigned him the task of teaching at their university the subject of mathematical sciences, from geometry to perspective, from arithmetic to astrology. The list also included music.

The other lists of studies continued to include writings about music, among which the compendium of Boethius and the personal reflections of Maurolico stood out. The last trace of the *corpus* Ω can be found in the list of manuscripts and books that Maurolicos heirs made up half way through the 17th century. This includes the item ‘A few things concerning the theory of music’.

2.2 The Puzzle of the Relationships Between Witnesses

The original *corpus* Ω cannot be the same as the Paris manuscript, *Par. Lat.* 7462 A6, included in our edition under the name of *Musica* because in it, the compendium from Boethius is totally missing today. This first part of Ω was finished in the Villacanense manuscript, which had a separate destiny from the group of manuscripts that arrived in Paris. Thus, the compendium from Boethius remained *in freto siculo*, on the Straits of Messina, where it could still be examined by Macrì at least until 1901, in the Villacanense manuscript. But it was subsequently dispersed, together with the manuscript, probably in the earthquake of 1908.

However, the compendium from Boethius must, or might, have been copied for the edition of *Opuscula Mathematica* in Venice, seeing that it is at the beginning of *Musicae traditiones*. Before going on to make a closer comparison between our edition of *Musica* and *Musicae traditiones*, let us briefly remember the events that had led up to the edition of 1575. These were initially related by Baron della Foresta and were then discussed by Rosario Moscheo and Jean-Pierre Sutto.

Maurolico was

largo in comunicar a gli amici le sue speculazioni, gelosissimo dell'altrui fama, non fu miga maledico, non rimproveratore, non ambizioso, anzi talmente sprezzatore di gloria, ed honore tereno, che solea ben spesso dire di stimar nulla, che l'opere da sé con sì lunga fatica composte, e lambiccate a viva forza di studio da qual raro cervello, d'altrui nome fregiate, si esponessero al mondo, pur che a pro dei mortali, e contezza del vero elleno fossero publicate. (Baron della Foresta 1613)³⁸

On 16 April 1569, Maurolico wrote to the general father of the Jesuits in Rome, Francisco Borgia, asking him to grant his patronage for the publication of certain compendia requested by his friends, who had forced him to start studying again. The letter reveals his fear that he might not succeed in terminating the work, and for this reason, he asks to be

helped by Clavius, 'whose involvement he requested, to examine and correct our work' (Moscheo 1988, p. 276; Sutto 1997, p. 106; D'Alessandro and Napolitani 2001). Maurolico would have liked to print all his works, and he was attempting to find help, both for the preparation and reorganisation of the folios and for the editing expenses.

At least two other provincial fathers from Sicily showed interest in the question, on 10 November 1569 and 6 December 1570, in support of Maurolico's requests.

C'è qui una persona molto versata nelle matematiche e molto anziana la quale ha scritto molte opere in queste discipline e noi l'abbiamo persuaso a scrivere un compendio matematico sulle cose più necessarie, allo scopo che lo si possa leggere in poco tempo, ed egli ne ha già fatta una buona parte. Desidererebbe che padre Clavius venisse per qualche mese onde portare a compimento il progetto. (Sutto 1997, p. 111)³⁹

Finally, in 1574, towards the end of April, Clavius arrived at Messina, where he stayed, albeit not uninterruptedly, until the beginning of September. Even if we do not know exactly what he did, Clavius must have worked on the manuscripts of Maurolico. Subsequently (in 1581), he wrote that he had had one of Maurolico's manuscripts before it was printed in *Opuscula Mathematica* in 1575 S7. This was the manuscript *De lineis horariis* (Sutto 1997, p. 116) and not the one copied from Ω , that is from the compendium on music from Boethius. However, the Jesuit must, in turn, have taken from the latter manuscript the brief summary included in the edition as *F. M. Boetianae musicae compendium* C13, which is discussed in detail below. Baron della Foresta, who gave hospitality to his uncle in those years, stated that there was a 'great familiarity' with Clavius and that he entrusted him with the original manuscripts on optics, to have them printed in Rome. On the contrary, other manuscripts, 'which were printed after his death', dealing with various subjects, from the calendar to the five regular bodies, and also including music, were entrusted to Comenzino, who was going to Venice. But here, due to a question of debts, they were finally printed by Francesco De Franceschi as *Opuscula Mathematica*. It is curious that destiny led the musical theory of Maurolico to arrive at the same editor as the most famous theoretician of music of the period, that is to say, Gioseffo Zarlino. Last, the whole story has more recently been discussed by Moscheo (Moscheo 1998, pp. 185–232).

Let us now construct a table summarising which parts of the original *corpus* Ω have passed into our *Musica*, that is to say, into A6, which into *Musicae traditiones* of 1575 S7, and which into the summary of Clavius, C13. (Table 1)

The most complete collection of writings about music left to us by Maurolico is therefore our *Musica*. However, the other missing pages on the subject can be found either in S7 or in C13. In particular, the compendium from Boethius, which was dispersed with the Villacanense manuscript, had been printed, it is true, in *Musicae traditiones*, but a comparison with C13 of Clavius arouses doubts about its perfect conformity with what Maurolico wrote in Ω and may have recopied. Of the approximately 30 foreseeable propositions about sound and musical proportions, only 20 are left in *Musica*, together with a small fragment; they can, however, be completed to good purpose, with the 32 printed in 1575 in S7. The list of '*Tropi*' can be recovered, thanks to the summary of Clavius C13. As regards, the

authores, briefly mentioned in *Musica*, we have the whole page of S7. Last, we have to be satisfied, as regards the duration values of notes, with the small figure 38, which has remained in *Musica*.

Between Maurolico's *Musica* and *Musicae traditiones*, there are not only differences in quantity and in the subjects discussed but the most important difference concerns Boethius, the compulsory reference point for all theoreticians of music of the period. The fact that there was no compendium from Boethius in *Musica*, whereas *Musicae traditiones* placed it at the very beginning, did not depend on the studies, or the intentions, of Maurolico but depended on the fortunes of the manuscripts. However, we cannot ignore the fact that Boethius was heavily criticised in *Musica* for his mistake of treating the terms of a geometrical sequence as if they were, on the contrary, part of an arithmetic sequence. To correct him, Maurolico invented another very brief and elegant procedure, which has been described in section 1.2. In *Musicae traditiones*, instead, there is only a brief mention of this at the end, which completely leaves out Maurolico's new demonstration. Thus, the main contributions of Maurolico do not appear in *Musicae traditiones*, while they can now be read in *Musica*.⁴⁰

Boethius had judged it impossible to divide the tone into two equal parts. If nothing else, Maurolico performed calculations to verify this and left the proportional mean between 8 and 9 to fol. 27v.

As a result, *Musicae traditiones* assumed the aspect of a text for the teaching of current theory, while *Musica* is now able to show us what Maurolico added to the Pythagorean tradition. It also offered the *Regula compositionis* and the *Regula subtractionis* as a general procedure to calculate the proportions of music. In *Musicae traditiones*, only the results remain summarised in the final table. It should also be noticed that his contributions showed a greater mathematical engagement. For these reasons, therefore, *Musica* appears to be a richer and more interesting text than *Musicae traditiones*.

Vice versa, however, in the subjects discussed, *Musicae traditiones* is necessarily more complete than *Musica*. In the introduction on sound and musical proportions [II], *Musica* is mutilated. It stops at point 20 in the middle of a sentence; a few more lines can be recovered at the bottom of fol. 31r. *Musicae traditiones*, on the contrary, goes on as far as point 32. The mention of Mercury is equally insufficient. In *Musicae traditiones*, the whole of page 159 was reserved for the subject of *authores*, from Mercury to Pythagoras. The only small chapter that is present in its entirety, both in *Musica* and in *Musicae traditiones*, is the one dedicated to precepts for composition.

There may be various reasons for these differences. The original *corpus* cannot have been composed in a homogeneous manner due to its elaboration, which took up a long time of about 30 years. Thus, together with parts that are well arranged and well written, it is possible to find others that have only been begun, and outlined, with repetitions and calculations in the margin. Ω , therefore, could not be published as it was, even if certain subjects appear already to be in order. These included the compendium from Boethius, the numbered propositions about sound and musical proportions, the rules for composing

well, and the *scholium* on Boethius. On the contrary, the pages about the modes and Guido D'Arezzo's eicosichord must have needed readjusting, completing, simplifying, or summarising.

When the possibility of the desired publication became a reality, someone must have worked on the organisation of the edition. Who? Maurolico by himself? Maurolico with the help of Clavius? Clavius by himself? Other anonymous editors (probably Jesuits) between Rome and Venice? At all costs, besides the reorganisation for the Venetian edition S7 of 1575, Ω also underwent all the dismemberment, the losses, the new page make-ups, and the disorganisation due to the events that transferred it, in part, from the *freto siculo* to the library in Paris, where it is nowadays kept. It cannot be excluded that librarians worked on it, even here.

Finally, we must ask where Clavius found his *F. M. Boetianae musicae compendium* and how faithful it was to the source.

As *Boetianae musicae compendium* is found both in S7, printed at Venice in 1575, and in C13, as copied by Clavius in the manuscript in Rome and in the Villacanense manuscript, which remained at Messina until 1901, we are obliged to imagine that at least one copy must have been made of it. We should come to the same conclusion as regards those parts of Ω that, on the one hand, finished up in *Par. Lat. 7462* in Paris A6 and, on the other hand, were also printed at Venice in S7, as *Theoria musices proportiones, Regulae contextendi Symphonias*, the results of the *Systematum calculus*. In this last case, the direct link between *Par. Lat. 7462* A6 and S7, as printed at Venice, is confirmed by a mistake that Maurolico made in figure 31 of *Musica*, which is also found in the identical figure 1 of S7, printed at Venice.

It would therefore be more simple and linear to imagine that certain parts of Ω were copied and entrusted to the editor for the edition printed at Venice. Yet, in a situation in which the documentation is generally insufficient, one of the few elements that remain seems to make this reconstruction contradictory. In the letter in 17 September 1574, in which Doménech, the most important Jesuit father in Sicily, gave the news of the return of Clavius to Rome, we read:

Bisognerebbe scriver a Venezia alli nostri [i gesuiti] che havessero per raccomandata certa stampa di alchuni libri del Abbate [Maurolico] ... Sono ben informati li nostri. Il libraro se chiama Io. comisino il quale tene botega in Messina et hebbe detti libri delli quali non è restata copia et importano per questo nostro intento. (Clavius 1992, p. 8; Moscheo 1998, p. 223)⁴¹

The pieces that have remained do not combine, therefore, to form a simple, consistent picture. Because if no copies had been made, how could the compendium from Boethius have arrived at Venice, when Macrì still listed it in the Villacanense manuscript in 1901? Somebody should have taken it back after it was printed. A highly unlikely eventuality. Furthermore, if it had not been at least partly copied, how would *Par. Lat. 7462* A6 have arrived in Paris? In that case, it could only have happened after being printed (in part) at Venice.

The alternative would be not to consider reliable either the description that Macrì gave of the Villacanense manuscript or the more direct passage of *Par. Lat.* 7462 A6 from Messina to Paris, as Clagett believed; or else was Doménech badly informed and did he not know of the existence of copies? Someone, however, might have made a mistake. Finally, we have the documentation of at least one figure who had copied and summarised parts of Ω : Clavius C13. Therefore, before discussing the brainteaser, let us take a good look at what Clavius left us.

Part III

3.1 The Witness of Clavius Compared With the Others

From the original *corpus* Ω , Clavius first of all copied *F. M. Boetianae musicae compendium*. This was followed by *Repastinatio et Appendix*. Therefore, Clavius must undoubtedly have seen also parts of Ω directly and not only, possibly, the printed edition of 1575, or else he might have seen what might have been copied from Ω for printing. For the sake of convenience, I call this hypothetical copy C_1 , seeing that it will come into the following discussions. We find in the manuscript of Clavius (with a few variants that are discussed later) also what we can read only on folios 31r, 31v, and 32r of our *Musica* but not in *Musicae traditiones*. The subsequent *Icosichordum Guidonis* of Clavius is found both in the printed edition of 1575 (Maurolico 1575a, p. 154) and in the manuscript of Maurolico (M. table 6). However, in his copy, Clavius only presented the naked table, without any explanation. Clavius finished rapidly with *Octo modulatum, sive tonorum proprietates*. Of these, the four *Tropi* (*Protus*, *Deuterus*, *Tritus*, and *Tetradus*, which contained the first and second, the third and fourth, the fifth and sixth, and the seventh and eighth modes, respectively) are missing, both in *Musica* and in the edition of 1575. Last, the final description of the eight modes with their effects can be found today (with variants) both in the printed edition (Maurolico 1575a, p. 157) and in manuscript A6 of Maurolico on folios 35v (M. I, 24) and 36v (M. IX, 17–25).

