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DARRIGOL, O.
2010

The Analogy between Light and Sound in the History of Optics from the Ancient Greeks to Isaac Newton. Part 1 + 2

OLIVIER DARRIGOL*

Abstract. Analogies between hearing and seeing already existed in ancient Greek theories of perception. The present paper follows the evolution of such analogies until the rise of 17th century optics, with due regard to the diversity of their origins and nature but with particular emphasis on their bearing on the physical concepts of light and sound. Whereas the old Greek analogies were only side effects of the unifying concepts of perception, the analogies of the 17th century played an important role in constructing optical theories by imitation of acoustic theories, or vice versa. This transition depended on several factors including the changing relations between optics, music, mathematics, and physics, the diversity of early modern concepts of sound, and the rise of a new physics based on experimentation and mechanical explanation.

Keywords. Acoustic analogies, history of acoustics, history of optics

The idea that visual perception involves a medium somehow relating the beheld to the beholder is as old as ancient Greece. It found a precise physico-mathematical expression in Augustin Fresnel's and James Clerk Maxwell's theories of light in the 19th century. This medium-based optics is commonly believed to have resulted from analogy with a better established science of sound. But historical scrutiny leads to a more nuanced picture: although there is no doubt that the acoustic analogy played an important role at some stages in the history of optics, this analogy was more problematic and less productive than imagined by the authors of modern physics textbooks.

The main reason for this limited efficiency of the acoustic analogy is that there never was a time in which the science of sound was much ahead of the science of light. Typically, new wave-theoretical concepts entered acoustics shortly before they entered optics; but the reverse also sometime happened, especially in the 19th century.¹ In some cases, such as the introduction of the concept of interference, the evolution of the two theories is best described as symbiotic. As acoustics was long an underdeveloped science, it could not easily serve as a paradigm for other wave phenomena. The reasons why it nevertheless did so at important stages of the history of optics are worth a historian's scrutiny.²

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The broader purpose of the present paper is to trace the history of the analogy between light and sound, or between seeing and hearing, over the period extending from Greek antiquity to Isaac Newton's *Opticks*. The period extending from Newton to Thomas Young will be covered in another paper. The rationale for the *longue durée* of this study is the existence of long-term correlations in the evolution of the relevant parts of physics. Some Greek physics still informed debates on the similarities or dissimilarities of light and sound in the 17th century; Thomas Young heavily relied on Newton's writings and even liked to see in Aristotle a precursor to some of his ideas.

Analogies between hearing and seeing occurred at phenomenal, metaphysical, mathematical, physical, and physiological levels. Although this paper's emphasis is on physical analogies and physical theories, the various levels of analogy are historically entangled. It will indeed be seen that analogies at the physical or mathematical levels often derived from analogies at the other levels. Besides, the distinction between different levels of analogy depends on the period considered and on the general philosophy of the actors. For example, in Galen's philosophy the physiological and physical levels are intertwined; in Kepler's, a mathematical analogy is *de facto* a physical analogy; in Francis Bacon's, a phenomenal analogy indicates a physical analogy.

The use of acoustic theory as a template for optical theories only began to occur in the 17th century. In the long period covered in this paper, the analogies between hearing and seeing did not necessarily go from the former to the latter sense. They sometimes involved more physiology of perception than physics, as was the case in some Greek theories. They did not necessarily play a constructive role. They could instead result from the author's desire to include both kinds of perception within a homogenous philosophy of nature. The structural content of the analogy varied considerably, and it did not involve the modern concept of wave propagation until the late 17th century.

These anticipatory remarks show the necessity of keeping an open mind when dealing with pre-modern analogies between hearing and seeing. They also raise a few questions regarding the origin of the modern exploitation of acoustic analogies:

To what extent did the early medium theories of light rely on acoustic analogy? What were the resources of the contemporary physics of sound and music? What prompted the analogy? Which structural or ontological components did the analogy carry along? A detailed answer to these questions is given in the concluding section of this paper. One remarkable fact is the bewildering multiplicity of concepts of sound from antiquity to the early 19th-century. Another is that some of the most fruitful acoustic analogies of the history of optics depended on now antiquated concepts of sound. To give only one example, Thomas Hobbes's influential medium-based derivation of the sine law of refraction depended on the old 'breath' conception of sound, according to which air moves from the sounding body to the ear at each stroke of this body. This remark explains the inconsistencies that historians of optics have detected in Hobbes's optics.³

This study sheds light on a rarely addressed aspect of analogical thinking: the importance of the state of development of the basis for the analogy. In many formal

analogies of physics, for instance in the mechanical analogies of 19th-century physics, or in the classical analogies implied in the genesis of quantum mechanics, the basis was a well-understood, fully articulated theory of physics. Most literature on analogy focuses on such cases, although it is sometimes noted that the analogy can bring new structure to the theory on which it is based.⁴ Little attention is given to difficulties inherent in the underdevelopment of the analogical basis. In the case of sound and light, it turns out that an insufficient understanding of sound propagation often determined the course of history. For instance, erroneous views on the diffraction of sound could exclude a wave-theoretical explanation of the rectilinear propagation of light. Had Newton known modern acoustics, he would have lost the main motivation behind his idiosyncratic mixture of light corpuscles and ethereal vibrations.

The first section of this paper is devoted to ancient Greek theories of seeing and hearing. As in these theories light was not considered to be something that could be transmitted from visible objects to the eye, there could not be any analogy between sound and light proper. There were nonetheless analogies drawn between conceptions of hearing and seeing, both by the atomists and by Aristotle. These analogies resulted in part from the existence of a common conceptual framework for all forms of perception, and also from the Pythagorean idea of universal harmonies in the various realms of perception. They did not serve to model seeing on hearing or vice versa.

The two following sections do not deal directly with the analogy between hearing and seeing. Rather, they summarize medieval, Renaissance, and 17th century developments that ultimately permitted constructive analogies between light and sound. Special attention is given to the little known evolution of music theory and concepts of sound, which played an important role in this process.

The brief second section recalls some moments in the evolution of optics and music theory in the Middle Ages and in the Renaissance, especially the Arabic invention of light as an entity traveling from visible objects to the eye. This step was a necessary precondition to any direct analogy between light and sound, which however did not happen in this period. Until the Renaissance, the physics of sound long remained confined to three Greek rudiments: the breath conception of sound, the stoic analogy with water waves, and interpretations of pitch by speed or frequency. Music theory had almost no need of physics, for it was usually based on Pythagorean ratios. The situation changed in the late Middle Ages and in the Renaissance, as the rise of polyphonic music implied new consonances and new attunements. A physical justification of harmonious ratios now became necessary. It was found in the periodicities of superposed sounds. The contemporary rise of mechanical thinking favored this view and prompted inquiries into the vibrational nature of sound.

The third section is devoted to the many concepts of sound in the 17th century. Although the correspondence between pitch and frequency became universally accepted, opinions varied on whether a medium was needed, on whether this medium was air or some other substance, on whether sound propagation involved compression of the air or

not, on whether it occurred in pulses or in a smooth oscillation. In the last third of the century, Baconian experimentation on sound and the progress of Newton's mechanics favored the idea that sound consisted of compression air-waves with no net transport of air. In earlier times, there was no theory of sound that could serve as a paradigm for other wave theories. There only were diverse, controversial concepts of sound. On the one hand, the physical and mechanical character of these concepts eased their bridging with contemporary optics. On the other, their diversity called for prudence in developing such analogies.

The fourth section is a discussion of the many theories of light put forward during the 17th century and their connection with or disconnection from the new physics of sound. As is well known, the notable characteristic of most of these theories is their mechanical character. The authors of some mechanical theories, especially René Descartes, insisted on the dissemblance between sound and light and developed a specific model for the propagation of light. Others like Marin Mersenne and Athanasius Kircher developed analogies that proved of little use, unless they went from light to sound. Still others, such as Thomas Hobbes and Robert Hooke, partly based their optical theories on an analogy with sound, though in a mostly implicit manner. After the compression-wave theory of sound gained ground, Gaston Pardies and Christiaan Huygens explicitly based their optics on an analogy with sound, although in Huygens's case much of the contents of the theory derived from other considerations. In the first half of the century, there were also neo-atomist theories in which light and sound were both thought to be composed of streams of particles.

The fifth section of this paper is about Newton's optics, its early development, Newton's debate with Hooke, and the treatise of 1704. Even though Newton favored the corpuscular view of light, the acoustic analogy played a more significant role in his optical writings than in those of any of his contemporaries: as a means to discard the pure wave theory (arguing that light, if it were similar to sound waves, would not behave as it should), as a means to improve the pure wave theory over Hooke's version (by suggesting the correspondence between color and frequency), as a model for the ether waves that Newton held responsible for the colors of thin plates, and as the basis of his musical division of the spectrum. This explains why Newton's writings were an important source for later wave theories of light, even though Newton himself provided the most influential objection to the wave concept of light.

The sixth and last section contains a few conclusions regarding the evolving nature of the analogies between light and sound. It involves the distinction between descriptive and constructive analogy, and, within the latter category, the distinction between positive, negative, neutral, silent, and soft analogy as an extension of Mary Hesse's well-known terminology. The three last sections will appear as 'Part 2' in the next issue of *Centaureus*.

The present research would not have been possible without the abundant resources of earlier histories of optics and acoustics. Especially relevant to this study were David

Lindberg and Gérard Simon on older theories of vision, Alan Shapiro on 17th-century optics, Hendrik Floris Cohen and Benjamin Wardhaugh on music and the physics of sound in the 16th and 17th centuries. The following text includes much of these and other historians' results in order not to confine the readership to experts in the history of ancient and early modern optics and acoustics. Its main originality resides in the attention given to the historical mechanisms that permitted or inhibited analogies between two domains of physics that most historians have treated separately.

1. Greek Theories of Seeing and Hearing

In the sixth century B.C. a few Greek thinkers pioneered a philosophy of nature in which simple causal explanations replaced the traditional appeal to supernatural forces. They trusted reason and observation more than myths and religious authority. They held diverse views and were eager to debate with their opponents. Whereas the Milesians sought to reduce phenomena to a single material principle (water, air, or 'the boundless'), the Pythagoreans believed that numbers were the principle of all things. In the fifth century, the monistic philosophies collided with Parmenides's paradox of change, according to which nothing can come to be from not being; Empedocles explained apparent change in nature through the variable sympathies or antipathies of four stable 'roots' (earth, water, air, and fire); Leucippus and Democritus reduced natural phenomena to the variable configuration of stable atoms of various shapes. In the fourth century, Plato adopted a doctrine of matter that was a compromise between Empedocles's, Pythagoras's, and the atomists' views: namely, he analyzed Empedocles's elements into atoms of Pythagorean polyhedral shape. Aristotle retained the Empedoclean elements, in a less rationalist guise in which sensible qualities played a central role (Lloyd, 1970).