In his *Boetianae musicae compendium*, Clavius took a position close to *Boetianae musicae epitome*, but even if it sometimes seems to be a word-for-word compilation, his copy also presents numerous variants here and there. These are almost all of a lexical nature, such as, for example:

- Substitute conjunctions (*vel* [or] for *ac* [and]), adverbs (*etiam* [also] for *quoque* [also]), synonyms (*mutuo* [mutual] for *reciproco* [reciprocal]), idiomatic expressions (*de quo inferius* for *ut postea patebit*)⁴², etc.
- Add adverbs (*vero* [truly]), etc.
- Omit adverbs (*itaque* [so]), etc.
- Change the construction of sentences ('*Subtracta vero Diesi de integro tono, superest ἀπὸ τοῦ ἐ sive maius semitonium sub hac proportione . 2187 . 2048*' instead of '*Porro*

dieseos ab integro tono differentiam esse apotomen, quae semitonium maius dicitur, terminos habens . 2187 . 2048')⁴³, etc.

- Use Greek letters ($\delta\iota\alpha\pi\acute{\epsilon}\nu\tau\epsilon$ for *diapente*), etc.

At two points, Clavius omitted whole sentences. After writing about Pythagoras, we read in *Musicae traditiones*:

ut praxis speculationi et experimentum arti respondeat. Quod autem infinitatem vocum humana ratio terminaverit, necessarium est. Omnis enim artis, non tantum musicae, subiectum infinitum cum sit: opera ... statuit. Solus enim Deus infinitus. (Maurolico 1575a, p. 147)⁴⁴

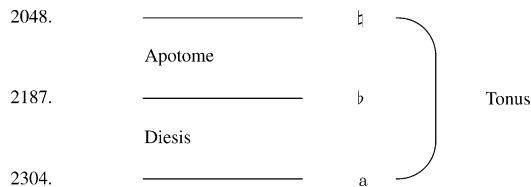
Clavius, on the contrary, jumped to

Sed cum omnis artis, non tantum musicae subiectum sit infinitum: opera ... statuit.⁴⁵

He then omitted the sentence about God.

In *Musicae traditiones*, the statement that musical notes should stand in commensurable ratios was followed by the explanation: '*Nam incommensurabilitas non recepit consonantiam, nec vocis scitum terminum, cum sit ignota*' (Maurolico 1575a, p. 149).⁴⁶ This sentence was completely left out by Clavius.

Before the statement that the tone could not be divided into two equal parts, Clavius inserted a '*Sectio toni*', which does not exist in *Musicae traditiones*. (Maurolico 1575a, p. 148)



Haec est toni in semitonium minus et maius divisio, secundum dictam superius proportionem.⁴⁷

The most significant variants, two in number, both are directly connected with Boethius. The first one precedes a table printed in *Musicae traditiones* (Maurolico 1575a, p. 148).

Hic est ordo, haec series, haec proportio, et processus naturalis. Nervorum Graeca vocabula, aut characteres nihil ad speculationem conferre. Exponatur nunc cum suis intervallis et proportionibus octochordum: quod theoriae satis esse potest.⁴⁸

Clavius instead wrote:

Hinc multum sudat Boëtius in vocabulis nervorum Graecis, et in processis characteribus; et in coaptatione consonantiarum, et proportionum vocalium quae (...habeatur Icosichordi Guidonici) non sunt necessaria. Omnis enim musicae praxis et speculatio constat in ordine, ac proportione vocum, in processu Diatonico (admissa tonorum singulorum in semitonia divisorum dispone). Sed eccum hinc octochordi naturalem per suas proportiones processum.⁴⁹

Also, the following table of *Musicae traditiones* (Maurolico 1575a, p. 156) was different from the one presented by Clavius. Clavius' table (figure 3) was more similar, in its

☾ .	216 .	——	g .	Nete
		Tonus		
☿ .	243 .	——	f .	Paranete
		Diesis		
♀ .	256 .	——	e .	Paramese
		Tonus		
☼ .	288 .	——	d .	Mese
		Tonus		
♂ .	324 .	——	c .	Lichanos
		Diesis		
♂ .	341 $\frac{1}{3}$.	——	b .	Parhypate
		Tonus		
♂ .	384 .	——	a .	Hypate
		Tonus		
*	432 .	——	Γ	Proslambanomenos

Fig. 3. From Clavius' manuscript, the correspondence between numbers, notes, and stars.

numbers, to that of fol. 14r of *Musica* (figure 14) and, in its arrangement, to table 9 (M. IX, 17). It should also be noted that the numbers on the table of Clavius' manuscript are obtained by multiplying by 36 those contained in *Musicae traditiones*. Also, in *Musica*, Maurolico had done something similar, choosing for the various tables different numbers, some of which were very large, as in figure 16.

When he presented the number of commas contained in the various tones and semi-tones, Clavius was departing, in a particularly significant manner, from *Musicae traditiones* (Maurolico 1575a, p. 148). While this work was Boethian in stating that the minor semitone was '*maius tribus commatibus*', Clavius, on the contrary, affirmed that it '*maius quidem esse tribus commatibus ac $\frac{1}{2}$* ',⁵⁰ in line with the modification of Maurolico in *Musica*. The printed version continued with the fact that the apotome '*maiozem esse quam quatuor commata*', whereas Clavius insisted: '*Et propterea Apotomen maiozem esse quam 4^{or} commata et $\frac{1}{2}$* '.⁵¹ These statements were confirmed in *Musicae traditiones* by a: '*ut constat rationes componenti, aut subtrahenti*'.⁵² But we read in Clavius manuscript: '*sicut per numeros differentiarum Boetius concludit*'.⁵³

If the differences between the hand-written *Compendium* of Clavius and the printed version of *Epitome* of 1575 had been negligible or insignificant, we could have concluded that the original *compendium* of Maurolico from Boethius (which was dispersed with the so-called Villacanense manuscript) had passed faithfully into *Musicae traditiones*. The possibility could not even have been excluded that the possible copy made of Ω for Venice corresponded to the copy of Clavius himself, which, for the sake of convenience, we shall call C_2 . However, the differences between the two texts are too great and too important. What are they due to? Either the editors of the Venetian edition had introduced them or Clavius had taken the liberty of making the changes. Another possible alternative would lead us to imagine that Maurolico modified his original *compendium* for the printed version. But why did he change it so much? And why bring it closer to Boethius? We would

be greatly helped in solving this problem if we had the Villacanense manuscript, which, according to Macrì, contained the *compendium* in question. Without it, we can only advance more or less probable hypotheses, which we shall leave to the end. Let us start, however, by pointing out that the title given by Clavius, '*Compendium*', coincides with the original title chosen by Maurolico as early as in 1540.

In '*Repastinatio*',⁵⁴ Clavius followed very closely, almost word for word, Maurolico's *Musica*: folios 31r, 31v, 32r. The variants of the former compared with the latter are above all of the following types:

- Adding an adverb (*igitur [therefore]*), small sentences (*ut dictum est [as said above]*), short explanations (*quae proportio sesquialtera est [which is a sesquialter proportion 3 : 2]*).
- Changing a word (*quoniam irrationales [as irrational]* for *quae irrationales [which irrational]*).
- Inserting an Arabic number (*3ⁱⁱ* for *tertii*), or vice versa, putting it into Latin letters (*Decima* for *10^{mam}*).
- Substituting with synonyms (*nervis* for *chordis*)⁵⁵.
- Varying the construction of a sentence.

Et si his singulis rursus apponatur diapason, gignentur totidem consonantiae 3ⁱⁱ ordinis, scilicet, Quindecima, seu Disdiapason, Septemdecima, Decimaoctava, Decimanona, Vicesima. Et si complectur Icosichordum totum Guidonis complexum tertii ordinis symphonias.⁵⁶

instead of

Adnectatur 13^{ae} alia diapason. Et conflabitur 20^a quae complet Icosichordum totum Guidonis quod complectitur 3ⁱⁱ ordinis systemata, scilicet disdiapason .17^{am}.18^{am}.19^{am}. 20^{am} (M. IV, 13).⁵⁷

In Clavius' manuscript, however, some sentences were omitted. Where Maurolico had written on fol. 31r: (M. IV, 3–6)

Quae duae efficiunt proportionem .2. ad .4. quae diapason est. Ex his eliciuntur spacia tonorum et semitoniorum, quae faciunt diatonicos et naturales gradus vocum ascendentium et descendentium. Tonus enim est spacium sesquioctavae proportionis, quae scilicet differentia est ipsarum diapente et diatessaron hoc est sesquiterciae et sesquiquartae, (sic! sesquialterae et sesquiterciae) sicut constat in his numeris .9.8.6. Tonus autem bis ablati a diatessaron relinquit semitonium minus, sive diesim. Ideo ascendimus in cantu naturali per tonum, tonum et diesim. Et rursus per tonum, tonum et diesim⁵⁸

On the contrary, Clavius only repeated:

Unde ex his duabus consu[m]matur] proportio Diapason, sic .2.3.4. Ex differentia vero eorundem fit toni proportio sesquioctava, ut patet [?] in his numeris .9.8.6. Tonus autem bis ablati a diatessaron relinquit semitonium minus, sive diesim. Ideo ascendimus in cantu naturali per tonum, tonum et diesim.⁵⁹

And that was a pity because, as we have pointed out in the edition of *Musica*, at this point, Maurolico had made a mistake when he wrote '*diapente et diatessaron hoc est*

sesquiterciae et sesquiquartae’ instead of *sesquialterae et sesquiterciae*, which are the correct proportions for the fifth and the fourth. If Clavius, too, had copied the same mistake, this would have been convincing proof that he had seen our *Musica*. Of course, the opposite is not true, and it is also likely that Clavius had immediately realised this, limiting himself to re-elaborating the passage, so as to omit the mistake.

Further on, on fol. 32r, Maurolico had written: (M. IV, 12)

Nam diapason addita semper facit eiusdem qualitatis symphoniam. Itaque 13^{um} spacium continebit 2^{i} ordinis symphonias scilicet diapason . 10^{mam} . 11^{am} . 12^{am} . 13^{am} .⁶⁰

In his manuscript, Clavius only put:

scilicet in primis ipsa diapason, sive Octava, Decima, Undecima, Duodecima, Tredecima.⁶¹

The manuscript of Clavius includes two long passages, of which there is no trace in *Musica*. The first appears immediately at the beginning of ‘*Repastinatio*’.

Coetera, quae tractat Boëtius, versantur circa intervalla et proportiones vocum et symphoniaram quae omnia comprehendunt in Icosichordo Guidonis. Unde et praxis canendi et instrumentorum dispositio propagat. Itaque nunc, premissis quibus . . . preambulis, Icosichordum versum exponemus et erit peragenda quaedam praemissorum repastinatio.⁶²

On fol. 32r, Maurolico had written: (M. IV, 14)

Itaque deinceps fieri potest in infinitum. Hinc patet origo numeri harum vocum hexachordum constituentium scilicet .*ut.re.mi. fa.sol.la*.⁶³

But in Clavius’ manuscript, a variant of this sentence is followed by others, which is not found in *Musica*.

Quae quidem appositio diapason fieri potest 4^0 , 5^0 et deinceps in infinitum, sicut patet in Harpichordis, organis et magnis instrumentis. In quibus . . . Diatonicus et chromaticus excedit manum Icosichordum Guidonis.

Hinc ergo derivatur Etymologia nostris diapason. quoniam scilicet sic semel, bis, ter et quotiescumque applicata consonantiis singulis generat eiusdem cum ipsis simplicibus generis consonantias singulas.

Denique non minus hinc notescit numeri harum 6-vocum hexachordum constituentium . . . et origo, quae sunt .*ut.re.mi. fa.sol.la*.⁶⁴

Here, Maurolico had carried on as follows: (M. IV, 14)

Octo autem literae $\Gamma.a.b.c.d.e.f.g.$ statutae sunt ut earum unaquaeque octavo quoque loco repetita diapason consonantiam in proportionem dupla semper indicet. Quod numeri in singulis chordis Icosichordi dispositi, sicut omnes alias consonantias et spacia ostendunt.⁶⁵

In Clavius’ manuscript, we also find some slight variations:

Octo autem literae $\Gamma.a.b.c.d.e.f.g.$ statutae sunt sub tali numero, ut earum unaquaeque octavo quoque loco repetita diapason consonantiam in proportionem dupla semper indicet. Quod numeri in singulis nervis Icosichordi dispositi ostendunt, sicut alias consonantias et systemata. Nunc repetam calculum.⁶⁶

But it is at this point that the two manuscripts become too divergent, making it difficult to compare them. The manuscript of Clavius goes through the stages with the calculations to obtain the proportions of the tone, of the diesis, and of the comma.

Si utralibet duarum consonantiarum, Diapente, ac Diatessaron, subtrahatur de Diapas ω n superest reliqua. 4 . 3 . 2 .

Si Diatessaron subtrahatur a Diapente relinquitur Tonus. 9 . 8 . 6 .

Si duo toni subtrahatur a Diatessaron, residuatur Diesis. 324 . 288 . 256 . 243 .

Unde patet quidem si tres toni auferantur de Diapente, superit Diesis.

Si Diesis detrahatur a tono, superest Apotome, sic . 2304 . 2187 . 2048 .

Demum, si Diesis abscondatur ad Apotome, relinquitur comma. Ut patet per hos numeros . 559872 . 531441 . 524288.⁶⁷

These numbers are, of course, the same ones presented several times in *Musica*, but there they are offered in a different manner, as in the figures 5–8 (M. IV, 19–21). Above all, in Maurolicos manuscript, as has already been said, this is followed by the statements of Boethius about the number of commas in the diesis, the apotome, and the tone. Instead, Clavius manuscript immediately presents the more narrow inequalities of Maurolico.

Diesis excedit tria commata et dimidium. Apotome maior quam $4\frac{1}{2}$ commata. Unde Tonus integer excedet 8 commata et minor est quam 9. Quae omnia constant per regulas componendi et subtrahendi proportionibus quae regulae sunt similes aut eadem cum regulis fractionum in numeris.⁶⁸

Now, Clavius expounded the eicosichord directly, but in words that are difficult to compare with those found in our edition of *Musica* (M. VIII, 1–12).

Nunc autem exponam Icosichordum Guidonis in quo per proportionibus numerorum et ordinem Diatonicum naturaliter[?] procedentem, receptam (ad temperandum Tritonum harmonicum) chromatica divisione, representantur omnes Musicalium vocum proportionibus, scilicet, tonorum, semitonium minorum et maior quae dicuntur Diesis et Apotome, Diapas ω n, Diapente, Diatessaron, Tritonus et . . . tematum ab ipsis per compositionem (ut dictum est) propagatorum. Constat autem totum Icosichordum ex 14 tonis et 5 Diesibus.⁶⁹

Thus, departing from *Musica*, Clavius' manuscript seems to come closer here to *Musicae traditiones* because its table of the eicosichord is also very similar to the one that is printed there (Maurolico 1575a, p. 154, table 2). Of course, nobody could have repeated the table on folios 6v and 7r in A6, table 6, as it was too messy and some action on the part of the editors would have been necessary in any case. The left-hand side of our table 6 (M. VIII, 4) coincides almost perfectly both with that of *Musicae traditiones* and with the one found in Clavius' manuscript.

Yet, certain difference of details are significant. All three give the geometrical progression from 4 to 27. However, in *Musicae traditiones*, the proportions to divide the tone $21\frac{1}{3} : 24$ into apotome and diesis are printed wrongly as ' $25\frac{22}{32}$ ' instead of ' $22\frac{25}{32}$ ', as both Maurolico and Clavius rightly wrote. The same (printing?) mistake was repeated between $10\frac{2}{3}$ and 12, with the number ' $12\frac{25}{64}$ ' instead of the correct value ' $11\frac{25}{64}$ ', which is present in both manuscripts. While these two mistakes, then, are not present in Clavius manuscript, it is strange that it contains a new one: ' $5\frac{1}{15}$ ' instead of ' $5\frac{1}{16}$ '. Both Clavius and *Musicae*

traditiones divided the tone into ‘*Apotome*’ and ‘*Diesis*’, whereas Maurolico had used, in his table 6, the terms ‘*Maius*’ and ‘*Minus*’. Statistics would therefore seem to bring Clavius manuscript slightly (slightly?) closer to that of Maurolico because it appears to be much easier to substitute a word with a synonym than to perform the calculations again, even if an attentive reader can easily note the two wrong numbers printed in *Musicae traditiones*.

At the left of the table in question, but rotated by 90 degrees from the vertical, Clavius had placed the following numbers in a triangle:

$$\begin{array}{ccccccc} & & 8 & . & 12 & . & 18 & . & 27 \\ & & & & 4 & . & 6 & . & 9 \\ & & & & & & 2 & . & 3 \\ & & & & & & & & 1 \end{array}$$

The highest line of these is at the same level as the numbers . 8 . 12 . 18 . 27 of the geometrical progression in the table. The lower numbers are obtained from the differences between the higher ones. In the geometrical progression (by fifths, that is to say, by a factor of $\frac{3}{2}$), it is clear that the differences are not constant but increase proportionally. Was this the verification of the mistake made by Boethius? These numbers are not found either in *Musicae traditiones* or in Maurolico’s *Musica*.

Clavius entitled the last part of the manuscript ‘*Octo modulatum, sive tonorum proprietates*’. The following four *Tropi* contained in it did not appear, as has been said, either in *Musicae traditiones* or in Maurolico’s manuscript.

Tropi sunt 4^{Or}. Protus. Deuterus. Tritus. Tetradus, quorum singuli continent duos tonos.
 Protus continet Dorium et Hypodorium. 1^m et 2^m.
 Deuterus continet Phrygium et Hypophrygium. 3^m et 4^m.
 Tritus continet Lydium et Hypolydium. 5^m et 6^m.
 Tetradus continet Mixolydium et Hypomixolydium. 7^m et 8^m.
 Ex his 1^{us} . 3^{us} . 5^{us} . 7^{us} dicuntur Autentici 2^{us} . 4^{us} . 6^{us} . 8^{us} placales.⁷⁰

Clavius went on to list the eight modes, as they are found also in *Musicae traditiones* (Maurolico 1575a, p. 157), with a few small variations mainly in the order 1, 2, 3, 4, 5, 6, 7, 8 instead of 1, 3, 5, 7, 2, 4, 6, 8.

Primus exordium fit in d . sol . re.
 Tertius exordium fit in e . la . mi.
 Quintus exordium fit in f . fa . ut.
 Septimus exordium fit in g . sol . re . ut.
 Secundus exordium fit in a . re.
 Quartus exordium fit in ḅ . mi.
 Sextus exordium fit in c . fa . ut.
 Octavus exordium fit in d . sol . re.⁷¹

However, in that precise form, the list is not present today in *Musica*. In its place, we find table 8 on fol. 8r, with a long explanation (M. IX, 1–16).

Last, the following lines may be synoptically compared with all three texts under discussion. On fol. 35r, Maurolico had written: (M. I, 23)

Ascendunt autem autentici, ac descendunt per diapason, placalis autem a loco sui autentici ascendit per diapente, descendit per diapason, et rursum per diatessaron scandens in locum dictum definitum.⁷²

Clavius:

Autentici ascendunt et descendunt per spacium diapason: placales indidem ascendunt per diapenten et descendunt per diapason. Et rursus ascendentes per diatessaron in locum singuli suorum autenticorum (unde sumebant exordium) desinunt.

Horum tonorum 1^{us} datur Soli: 2^{us} Lunae: 3^{us} Marti: 4^{us} Mercurio: Quintus Jovi: Sextus Veneri: 7^{us} Saturno: 8^{us} Firmamento.⁷³

In *Musicae traditiones*: (Maurolico 1575a, p. 157)

Formantur autem autentici a loco proprio ascendendo per diapente et diatessaron, hoc est, per diapason: et inde tantundem descendendo. Placales autem a sede sui quisque autentici per diapenten ascendunt: et inde per diapenten ac diatessaron descendunt: unde rursus per diatessaron ascendunt, et in locum autenticorum simul desinunt.⁷⁴

This was followed by the effects of the eight modes on the soul and on the behaviour. Here, Clavius and *Musicae traditiones* (Maurolico 1575a, pp. 157–158) proceeded in parallel, using almost the same terms and with few variations.

Clavius:

Primus sonnolentiam, pigritiamque expellit; verbisque iocosus, lepidus, ac facetis convenit.

Secundus somnum quietum ac lenem inducit: quo utebantur Pythagorici, cum continuas curas quiete, aut somno temperabant. Et est moestus, flebilis ac libertatis amicus.

Tertius est incitativus, severus, asper, iracundus, bellicosus.

Quartus blandus, garrulus, lascivus, adulatorius, mitigativus, exhortatorius.

Quintus delectabilis, hilaris, modestus, nonnihil petulans, consolatorius, encomiasticus.

Sextus lacrymabilis, pius, devotus, amatorius, compassionem aut laetitiam inducens.

Septimus varius, querulus, audax, et proprietates tertii, quarti et quinti habet.

Octavus tristes ac lentos excitat, suavis moratus, deprecativus, aptus ad implorandum, agit de rebus profundis, de contemptu inferiorum, de coelestibus rebus.⁷⁵

In *Musica*, on the contrary, Maurolico had followed a different course, on fol. 35v (M. I, 24), with a far more concise description and without numbering the modes.

Dorius lepidus est ac iocosus, sonnolentiam expellens. Hypodorius somnum inducit, flebilis, liber. Phrygius severus, iracundus. incitat, exasperat. Hypophrygius blanditur, lascivus, mitigativus, hortatorius. Lydius hilaris, petulans, laudat, consolatur. Hypolydius compatitur laetificat, pius, lacrymabilis. Mixolydius varius, querulus, audax et qualitates tertii, quarti et quinti habens. Hypomixolydius excitat precatur, de contemptu rerum, et de coelestibus agit.⁷⁶

However, on fol. 36v (M. IX, 18–25), these effects were repeated with a few words added.

3.2 Routes and Scenarios

The original *corpus* Ω of the folios written by Maurolico about music appears to have been composed over a period of 30 years. It is divided up by subjects: Boethius, Guido D'Arezzo, the *authores*, the Modes, the *Regulae*, etc. Some were complete, well arranged,

and well written and others were incomplete, only outlined, or even only planned. There is no single manuscript today or any place where all the Ω are contained. Most of Ω is to be found in *Musica*, from the Paris manuscript, *Par. Lat.* 7462; other parts are in the edition printed at Venice in 1575, and last, a small part is in Clavius manuscript in Rome. The fragment of the San Pantaleo manuscript only contains one diagram and a few calculations that are already present in the edition of *Musica*.

What events led up to the present situation? We can be fairly sure about some of them, but we have little information about others. Now we are faced with two possibilities. A historian could stop at this point, at least until, possibly, the Villacanense manuscript is found. Thus, anybody who prefers to limit himself only to the bare written texts, not believing that they represent precious memories and traces of more complex realities to be studied, will do well not to go on reading. Or else, we can start to see in how many ways the pieces of the puzzle that we possess fit together, excluding cases that are not compatible with the present information. Consequently, as we do not give up the chance of understanding better, let us pass on to describe the possible scenarios of our events, evaluating their relative probability. When, and if, we are presented with other pieces to fit into the picture, it will be sufficient to exclude cases that are still contemplated today but become incompatible in the light of the new developments.

These scenarios also depend on the various elements we choose, as important and relevant for our historical reconstruction. Among these, a non-marginal role might even be played by the character that we decide to delineate for Maurolico himself, seeing that this determined his behaviour in the events discussed.

The original *corpus* Ω was undoubtedly dismembered in some of its parts. The *Boetianae musicae compendium* seems to have stayed in Sicily, in the Villacanense manuscript. Macrì speaks of it as '*Musica Boetii*'. The other part, the larger one, was still kept by the heirs for most of the 17th century. Thus, it, too, initially remained in Sicily, but then this second part reappeared in the catalogue of the Colbert Library in Paris, as (subsequently) manuscript 7462. Another part of Ω was taken to Venice to be printed and published in the 1575 edition. Doménech wrote of this work that 'no copy has remained' in Sicily (par. 2.2). At this point, the problems begin.

From Table 1, we can see the coincidences between the folios that ended up at Venice and those that ended up in Paris: a large part of 'Theoria musices proportionēs', 'Calculus Boetii', 'De octo modis ...', 'Regulae contexendi symphonias'. Then, there is the overlap between the folios that ended up at Venice and the musical writings that remained in the Villacanense manuscript, that is to say, 'Boetianae musicae compendium', together with 'Musica Boetii', according to Macrì. Therefore, the parts of Ω that were sent to Venice to be printed must have returned to Sicily, where the 'compendium' entered into the Villacanense manuscript (seen by Macrì at least until 1901). The rest took the direction of Paris.

If Doménech was well informed, therefore, we must necessarily imagine a return of the musical writings from Venice *in freto siculo*. This is difficult to believe because manuscripts were generally treated badly and divided up at the printers. Furthermore, who could have

been interested in taking the folios away from the printers, to take them back to Sicily? But if we do not consider the return journey likely, then we have no choice but to deny the other testimonies. Consequently, certain dilemmas remain to be solved. The first of these is whether copies were made or not.

Case A: no copies were made. Due to the coincidence in the texts, another bifurcation is created here, between the return to Sicily and not.

A_1 : the part taken to Venice returned to Sicily, either to become a part of the Villacanense manuscript or to be kept by the heirs and then transferred to Paris.