Among these founders of Greek science, the early Milesians accepted the testimony of senses, Parmenides belittled it as mere illusion, and all the others recommended a critical approach in which the senses should be used under the control of reason. Empedocles, Pythagoras, Leucippus, Plato, Aristotle, and their successors all gave causal accounts of perception that enabled them to distinguish true observations from illusions. These accounts varied considerably, despite some mutual connections. They depended on the broader philosophical outlook of the author, on casual observation, and on popular beliefs about the nature of vision and hearing. They usually treated the five senses in parallel, although sight and hearing by far received the most attention.⁵

The scarcity and indirectness of textual records makes it impossible to reconstruct the Greek theories of perception with much precision and authenticity. There are a few cases, however, in which enough is known to compare approaches to visual and auditory perception. As these cases are those that matter for the later history of physics, their discussion is sufficient for our purpose.⁶

1.1 The Atomists

In the fifth century B.C., Leucippus and Democritus had hearing and seeing depend on fluxes of atoms received by the ear and the eye. As only a few fragments of their writings have survived and as their early commentators often disagree, it is convenient to rely on the much later poem of Lucretius (ca. 50 B.C.), which contains a coherent, detailed account of Epicurus's derived atomist doctrine (ca. 300 B.C.) and which inspired much of later atomist philosophy.⁷

According to the *De rerum natura*, material bodies constantly emit thin films that preserve the configuration of their atoms when they travel through the air (Leonard, 2004, p. 105):

And thus I say that effigies of things,
And tenuous shapes from off the things are sent,
From off the utmost outside of the things,
Which are like films or may be named a rind,
Because the image bears like look and form
With whatso body has shed it fluttering forth.

These effigies (*simulacra*) strike the eye and convey to it all the information needed to recognize the shape and colors of bodies. Figure corresponds to the arrangement of the atoms of the effigy, and color to their shape. Although this idea of traveling effigies may seem fanciful to the modern reader, it is hard to imagine any other emissionist explanation of the perception of images without the modern understanding of the eye as an optical instrument.

Lucretius addressed two evident difficulties with this explanation. Firstly, it seems to exclude the eye perceiving the entirety of the effigy of a large object. Lucretius mysteriously suggested that a small part of the effigy was enough to convey all its properties, just as the hardness of a big stone can be inferred by touching only a small part of it. Secondly, this picture does not explain why objects are only seen in the presence of a source of light, namely, the sun or a flame. Lucretius solved this difficulty by admitting the continuous emission of subtler atoms of light from the luminaries. These atoms, being able to enlarge the pores between the atoms of air, permitted the free traveling of the effigies from the object to the eyes. Hence the atomists' vision required two entities, the light from the sun, and the more material effigies from the observed bodies (Leonard, 2004, pp. 114–115).

Lucretius also assumed the corporeal, atomist nature of voice and sound, and had them flit through the air until they struck the ear (Leonard, 2004, p. 120):

Firstly, a sound and every voice is heard,
When, getting into ears, they strike the sense
With their own body. For confess we must

Even voice and sound to be corporeal,
Because they're able on the sense to strike.

He regarded the soreness of a yelling throat as a proof of this materiality, and the roughness of a sound as a consequence of the roughness of the corresponding atoms (Leonard, 2004, p. 120):

Besides voice often scrapes against the throat,
And screams in going out do make more rough
The wind-pipe—naturally enough, methinks,
When, through the narrow exit rising up
In larger throng, these primal germs of voice
Have thus begun to issue forth.

For sound as for effigies, he interpreted reflection as the bouncing of their atoms over a wall or mirror. He believed that sound could travel along curved lines, owing to the interactions between the atoms of sound and the atoms of air. In contrast, he asserted that the effigies had to travel along straight lines in order to preserve their integrity until they reached the eye (Leonard, 2004, pp. 114–115)⁸:

Again,
One need not wonder how it comes about
That through those places (through which eyes cannot
View objects manifest) sounds yet may pass
And assail the ears. For often we observe
People conversing, though the doors be closed;
No marvel either, since all voice unharmed
Can wind through bended apertures of things,
While idol-films decline to—for they're rent,
Unless along straight apertures they swim.

In sum, the Greek atomists believed that seeing and hearing (and smelling) involved the traveling of atoms (at finite speed) from the perceived object to the perceiving organ and that the form of the atoms conveyed information (about color in one case, about roughness of sound in the other). The analogy was incomplete, because specific facts of vision required specific mechanisms: the vision-inducing atoms had to be arranged according to effigies, and the additional entity of light was needed to permit the rectilinear, figure-preserving propagation of the effigies.

1.2 Plato

Plato's *Timaeus* contains detailed considerations on seeing and hearing, which draw on multiple sources including Greek popular ideas on vision, Democritus's atomism, and the Pythagorean conceptions of Plato's friend Archytas. In agreement with the popular

view reflected in Greek poetry and theater, Plato believed that vision required a fire from the eye. This fire served to probe visible objects, in analogy with touch and in conformity with the attention needed to observe objects. In order to explain the role of the sun or flames in making bodies visible, Plato assumed another fire from them. This fire coalesced with the fire from the eye to form a coherent, homogenous, percipient body between eye and sighted object (Archer-Hind, 1888, p. 157)⁹:

The fire within us, which is akin to the daylight, [the gods] made to flow pure smooth and dense through the eyes. . . . Whenever this visual current is surrounded by daylight, then it issues forth as like into like, and coalesces with the light to form one uniform body in the direct line of vision, wherever it strikes upon some external object that falls in its way. So the whole from its uniformity becomes sympathetic; and whenever it comes in contact with anything else, it passes on the motions thereof over the whole body until they reach the soul, and thus causes that sensation which we call seeing.

What came in contact with this percipient body was not the seen object itself, but a third fire emanating from the surface of this object. Here Plato probably followed Democritus, although he preferred to explain the perception of images by extended sensitivity rather than by traveling effigies. As was mentioned, Plato also shared the Democritean idea of an atomic constitution of the elements, with some Pythagorean additions such as the regular polyhedral shape of atoms. In order to explain contrast and brightness, he assumed different sizes and velocities for the particles of the fire emanating from bodies. When these particles were smaller than those of the visual current, they dilated this current and caused the sensation of whiteness. In the contrary case, they caused the sensation of blackness (Archer-Hind, 1888, p. 248):

The particles which issue from outward objects and meet the visual stream are some of them smaller, some larger, and some equal in size to the particles of that stream. Those of equal size cause no sensation, and these we call transparent; but the larger and smaller, in the one case by contracting, in the other by dilating it, produce effects akin to the action of heat and cold on the flesh, and to the action on the tongue of astringent tastes and the heating sensations which we termed pungent. These are white and black, affections identical with those just mentioned, but occurring in a different class.

The swifter particles are able to travel against the visual current and reach the eye to cause the sensation of brightness. Colors (other than black and white) result from the mixture of the two fires (from the eye and from the object) within the eye in various proportions and speeds.

Plato defined sound as ‘the stroke inflicted by the air on the brain and on the blood, traveling through the ears and transmitted to the soul.’ Like Archytas, he identified the collision of two bodies (or of two portions of air) as the cause of sound and he imagined a related rush of air from the site of the collision to the ear. This conception of sound is usually called the ‘missile theory’ of sound, because Archytas compared this rush with a missile. An evident precondition of this view is the materiality of the air, which the

phenomena of winds, bellows, and siphons amply suggested. Plato further agreed with Archytas that pitch corresponded to the rapidity of the motion that caused the sound or the induced aerial motion, an idea gained from hearing the sound produced by moving a stick through the air with varying speed (Archer-Hind, 1888, p. 247):

A rapid motion produces a shrill sound, a slower one a deeper sound; regular vibration gives an even and smooth sound, and the opposite a harsh one; if the movement is large, the sound is loud; if otherwise, it is slight.

Accordingly, Plato identified the Pythagorean ratios of musical harmony with speed ratios. He defined the consonance of two sounds as the condition that the slower sound should reach the ear when the disturbance created by the faster one has decreased to the level that the slower sound would induce by itself (Archer-Hind, 1888, p. 301)¹⁰:

[For a harmonious combination of sounds], the slower sounds overtake the motions of the first and swifter sounds, when these are already beginning to die away and have become assimilated to the motions which the slower on their arrival impart to them: and on overtaking them they do not produce discord by the intrusion of an alien movement, but... they form one harmonious sensation by the blending of shrill and deep. Thereby they afford pleasure to the foolish, but to the wise joy, through the imitation of the divine harmony which is given by mortal motions.

Compared to the atomist doctrine, Plato's implies a lesser analogy between hearing and seeing. The only similarities between the two senses are those related to the shared idea of an emanation from the seen object. This emanation is somewhat comparable to the rush of air from a sounding body, although it is not directly perceived by the eye. Brightness corresponds to the speed of (the particles) of this emanation, as pitch corresponds to the speed of the moved air. Contrast corresponds to the size of the particles, as loudness corresponds to the quantity of the displaced air. Otherwise, Plato's doctrine implies much dissemblance between hearing and seeing. Whereas aerial motion suffices to convey sound, vision requires no less than three fires: from the eye, from the sun, and from the object. The visual fire serves to extend human sensitivity beyond the human body, whereas auditory sensitivity remains confined to the ear.

There is, however, one important kind of analogy between visual and auditory perceptions that Plato did not inherit from the atomists: he believed that divine harmony, expressed in Pythagorean ratios, occurred in at least three different realms: celestial motion, music, and the motions of the soul. The perception of celestial and musical harmonies both depended on their agreement with the inner harmonies of the soul (Archer-Hind, 1888, p. 165).

1.3 Aristotle

Aristotle's considerations on vision strongly departed from those of his predecessors. He rejected Democritus's idea of effigies as an erroneous inference from the fact that

a small picture of an object can be seen in the eye of another person (he correctly interpreted this fact as a case of mere reflection). He equally rejected Plato's visual fire, for he judged the coalescence of this fire with daylight or its quenching by darkness to be absurd notions. He questioned the ability of visual rays to reach remote stars, and brushed away the suggestion that the light from the stars might only meet the visual rays in the vicinity of the eye. (Aristotle, *De sensu*, 437a–438a).¹¹

In his own theory of vision, Aristotle retained from Plato the idea that a homogenous medium was required between the eye and the object of sight:

Seeing is due to an affection or change of what has the perceptive faculty, and it cannot be affected by the seen color itself; it remains that it must be affected by what comes between. Hence it is indispensable that there be *something* in between—if there were nothing, so far from seeing with greater distinctness [as the atomists would think], we should see nothing at all. (Aristotle, *De anima*, 418b)

This medium no longer involved the visual fire and no longer offered an extended sensitivity. It was just the 'ether' contained in air or in any other transparent body, and its sole function was to permit the transmission of some intrinsic qualities of the objects of sight, named colors (including black and white). Just as Plato's theory required sunlight for visibility, the transparency of Aristotle's medium needed to be activated by fire from the sun or burning bodies. By Aristotle's definition, light is activated transparency (whereas for Plato and the atomists, light was a fire from the sun):

Every color has the power to set in movement what is actually transparent; that power constitutes its very nature. That is why it is not visible except with the help of light. . . . Light is the activity [of the ether]. Light . . . exists whenever the potentially transparent is excited to actuality by the influence of fire or something resembling the uppermost body. (*De anima*, 446b)

Aristotle regarded both the activation of transparency and the transmission of colors as instantaneous, global processes involving no displacement of the parts of the medium: 'Light has its *raison d'être* in the being of something, but it is not a movement.' He did refer to the transmission of colors as a 'movement' through the lit transparent body; but by movement he only meant qualitative change, which can 'conceivably take place in a thing all at once. . . e.g. it is conceivable that water should be frozen simultaneously in every part' (*De anima*, 447a).¹²

Somewhat like Plato, Aristotle reduced color to a mixture of black and white so fine as to elude separate perceptions of its components (Pseudo-Aristotle, *De coloribus*, 791a–799b). For instance, he obtained crimson by mixing dusky black with sunlight and purple by mixing feeble sunlight with thin dusky white. He further suggested that pleasant colors corresponded to mixtures made according to simple ratios, by analogy with Pythagorean harmony in music¹³:

It is conceivable that the white and the black should be juxtaposed in quantities so minute that either separately should be invisible, though the joint product would be visible; and that they

should thus have the other colors for resultants. . . . We may suppose that many [colors] are the result of a [numerical] ratio; for the blacks and whites may be juxtaposed in the ratio of 3 to 2, or of 3 to 4, or in ratios expressible by other numbers. . . . Accordingly, we may regard all these colors as analogous to the sounds that enter into music, and suppose that those involving simple numerical ratios, like the concords in music, may be those generally regarded as most agreeable; as, for example, purple, crimson, and some few such colors, their fewness being due to the same causes which render the concords few. (*De sensu*, 439b–440a)

Like Archytas and Plato, Aristotle taught that the production of sound required the hitting of a body by another, and that its propagation implied a concomitant movement of the air. However, he held a different view of this movement as he required some solidarity of the air between the sonorous body and the ear:

Hearing occurs in air, and in water too, though to a smaller degree: for neither air nor water is responsible for sound, but rather there must occur an impact of solid things against one another and against the air. This happens when the air stands fast when it has been struck, and is not dispersed. Hence if it is struck quickly and vigorously, it makes a sound; for it is essential that the movement of the striker should forestall the fragmentation of the air, as when something in rapid motion strikes a heap or an eddy of sand. . . . A thing is productive of sound, then, if it can move air as one unified thing continuously as far as the hearing. . . . Air as such is soundless, because it is readily fragmented: but when it is prevented from fragmenting, its movement is sound. (*De anima*, 419b–420a)

The Aristotelian author of the *Problemata* further indicated a propagated action, by analogy with Aristotle's idea that a projectile is kept in motion by the air that rushes into the gap behind it¹⁴:

Sound is made by air in motion; and just as sound is first made by that which moves the air, so the air in its turn must do the same, and some air must be the mover, some the moved. . . . For a continuous vocal sound occurs when air is propelled by air, while a missile travels when a body is moved by air. (*Problemata*, book 11, query 6)

Broadly calling 'breath theory' any theory of sound in which air moves from the source to the ear, two varieties of this theory must now be distinguished. The first variety is Archytas's 'missile theory' in which an excess of air travels like a missile from the source to the ear. The other, first formulated in Aristotle's and peripatetic writings, may be called the 'pestle theory' (some historians call it the 'shunt theory'), for in it the air between the source and the ear moves globally owing to the mutual impenetrability of its parts. In the missile theory, the velocity of the air is also the velocity of propagation of sound. In the pestle theory, these two velocities are independent of each other.¹⁵

Like Plato, Aristotle related pitch to speed and loudness to displaced mass: 'In sound what is swift is high-pitched'; 'Large voices occur when what is moved is a great quantity absolutely' (*De generatione animalium*, 787a). Similarly, the author of the *Problemata* wrote: 'A large voice occurs when one moves much air, a high-pitched one when one moves it swiftly, and a low-pitched one when one moves it slowly' (*Problemata*, book 9,

query 3). In a discussion about the simultaneity of sensations, Aristotle wondered: 'Are people right or wrong about concord, when they say that the sounds do not arrive simultaneously, but only seem to, and that this is why the time involved is imperceptible?' (*De sensu*, 448a). The missile theory and the correspondence between pitch and velocity indeed suggested that high-pitch sounds traveled faster than lower-pitch ones. As we saw, Plato assumed so much in his explanation of consonance. Aristotle's concept of propagation through a unified mass of air left the question open. It lent itself to variants in which the propagated property no longer was a displacement. Aristotle's successor as head of the Lyceum, Theophrastus, denied that pitch was a quantity. In particular, it could not be a speed: 'The high note [in a concord] cannot be distinguished by its speed: for then it would occupy the hearing first, so that a concord would not arise' (Barker, 1989, vol. 2, p. 116). Another of Aristotle's disciples, Aristoxenus, represented pitch as a point on a line and rejected any numerical interpretation. In his influential *Elementa harmonica*, he rejected Pythagorean ratios in favor of musical intervals phenomenologically defined by adding equal tones and fractions of a tone. He believed that the theory of music should be founded on rationalized musical experience, not on any dubious physics of sound.¹⁶

Being more Pythagorean and more Platonist than these successors, Aristotle did not refrain from musical ratios and addressed the physical nature of pitch. Additional remarks on the physics of sound appear in his discussions of voice and speech, which seem to have concerned him more than music. He compared echoes to the reflection of a ball¹⁷:

An echo occurs, when, a mass of air having been unified, bounded, and prevented from dissipation by the containing walls of a vessel, the air originally struck by the impinging body and set in movement by it rebounds from this mass of air like a ball from a wall. It is probable that in all generation of sound echo takes place, though it is frequently only indistinctly heard. What happens here must be analogous to what happens in the case of light; light is *always* reflected—otherwise it would not be diffused and outside what was directly illuminated by the sun there would be blank darkness. (*De anima*, 419b)

This extract involves an analogy between the diffusion of light (here a fire from the sun) and the reflection of sound, which should not be confused with an analogy between mirror images and echoes (for images do not involve the rebound of anything according to Aristotle). Aristotle saw more important analogies between hearing and seeing. He emphasized that the perception of colors and pitches both implied a medium, as well as their being actualized by something, which was light in the case of colors and another body in the case of sounds:

The distinctions between different sounding bodies show themselves only in actual sound; as without the help of light colors remain invisible, so without the help of actual sound the distinctions between acute and grave sounds remain inaudible. (*De anima*, 420a)

Aristotle based his whole theory of sensations on the distinction between the actual and the potential, as he wanted to show that perception revealed properties that existed independently of their being observed.¹⁸

Within this unifying frame, Aristotle admitted much dissemblance between the various senses. He made clear that movement was involved and propagation took time in the case of sound (and smell) but not in the case of light. The author of the *Problemata* agreed with the atomists that sound could travel along curved lines whereas sight always proceeded along straight lines:

Why is it that the sight cannot pass through hard objects, but the voice can do so? Is it because the course of the sight can only take one direction, namely, a straight line (as is shown by the rays of the sun and the fact that we can only see what is directly opposite to us), whereas the voice can take many directions, since we can hear from everywhere? (*Problemata*, 905a)

In the same vein, he asked:

Why is it that light cannot penetrate through dense objects, whereas sound can do so, although light is rarer and travels farther and quicker than sound? Is it because light travels in a straight line, and so, if anything blocks its direct course, it is completely cut off, but sound, because it is a breath, can also travel in a line that is not direct? (*Problemata*, 904b)

On the one hand, this Aristotelian author admitted the rectilinear propagation of light without explanation. On the other, he believed that sound, being a 'breath' or a movement of air, could circumvent obstacles just as the stream of a river skirts round the pillars of a bridge.¹⁹

1.4 The Stoics

Chrysippus (third century B.C.) and other stoic philosophers shared with Plato and Aristotle the idea of a medium relating the seen object to the eye. Plato generated this medium from daylight and visual fire. Aristotle took it to be the ether activated by daylight. The Stoics identified it with air properly modified by the joint action of daylight and of a visual *pneuma* (combination of air and fire) emitted by the eye. They followed Plato in retaining the popular idea of an emission from the eye and in imagining a percipient body akin to the tentacles of an octopus between the eyes and the object. As Cicero put it, 'the air itself sees together with us' (Sambursky, 1959, p. 28). The Stoics nonetheless rejected Plato's idea of an emanation from the surface of bodies and imagined a direct transmission of the pattern of the object through stressed air. So reported Alexander of Aphrodisias (Sambursky, 1959, p. 124):

[The Stoics] explain vision by the stress of air. The air adjoining the pupil is excited by vision and formed into a cone which is stamped on its base by an impression of the object of vision, and thus perception is created similar to the touch of a stick.

In an influential variant of the stoic view, the Greco-Roman physician Galen (second century A.D.) rejected the walking-stick analogy and compared the illuminated air to an extended nerve. He held the same principle, the visual pneuma, responsible for the sensory power of both the nerves and the illuminated air. Drawing on anatomic observations, he identified the crystalline humor as the part of the eye through which this pneuma was emitted after traveling from the brain through the optical nerves.²⁰

Whatever its precise sensory meaning, the notion of stressed air conveniently unified stoic ideas of seeing and hearing. A fragment of Cleomedes (first century A.D.) illustrates this point (Sambursky, 1959, p. 41):

Without one binding tension and without the all-permeating pneuma we would not be able to see and hear. For the sense perceptions would be impeded by the intervening empty spaces.

Famously, Diogenes Laertius (third century A.D.) compared the propagation of sound with waves on water (Hülser, 1987–1988, p. 541):

Hearing takes place when the air between the source and the receiver of sound is set into spherical vibrations, then expands in waves until it presses the ear, just as the water in a container forms circular waves, when a stone is thrown into it.

As the ancients lacked any precise understanding of water waves, not much can be drawn from this comparison. It is not clear, for instance, whether Diogenes meant the spherical waves of sound to carry the substance of air with them. Perhaps tension was more important, as implied in this extract from Seneca: ‘What is indeed sound if not a tension of the air caused by a stroke of the tongue in order to be heard?’ (Hülser, 1987–1988, p. 543). Another stoic metaphor, the spider’s web, makes the analogy between hearing and seeing more evident: the soul feels the presence of external bodies through those two senses just as a spider feels the presence of trapped insects through the vibrations of its web (Sambursky, 1959, p. 24).

1.5 Euclid and Ptolemy

None of the above-mentioned theories of perception involved mathematical reasoning. In the case of optics, both the atomists and the medium-theorists (Plato, Aristotle, and the Stoics) had a global understanding of the perception of images, either based on the traveling of effigies or based on comprehensive transmission through the medium. There was, however, a Greek geometrical optics according to which the perception of images depended on the rectilinear propagation of some entity. Common sense excluded the possibility of this entity emanating from the object, since the eye was a priori unable to distinguish between rays issuing from various points of the object. Instead Greek geometers implicitly assumed that any observer was aware of the directions in which

the eye emitted visual rays. This means that the angular distribution of the rays' contact points with the object could be appreciated.

The earliest extant text based on this simple idea is Euclid's *Optics*, written ca. 300 B.C. (Ver Eecke, 1959). It is mainly a treatise on angular perspective regarded as an application of Euclidean geometry. Its typical problems concern the apparent ratios of lengths situated at various distances from the eye or at various inclinations with respect to the visual axis. It does not include other aspects of vision (not even the appreciation of distances), and it says very little about its physical foundations. Euclid's optical axioms nonetheless make clear that the rays issue from the eye, except in the case of shadows for which rays from the sun are blocked by opaque bodies. This double recourse to visual rays and rays of light suggests a vague Platonic framework. In general, however, geometry dominated Euclid's optics.²¹

Similarly, arithmetic dominated Euclid's theory of music. His *Sectio canonis* was a Pythagorean treatise on harmonics, the main purpose of which was to justify the Pythagorean relation between consonance and simple ratios of numbers and to explain the associated rules of music making (Busch, 1998). Physics and physiology played no role in this reasoning. According to a myth popularized by Nicomachus and Boethius, Pythagoras discovered the correspondence between numerical ratios and concords when hearing the sounds produced by hammers of different weights in the shop of a blacksmith (Meyer, 2004, pp. 46–51). More plausibly, he was aware of the relation between the pitch and the length of a musical chord (*ceteris paribus*). Yet the true basis of Pythagorean harmony is likely to have been independent of any observation of this sort. As Euclid explains in probable agreement with earlier Pythagorean ideas, it is natural to associate the most pleasing consonances (our octave, fifth, and fourth; the third not being regarded as a consonance by the Greeks) with the simplest ratios (multiple ratios and *superparticular* ratios involving two numbers differing by a unit only) based on the numbers 2, 3, 4, namely, the ratios 1:2, 2:3, and 3:4. As the double fifth and the double fourth are empirically dissonant, they cannot be multiple ratios (the square of a multiple ratio being also a multiple ratio). Therefore, the fifth and the fourth are superparticular, whereas the octave is multiple because a double octave is consonant and a squared superparticular ratio is neither superparticular nor multiple. These two facts, together with the fact that the fourth, fifth, and octave are the first three consonant intervals in growing order of pitch, imply that 1:2 corresponds to the octave, 2:3 to the fifth, and 3:4 to the fourth. This reasoning relies only on primitive musical experience and simple arithmetic.²²

Nevertheless, Euclid introduced his treatise with a few remarks on the physics of sound. This introduction begins with a rehearsal of the usual relation between sound and blow (Cohen and Drabkin, 1948, p. 291):

If there were complete rest and immobility there would be complete silence. And if there were silence and nothing moved, nothing would be heard. Therefore for anything to be heard there must first be a blow and motion.