A_2 : nothing returned to Sicily. The folios passed directly from Venice to Paris. In this case, we must necessarily deny the testimony of Macrì about the musical part of the Villacanense manuscript. We must also reject Clagett's hypothesis about the transfer of the papers from Sicily to Paris (Clagett 1974).

Case B: copies were made. We must necessarily deny what Doménech wrote. In this case, the bifurcations regard who copied the papers.

B_1 : Maurolico.

B_2 : Clavius.

B_3 : other anonymous hands (Jesuits?)

The C_1 copy is no longer extant, but we do possess Clavius' manuscript, C_2 , which is discussed in section 3.1.

Let us now examine the *pros* and the *cons* of each case to evaluate their relative probability. In case A (no copies were actually made), the pieces of the puzzle would fit together easily only if the folios really returned to Sicily. But this would be an exceptional event and is highly unlikely. This is the problem with A_1 . Otherwise, in case A_2 , we are obliged to conclude that Macrì is unreliable. This question would be solved if we could verify whether the Villacanense manuscript contains the compendium from Boethius or not. We are also forced to refute Clagett's hypothesis about the direct transfer between Sicily and Paris. Clagett hypothesised that Maurolico's manuscripts, which ended up in Paris, had been collected *en bloc* and carried away, on Colbert's orders, during the revolt of Messina, which saw the presence of French soldiers on the straits of Messina at the end of the 17th century (Clagett 1974). But Clagett's arguments are weakened by Moscheo, who, after analysing the catalogues, seems to advance the hypothesis that the manuscripts found their way into the library in different periods. Furthermore, the emissaries of the powerful French minister, who included his brother, wrote, in reality, that they had not found any of the things they had been ordered to collect (Macrì 1901, pp. 98–100; Moscheo 1988, pp. 143–148, 455–476, 467). Thus, the route followed by a part of Ω in the direction of Paris would appear to be more uncertain than we might have imagined at first sight. Furthermore, why did not, the French soldiers, take possession, by fair means or foul, also of the Villacanense manuscript? Clagett's hypothesis appears to be backed up by the general historical circumstances of the presence of French troops at Messina. However, it is made less certain by the emissaries' letters and also by Moscheo's studies on the catalogues of the royal library.

Therefore, in case A, without any copies and abandoning Clagett's hypothesis, we may imagine that the musical writings taken to Venice were partly (chosen and) printed in 1575 and partly took the direction of Paris, while the compendium from Boethius was lost at the printers. Even if we do not know how the remaining papers arrived in Paris, this solution to the puzzle may seem more likely than the return of the manuscript *in fretu siculo*. But only provided that we confirm the absence of the compendium from Boethius from the Villacanense manuscript, which was still present in Sicily until 1901. Otherwise, having solved the overlaps between Venice and Paris (without copies), we would fall down on the coincidence between Venice and Messina. As we cannot do this *de visu*, we must shift our investigation to consider the reliability of Macrì. How far can we trust him? Did not the various disputes about the lessons held at Messina for the University under construction weaken the testimony of Macrì? Why is the Villacanense manuscript no longer to be found? Who else says that he had examined it *de visu*? As the Villacanense manuscript cannot be produced, it would be legitimate to advance doubts also about what Macrì wrote of it. If Macrì became unreliable, the other pieces of the puzzle would fall into place, with the departure of the original manuscript from Boethius for Venice, and its loss amid the mists of the lagoon, and consequently, its non-arrival in Paris.

The reader who is interested in solving the question can choose the solution that he considers to be most likely by himself or else he may hope that he succeeds in finding the Villacanense manuscript.

The present author is led to consider, within route A of the bifurcation, A_2 more probable. However, these probabilities are relatively low because they are the result of the combination (product) of probabilities, which, in turn, are very low: that Macrì imagined musical pages that did not exist in the Villacanense manuscript and that the French troops did not take the manuscripts to Paris. Compared with all this, there is a far greater probability of the existence of copies. Let us therefore examine the reliability of the written statement of Doménech and follow route B of the fork.

Let us start by comparing the Paris manuscript with the text printed at Venice to examine in detail the elements in the common parts of the manuscript and the printed version, which tend to confirm a direct link between the two and those that, on the contrary, suggest the intermediation of a copy.

As regards '*Theoria Musices proportiones*', the manuscript of *Musica* cannot have been copied directly at the printers in Venice because the variants between it and the printed version of 1575 are too numerous and, above all, too significant. The manuscript already contains integrations and corrections, perhaps introduced later, because the quality of the ink appears to be different. The integrations are generally to be found also in the printed version but not always at the same point indicated by the manuscript or with the same Latin declinations. Some words have been added, and others have been eliminated. A '*unde necesse est ut tam*' becomes '*unde sequitur*', which is shifted inside the first corollary. This was also rearranged. Point 8 becomes a corollary in the printed version, and consequently, from this point on, the numbering no longer corresponds. A few words have been erased

from point 9 in the printed version. 'A *nervo*' is repeated twice, in point 8 of the printed version; and this is a typical printers mistake. The numbering has been corrected, starting from number 11 in the manuscript, as if certain integrations had forced Maurolico to re-organise it. But in the printed version, it was modified, turning number 8 into a corollary, as mentioned above. As a result, from this point on, the numbers in the manuscript (for the same sections) are one higher than those of the printed version. The construction of the accusative and the infinitive in point 12 becomes, in point 11 of the printed version, a nominative and an indicative and so on. Point 14 in the manuscript was integrated, in point 13 of the printed version, with '*adhuc minus suavem, adeo ut dubium sit an consonantiis sit adnumeranda: cum a Ptolomaeo solo admittatur*'.⁷⁷ Another significant addition can be found in point 15: '*Quoniam sesquialtera cum sesquitercia proportionibus componunt duplam*'.⁷⁸ The figures 2 and 3 in the manuscript (M. II) become only a succession of numbers in the printed version.

At this point, in the manuscript, we can find an inversion of the reasoning. We read in the corollary that the diapason is made up of five tones and two dieses, while the definition of diesis follows only in point 17. In reality, the manuscript appears to be messy here, with a deletion, the diagrams of the proportions, and the head of the dragon.⁷⁹ Maurolico might have added the corollary where he found space for it. Anyway, in the printed version, the logical order was restored, putting the derivation of the diesis first, at numbers 16 and 17, and (instead of the corollary) the composition of the octave only at number 18, with the usual variants in the wording. The attribution in the printed version of number 18 to the paragraph that appears as a corollary in the manuscript restores the same numbering from number 19 on.

In '*Calculus Boetii*', figure 10 of the manuscript (M. VI) would appear to be the same as the last figure of *Musicae traditiones* (Maurolico 1575a, p. 160), if three mistakes had not been introduced here in the printing. Two concern figures may have easily been introduced by the printers. The exchange of the apotome with the comma may have the same origin, or maybe not. In the comment on the figures, however, there is a particularly significant mistake, which is only present in the printed version. In the manuscript, we read: '*Hic est Calculus Boetii in 3^o Musicae Suae ...*'. In the printed version: '*Ex hoc ultimo calculo Boëtius in 3 Arithmetiae ...*'.⁸⁰ The passage of Boethius quoted is to be found only in *De Musica* (Boezio 1867, pp. III, 15, 25). *De Arithmetica* only contains two chapters. The rest of the comment has been completely re-elaborated for the printed version.

Also, the pages with the effects of the modes on the soul have been re-elaborated for the printed version. The '*Regulae contexendi symphonias*' become '*Praecepta*' in the printed version. Some rules were simplified and shortened in the printed version. One, which was not numbered, was completely eliminated.

Last, in '*Calculus vocalium proportionum*' in figure 1 of the printed version (Maurolico 1575a, p. 160), the same mistake remained as in figure 31 of *Musica*. Here, Maurolico had confused the diagram of 'multiplication' between proportions with that of 'division' (M. XII).

Patet, it is evident that between the manuscript and the printed version other hands had been at work. We discuss below the people to whom these hands might belong. In any case, these hands at times copied word for word, and once even a mistake, and at other times, they took the liberty of introducing variants, modifications, and cuts of all kinds. They must also have been guided by people who were considered to be experts on the subject, if they replaced the correct quotation from Boethius, as given by Maurolico, with another quotation, which was wrong! A better understanding would have been displayed if they had restored the logical order of points 17 and 18 in '*Theoria Musices Proportiones*'.

If these changes had been made *in fretto siculo*, they would have been difficult to reconcile with Doménech's statement that 'no copy has remained'. As a final attempt to save him, the only thing that could be suggested is that the changes had been so radical as to produce not a copy but a whole new text. This would then depart for Venice, together with Comencino (par. 2.2).

Or else the changes were made at Venice. In this second case, we would be able to go on believing Doménech's statement that no copies had remained in Sicily. But at the same time, we should conclude that Macrì had invented the compendium from Boethius in the Villacanense manuscript. The dilemma amounts to choosing between Maurolico (Sicily) and the others (Jesuits in Rome, Venice) as authors of the copies and re-elaborations. However, we know for certain, from other sources, that some copies (at least of certain parts) had been made. Now, therefore, it is Doménech who proves to be *badly* informed. Actually, a non-autograph copy of *Arithmeticonum Libri Duo*, subsequently also printed at Venice in 1575, is extant in *Vat. Lat.* 3131 C14. Indeed, in the dedication with the date 1 December 1568, to cardinal Marco Antonio Amulio, Maurolico said that his nephew Francesco, the future Baron della Foresta, had made the copy, but he had not written it as well as he would have liked (Moscheo 1988, p. 277 ff.; Moscheo 1998, pp. 181–182). Furthermore, Cristoforo Grienberger wrote in a letter to Clavius on 20 January 1608 that Silvestro Marolì had made copies of every book (Clavius 1992; Moscheo 1988, p. 62). It is, however, possible, after more than 30 years, that in this second case, he was referring to other books and not to those discussed here, which by now had been taken to Venice.

These circumstances greatly increase the probability that a copy of the musical writings, which we have called above as C_1 , was made. Even if this has not been preserved, because it ended up at Venice in a printer's works, another sure copy of the musical writings has remained; the one made by Clavius, which is discussed in section 3.1 (called C_2).

At this point, leaving aside a Doménech who is so badly informed, we follow route B to face up to the other bifurcations. The overlaps and the comparisons between the extant texts and the testimonies allow us to be certain that the musical writings were copied. The compendium from Boethius had been copied, together with other papers, by Clavius, who has left us his autograph copy. It is highly likely that another copy was made, to be sent to Venice for printing, seeing that it is less probable that the original returned from Venice to Sicily, where it was seen by Macrì at the beginning of the 20th century.

The forks that we meet along route B regard the people who made the copies, and perhaps introduced modifications, and those who chose the parts that were suitable for publication. The first scenario, B_1 , is set in *freto siculo*, that is to say, on the Straits of Messina. Here, it would have been Maurolico himself who set to work on the musical writings, choosing what to publish and what to copy. He could also have summarised the wordy parts or completed subjects that had been left incomplete. As the title, *Musicae traditiones*, indicates, the Jesuits must have asked him to deal with the subject in a manner suitable for teaching. It transpires from Maurolico's letter to Francisco Borgia, written on 16 April 1569, that he had become involved in the project (Moscheo 1998, pp. 164 ff, 318; D'Alessandro and Napolitani 2001). It was probably for him that Maurolico had prepared the '*Ordo compendii*' of 17 March 1569, contained in *Musica* on fol. 20v (M. VIII, 53). Furthermore, it would seem to be likely that he had been helped and advised in this by Clavius, who had arrived at Messina partly for the purpose of reorganising the manuscripts, in view of their publication. The surest proof of this is the summary that the Jesuit left, which is discussed in section 3.1. Therefore, Clavius must necessarily have seen the musical writings Ω . This scenario is probable in view of:

- (1) all the parts of Ω that stayed in Sicily, which Maurolico would no longer need to send to Venice to be printed, since they had been copied, re-elaborated, and arranged to create another text for this purpose
- (2) an easier direct transfer from Messina to Colbert's library, as hypothesised by Clagett.

Yet, this scenario is weakened by the extant descriptions of the character that our man from Messina revealed in other similar activities. The differences that can be found, and have been pointed out at the beginning, between our edition of *Musica* and the printed edition of 1575 and also between the latter and the Clavius copy lead us to believe that other interventions must have taken place in this scenario. Why should Maurolico have renounced publishing his own contributions? Why should he have left more space for Boethius, when he knew the latter had made a mistake over a question of mathematics, which was what he considered most important? The possibility that in the end, the Venetian editors had given the text a character that was less critical of Boethius can easily be understood: he still represented orthodoxy on the subject of music. Furthermore, he had been reprinted, and what's more, at Venice, not long before. On the contrary, it would appear to be difficult to give all the responsibility for the printed edition directly to the pen of our Sicilian mathematician. We have already recalled that his nephew, Baron della Foresta, described him as: 'unstinting in communicating his speculations to his friends, extremely jealous of the fame of other people', desirous that 'the works that he had so painstakingly composed ... should be presented to the world ... provided that they were published for the benefit of mankind, and for knowledge of the truth', 'so keen in his studies that several times, he spent the night awake with his eyes on his books and his mind on the secrets of Nature, speculating on other scholars doctrines and censuring their mistakes'. This Maurolico seems to be very different from a scholar who has, with the passing of time, become

indifferent to the destiny of his works; on the contrary, he had taken considerable pains to have them printed (Moscheo 1988, pp. 275–276; Sutto 1997, pp. 105–107).

Maurolico introduced himself to Vega in 1556 as follows:

...ad haec me studia non spes aliqua lucri, non ulla famae(?) vel honoris aut pecuniae, non inanis superstitio unquam traxit. Sola speculationis jucunditate ac veritatis amore cuius scopus nullo certiori telo quam his studiis attingitur, allectus huc veni. (Macrì 1901, p. lxxiv; Moscheo 1998, p. 304)⁸¹

But then, at the end of his earthly existence, would he have desired only to remain on the sidelines? Not at all. In his letter of 1571 to his pupil, Pietro Barresi, he wrote that the prince should not go away from Sicily if he wanted to study ‘the arduous and deep sciences’. For he knew very well

chi habbia scritto profunda scientificamente di arithmetica, di prospettiva, de li diafani, de Iride et altri importanti passi de la math.ca facultà . . . chi sia stato laudato et nominato non dirò in Sicilia, ma per tutta Italia et Europa, di cui l’opere siano state celebrate in Roma, Venetia, Parigi, Basilea et altre celebri città di Germania, Francia et Ispagna. (Macrì 1901, pp. 81–82)⁸²

Did he retreat when he encountered positions that were different from his own? Quite the opposite. In his introduction to *Computus Ecclesiasticus* in *Opuscula Mathematica*, he wrote: (Maurolico 1575a, p. 26)

Toleratur et Nicolaus Copernicus, qui solem fixum ac terram in girum circumverti posuit: et scutica potius aut flagello quam reprehensione dignus est.⁸³

He was equally heavy-handed against the heliocentric position in the Villacanense manuscript (Macrì 1901, p. 199).

Sed quid mirum, cum sint quidam adeo stulti, ne dicam insani, qui Solem stare, ac moveri terram asserere conati sunt?⁸⁴

He also criticised Oronzo Fine for the mistake he made in determining the ratio between the circumference and the diameter of the circle (Moscheo 1998, p. 97). Other criticisms of this or that figure are found in the letter to cardinal Marco Antonio Amulio of 1 December 1568 (Moscheo 1988, p. 278). Macrì describes him as a person without hypocrisy, who ‘used to confute opposing opinions spiritedly’ (Macrì 1901, p. 237).

This does not seem anything like the character of a person who would have preferred to avoid criticising Boethius. Only the academic, religious, and political context (where the three adjectives were intertwined and mingled in the period, and often referred to the same person) could influence him, however. Maurolico was always in contact with the Jesuits and benefited from this (Macrì 1901, p. 36; Moscheo 1998, *passim*). His constant interlocutors were the most prominent nobles and the leading figures in Sicily. The various Ventimiglia, Barresi, and Vega were strategus at Messina and viceroy at Palermo. When the manuscript of *Sicanicarum rerum compendium* had been printed, ‘several passages that, if published in 1562, would have caused him great trouble’ were omitted (Macrì 1901, p. 98). But also these parts that had been omitted had subsequently found their way into Colbert’s

library, where they were published by Stefano Baluzio. Therefore, we may be sure that, at least in one other case, not all of a manuscript had been published. Rather than by choice of Maurolico himself, it seems that the elimination of the passages subsequently published by Baluzio was the work of the Jesuits. In the letter of Giacomo Croce, written in 1560, we read '[the viceroy] commanded it to be given to ours [the Jesuits] so that it could be corrected' (Moscheo 1998, p. 136). It is strange, however, that Moscheo does not detect any censorial intervention in the text but underlines the praise of the Jesuits contained in it. As if a censor could publish criticisms of himself. On the contrary, he would have been expected to evaluate as certain elements for his intervention the differences between the manuscript that had remained in Paris and the printed text of 1562 (Moscheo 1998, p. 137).

Can we imagine something similar also for the papers about music? Should we made a distinction between his attitude towards the powers of his island and his attitude towards Boethius, the famous victim of his emperor? Certainly, it cannot be excluded that a part of the re-elaboration of the musical writings Ω in view of their printing in 1575 was due to the direct work of Maurolico. However, he cannot have had the last word, and consequently, we should not exclude other interventions on the part of people who were undoubtedly involved in the printing process. If we could consult the text '*Musica Boetii*' of the dispersed Villacanense manuscript, we would have a solid basis to clarify the question. As this text is missing, we are forced to base our judgement on the copy made by Clavius, which has been called C_2 and discussed in section 3.1.

We have seen that this does not coincide at all with the printed version of 1575 and that it is above all more critical of Boethius. Indeed, in the compendium itself, the figure given for the number of commas in the semitones was the more narrow of $\frac{1}{2}$ inequality provided by Maurolico. We are thus faced with another dilemma. We may consider Clavius' copy, C_2 , a faithful copy of Maurolico's writings. In this case, the differences between what was copied from the musical *corpus* Ω under the supervision of Maurolico and Clavius, C_1 , and the printed version of 1575 would be due to heterogeneous interventions beyond the control of the author, who in the meantime had died. In this case, C_2 would become a good approximation of C_1 , at least as regards the parts that are in common with the printed version of 1575. The second hypothesis would be to consider Clavius as responsible for the variants discussed above. How significant are Clavius' interventions? Let us take another look at those mentioned in the previous section.

First of all, the summary prepared by Clavius is even shorter than *Musicae traditiones* compared with *Musica*. In *Boetianae musicae compendium*, Clavius remained close to *Boetianae musicae epitome* of *Musicae traditiones*. But even if he proceeded in parallel, and at times word for word (with a few lexical variations), there are some significant differences. A '*Sectio toni*' that Clavius repeated is missing in *Musicae traditiones*. Then, Clavius wrote of a Boethius, who '*multum sudat*', an expression that does not exist in *Musicae traditiones*. The table of Clavius with the proportions for the musical notes and the planets is different in its choice of numbers from that of *Musicae traditiones*. In its order and its numbers, it is closer to figure 14 and to table 9 of *Musica* (M. VIII e IX).

Last, and this is the most important difference, Clavius wrote that the minor semitone '*maius quidem esse tribus commatibus ac $\frac{1}{2}$* ' and the apotome '*maiozem esse quam 4^{or} commata et $\frac{1}{2}$* '. Where Boethius had given the poorer inequality without $\frac{1}{2}$.

In his '*Repastinatio*', Clavius was now very close, word for word, to *Musica*, in certain parts, which are found only there, and not in *Musicae traditiones*. However, also in '*Repastinatio*', we find more or less lexical variants and the absence of a couple of sentences. Two long passages that are found in Clavius' manuscript are completely missing in our edition of *Musica*. Furthermore, Clavius expounded the eicosichord of Guido D'Arezzo in a highly simplified manner compared with *Musica*, with a table that is not so messy or complex as that of *Par. Lat. 7462* and *Musica*. The eicosichord described by Clavius, in its structure, almost coincides with that of *Musicae traditiones*, were it is not for the fact that the former does not contain two mistakes that are found in the latter, which are absent in the corresponding table 6 of *Musica*. In any case, both that of Clavius and that of *Musicae traditiones* came from the left-hand side of our table 6 (M. VIII).

The modifications, additions, and erasures are a bit too numerous to be able to attribute them all to Clavius alone, without any contribution from Maurolico. After all, didn't the Jesuit go to Messina partly to help Maurolico put his papers in order? Clavius' copy, therefore, was probably the result of their collaboration. Otherwise, when and where would he have had the opportunity to make himself a copy? It cannot in any case come from the printed version.

An analysis of the last short chapter left to us by Clavius, '*Octo modulatum, sive tonorum proprietates*', confirms the idea of Maurolico who re-elaborates his initial projects about music, with additions, modifications, and summaries. Yet, this is still not the whole story: the part about the '*Tropes*' is found only in Clavius, and it is missing both in *Musica* and in *Musicae traditiones*. The eight modes are found in the same form as in Clavius in *Musicae traditiones*, but with a change in the order, whereas in their place, *Musica* contains table 8, together with a long explanation. Only a part of the description remains common, and it fully bears a synoptic comparison between the three versions. Last, the effects of the eight modes described by Clavius appear, in a long numbered list, almost in the same terms and with few variations, in *Musicae traditiones*, whereas *Musica* gives two concise versions without numbers. All this has already been explained in the previous section.

Thus, the idea emerges from the significant differences between the Paris manuscript and the Clavius' copy, that both Maurolico and Clavius worked on the latter text. The further important differences between Clavius' copy and the Venetian edition suggest that also the Jesuits in Venice (who were invited by Doménech to take care of the printing, as we have seen in section 2.2) played an active role in modifying the final text. Who else could be accused of making the mistake (mentioned above) at the end of *Musicae traditiones*, which confused *Arithmetica* with Boethius' *Musica*? The Venetian editors, experts *ma non troppo*, are the best candidates, considering that it is difficult to attribute it either to Maurolico or to Clavius. Finally, the largest modifications concern the systematic

elimination of the mathematical novelties that are the work of our Maurolico. Who might be responsible for this? It is more likely that this was done by the Jesuits at Venice, rather than by Maurolico at Messina. Unless we imagine a case of self-censure, the author neither have any interest in doing it nor was this in his character. The Venetian editors, instead, had an institutional role, in the climate of the counter-reformation, to verify the orthodoxy of a text that was declared to be destined for teaching, an activity in which the order of the Jesuits was particularly interested. It is well known that music was often present in the discussions of the Council of Trent.

The information that we possess today leads us to construct scenarios, where the various differences between the manuscripts and the printed edition derive from a plurality of people and reasons. Apart from Maurolico, who cannot, of course, be excluded, the interventions of Clavius and the others can be modulated in various ways, depending on who we prefer to trust. The extreme scenarios are as follows: in the first, we could consider Clavius responsible for all the variants contained in the summary compiled by him. In another, it would be our mathematician from Messina himself who re-elaborated his work for the 1575 edition. Even if we may not like this Maurolico, too much at the mercy of circumstances and political power. In a third scenario, it would have mainly the Jesuits, for the educational purposes of order, in the climate of the Council of Trent, who purged the manuscript for printing.

We may have different ideas of our Maurolico, seeing that he can be interpreted, within certain limits, in accordance with our different conceptions of history, scientific research, and scientific community. We can even pronounce his name in two different ways: at Messina, his birthplace, they put the accent on the *ì*, but in the academic environment, many people prefer to pronounce it in the Greek style, with the accent on the first *ò*. Different solutions to the problems examined here remain possible. In the Appendix, we provide the diagrams of the alternative scenarios. Some can already be excluded, but their number is inversely proportional to the amount of information available. The present writer has presented arguments in favour of the first one.

Even those who were to change the evolution of sciences during the 17th century would continue to write about music (Cohen 1984; Gozza 1989; Coelho 1992; Knobloch 1992; Bailhache 1993; Knobloch 1995; Bailhache 1996; Settle 1996; Tonietti 1999; Tonietti 2000). A comparison in this case of their conceptions with that of Maurolico or of Giorgio Valla (Valla 1501) allows us to introduce a new subject to continue to discuss one of the most interesting and controversial chapters of history. If for no other reason, these pages of the theory of music should not be underestimated. In any case, they reveal the mathematical conception of the author (Tonietti 2002; Tonietti 2003; Tonietti 2004). But even taken by themselves, the musical writings of Maurolico have revealed new capacities in the use of symbolism and new abilities in demonstration. They have even succeeded in giving us new details about scientific life and European culture in the 16th century.

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BIBLIOGRAPHY

Aristoxenus.

1954: *Elementa Harmonica*, (R. Da Rios, ed.), Roma: Istituto poligrafico dello stato.

Bailhache, P.

1993: "Cordes vibrantes et consonances chez Beeckman, Mersenne et Galilée", *Sciences et Techniques en Perspective* 23, 73–91.

1995: "Deux Mathématiciens Musiciens: Euler et D'Alembert", *Physis* 32, 1–35.

1996: "Sciences et musique: quelques grandes étapes en théorie musical", *Littérature, médecine, société* 13.

Baron della Foresta, (Francesco Marolì)

1613: *Vita dell'Abbate del Parto D. Francesco Maurolico*, Messina. Now available at <http://www.maurolico.unipi.it>

Benedetti, G. B.

1585: *Diversarum Speculationum Mathematicarum et Physicarum Liber*, Torino: Apud Haeredem Nicolai Bevilaquae.

Boethius, S.

1867: *De institutione arithmetica libri duo - De institutione musica libri quinque* (G. Friedlein, ed.), Leipzig: Teubner.

Cicero, M. T.