More originally, Euclid associated pitch with frequency:

Some motions are more frequent, others are rarer, and the more frequent produce the higher pitched sounds while the rarer produce the lower pitched. From this it follows that some sounds are higher pitched, being composed of more frequent and more numerous motions, while others are lower pitched, being rarer and composed of less numerous motions.

From this insight, Euclid concluded that pitch was a quantitative notion:

Therefore sounds must be said to consist of parts, since they reach their proper pitch by addition and subtraction. Now all things that consist of parts may be spoken of as in numerical ratio to one another.

Euclid's nearly exclusive interest in the mathematical aspects of optics and music prevented any physical analogy between these two fields. His few remarks about the nature of vision and sound implied a strong dissemblance, as there was no acoustic counterpart to the notion of visual rays, and no optical counterpart to the frequency of a sound. The only analogy between hearing and seeing was the existence of measurable quantities in both fields: sighting angles in one case, pitch in the other. The relevant mathematics differed: it was geometry in one case and arithmetic in the other, although Euclid bridged them through the Eudoxean theory of ratios.

Similar remarks can be made about Ptolemy's *Optics* and *Harmonics*, written in the second century A.D. (Mark Smith, 1996). In both fields, Ptolemy's approach was mainly mathematical, although he dwelt more on physical, experimental, and psychological aspects than Euclid. In optics, Ptolemy focused on geometric perspective and therefore favored visual rays. He rejected Plato's idea of emanations, and roughly followed Aristotle in regarding colors as intrinsic properties of bodies activated by daylight. He also applied ray propagation to daylight in order to explain the casting of shadows. He had no use for Plato's or Aristotle's medium, since the visual rays by themselves conveyed all the needed visual information (even distance from the touched object in his opinion). He carefully measured the reflection and refraction of visual rays, and developed the theory of the resulting illusions much further than Euclid had done.²³

Ptolemy's discussion of the physical foundation of harmonics was more detailed but also more confused than Euclid's. After describing sound as 'a condition of beaten air,' Ptolemy identified various factors upon which the volume and pitch of sound depended: the force of striking agents, the density of the struck bodies, and their dimensions. He concluded, like Euclid, that pitch was a quantitative notion without making clear what the relevant quantity was (Solomon, 2000, pp. 2–16). He defined consonance through pleasantness of the resulting impression, or through closeness to homophony (being heard

as a single sound) (Solomon, 2000, p. 22). In his lengthy discussion of the principles of harmony, he favored the Pythagorean approach over the Aristoxenian one. In true Pythagorean and Platonic spirit, he devoted a whole book of his *Harmonics* to harmonies in general, including the harmonies of the soul and those of celestial motion.²⁴

Besides the shared harmonies, Ptolemy perceived two global analogies between seeing and hearing. He agreed with Aristotle that the objects of these two sensations, colors and sounds, required activation by light and blows to be perceived. And he agreed with the peripatetic distinction between continuous and discrete quantity, applied to both pitch and color (Solomon, 2000, p. 15):

Continuous sounds have their places of transition into one another obscured. . . , like the colors of the rainbow. Such sounds blend together during tightening and loosening movements, ceasing at the lower end with the lowing of cows and at the upper end with the howling of wolves. Discrete sounds are those which have their places of transition evident whenever their isotonic parts remain at a perceivable interval, like the noticeable juxtaposition of unmixed, unblended colors.

Beyond these broad analogies, Ptolemy's optics and acoustics necessarily differed in essential ways, because the connection between the perceiving organ and the perceived object was of a quite different nature in the optical and acoustic cases: it was aerial motion for sound, percipient rays for colors.

To summarize, for the ancient Greeks there could not be any direct analogy between light and sound, for only sound was regarded as an entity traveling from the perceived object to the perceiving organ. For the atomists and for Plato, it was not light that emanated from the object, but some specific matter or fire. For all other thinkers, nothing material emanated from luminous objects. What permitted vision was an instantaneous and global activation of the transparency of the medium, or the sensitivity of visual rays. The name of light was usually reserved for an entity emanating from the sun or from flames and activating the visual process, except for Aristotle, who called light the activated medium.²⁵

This dissemblance between light and sound is surprising for modern readers accustomed to the idea that the diffuse reflection of light by bodies is the cause of perceived images. For the ancient Greeks who ignored the internal optics of the eye, it was more natural to imagine visual rays probing the object, or effigies emanating from the object, or else a medium somehow transferring the figure and color of the object to the eye.

Analogies could be drawn between visual and auditory perception, to the extent that both faculties depended on the same general philosophy of perception. The atomists built sounds and effigies from traveling particles, and associated the size, shape, and speed of these particles with the qualities of sounds (roughness and loudness) and effigies (color and brightness). The medium-theorists believed that both the perception of sounds and the perception of colors depended on the state of the medium at one end being reproduced

at the other end, although both the medium and the kind of its states strongly differed in the two cases. Whereas the medium for sounds always was air and the kind of state usually was a motion of its substance, the medium for colors could be the ether (for Aristotle), a compound of visual fire and light (for Plato), or a compound of air, pneuma, and light (for the stoics); and the states of this medium did not imply any motion of its parts. Moreover, the medium for colors could be percipient (for Plato and for the stoics), whereas the air was purely material. There is a last sort of Greek analogy between vision and audition: Plato, Aristotle, and Ptolemy saw the same Pythagorean harmonies and the same dichotomy between continuity and discontinuity in both kinds of perceptions. Much of the remaining differences depended on the need of a theory of images that had no counterpart in the sense of hearing. For the atomists, the particles of effigies had to travel in straight lines to preserve the figure of the body from which they emanated, whereas the atoms of sound could be scattered by the atoms of the air. For other thinkers, the visual rays had to be straight in order to define angular perspective, whereas sound, being displaced air, could turn around obstacles. When they were mathematized, vision and audition called for different mathematics: perspective required geometry, music required arithmetic.

From the above given quotations, it is clear that Greek authors explicitly discussed analogies between visual and auditory perception. Yet the analogies were not used as heuristic devices to explain the working of one sense from the working of the other. Rather, they derived from a common conceptual framework, or from analogy with the most elementary sense, touch. As part of their rejection of supernatural explanations, the Greeks did not admit direct action at a distance, and all their theories of perception relied on contact action. Reduction to contact required a mediating entity, which could be traveling atoms, visual rays, or a medium of some sort. For the atomists, all the qualities of the sensations depended on generic properties of the contact such as roughness and speed, both in the visual and in the auditory case. For all other thinkers, some qualities depended on the precise nature of the contact, which varied with the sense considered; the analogy between hearing and seeing was therefore weaker.

2. *A Few Medieval and Renaissance Developments*

2.1 *Optics*

Despite the vicissitudes of their transmission, Greek sources kept feeding discussions about optics until the Renaissance. The diversity of Greek theories of vision was preserved, perhaps because different motivations favored different theories. Astronomers and geometers naturally focused on Euclid's and Ptolemy's theories; philosophers and theologians on Plato's or Aristotle's; physicians on Galen's stoic theory. In the Christian world, the general decline of natural philosophy prevented any creative departure from

received ideas; the most widely accepted theory of vision remained Plato's until the 12th century (it still played a central role in Robert Grosseteste's metaphysics). In the Arabic and Persian worlds, every Greek theory of vision found clever supporters and critics when the systematic study of Greek texts began in the ninth century. Most importantly, around 1000 A.D. the Cairo-based mathematician Ibn al-Haitham (latinized as Alhazen) managed to justify and extend Euclidian perspective within an intromissionist framework.²⁶

Like Aristotle and several of his Arabic contemporaries, Alhazen rejected visual rays as well as the atomist idea of effigies. His basic idea was to analyze objects of vision into point-like centers of ray emission, and to establish a one-to-one correspondence between these points and the points of a receptor within the eye. For the latter purpose, he relied on Galen's observation that the eye involved a series of (roughly) concentric tunicae and various humors between them. He assumed that the only rays that penetrated the eye crossed these tunicae perpendicularly. Consequently, the crystalline humor, which he believed to be sensitive, was crossed by a conic beam of rays converging at the center of the eye and issuing from the various points of the object. Except for the reverted direction of propagation, this scheme reproduced the geometric rules of Euclidean perspective (Sabra, 1989; Mark Smith, 2001).

This new sort of intromission implied that light could now be regarded as a single entity, originating from the sun or flames, traveling along rays susceptible to a number of alterations (diffuse reflection, mirror-reflection, refraction), until it penetrated the sensitive component of the eye. This is essentially the modern concept of light, although Alhazen's understanding of the perception of images widely differed from ours. He was still Aristotelian in his understanding of colors, as he regarded them as modifications of the reflected light by some intrinsic property of the reflecting surface. He was more Democritean in his intromissionism, and more Galenian in his anatomy of the eye and in having a visual spirit connect the brain to the sensitive part of the eye. His main concern was not the deeper philosophy of light but an efficient, geometrized understanding of vision (also burning mirrors and lenses).

Twelfth- and thirteenth-century Europe experienced a major revival of interest in Greek and derived Arabic learning. Albert the Great promoted and expanded Aristotle's doctrine. Roger Bacon's treatises offered a syncretic theory of vision, adding Platonic and Aristotelian elements to Alhazen's doctrine. The Polish friar Witelo offered the most detailed and faithful account of Alhazen's theory, although his encyclopedic *Perspectiva* also presented earlier theories and the Baconian synthesis. In the two following centuries, scholasticism seems to have dampened interest in Alhazen's optics. This theory once again played a privileged role in various Renaissance contexts, including artistic perspective, ocular anatomy, astronomy, and natural magic, although visual rays still had adepts (Lindberg, 1976, chaps. 6–8).

2.2 Music

In the science of sound as in optics, Greek writings remained the main source for most of the Middle Ages. On the nature of sound, there were essentially two Greek views: the atomist identification of sounds with streams of atoms, and the much more common view of a breath of air from the source to the ear, either in the Pythagorean missile version or in the Aristotelian pestle version. The stoic view of spherical, tensional waves expanding from the source may be seen as a variant of the latter view in the absence of any precise understanding of the waves. The breath view, supplemented with the stoic waves, is reflected in the most influential Roman and medieval writings on architecture and music. For instance, Vitruvius (1st century B.C.) wrote:

Voice [or sound] is nothing but the breath which, having been strongly expelled, impresses the organ of hearing by means of the air which it has struck, and the agitation of which forms an infinity of circles that grow indefinitely from the center. (*De architectura*, book 5, chap. 3, §3)

Vitruvius noted the care with which architects avoided obstacles perturbing this propagation. He also explained how the acoustics of a theater could be improved by means of resonant vases and other devices (Truesdell, 1960, pp. 15–16; Hunt, 1978).