1992: "Somnium Scipionis", in A. Resta Barrile (ed.), *Dello Stato*, Milano: Mondadori, pp. 185–205.

Clagett, M.

1974: "The Works of Francesco Maurolico", *Physis* 16, 148–198.

Clavius, C.

1992: *Corrispondenza*, (U. Baldini and P. D. Napolitani, eds.), Pisa: Dipartimento di Matematica, Vol. II.

2006: *F. M. Boetianae Musicae Compendium*, Manuscript, Roma: Biblioteca Gregoriana. APUG Fondo Curia 2052, to be published at <http://www.maurolico.unipi.it>

Coelho, V., ed.

1992: *Music and Science in the Age of Galileo*, Dordrecht: Kluwer Academic Publishers.

Cohen, H. F.

1984: *Quantifying Music*, Dordrecht: Reidel.

- D'Alessandro, P. and Napolitani, P. D.
2001: "Primi contatti fra Maurolico e Clavio: una nuova edizione della lettera di Francesco Maurolico a Francisco Borgia", *Nuncius* 16, 511–522.
- D'Arezzo, Guido
1963: "Opuscula de Musica", in M. Gerber (ed.), *Scriptores ecclesiastici de musica*, Hildesheim: Olms V., Vol. II, 1–61.
- Descartes, R.
1618: *Compendium Musicae*. It. tr. *Breviario di musica*, (Luisa Zanoncelli, ed.). Corbo e Fiori editori.
- Euclid
1557: "Rudimenta musices" (G. Pena, ed.), Andrea Wechelo, Parigi. It. tr. in Bellissima, F., 2003. "La Sectio Canonis di Euclide e il suo errore logico", *Bollettino di Storia delle Scienze Matematiche* XXIII, 6–45.
- Euler, L.
1739: *Tentamen novae theoriae musicae*, St Petersburg. Repr. *Opera Omnia* series III, Vol. I, Teubner, Leipzig 1926. Fr. tr. *Musique Mathématique*. Librairie Scientifique et Philosophique, Paris 1865.
- Faber Stapulensis, (Lefèvre d'Étaples)
1496a: *Elementa Musicalia*, Parigi.
1496b: *Arithmetica*, Parigi.
- Galilei, Galileo
1638: *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, Leiden. Rist. *Opere*. 1964. Torino: Utet.
- Galilei, V.
1581: *Dialogo della musica antica et moderna*, Reedited (F. Fano, ed.), Roma: Reale Accademia d'Italia, 1934.
- Gozza, P., ed.
1989: *La musica nella rivoluzione scientifica del seicento*, Bologna: Il mulino.
- Huygens, C.
1940: *Oeuvres Complètes*. La Haye, Martinus Nijhoff: Vol. XX, *Musique et Mathématique*.
- Kepler, J.
1619: *Harmonices Mundi Libri V*. J. Plank, Linz. Reprinted, Forni, Bologna, 1969. German translation by, *Welt-Harmonik*. (M. Caspar, ed.), München, 1939; repr. R. Oldenbourg, München, 1990. English translation by, *The Harmony of the World* (E. J. Aiton, A. M. Duncan, and J. V. Field, eds.), Philadelphia: American Philosophical Society, 1997.
- Knobloch, E.
1992: "Rapports historiques entre musique, mathématique et cosmologie", in *Quadrivium, Musique et Sciences*, Paris: Éditions IPMC, pp. 123–167.
1995: "Harmony and Cosmos: Mathematics Serving a Teleological Understanding of the World", *Physis* 32, 55–89.
- Leibniz, W.
1666: "Dissertatio de arte Combinatoria", in G. I. Gerhardt (ed.), *Mathematische Schriften*, Reprinted Hildesheim: Olms, 1962, Vol. IV, pp. 27–104.
- Macri, G.
1901: *Francesco Maurolico nella vita e negli scritti*, Messina: Tipografia D'Angelo.
- Maurolico, F.
1528: *Grammaticorum rudimentorum libelli sex*, Messina: Petruccio Spira.
1543: Dedication to Pietro Bembo, 1540, in *Cosmographia*, Venezia.
1558: *Theodosii Sphaericorum elementorum libri III*, Messina: Pietro Spira.
1575a: *Opuscula Mathematica*, Venezia: Francesco De Franceschi. Now available at <http://www.maurolico.unipi.it>

- 1575b: *Arithmeticonum libri duo*, Venezia: Francesco De Franceschi.
 - 2000: "Musica", in T. M. Tonietti (ed.), *Opera Mathematica*. Now available at <http://www.maurolico.unipi.it>
 - 2001: "Frammento" in T. M. Tonietti, (ed.), *Opera Mathematica*. Now available at <http://www.maurolico.unipi.it>
 - 2006: *Opera Mathematica*, Pisa (O. Besomi, E. Giusti, C. Maccagni, P. D. Napolitani, J. P. Sutto, et al., eds.). Now available at <http://www.maurolico.unipi.it>
- Mersenne, M.
- 1636: *Harmonie Universelle*, Paris: Sebastien Cramoisy.
- Moscheo, R.
- 1988: *Francesco Maurolico tra Rinascimento e scienza galileiana* Messina: Società Messinese di Storia Patria.
 - 1998: *I gesuiti e le matematiche del sec. XVI*, Messina: Società Messinese di Storia Patria.
- Pacioli, L.
- 1494: *Summa de Arithmetica*, Venezia. Reprinted faxes. Roma: Istituto poligrafico dello stato, 1994.
- Platone
- 1994: *Timeo* (G. Lozza, ed.), Milano: Mondadori.
 - 1999: *La Repubblica*, (F. Sartori, M. Vegetti, and B. Centrone, eds.), Bari: Laterza.
- Ptolemaeus, C.
- 1682: *Harmonicorum libri tres*, (J. Wallis, ed.), Oxford.
- Pugliatti, S.
- 1968: "Le Musicae Traditiones di Francesco Maurolico", *Atti Accademia Peloritana dei Pericolanti* (Messina) 48, 313–399.
- Settle, T.
- 1996: *Galileo's Experimental Research*, Preprint. Berlin: Max Planck Institute for History of Science.
- Shea, W.R.
- 1991: *The Magic of Numbers and Motion. The Scientific Career of René Descartes* Nantucket, Massachusetts: Watson Publishing International.
- Stevin, S.
- 1585: "L'Arithmetique", in D. J. Struik (ed.), *The Principal Works of Simon Stevin*, Amsterdam: Swetz and Zeitlinger, 1958, Vol. II, pp. 457–739.
 - 1966: "Vande Spiegheling der Singconst", in A. D. Fokker (ed.), *The Principal Works of Simon Stevin*, Amsterdam: Swetz and Zeitlinger, Vol. V, pp. 413–464.
- Sutto, J. P.
- 1997: "Francesco Maurolico, mathématicien italien de la Renaissance", Thèse, Université Paris VII
- Tonietti, T. M.
- 1999: "Verso la matematica nelle scienze: armonia e matematica nei modelli del cosmo tra Seicento e Settecento", in M. Mamone Capria (ed.), *La costruzione dell'immagine scientifica del mondo* Napoli: La città del sole, pp. 155–219.
 - 2000: "Does Newton's Musical Model of Gravitation Work?", *Centaurus* 42, 135–146.
 - 2002: "Is Music Relevant for the History of Science?", in P. Cerrai and S. Monteizo (eds.), *The Applications of Mathematics to the Sciences of Nature*, Dordrecht: Kluwer, pp. 281–291.
 - 2003: "The Mathematics of Music During the XVI Century: The Cases of Francesco Maurolico, Simon Stevin, Chéng Dàwèi, Zhu Zàiyù", *Ziran kexueshi yanju [Studies in the History of Natural Sciences]* 22, 223–244.
 - 2004: *Nuvole in silenzio. Arnold Schoenberg svelato*, Pisa: Edizioni Plus Università di Pisa.
- Valla, G.
- 1501: *De expetendis et fugiendis rebus opus*, Venezia.
- Zarlino, G.
- 1558: *Le istituzioni harmoniche*, Venezia.

Appendix

Ω : The original musical corpus.

A: Autograph *Par. Lat.*, A6.

S: Printed version of 1575, S7.

C_1 : Possible copy of Ω .

C_2 : Clavius' copy, C13. (A6, S7, C13 are the abbreviations used in the Conspectus Siglorum of *Opera Mathematica*).

Scenario 1: Clavius faithful, editors unfaithful.

Scenario 1bis: Clavio unfaithful, editors unfaithful.

Also, Francesco Maroli, Baron della Foresta, may have played a role in copying parts of Ω to C_1 , as he had done for *Arithmeticonum Libri Duo*.

Scenario 2: Maurolico a succubus and self-censor, Clavius unfaithful. It is not possible to consider a scenario 2bis with Clavius faithful. In that case, C_2 should have been similar to S. C_1 coincides with S.

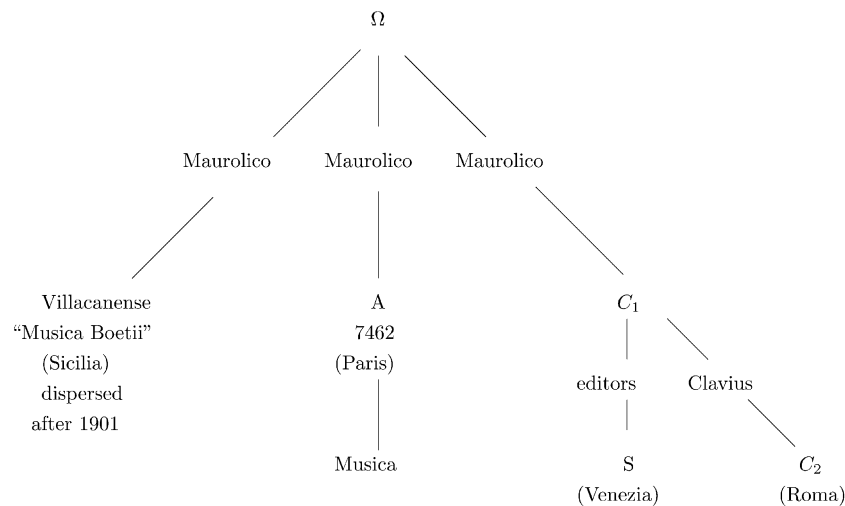
Scenario 3: To be excluded because otherwise C_2 should be similar to S, or because otherwise Clavius should be faithful on the one hand and unfaithful on the other hand, or improbable because Clavius would have been unfaithful in different ways and at different times.

Scenario 4: Editors unfaithful, Clavius faithful.

Scenario 4bis: Editors unfaithful, Clavius unfaithful.

Scenario 5: Maurolico a self-censor, Clavius faithful. C_1 coincides with S.

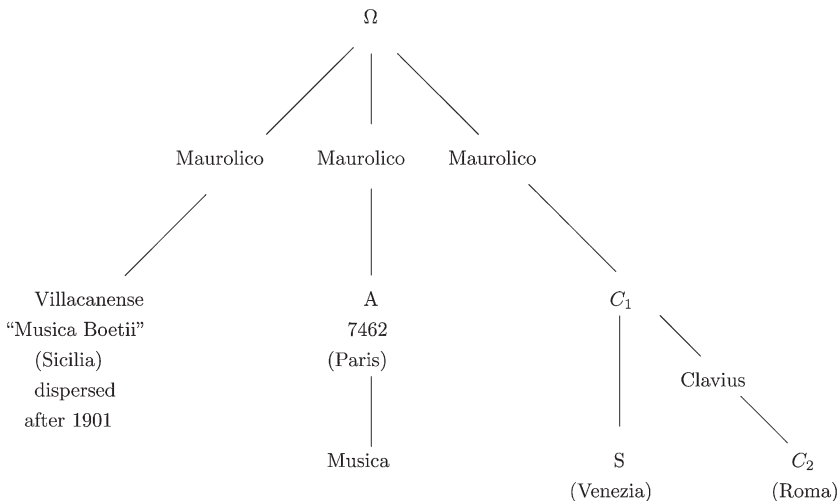
Scenario 5bis: Maurolico a self-censor, Clavius unfaithful.



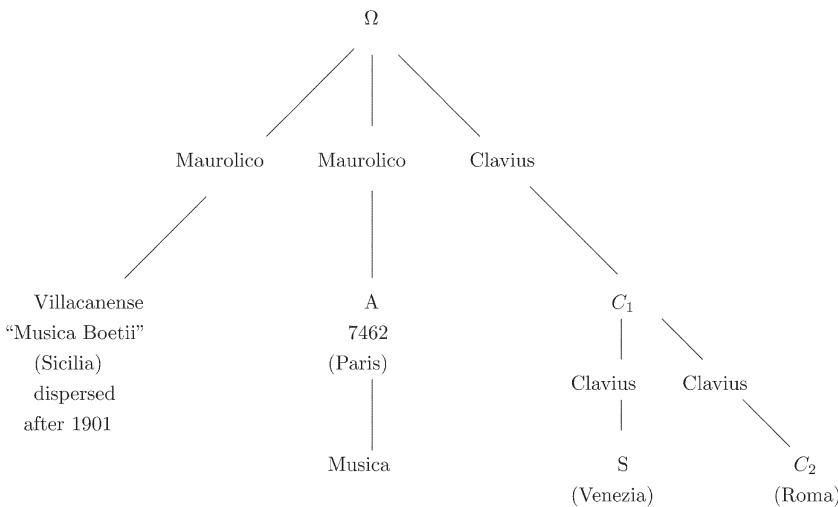
Scenario 1: Clavius faithful, editors unfaithful.

Scenario 1bis: Clavio unfaithful, editors unfaithful.

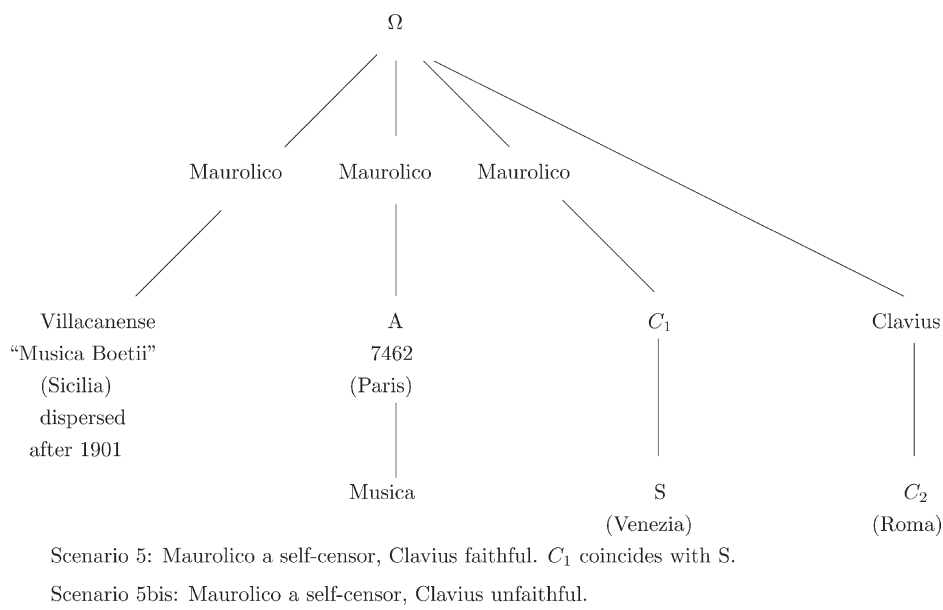
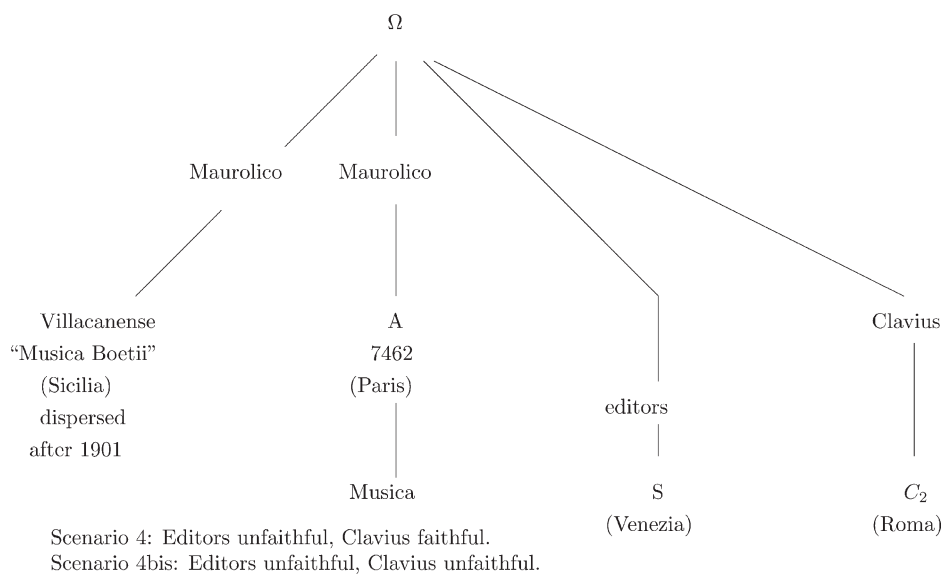
Also Francesco Maroli, Baron della Foresta, may have played a role in copying parts of Ω to C_1 , as he had done for *Arithmeticonum Libri Duo*.



Scenario 2: Maurolico a succubus and self-censor, Clavius unfaithful. It is not possible to consider a scenario 2bis with Clavius faithful. In that case, C_2 should have been similar to S. C_1 coincides with S.



Scenario 3: to be excluded, because otherwise C_2 should be similar to S. Or because otherwise Clavius should be faithful on the one hand and unfaithful on the other. Or improbable because Clavius would have been unfaithful in different ways and at different times.



NOTES

1. The edition of his *Opera Mathematica* is being completed in Maurolico 2007. Rosario Moscheo described it for the first time in Moscheo 1988.
2. We would like to thank Rosario Moscheo for supplying copies of the relative folios.
3. 'Mathematics, that is to say, the discipline, includes arithmetic, geometry, music and astronomy Music [deals with] the ratios and the qualities of the notes'.
4. The numbers refer to the chapter and verse of the electronic edition, Maurolico 2000, henceforth indicated as M.
5. 'The difference between the apotome and the diesis is also called a comma; it is obtained by subtracting one ratio from the other, as is shown below'.
6. 'It will also be found that the diesis is greater than three commas, but less than four, while the apotome is greater than four commas, but less than five. Hence also the tone is greater than 8 and less than 9 commas, which are things that can all be verified by means of a long calculation with several figures. See the elements of music in Boethius and Faber'.
7. 'But he made a mistake in this calculation, because equal differences were used in the division of the ratio. Instead, proportional differences should have been used: this is how ratios are multiplied, and not by means of equal differences. In this way, in actual fact, he avoided the toil of carrying out the multiplications. And yet he ended up with a correct result, and consequently he may be excused'.
8. 'And yet, as we have proved by calculating proportionally, Boethius achieved his purpose of truth'.
9. 'Take these numbers 64 65 66 67 68 69 70 71 72. Now let the ratio of the tone be equal to 72:64; it is clear that ratio 72:71 is less than the eighth part of the tone. But 531441:524288, the ratio of the comma is like 72:71 $\frac{16425}{531441}$, that is to say, it is less than 72:71. A fortiori, therefore, it will be less than the eighth part of the tone'.
10. For example, Faber Stapulensis (1496b, X, 61).
11. Zarlinos scale contains two tones, 9:8 the major and 10:9 the minor; the syntonic comma is their difference, that is to say, $\frac{9}{8} \times \frac{9}{10} = \frac{81}{80}$. Subsequently, others were to use logarithms. Among the first of these was Christiaan Huygens (Huygens 1940, p. 12).
12. 'But if anyone wants to turn the fractions into whole numbers, he should multiply the single terms by 384; for this number contains all the fractions of the terms'.
13. 'And the looser sinew makes a deeper sound, but that of the tighter one is higher'.
14. 'The denser body quivers more quickly, just as a bronze string quivers more than a sinew [of an ox] even if the tension is less'.
15. 'Consonances consist of commensurable proportions: it is impossible to put incommensurable sounds in concordance. In the same way, it is impossible to make the quiverings of incommensurable speeds correspond, seeing that concordance, like unison or consonance, is born from the correspondence of beats'.
16. 'The consonances of musical melodies consist not only of commensurable proportions, but also of particular numbers. For incommensurable proportions (which are irrational and unknowable) always produce dissonance: seeing that the melodies composed of these proportions respond to each other, as a result of their incommensurable nature, not by means of well-ordered beats, but beats that are always different (and this diversity generates the disharmony)'.
17. In this case, as regards, the dispute between Beeckman and Descartes about how much the latter learnt from the former concerning music, it should be added that the Frenchman's refusal to prize the Dutchman for his discovery (that the theory of the coincidence of beats might serve to explain the difference between consonances and dissonances) was to some extent well founded. Better founded, at any rate, than Cohen and William Shea are prepared to grant. The latter science historian follows Cohen in comparing Descartes (Descartes 1618) only with Beeckman and Galileo

- Galilei (Galilei 1638), ignoring Maurolico and Benedetti (Cohen 1984, pp. 190–202; Shea 1991, cap. IV).
18. 'The tone cannot be divided into equal parts because the ratio of the sesquioctave tone [9:8] is not that of a square to a square number: therefore, it does not admit a proportional mean number, which divides the proportion into equal parts. In this way, no consideration is given to Aristoxenus, who affirms that it must be possible to divide the tone into equal parts'.
 19. 'Therefore we should not pay any attention to Aristoxenus, who has divided the tone into equal parts, or to Philolaus, who has divided it in another way'.
 20. 'As the proportional mean number between 2187 and 1944 is greater than 2061, the proportion 2,048:1,944 of the minor semitone is, as a result, less than half of the tone. And the proportion 2,187:2,048 of the major semitone is greater than half of the tone'.
 21. 'For only God is infinite'.
 22. 'These first eight sinews are also called *Proslambanomenos*, *Hypate*, *Parhypate*, *Lichanos*, *Mese*, *Paramese*, *Paranete*, *Nete*'.
 23. '*Lichanos*, which is placed over the following index finger. The etymology of the other strings is patently clear'.
 24. 'The Etymology of these words is readily apparent'.
 25. 'Boethius toils hard here over the Greek words ...'.
 26. 'I understood Boethius' elements of music and arithmetic by myself, without anybody previously teaching me'.
 27. 'Some little lucubrations of mine ... in the second section ... a compendium of the music of Boethius. From Guido D'Arezzo and from other authors, the compendium of speculative and practical music, in which the ratio of consonant and dissonant sounds is discussed in detail'.
 28. 'Here [Abbadia del Parto, Castelbuono] the good Shepherd dwelt with his beloved flock, singing psalms in a choir with them, and dedicating himself in his room to mathematical speculation', 'in which [church of San Giovanni Battista of the Cavalieri Gerosolimitani at Messina, where he was later buried] he was often heard to sing, sitting in the choir, with a joyous expression'.
 29. 'About music ... a compendium of the music of Boethius with a few comments relating to the proportion of intervals'.
 30. 'The arithmetic of Boethius (pp. 208–212). The most useful theories of Boethius reduced to a compendium which finishes with the words: Catania, 28 gennaio 1554. Farewell, O reader; the other things on which Boethius speculates give more troubles than pleasure; therefore we have reckoned that they should be omitted'.
 31. '... and although Euclid, starting from the continuum, and our Boethius, starting, on the contrary, from the discrete, disagree ... [Euclid, Boethius and Giordano disagree about linear, plane and solid numbers]' ... 'all the same, the quantity, whether discrete or continuous, applied to these or other things, generates this or the other science, which lies at the basis of arithmetic, such as counting, rhythm and music, or at the basis of geometry, such as astronomy, geography, chronography, perspective, of which we shall say more below'. ... 'In the books about arithmetic by Giordano and Boethius' ... 'without which [*Sferica*] nobody can adequately examine the origins of astronomy, like music without the books about arithmetic, if pure mathematics is to be placed before material sciences'. ... 'What evil will I commit, I ask immortal God, if I create a unity out of definitions, concepts, postulates, problems and theorems of music, perspective, astronomy and the mechanical inventions found in Euclid's *Elements*, in the *Sphericals* of Theodosius and Menelaus, in the *Conicals* of Apollonius, in the *Cylinders* of Serenus, in the works of Archimedes, in the *Arithmetics* of Giordano?' ... 'Oh happy century, ... not only are there clearly large numbers of sculptors ... architects, artists, painters, flautists, and excellent musicians, but the works of orators and philosophers are also investigated for the common good'.