In the Middle Ages, music theory was the main context for expressing ideas on the nature of sound. Other related topics, such as the nature of the human voice, the anatomy of the ear, or the physics of musical instruments played a negligible role before the Renaissance. The most popular text on music theory for most of the Middle Ages was Boethius's *De institutione musica*, written around 500 A.D. Paraphrasing Euclid, Boethius wrote (Meyer, 2004, p. 35):

Sound cannot be produced without a pulse [*pulsus*] and a shock [*percussio*], and there could not be any pulse and shock without a preceding motion. . . . This is why sound is defined as a shock of the air that reaches the ear.

Boethius also described the stoics' waves (Meyer, 2004, p. 55):

What happens to sounds is usually compared to what happens to a stone when thrown into a pond or calm water. The stone first causes a wave in a very small circle, then enlarges the circles of the waves until the motion quiets down. . . . The waves expand under a smaller impulse as they grow. Similarly, when the blown air has made a sound, it pushes some other air nearby and somehow causes a round flux of air. Thus, this air spreads around and strikes all at once the ears of the people around. And the sound is dimmer for those who stand further, because a weaker wave of blown air reaches them.

Note that Boethius believed that the waves carried the air with them, in agreement with the Aristotelian pestle theory (Chadwick, 1981).

Probably inspired by his reading of the Euclidean *Sectio canonis*, Boethius clearly expressed the relation between pitch and frequency: 'If the motion is slower and

rarer, the sounds produced must be grave. ... If the motion is faster and denser, the sounds produced must be acute' (Meyer, 2004, p. 35). He illustrated this notion by hand waving (literally) and by vibrating strings, making clear that frequency and speed were correlated (implicitly at constant amplitude). Lastly, Boethius rejected Plato's explanation of consonance and supported one he attributed to Nicomachus (Meyer, 2004, p. 89):

A single impulsion is not enough to produce a simple measure of sound [i.e. a tone]. A string struck once beats the air several times and produces many [successive] sounds. However, the rapidity of this beating is such that one sound overlaps the other, the distance between them is not felt, and a single sound is heard. Consequently, if the beats of a graver sound are commensurable with the beats of a sharper one, in the [Pythagorean] proportions earlier advocated, there is no doubt that this commensuration implies fusion and the consonance of the two sounds.

Boethius here related consonance to the commensurability of implied frequencies, without giving a precise mechanism for the blending of the two periodic series of pulses.²⁷

In the 12th and 13th centuries, a scholastic concept of sound began to compete with Boethius's views as a result of the resurgence of Greek writings and derived Arabic sources. Although Aristotle himself favored the breath concept, his Arabic and late-medieval followers rather emphasized a general feature of Aristotle's theory of perception: that any sensation implies the conveying of a change of state through a medium. This change of state could be interpreted as the imprint of immaterial sonorous qualities by the source on the next element of air, and so forth until the imprint on the last element of air was transferred to the ear. *Multiplicatio specierum* is the name Roger Bacon gave to such replication of forms. This notion, as well as Aristotle's broader concept of a medium, applied both to seeing and hearing. It therefore made light more analogous to sound.²⁸ (Burnett, 1991, pp. 43–70)

On the whole medieval theorists of music, whether they were followers of Boethius or scholastic authors, did not add much to the Greek concepts of sound. The reason is that they did not believe that either the physics of sound or the physiology of hearing mattered much for music. Following Euclid's and Ptolemy's example, they only addressed these questions to the extent needed to prove the quantitative nature of pitch. The bulk of their considerations depended on Pythagorean ratios, justified in a manner similar to that of Euclid's *Sectio canonis*. This means that music had a privileged relation to arithmetic, and nearly none to physics and physiology. From an academic point of view, the joint teaching of music and arithmetic in the *quadrivium* rigidified this state of affairs (Floris Cohen, 1984, chap. 1).

In the late Middle Ages, the arithmetic concept of consonance began to suffer from the growing discrepancy between Pythagorean theory and musical practice. The Pythagorean predilection for the ratios 1:2, 2:3, 3:4 probably resulted from a method of tuning string instruments based on combining fifths and octaves (or fifths and fourths, since a fourth is

an octave minus a fifth), because the twelve notes of the diatonic scale can be generated by a concatenation of fifths ($C \rightarrow G \rightarrow D \rightarrow A \rightarrow E \rightarrow B \rightarrow F^\# \rightarrow \dots$ modulo a variable number of octaves). As twelve fifths slightly exceed seven octaves (by a *Pythagorean comma*), one of the fifths obtained by this method of tuning is necessarily dissonant. The resulting tone (e.g. C–D) has the ratio 8:9, and the third (two tones) has the ratio 64:81, which also yields a dissonance. These dissonances did not affect Greek and medieval monodic musicians, who accepted only fifths and fourths as consonances within an octave and did not transpose or modulate enough to encounter the ‘wolf’ fifth.²⁹

From the 13th century onwards, polyphonic music began to recognize pure major thirds (4:5) and minor thirds (5:6), as well as the pure major sixths (3:5) and minor sixths (5:8) obtained by combining the former intervals with an octave (1:2). This practice ultimately led to the ‘just intonation’ theorized in Gioseffo Zarlino’s *Istitutioni harmoniche*, published in Venezia in 1558. Zarlino’s attunement allows ratios to be built from the numbers 1 to 6—hence the name *senario*. It therefore has more pure consonances than the Pythagorean one in the reference key. It is however impractical for instrumental music, since it does not allow any key change without retuning. Moreover, it is vocally unstable because a cycle of just intervals does not in general bring the singer back to the initial note. For instance, C–G–D–A–E–C has the ratio $(2:3)(4:3)(2:3)(4:3)(5:4) = 80:81$, which may lead to significant key shifts through repetition³⁰ (Wardhaugh, 2006, pp. 32–34).

Around the same time, the growing use of fretted instruments such as lutes and viols favored the equal temperament for which all semi-tones in the 12-note scale are equal. As each of the frets is shared by all the strings of the instrument, all intervals that are consonant for one string correspond to shifted consonant intervals for another string. Equal temperament is the only simple way to place the frets in conformity with this constraint. It implies a (logarithmic) commensurability of the musical intervals with a tolerable departure from the consonance associated with simple ratios. Its popularity grew in the 17th century, when the baroque aesthetics began to require modulations that other temperaments did not permit. In addition to Pythagorean attunement, just intonation, and equal temperament, musicians and instrument makers developed various tuning systems that compromised between the desire to maintain a number of pure or quasi-pure intervals and the possibility of key change. For instance, the ‘meantone’ temperament had pure thirds and slightly tempered fifths. The qualification ‘well-tempered’ usually pointed to one of those systems rather than to strictly equal temperament.³¹

The introduction of new consonances and tempering practices probably favored reflection on the meaning of consonance (Palisca, 1961). As we saw, Ptolemy associated consonance with the harmonious blending of two sounds and dissonance with their being heard separately. Boethius further suggested that the blending had to do with the commensurability of the frequencies of the pulses belonging to the two sounds. Much earlier, the peripatetic author of the *Problemata* (book 11, query 39) had related the

pleasantness of the concord of the sounds made by two strings tuned an octave apart to the coincidence of every other blow of one string with the blows of the other. As this author defined pitch by the velocity of each separate blow, he probably did not mean to define consonance through the coincidence of pulses. Rather, he implied that consonant sounds had a pleasant micro-rhythmic pattern besides their being harmonious. In 1585, the Turin-based mathematician Giovanni Battista Benedetti took the more decisive step of defining consonance as the frequent coincidence of the successive pulses of the two sounds (Benedetti, 1585, p. 283). The sources of his inspiration are obscure. Perhaps he reinterpreted the query of the *Problemata* in light of Euclid's and Boethius's identification of pitch with frequency.³²

The contemporary rise of mechanical thinking favored this coincidence theory of consonance. As the Pythagorean reference to universal harmony failed to provide sufficient guidance in music theory, and as the mechanics of oscillatory motion became better understood, it became natural to seek an explanation of consonance based on the vibratory character of sound. Galileo Galilei, Isaac Beeckman, René Descartes, and Marin Mersenne adopted the coincidence theory in the early seventeenth century. It became the dominant explanation of consonance for about two centuries, although it is not without problems. As Mersenne noted in the 1630s, the resulting order of consonances does not match their empirical degree of sweetness; even worse, it implies the consonance of some patently dissonant intervals: for instance, it would make 7:4 better than a minor third (6:5). As Isaac Newton further noted in 1677, the theory assumes an unwarranted synchronization of the pulses of the two sounds; and the slightest shift of the frequencies would destroy the synchronization after a sufficiently long time.³³

3. *The Many Concepts of Sound in the 17th Century*

Despite its sketchy character, the coincidence theory succeeded in rescuing the theories of music based on simple ratios. It also favored the development of a physics of sound based on periodic vibrations—although one should not infer that the modern understanding of sound as a wave of condensation through an elastic medium immediately followed. To the contrary, the historian Benjamin Wardhaugh has shown the bewildering variety of concepts of sound in the 17th century. The coincidence theory only required that musical sounds (tones) should cause a periodic series of impressions on some sensitive part of the ear. The means by which these impressions occur were left to the imagination of natural philosophers³⁴ (Wardhaugh, 2006).

3.1 *Around 1600*

On the nature of sound, the least committed natural philosopher of the early 17th century was Francis Bacon. In conformity with his inductivist methodology, Bacon produced a

natural history of sound and light as a basis for a true philosophy which he never produced. His main observations were that sound does not imply any sensible motion of the air, that its loudness depends on the strength of the generating percussion, that it can travel along arched lines, that it ‘spreads round (so that there is an orb or spherical area of the sound)’ with different intensity in different directions and with a finite velocity, and that sounding bodies have lasting trepidations (Bacon, 1627, §115). He had no clear understanding of pitch (at some point, he correlated it to the quantity of struck air), and he favored the empiricist, Aristoxenian approach to harmony over Pythagorean arithmetic (Bacon, 1627, §115). Judging that ‘the nature of sounds... is one of the subtlest pieces of nature,’ he contented himself with noting the correlation of sound with observable motion³⁵:

Whatever be this hidden motion which is sound, it does not appear to be generated without a manifest motion in the primary pulsation, and in turn it can be deviated or impeded by a manifest motion of the air. (Bacon, 1688, p. 657)

Bacon accompanied his remarks on sound with a systematic comparison with light. He argued that light and sound shared the following properties: their production does not involve the emission of a corporeal substance; ‘spiritual species’ rather than local motion occur where they pass (Bacon, 1627, §259); they both involve a medium; and they both may occur in harmonious proportions that please the ear or the eye (§111). Bacon perceived the following differences: audible species are more akin to local motion than visual ones (because of the deviation of sound by wind) (§268); light propagates along straight lines, sound along arched lines (§§262, 269); light is much swifter than sound (§§210, 273); light is refracted, sound is not (§254). Altogether, these views sound like the empirical skeleton of Aristotle’s *De sensu*. Bacon ignored Aristotle’s more speculative opinions on the nature of vision, colors, and sounds.³⁶

Some contemporary authors were more willing than Bacon to put forward an opinion on the nature of sound. They usually favored the Aristotelian pestle view according to which sound implies a wholesale motion of the air from the source to the ear. Early in the 17th century, the Padua anatomists Hyeronimus Fabricius and Julius Casserius asserted that such a motion was necessary to carry the audible species to the ear, whatever these species might be. Their explanations of the hearing mechanism did not require a more precise picture of sound. A contemporary British anatomist interested in the same question, Helkiah Crooke, also embraced the peripatetic theory:

The Aer being affected with the quality of the sound driveth and altereth that ayre that is next it, and so by succession till the alterations come to the Ayre that is next to the outward Eare. (Crooke, 1615, p. 610)

Like Theophrastus and some scholastic authors, Crooke believed that the ‘quality of the sound’ was not the motion itself. His theory of hearing had the eardrum filter this quality from the carrying motion (Wardhaugh, 2006, pp. 156–159).

These anatomists all mentioned the stoic analogy of the stone thrown into water, although they did not take it to imply anything precise on the process of propagation. Other authors emphasized the undulation of the air implied by this analogy. For instance, in 1604 the natural philosopher Thomas Wright defined ‘the best Philosophie’ of sound as follows:

The very sound it selfe. . . is nothing else but a certaine artificiall shaking, crispling [undulating, from the Latin *crispo*] or tickling of the ayre (like as we see in the water crispled, when it is calme, and a sweet gale of wind ruffleth it a little; or when we cast a stone into a calme water, we may perceive divers warbling naturall circles) which passeth thorow the eares, and by them unto the heart, and there beateth and tickleth it in such sort, as it is moved with semblable passions. (Wright, 1604, p. 170)

Wright’s pre-Galenian choice of the heart as the seat of consciousness probably betrays his reliance on peripatetic or stoic sources (Wardhaugh, 2006, p. 161).

3.2 Mersenne

In his highly influential *Harmonie Universelle* of 1636, Marin Mersenne generalized the comparison between sound and water waves by admitting spherical disturbances of the water:

It seems that we cannot better explain or understand the manner in which the air is moved, when it sounds, than through the manner in which water is moved by bodies that move in it and beat it with violence: because we must not only imagine the motion that we see on the water, when it makes ever growing circles from the place where the stone has been thrown, which acts as a center, up to the sides of the vessel that contains it: but we must also notice the similar motions it makes all the way to the bottom. (Mersenne, 1636–1637, vol. 1, p. 9)

Mersenne also used this analogy to refute the common objection ‘that two men could not hear the words they would say at the same time because the air cannot receive two contrary motions at the same time’:

When two or several men speak at the same time, the air takes the impressions which it receives from each of them, as calm water receives those of the stones thrown into it, because one can see that the stones make different circles which expand little by little until they reach the banks, and which yet are not as distinct nor as remarkable as when a single stone is thrown. (Mersenne, 1636–1637, vol. 1, p. 4)

At a more fundamental level, Mersenne argued that sound was motion only and did not involve the intentional species of some scholastic philosophers (as in Crooke’s case):

We must still conclude that intentional species are not required for Sound, since the motion of the air suffices. . . . This is why I shall not speak of these images or intentional species of Sounds, but only of the motion through which we grasp them: which will bring greater clarity and facility to our discourses and can be a greater satisfaction for the Reader. (Mersenne, 1636–1637, vol. 1, p. 4)

Mersenne mentioned the possibility that the propagation of sound would imply the compression of the air. However, he believed the air not to be porous enough for this purpose, and he favored the pestle view:

It is tempting to say that the other parties of the air are condensed to leave room for the impetuosity of the agitated part, although it is nearly impossible to imagine how the compression and condensation of the parts of the air could occur unless it contains vacuum. The difficulty will be eased if one does not toy with vacuum, or with rarefaction and condensation: because it can be said that when one part of the air has been struck, all the neighboring parts immediately succeed into its place, and that the whole mass of air moves when one of its parts changes place, as happens in bathtubs when the whole water moves at each movement of our body. (Mersenne, 1636–1637, vol. 1, p. 10)

Mersenne conceded that the finite velocity of sound, of which he performed pioneering measurements, failed to be explained in this picture. But he was much more interested in the musical aspects of sound. In this context, he vigorously asserted the correspondence between pitch and frequency, which was not yet universally recognized despite its ancient origins. He performed numerous experiments on vibrating strings, including an absolute frequency measurement with long strings, and he developed the coincidence theory of consonance with just criticism (Mersenne, 1636–1637, vol. 2).³⁷

3.3 *Descartes*

Mersenne's problems with compressibility may have had to do with René Descartes's philosophy of matter. While he was working on his monumental treatise, Mersenne repeatedly questioned his brilliant friend on the nature of sound. Descartes's authority in this domain derived from the reputation of his *Compendium musicae*, written in 1619 (but published only in 1650). On 16 December 1629, he wrote to Mersenne: 'Sounds. . . certainly are, as you say, a beating by to and fro motion, without the sound of a musket's bullet to present any difficulty' (Tannery, 1908, p.103). Mersenne had probably asked about the latter difficulty. According to Descartes's reply, the air of wind instruments, like the bullet, moves straight forward 'and yet causes an undulation of the air that strikes the ear, just as a stone entering water makes several successive circles despite the straightness of its descent' (Tannery, 1908, p. 103). In a letter written around October 1631, Descartes emphasized periodicity and its importance for the coincidence theory of consonance:

Sound is nothing but a certain vibration of the air that comes to tickle our ears, and. . . the cycles of this vibration are the prompter as the sound is sharper. . . . When two sounds strike the ear at the same time, they are the more consonant as their vibrations overlap more often with each other. (Tannery, 1908, p. 223)

The margin of this letter has an interesting note: ‘I have here overused the word vibration [*tremblement*] by which I mean each of the small jolts by which the vibrating body moves.’ Another letter of the following year (3 May 1632, in Tannery, 1908, pp. 244–248) makes clear that Descartes had in mind a periodic succession of pulses individually propagated by a wholesale displacement of the air. When Mersenne asked him why sound propagates faster in a wood beam than in air, Descartes used the picture of a spongy air in which the hard balls (of his third element) did not originally touch each other. In the wood beam, the contiguity of the balls permits instantaneous transmission of pressure, whereas in air some time is needed to compact the sponge before instantaneous transmission. In reply to another question by Mersenne, Descartes used this picture to explain why the speed of sound was much faster than that of air currents: the air from the source only has to move a small fraction of the distance to the ear in order to compact the air in the remaining space. This kind of reasoning did not involve the elasticity of the air, and implied a pushing of the air all the way from the source to the ear. Descartes did not develop this intuition in his writings on music or philosophy. There he simply restated that sound was a vibration or a periodic shaking of the air, that pitch corresponded to frequency, and consonance to the frequent coincidence of pulses (Descartes, 1662 [1633], pp. 413–414; Descartes, 1664 [1633], p. 313). His theory of music, mainly a justification of Zarlino’s just intonation, barely depended on these remarks.³⁸

3.4 Galileo

In the *Discorsi* of 1638, Galileo Galilei forcefully supported the interpretation of pitch as frequency. His father Vincenzo was a musician who had investigated the relation between the pitch and the physical build up of instruments in order to show the arbitrariness of Pythagorean ratios in music. In particular, Vincenzo had shown that the pitch of a string varied as the square root of its tension and not linearly as the Pythagorean legend had it. He used this result to argue that there was no more reason to represent a fifth by the ratio 2:3 than by the squared ratio 4:9. His son Galileo removed the ambiguity by arguing that frequency was the true parameter of pitch:

I say that the length of strings is not the direct and immediate reason behind the forms [ratios] of musical intervals, nor is their tension, nor their thickness, but rather, the ratio of the numbers of vibrations and impacts of air waves that go to strike our eardrum, which likewise vibrates according to the same measure of times. (Galilei, 1638, p. 104)

Galileo went on to explain how this definition of pitch allowed the explanation of consonance by the frequent coincidence of the impacts delivered to the eardrum. He also described two dubious but suggestive experiments: one about the ripples of the water surface in a sounding goblet, the other about the periodic stripes left by a sharp chisel quickly drawn on a metal plate to make a squeaking sound. Beyond his vague use of the wave metaphor, Galileo said little on sound propagation. His comparison between the mutual resonance of two musical strings and the excitation of a pendulum by puffs of air at the frequency of the pendulum suggests that he adhered to a variant of the breath theory.³⁹

Descartes's and Galileo's emphasis on pulses or impacts on the eardrum rather than continuous oscillation can be seen as a natural consequence of the coincidence theory, which favors the image of a sound as a discrete series of events. The link is explicit in Robert Hooke's experiments on toothed wheels in the 1670s and 1680s. Hooke showed that tones could be generated by having the successive teeth of the wheel hit a fixed obstacle and that the sounds of two wheels rotating at the same speed were consonant when the numbers of teeth on the wheels were in a simple ratio (Birch, 1756–1757, vol. 4, p. 96). In a diary entry of 15 January 1676, he insisted that the sound of vibrating bodies was the series of strokes caused by the vibrations, not the vibrations themselves:

To Sir Chr. Wrens, Dr. Holder and I discoursd of musick. . . . I told him but *sub sigillo* my notion of sound, that it was nothing but strokes within a determinate degree of velocity. I told them how I would make all tunes by strokes of a hammer. Shewd them a knife, a camlet coat, a silk lining. Told them that there was no vibration in a puls of sound, that twas a puls propagated forward, that the sound in all bodys was the striking of the parts one against the other and not the vibration of the whole. . . . Compared sound and light and shewed how light produced colours in the same way by confounding pulses. Eat cake and cheese, bread, ale and claret. (Hooke, 1935, p. 211)

We will return to the analogy that Hooke perceived between sound and light, which has little to do with his habit of recording the composition of every of his meals. As is well known, Hooke was extremely fond of mechanical devices and espoused a mechanical philosophy of nature. He had a special interest in vibrations, springs, and music. He often relied on musical metaphors to explain the vibrational processes he judged essential to most physical phenomena.⁴⁰

3.5 Other Views

Whereas the authors named so far all remained broadly within the breath tradition, there still were adepts of the scholastic concept of sound as a multiplied species. For example, the renowned Flemish physician Jan Baptist van Helmont claimed that sound had nothing to do with aerial motion:

What is sound? It is a *quality* generated in air by the collision of two bodies; this quality spreads out even in immobile air, in its sphere of activity. . . . Truly, sound propagates not *with* air but *in*

calm air. The similitude [with water-waves] is only in the mode of propagation, not with respect to material extension. (Helmont to Mersenne, 15 Jan. and 6 Feb. 1631, in Tannery, 1908)

Others tried to revive Greek atomism and its application to sound. The first to do so was Descartes's mentor, the Dutch natural philosopher Isaac Beeckman, who followed Epicurus in identifying sound with tiny particles of air sliced off by the sounding body and traveling to the ear (Floris Cohen, 1984, p. 122):

The very same air that is directly touched and affected by a hard thing is violently shocked and dispersed, and scattered particle-wise everywhere, so that the air itself that had received the impulse strikes our ear, in the way a candle flame spreads itself through space and is called *light*.