32. 'For the sixth book. On the subject of music. On sound, notes and melody. On the first intervals and the proportions of notes. On tone, the diesis, the apotome and their proportions. On Guido's eicosi-chord. The comma is the difference between apotome and diesis. The tone is less than 9 commas, and greater than 8. The diesis is less than 4 commas, and more than 3. The apotome is less than 5 commas, and greater than 4. Comment on the calculation of Boethius from which the diesis is found to be more than 3 and a half commas. The apotome, on the contrary, is greater than 4 and a half commas. Reasoning about consonances. The eight modes of melodies that are called Modes. On the diatonic, chromatic and harmonic genres. Rules for composing consonances. Instruments are to be constructed in accordance with the proportions of sounds. On organs, flutes, monochords, instruments with various strings. On the harp. Practical compendium of music'.
33. '...to hold, in the planned public halls of the city, the planned lessons of mathematics, geometry, arithmetic, theoretical astrology, music, theoretical perspective and all the other things and means connected with this mathematical science ...'.
34. 'On music ... Compendium of the music of Boethius with a few comments relating to the proportion of intervals. Music of Boethius. Of Greek authors, of Faber. To these is added a compendium of our own, the theory of notes and of consonances [of modes and melodies] which includes everything in brief ... The Arithmetic of Boethius and the music with a compendium of James Faber and a very brief treatise of ours ... September 17th, 1570'.
35. 'On music ... a compendium of the music of Boethius, with excellent theories and calculations and the reason for melodies and the proportions of systems'.
36. 'On music ... a compendium of Boethian music, with excellent theories and calculations'.
37. 'A few things concerning the theory of music'.
38. 'Unstinting in communicating his speculations to his friends, extremely jealous of the fame of other people, he was not at all slanderous, or a censor, not ambitious, indeed, he despised glory and earthly honour to such a point that he often used to say that he did not consider it at all important, if the works that he had so painstakingly composed, and elaborated through much study, should be presented to the world bearing somebody else's name, provided that they were published for the benefit of mankind, and for knowledge of the truth'.
39. 'There is, here, a person who is most expert in mathematics and very elderly, who has written many works about these disciplines, and we have persuaded him to write a mathematical compendium about the most necessary things, so that it can be read in little time, and he has already compiled a large part of it. He would like father Clavius to come for a few months, so as to complete the project'.
40. Also Salvatore Pugliatti, one of the first readers of the musical manuscript, underlined the novelty of Maurolico, compared with Boethius, but he did not appreciate the true mathematical nature of it (Pugliatti 1968, pp. 313–399).
41. 'You should write to Venice to ours [the Jesuits] that they may consider recommended the printing of some books by the Abbot [Maurolico] Ours are well informed. The librarian's name is Io. Comisino, who has a shop in Messina and obtained the said books, of which no copy has remained, and they are important for this purpose of ours'.
42. 'On this see below' 'as will be seen below'.
43. 'But having subtracted the diesis from the whole tone, what is left is the apotome, or major semitone which is this proportion 2187:2048'. 'Furthermore, the difference between the whole tone and the diesis is the apotome, which is called the major semitone, and is between the terms 2187 and 2048'.
44. '... as practice corresponds to theory and experience to ability. It is necessary, however, for man's reason to put a limit to the infinite number of notes. If the subject of every ability, and not only of music, were infinite: the work establishes ... only God is infinite'.
45. 'But if the subject of every ability, and not only of music, were infinite: the work establishes ...'.

46. 'For incommensurability does not allow consonance, or a known term of the note, seeing that it is unknown'.
47. 'This is the division of the tone into the major and minor semitone, in accordance with the above proportion'.
48. 'This is the order, this the series, this the proportion and the natural process. The Greek words for sinews or the letters do not add anything to the theory. Now let the octachord be presented, with its intervals and proportions: this may be sufficient as regards theory'.
49. 'Boethius toils hard here over the Greek words for sinews and the letters used in the processes, and over the adaptation of consonances and proportions for notes, which are not necessary (... we have Guido's Eicosichord). For all the theory and practice of music are composed of order and proportion of notes and of the Diatonic process (this regulates what is allowed for the semitones of the divisions of the single tones). But here is the natural process of the octachord with its proportions'.
50. 'Greater than three commas' 'was undoubtedly greater than three and a half commas'.
51. 'Was greater than four commas' 'And therefore the apotome is greater than 4 and a half commas'.
52. 'As is found with the ratios, the sums and the subtractions'.
53. 'As Boethius concludes by means of the numbers of the differences'.
54. 'Digging up again', that is to say, re-elaboration.
55. 'Sinews' 'Strings'.
56. 'And if the diapasen is added to these single consonances, all the consonances of the third order are generated, that is to say, the fifteenth or double diapasen, the seventeenth, the eighteenth, the nineteenth, the twentieth. And thus the whole eicosichord of Guido is completed, including the consonances of the third order'.
57. 'Let another diapasen be added to the thirteenth, also the twentieth will be obtained, which completes the whole eicosichord of Guido [D'Arezzo]. It includes systems of the third order, that is to say, the double diapasen, the seventeenth, the nineteenth and the twentieth'.
58. 'Together, these two form the proportion 2:4, which is the diapasen. From these are produced the intervals of the tones and semitones, from which the natural and diatonic degrees of the ascending and descending melodies are obtained. For the tone is the interval of the sesquioctave proportion [8:9], which means that it is the difference of the diapente and diatessaron, i.e. of the sesquialter [2:3] and of the sesquitercia [3:4], as is found in the numbers 9, 8, 6. On the contrary, the tone taken away twice from the diatessaron leaves the minor semitone, or diesis. Therefore, in the natural melody, we go up through one tone, another tone and a diesis. Again by one tone, another tone and a diesis ...'.
59. 'Thus the proportion of the diapasen is completed from these two, so .2.3.4. From their difference comes the sesquioctave proportion [9:8] of the tone, as is evident in these numbers 9.8.6. On the contrary, if the tone is taken away twice from the diatessaron, what remains is the minor semitone, or diesis. Therefore, in natural melody, we go up through one tone, another tone and a diesis'.
60. 'For the addition of the diapasen always produces a consonance of the same quality. And therefore the interval of the thirteenth will contain consonances of the second order, that is to say, the diapasen, the tenth, the eleventh, the twelfth and the thirteenth'.
61. 'That is to say, first of all the diapasen itself, or the eighth, the tenth, the eleventh, the twelfth and the thirteenth'.
62. 'The other things, which Boethius deals with, concern the intervals and the proportions of notes and consonances, which are all included in the Eicosichord of Guido. From which stems the practice of melodies and the planning of instruments. And thus now, with this introduction, leading towards the Eicosichord, we shall expound and a certain digging up of the premises will be necessary'.
63. 'And thus we could proceed continually ad infinitum. Hence the origin of these notes making up the hexachord is clear, that is to say, *ut, re, mi, fa, sol, la*'.

64. 'This addition of the diapason may be performed four or five times and again ad infinitum, as is clear in instruments with many strings, in organs and in large instruments. In which ... the diatonic and chromatic melodies exceed Guido's hand of the eicosichord. Hence, we thus find the etymology of our diapason, because when it is added once, twice, three times, or any number of times to the single consonances, it generates other single consonances of the same kind as the simple ones. Finally, from here, no less, the numbers become known of these six notes which make up the hexachord ... and their origin, which are *.ut.re.mi.fa.sol.la.*'.
65. 'Moreover, eight letters are fixed, $\Gamma.a.b.c.d.e.f.g.$ in such a way that any one of them, repeated in the octave, always indicates the diapason consonance in the double proportion. Where the numbers arranged on the single strings of the eicosichord actually reveal all the other consonances and the other intervals'.
66. 'Moreover, eight letters are fixed, $\Gamma.a.b.c.d.e.f.g.$ under this number, in such a way that any one of them, repeated in the octave, always indicates the diapason consonance in the double proportion. Where the numbers arranged on the single sinews of the eicosichord actually reveal all the other consonances and the other systems. Now I will repeat the calculation'.
67. 'If either of the two consonances the diapente or the diatessaron, is subtracted from the diapason, the other one remains. $.4.3.2..$ If the diatessaron is subtracted from the diapente, the tone remains. $9.8.6.$ If two tones are subtracted from the diatessaron, the diesis remains. $324.288.256.243.$ Hence it is also clear that if three tones are subtracted from the diapente, the diesis remains. If the diesis is subtracted from the tone, the apotome remains, thus $.2304.2187.2048.$ Lastly, if the diesis is subtracted from the apotome, the comma remains. As is evident from these numbers $.559872.531441.524288$ '.
68. 'The diesis is greater than three and a half commas. The apotome is greater than four and a half commas. Hence the whole tone is greater than eight commas and less than nine. All these things are verified by means of the rules for adding and subtracting proportions; these rules are similar, or the same as those for fractional numbers'. In this case, if Clavius had seen Maurolico's elegant demonstration in the manuscript, he entirely ignored it.
69. 'Now, moreover, I will expound Guido's eicosichord, in which, by means of the proportions of numbers, and in the diatonic order as it proceeds in nature, following the chromatic division (in order to temper the harmony of the tritone), all the proportions are represented of the musical notes, that is to say, of the tones, of the major and minor semitones, called apotome and diesis, of the diapason, diapente, diatessaron, tritone and ... of the following intervals by means of the composition of the same. Furthermore, the whole Eicosichord is composed of 14 tones and 5 diesis'.
70. 'There are 4 tropes. The first. The second. The third. The fourth, each of which contains two modes. The first contains the Dorian and the Hypodorian. 1^0 and 2^0 . The second contains the Phrygian and the Hypophrygian. 3^0 and 4^0 . The third contains the Lydian and the Hypolydian. 5^0 and 6^0 . The fourth contains the Mixolydian and the Hypomixolydian. 7^0 and 8^0 . Of these, $1^0 . 3^0 . 5^0 . 7^0$ are called authentic, and $2^0 . 4^0 . 6^0 . 8^0$ plagal'.
71. 'The first begins from *d . sol . re*. The third from *e . la . mi*. The fifth from *f . fa . ut*. The seventh from *g . sol . re . ut*. The second from *a . re*. The fourth from $\sharp . mi$. The sixth from *c . fa . ut*. The eighth from *d . sol . re*'.
72. 'While the authentic modes rise and fall by the diapason, the plagal one, on the contrary, rises from the place of its corresponding authentic mode by a diapente, descends by a diapason, and rises again by a diatessaron to the said place defined'.
73. 'The authentic modes rise and fall by the diapason interval: the plagal modes rise from the same place by a diapente and fall by a diapason. And, rising again by a diatessaron, they terminate in the relative authentic mode of each one (from which they had started). Of these modes, the 1^0 is assigned to the sun, the 2^0 to the moon, the 3^0 to Mars, the 4^0 to Mercury, the fifth to Jupiter, the sixth to Venus, the 7^0 to Saturn, the 8^0 to the firmament'.

74. 'Furthermore, the authentic modes are formed by rising from their own place by a diapente and a diatessaron, that is to say, by a diapason, and falling by the same intervals. The plagal modes, on the contrary, from the seat of the corresponding authentic mode of each one, rise by a diapente, and then fall by a diapente and a diatessaron; from there, they again rise by a diatessaron, and terminate together, in the place of the corresponding authentic mode'.
75. 'The first eliminates sleepiness and laziness; it goes together with humorous, playful, facetious words. The second induces quiet, restful sleep; the Pythagoreans made use of it when they tempered their continual anxieties with sleep and rest. It is also melancholic, moving and a friend of freedom. The third is provocative, severe, rough, angry, warlike. The fourth, mild, garrulous, lascivious, flattering, tender, exciting. The fifth, pleasant, joyful, modest, a little insolent, comforting, encomiastic. The sixth, sad, pious, devoted, lovable, compassionate, inducing gladness. The seventh, varied, querulous, bold, possessing also the properties of the third, the fourth and the fifth. The eighth spurs on the sad and the slow, it is sweet, temperate, pleading, suitable for imploring, dealing with deep, heavenly matters, and despising things that are inferior'.
76. 'The Dorian mode is pleasant and jocular, it drives away sleepiness. The Hypodorian, moving and unaffected, induces sleep. The Phrygian, severe and angry, incites and exasperates. The Hypophrygian, more tender, exhorts, attracts and makes exuberant. The Lydian, joyful and insolent, praises and consoles. The Hypolydian, compassionate and sad, gladdens and sympathises. The Mixolydian, varied, melodious and impudent, possesses the qualities of the third, the fourth and the fifth. The Hypomixolydian elevates, invokes, pays attention to heaven, despising [earthly] things'.
77. 'Even less sweet [the fourth], even with a doubt that it should be counted among the consonances; as is admitted only by Ptolemaeus'.
78. 'Seeing that the sesquialter and the sesquitercia form the double proportion'.
79. See figure d in the Introduction to *Musica*.
80. 'This is the calculation of Boethius in the third [book] of his *Musica*'. 'For this last calculation, Boethius in the third [book] of *Arithmetics*'.
81. 'I was not attracted to these studies by the hope of some gain, or of fame or honour or money, or some useless superstition. I was drawn to this solely by the pleasure of theory and the love of truth, whose purposes are assuredly not achieved by any other means than these studies'.
82. 'Who has written deeply and scientifically about arithmetic, perspective, diaphanous materials, the rainbow, and other important works of the faculty of mathematics ... who has been praised and renowned, I will not say in Sicily, but all over Italy and Europe, whose works have been celebrated in Rome, Venice, Paris, Basle and other famous cities of Germany, France and Spain'.
83. 'We tolerate also Nicola Copernico, who has placed the sun fixed and the earth that rotates around it, more worthy of the scourge or the whip than of being confuted'.
84. 'But what surprise is it, when there are certain foolish, not to say insane people, who go so far as to try to affirm that the sun stands still and the earth moves?'