Descartes seems to have adhered to this view when he presented Beeckman with a copy of his *Compendium* (Wardhaugh, 2006, p. 162). In mid-17th-century France, Descartes's opponent Pierre Gassendi similarly asserted that 'sound was nothing but corpuscles which, configured in a certain manner, transferred very quickly from the sounding object, penetrate the ear and make the instrument move in this manner, thus causing the sensation called hearing' (Gassendi, 1658, p. 414). The needed corpuscles belonged to a subtle component of the air. The frequency of their emission, not their speed nor their shape, determined the sharpness of a sound. Consonance depended on 'the variety of the agreements or disagreements of their strokes.' Gassendi thus integrated the coincidence theory in his doctrine. In 1654, his English follower Walter Charleton defined sound in a similar manner:

A Sound seems to be nought but the Aer, at least the subtler or more aethereal part of aer, extrite and formed into many small (Moleculae) masses, or innumerable minute Contextures, exactly consimilar in Figure, and capable of affecting the Organ of Hearing in one and the same manner: which configurated small masses of aer fly off from bodies compulsed or knockt each against other, with some violence; and progress by Diffusion in round. (Charleton, 1654, p. 212)

Like Gassendi, Charleton integrated the frequency interpretation of pitch and the coincidence theory of consonance in the atomist view. In 1672, the English physician and natural philosopher Thomas Willis still held a related view. He assumed that the sounding body set the 'sonorifick particles' of the air into motion and that this motion was propagated from one particle to the next. Unlike earlier atomists, he did not necessarily assume this motion to be translational: it could also be a 'contorsion' or a 'gyration' (Wardhaugh, 2006, pp. 166–167).⁴¹

Altogether, the neo-atomist concept of sound differed little from the breath concept. They both involved a transfer of aerial matter from the sonorous body to the ear, and they both existed in two guises: the missile concept (ballistic transfer) and the pestle concept (mediated transfer). In the pestle version, they both accommodated the stoic metaphor of waves, as waves were usually understood as involving a transfer of matter. The only difference was the continuity/discontinuity of the transferred matter. In Greek antiquity,

there was an additional difference: the atomists explained the qualities of sounds through the shape of the sonorific atoms, whereas most breath theorists reduced these qualities to the properties of the breath-like motion (velocity, suddenness, frequency, etc.). In the seventeenth century, this difference disappeared, since the neo-atomist philosophers adopted the frequency interpretation of pitch.

3.6 *The Spring of Air*

In 1660, Robert Boyle published an influential series of pneumatic experiments performed by means of Hooke's improvement of Otto von Guericke's air pump. In one of these trials, he observed that the ticking of a pocket watch ceased to be heard when the air was evacuated around it. Boyle prudently concluded that this experiment 'seem[ed] to prove, that whether or no the Air be the onely, it is at least, the principal medium of Sounds' (Boyle, 1660, pp. 208–209). Although Boyle's other experiments taught him much about the 'spring of the air' (which we now regard as the key to sound propagation), he refrained from any precise theory of sound propagation. He vaguely asserted that sound is 'either a certain undulating motion of the Aire, or, at least, is not produced nor does act independently from the motion of the Aire, or the Aire moveing after such a determinate manner' (Boyle, 1681, p. 44). And he made the manner of motion of the air unique cause of perceived sounds⁴²:

According as the strings, or other Instruments of producing sounds, doe tremble more or lesse swiftly, they put the Aire into a Vibrating motion more or less brisk, and produce those diversities of Sounds. . . . And though the Bodies from whence these sounds proceed may be of very differing Natures. . . yet provided they put the aire into the like waveing motions, the Sound and even the Note will be the same. (Boyle, 1676, p. 27)

Boyle's avoidance of a precise theory of sound propagation was part of his Baconian attitude. Similarly, he left the deeper nature of the experimental vacuum open, and he refused to commit himself on the deeper nature of light. In an unpublished manuscript on the mechanical production of light, he used the analogy between light and sound to illustrate the mechanical nature of light without deciding between the two leading mechanical theories of light, the Cartesian theory and the neo-atomist theory:

Just as some argue for the existence of very many aethereal or other almost inconceivably minute particles, whether constantly nestling among the pores or other larger interstices in the air and other transparent bodies (as the Cartesians have it), or abundantly pouring forth and spreading out like effluvia from the luminous bodies, which particles are especially apt to become luminous in the same way that aerial corpuscles are to becoming sonorous, so, too, certain other analogies between light and sound may therefore incline us to believe that the luciferous motion (if we may so call it) which excites this predisposed matter into actual light greatly resembles the undulating motion of the air when producing sound, save that it is faster and more vigorous. (Boyle, 2000[c. 1660?], vol. 14, pp. 7–8)

Both for light and sound, Boyle imagined some undulation but could not decide whether the relevant waves consisted in a modulated flux of particles or in periodic impulses through a medium.⁴³

While Boyle and several British authors relied on a vague wave metaphor for the propagation of sound (and light), others preferred the Cartesian idea of impulses transmitted through neighboring hard particles. In 1680, the French polymath and Academician Claude Perrault imagined that the grosser particles of the air promptly transmitted the small disturbance caused by the sounding body, as a contiguous alignment of hard balls transmits any small motion of the first ball to the last ball (Perrault, 1680, pp. 12–18).⁴⁴

Perrault nonetheless acknowledged the spring of the air and even sketched a wave theory of propagation in which the compression of one part of the air would imply compression and motion of the next part:

In order to promote this comparison [between water waves and sound], one could say that the Elastic virtue of the air, according to which it can be compressed and then return to its first state as a spring would do, enables it to do something similar to the undulation of the water, when, being compressed by the impulse [from the sounding body], this Elastic virtue makes it not only return to its first state but also go further: because this can cause a reciprocation such that an impulse whose immediate action occurs at one place only is passed from this place to another and thus travels very far. (Perrault, 1680, pp. 14–15)

Perrault ultimately rejected this theory because he believed it would contradict the fact that a single stroke of a body on another produces a single sound. The analogy with the stone thrown into water indeed suggests that a single stroke causes much more than one wave. Perrault avoided this possible difficulty by imagining a variation on the pestle theory in which the sound-transmitting particles of air all moved in a roughly equal manner at each impulse of the source on a line separating the source from the ear. He explicitly referred to Aristotle for the idea of a coherent mass of air between the sonorous body and the ear. He argued, like Descartes, that the finite speed of propagation resulted from the time lost in compressing the row of particles on the transmission line. As he also believed this speed to depend on the velocity of the particles' motion, he assumed the latter velocity to be the same for any sound and made the intensity of a sound depend on the number of moving particles.⁴⁵ (Perrault, 1680, pp. 23–25).

Having thus reduced sound to quick invisible shakes of the grosser particles of the air, Perrault needed to explain why the vibrations of sounding bodies were usually visible and involved velocities much smaller than the speed of sound. For this purpose, he assumed that the visible vibrations were not the direct cause of the aerial shakes and imagined that they caused a cascade of smaller-scale 'partial' vibrations, the smallest and fastest of which were able to excite sound (see Figure 1). As the frequency of these ultimate vibrations was proportional to the macroscopically observed frequency, the frequency interpretation of pitch and the coincidence theory of consonance remained



Fig. 1. Perrault's drawing of a vibrating string. From Perrault (1680, p.129). Besides the main global oscillation, Perrault imagined smaller vibrations that were the ones responsible for sound.

valid. In support of this view, Perrault evoked the hearing of harmonics and referred to the fact that tiny, inaudible vibrations of a string were made audible by touching the string with a finger-nail (Perrault, 1680, pp. 53–66). Perrault was not isolated in this speculation: Hooke similarly divorced the sound-causing vibrations of the particles of a body from its global vibrations:

Told them my experiment of the vibrations of a magicall string without sound by symphony [resonance] that touching of it which made the internall parts vibrate—caused the sound, that the vibrations of a string were not Isocrone but that the vibration of the particals was. (Hooke, 1935 [1676], p. 211)

Hooke probably meant that the global vibrations excited by resonance did not imply the smaller-scale vibrations necessary for the production of sound, unless the string was touched (by a hard object). As he believed that the frequency of the global vibrations of a string depended on the amplitude (lack of isochrony), he explained the constancy of the rendered pitch (during the decay of the sound of a plucked spring) by the isochrony of the much smaller vibrations of the particles of the string.⁴⁶

The first cogent theory of sound propagation based on Boyle's spring of air was the one Isaac Newton included in his *Principia* (1687, book 2, chap. 8). The novelty of this theory not only resided in the clear exploitation of the spring of air but also in the pioneering reliance on a fully geometrized dynamics. Before Newton, mathematics had hardly been used in acoustic considerations, except for the arithmetic theory of consonance or for the ray propagation of sound. Nearly as a modern physicist would do,

Newton considered the compression of successive slices of the air owing to the phase difference in the oscillation of the delimiting planes, and equated the resulting pressure gradient with the acceleration of the planes multiplied by the density. Unlike most of the earlier theories, Newton's did not imply a wholesale motion of the air from the source to the ear; it only involved oscillations of the particles of the air around their equilibrium. Newton was quite explicit on this point:

Since sounds are not propagated in a moment, the pulses must not be supposed to extend each of them at once from the sounding body to the ear in such manner that all the air in that interval be moved together first forward and then backward, and so forward and backward again so long as the sound lasts. But the pulses are rather to be conceived like so many spherical concentrick waves whose center is the sounding body and which arising continually from that center dilate an flow on from thence with that swiftness we find sound propagated till they arrive at the ear. (Newton to Roger North, 21 Apr. 1677, in Kassler, 2004, pp. 176–177)

Newton's theory was the first to relate the velocity of sound to other measurable properties of the air, as it made this velocity the square root of the ratio of its elasticity to its density, a formula upon which no one was able to improve before Laplace in the early nineteenth century. Toward the end of the 17th century, popularized forms of this theory tended to replace the old breath theory and the neo-atomist theory.⁴⁷

This late-century homogenization of the philosophy of sound was imperfect. Heterodox approaches can be found in a period extending as late as the close of the eighteenth century. Most importantly for our immediate purpose, it remains true that for most of the 17th century there was a great diversity of opinions on the nature of sound. Doubts and disagreements existed as to whether motion was involved, whether a medium was required, as to the nature of such a medium, the processes occurring in the medium, and the kind of motion involved in sounding bodies. The only consensual aspects were the relation between pitch and frequency and the coincidence theory of consonance. There was no well-established theory of sound that could serve as a model for other propagation phenomena.

[This article will be continued in the next issue of *Centaurus*].

Acknowledgments

I would like to thank Cathryn Carson for her hospitality at UC-Berkeley's Office for the History of Science and Technology, and the anonymous reviewers for helpful suggestions.

NOTES

1. Optical analogies may also have played a role in the history of mechanics: cf. Smith, 2008.
2. My use of 'acoustics' is partly anachronistic: this word was almost never used to name the physics of sound until Joseph Sauveur pointedly did so (defining *l'acoustique*) at the turn of the 17th and 18th centuries.

3. One apparent inconsistency is that Hobbes equates the index of refraction of light with a ratio of velocities, whereas he assumes the instantaneous propagation of light. Cf. Shapiro, 1973, note 56.
4. For instance, the formulation of mechanics evolved when it was used as an analogical basis for electrodynamics and thermodynamics.
5. Greek and medieval authors usually regarded the visual sense as superior to hearing (Aristotle only disagreed to the extent that he regarded speech as essential to the development of thought). In the early modern period, Francis Bacon departed from this tradition by regarding music as a more direct communication of souls. Cf. Frangenberg, 1991, pp. 71–74; Smith, 1999, p. 103.
6. For global studies of Greek theories of perception, cf. Beare, 1906; Sambursky, 1960. Asian sources need not be discussed here, since they were ignored in the West. The Mohist optics of ancient China knew ray propagation, reflection, refraction, and the pin-hole camera. It seems to have been mostly phenomenological and devoid of the Greek notion of visual ray: cf. Needham, 1954–2004, vol. 4:1, pp. 85–86.
7. Cf. Lindberg, 1976, pp. 2–3; Simon, 1988, pp. 36–38; Taylor, 1999. Epicurus's brief remarks on seeing and hearing in his surviving letter to Herodotus are compatible with Lucretius's more detailed account.
8. In his letter to Herodotus, Epicurus indicated a mutual interaction of the successive particles of the sound's current rather than an independent flight of these particles: 'Again, hearing takes place when a current passes from the object, whether person or thing, which emits voice or sound or noise, or produces the sensation of hearing in any way whatever. This current is broken up into homogeneous particles, which at the same time preserve a certain mutual connection and a distinctive unity extending to the object which emitted them, and thus, for the most part, cause the perception in that case or, if not, merely indicate the presence of the external object. For without the transmission from the object of a certain interconnection of the parts no such sensation could arise. Therefore we must not suppose that the air itself is molded into shape by the voice emitted or something similar; for it is very far from being the case that the air is acted upon by it in this way. The blow which is struck in us when we utter a sound causes such a displacement of the particles as serves to produce a current resembling breath, and this displacement gives rise to the sensation of hearing.' Cited in Hicks (1910, p. 238).
9. Cf. Lindberg, 1976, pp. 3–6; Simon, 1988, pp. 29–30. On the popular view as expressed in Homer and other Greek (pre-philosophical) poets, cf. Mugler, 1964, pp. 7–13 (Introduction), ἀχτις and ὄψις entries.
10. On Archytas's views, cf. Porphyry's citation in Cohen and Drabkin (1948, pp. 286–288), or in Barker (1989, vol. 2, pp. 39–42). On Greek interpretations of pitch, cf. Barker, 1989, 'Introduction' (pp. 1–27), pp. 9–10; Barker, 2002). Archytas did not clearly distinguish between pitch and volume, as a probable consequence of the ambiguous metaphors used to denote pitch in ancient Greek.
11. In Ross (1908–1952, vol. 3). Cf. Lindberg, 1976, pp. 6–9; Simon, 1988, pp. 42–45.
12. In Ross (1908–1952, vol. 3). Aristotle introduced the ether as a fifth element in which natural motion was circular and unimpeded, as was required for celestial bodies.
13. In Ross (1908–1952, vols. 3, p. 6).
14. In Barker (1989, vol. 2, pp. 77–78) (Aristotle here explains hearing by the trapping of the sonorous breath in the cavity of the ear), pp. 86–87. See also Pseudo-Aristotle, *De audibilibus*, 800a–804b, in Barker (1989, vol. 2, p. 99): 'It is a fact that all voices and sounds occur when either bodies collide with bodies or the air collides with bodies, not by the air being shaped, as some people think, but by its being moved in just the same way... as the result of impacts made by the breath or the strings. For when the breath that impinges on the air strikes the air next to it, the air is at once forcibly moved, pushing forward in the same way the air continuous with it, so that the sound is stretched out and remains the same throughout, to the limit of the distance to which the movement of the air goes on.'

15. In both cases, the effective aerial motions are directed toward the ear. Consequently, only the properly directed half of the strokes of a vibrating string contributes to the heard sound.
16. In Barker (1989, vol. 2, pp. 76–77, 81–82, 116, 119–125).
17. In Ross (1908–1952, vol. 3).
18. In Ross (1908–1952, vol. 3). Aristotle's discussion of hearing may have served as a paradigm for various aspects of his general treatment of perception: cf. Towey, 1991, pp. 7–18. On Aristotle's understanding of mirrors, cf. Simon, 1988, p. 47.
19. In Ross (1908–1952, vol. 6).
20. Cf. Lindberg, 1976, pp. 9–10; Simon, 1988, pp. 31–32; Hülser, 1987–1988. In stoic physics, the *pneuma* was held responsible for the coherence of material objects, plants, animals and even of the whole universe.
21. Cf. Lejeune, 1948; Lindberg, 1976, pp. 12–14; Simon, 1988, pp. 63–72
22. The *kanôn* is a ruler placed under a monochord, or the monochord itself by metonymy. I have slightly modified Euclid's reasoning without altering its spirit. The authenticity of (some parts of) the *Sectio canonis* has been a matter of discussion: cf. Barbera, 1991; Barker, 2007, chap. 14.
23. Cf. Lejeune, 1948; Lindberg, 1976, pp. 15–17; Simon, 1988, pp. 83–91; Mark Smith, 1996.
24. Ptolemy's hesitations on pitch may have resulted from his depending on the relevant semantic network in old Greek: cf. Barker, 2002.
25. Simon (1988, chap. 1), rightly insists that light is not a protagonist in Greek theories of vision.
26. On medieval optic, cf. Lindberg, 1976, chap. 5. On Arabic optics, cf. Lindberg, 1976, chaps. 2 and 4; Rashed, 1992, 1997.
27. Nicomachus (ca. 60–120 A.D.), in Levin (1994, p. 173), followed Ptolemy in asserting the blending of consonant sounds: 'Systems are consonant when the notes comprising them, though they be different in compass, commingle with one another when played together or are somehow sounded simultaneously, in such a way that the sound produced from them is of a oneness like a single voice. Notes are dissonant, however, when the sound emanating from both of them is heard to be disparate in some way an unblended.' Nicomachus accepted the breath concept: 'We say that sound in general is a percussion of air that is unbroken in its progress to the ear,' and he associated pitch with the velocity of the blows (Levin, 1994, p. 61). Boethius's discussion of consonance seems closer to that found in the peripatetic *De audibilibus*, 804a (Ross, 1908–1952, vol. 6) which also refers to the inseparability of successive blows (for one note) and to the synchronicity of blows (for two notes).
28. On Roger Bacon's *multiplicatio specierum*, cf. the extracts of his *Opus Majus* in Grant (1974, pp. 394–399, esp. p. 394): 'But a species is not body, not is it moved as a whole from one place to another; but that which is produced [by an object] in the first part of the air is not separated from that part, since form cannot be separated from the matter in which it is unless it should be mind; rather it produces a likeness to itself in the second part of the air, and so on. Therefore there is no change of place, but a generation multiplied through the different parts of the medium.' *Multiplicatio specierum* is usually translated as 'multiplication of species,' although 'species' should here be understood as 'form.'
29. Cf. Barbour, 1951, 2004; Floris Cohen, 1984, pp. 34–45; Lehman, 2005; Wardhaugh, 2006, chap. 1. For a clear and concise exposition of the theory of tuning and temperament, cf. Berg and Stork, 2005, chap. 9.
30. Such difficulties prompted some authors (including Vincenzo Galilei, Francis Bacon, and Simon Stevin) to reject the arithmetic approach altogether and to return to Aristoxenian notions.
31. Cf. Floris Cohen, 1984, p. 111; Wardhaugh, 2006, pp. 34–39. Adoption of equal temperament (or any other temperament) did not at all imply the rejection of simple ratios in the theory of harmony, since this temperament was most commonly regarded as an approximation of just intonation: see, e.g. Rameau, 1737).

32. Cf. Palisca, 1961, pp. 104–110; Floris Cohen, 1984, pp. 75–78. Benedetti reasoned on the ‘percussions’ or the ‘waves of air’ corresponding to two complementary portions of the string of a monochord. His purpose was to justify Zarlino’s consonant ratios.
33. Cf. Floris Cohen, 1984, chaps. 3–4; Dostrovsky, 1975, pp. 174–183. Newton’s objection is in his letter to John North of 21 Apr 1677 about his brother’s Francis North *A philosophical essay on musick* (London, 1677), in Turnbull *et al.* (1959–1977, vol. 2, pp. 205–207). Cf. Kassler, 2004, pp. 175–178.
34. See also (Crombie, 1990). For a competent overview of the history of acoustics, cf. Dostrovsky *et al.*, 2001.
35. Baconian empiricism shaped much of British 17th-century acoustics: cf. Gouk, 1982. Bacon’s followers, however, did not imitate his rejection of Pythagorean ratios.
36. See also (Bacon, 1653).
37. Cf. Ludwig, 1935; Truesdell, 1960, 29–33; Dostrovsky, 1975, pp. 185–188, 197–199; Wardhaugh, 2006, pp. 57–61, 221. On Mersenne’s measurements of the speed of sound, cf. Hunt, 1978, pp. 85–87, 95–100.
38. On Descartes’s musical theory, cf. Floris Cohen, 1984, pp. 161–175. Descartes was aware of the elasticity of the air (see the fourth part of his *Principia*), even though he ignored it in his explanation of the propagation of sound.
39. Cf. Dostrovsky, 1975, pp. 178–182. On the metal-plate experiment, cf. Baskevitch, 2007.
40. Cf. Gouk, 1980, 1999, chap. 6; Wardhaugh, 2006, pp. 235–240. In his young age, Hooke was a chorister and played the organ at Christ Church in Oxford: cf. Westfall, 1972, pp. 481–488; Gouk, 1980, p. 575. Hooke’s MS on the causes of effect of music, reproduced in Gouk (1980, pp. 597–605), included a vague and fairly standard definition of sound: ‘Sound being nothing els but a tremulous motion of the drum & organ of the ear, excited by the like motion of the sonorous medium, w^{ch} received its motion from the Sounding Body’ (Gouk, 1980, p. 601).
41. On Gassendi, cf. Brett, 1908, pp. 74–78. On Beekman, cf. Floris Cohen, 1984, pp. 116–161 (on Beekman).
42. Cf. Wardhaugh, 2006, pp. 172–174. Athanasius Kircher (1650, vol. 1, pp. 11–13) reported that a bell could be heard ringing through Torricelli’s alleged vacuum, and used this fact as a refutation of the vacuum. Boyle failed to observe any appreciable decrease of the bell’s sound in his version of this experiment. He suggested that this failure implied either the intervention of a subtler air or (rather) the residual air allowed by the imperfections of his pump and receiver. Francis Hauksbee managed to suppress the bell’s sound in improved experiments published in 1705. Cf. Hunt, 1978, pp. 112–121, for a discussion of 17th-century experiments about sound and vacuum by Sagredo, Kircher, Guericke, Boyle, and Hauksbee.
43. The citation is from the undated MS (probably anterior to Hooke’s *Micrographia* of 1665) ‘Commentarii experimentales de mechanica productione lucis,’ transl. in Boyle (2000, vol. 14, pp. 5–54). A similar vagueness is found in Boyle (1685, pp. 55–68), although Boyle compares the aerial undulation to ‘so many little swimming hammers and flying bullets’ (Boyle 1685, p. 68).
44. Cf. Wardhaugh, 2006, pp. 168–171. This Perrault was the brother of Charles, the author of *Cendrillon*. Originally a physician, he translated Vitruvius, and designed the neoclassical colonnade of the Louvre. His variety of Cartesianism implied the explanation of cohesion by compression through subtle matter. Cf. Picon, 1988, pp. 75, 90.
45. Not every Cartesian adopted the pestle concept. In his widely read treatise (Rohault, 1676, p. 259), Jacques Rohault imagined a trembling and boiling motion of the air, with intimate division of its parts, and compared the emission of sound with the turbulence generated by the shaking of a stick in a bowl of water (something analogous to the modern understanding of diffusion processes).
46. Fontenelle approved Perrault in *Histoire de l’Académie Royale des Sciences*, vol. 1 (1677), pp. 223–229. The strange similarity between some of Hooke’s and Perrault’s ideas on sound perhaps had to do with their both being architect.

47. Cf. Dostrovsky, 1975, pp. 211–218; Truesdell, 1955, pp. XIX–LXXII, XXXII–XXXIII. In the cited letter, Newton blamed Francis North for implicitly assuming the pestle view when he argued that resonance between two strings implied the synchrony of their pulses.

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