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A mistake and its meaning

BY TITO M. TONIETTI†

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1. Introduction

Historical studies devoted to Isaac Newton and editions of his works are numerous – recently even an edition of his comments on the Apocalypse has appeared (Newton 1994). It is not my purpose here to review details of this large literature, but rather to examine one of Newton's writings that I believe has had inadequate attention and understanding.

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2. The text at issue

Between the first edition of the *Philosophiæ Naturalis Principia Mathematica*, published in 1687, and the second, which appeared in 1713 (Newton 1972), Newton wrote a series of comments on the text, the so-called classical *scholia*, which are undated but were probably written in the 1690s. In the end, these comments were not included in the 1713 edition, being replaced by the famous general *scholium* that closes the book. In recent years Paolo Casini published them in the original Latin (Casini 1981; Casini 1984).

Propositio VIII in the Third Book of the *Principia* enunciated the well-known law according to which the gravitational attraction of matter decreases with the inverse of the second power of the distance. In the appendix to the present paper I give my English version of the *scholium* related to that proposition. I suggest reading this translation – perhaps a case of *traduttore traditore* – for aid in following the argument and as showing my interpretation of the *scholium*.

In the *scholium* Newton, in trying to support his law of gravitation, referred it to the Pythagorean tradition. These ancients, he wrote, had already discovered the law but had concealed it within another theory of theirs – in a discovery they had made concerning music.

Several other scholars (on whose works I will comment later) have already considered the same *scholium* or the musical papers of Newton, even in connection with the more general question of the relationship between music and science (McGuire Rattansi 1966) (Dostrovsky 1974/75) (Jeans 1980) (Gouk 1986). However, as I believe these scholars have not fully understood Newton's argument, I repeat it in full.

Newton, in order to make the case that the ancients “*adumbrassent*” (had foreshadowed) his law of gravitation, built a musical model of attraction. He based this model on the relation of the inverse proportion between the pitch p of the sound and the length l of the string. The heirs of the symbolism introduced by Descartes and Leibniz would write: $p \propto \frac{1}{l}$. Newton used also the direct relation between the pitch of the sound and the square root of the weight (tension) w : $p \propto \sqrt{w}$ – even though this latter relation might be ascribed to the tradition of the Pythagorean sects only with much difficulty (Casini 1981, 1984, footnote 39). Consequently, to get the high octave of a tone, that is the double frequency, Newton would either halve the length of

the sounding string, leaving it with the same weight, or stretch it with a quadruple weight, leaving it at the same length:

$$2p \propto \frac{1}{\frac{l}{2}} ; 2p \propto \sqrt{4w}.$$

But when he specified the relation that would provide unison, he wrote that the weight should be “inversely proportional” to the square of the length. That was an error, because to counterbalance the lowering of the pitch with the lengthening of the string, the weight would need to increase with the square of the length:

$$p \propto \frac{\sqrt{4w}}{2l} \propto \frac{\sqrt{9w}}{3l} \dots$$

What was difficult to capture in Latin terms would have come more easily with the symbolism we are now accustomed to.

What I suppose to be a mistake on Newton's part hinges on the term “*reciproce*” appearing on sheet 12r of the Latin manuscript on which Casini's edition is based. Could this critical word be missing in the original? I have checked a photocopy of the Newton's original manuscript now at the Royal Society of London: Mss Gregory 247, 6–14¹. The word “*reciproce*” is evident on the manuscript.

It is also possible that “*reciproce*” might have a different meaning from the one assumed in my translation. For instance, it might be meant to convey dependence between weights and square lengths as well as “reciprocally” between lengths and weights, without specifying that it would be an inverse relation. If that were the sense of the text, Newton would have given the rule to make the conditions of unison coherent with his knowledge of acoustics. In that case, however, he would not have been able to obtain the same proportion stated for gravitation. So his musical model would not have justified his contention that the ancients had “foreshadowed” his own law of attractive relation between the sun and its planets.

Another way of assessing what Newton meant by that “*reciproce*” is to look at other passages where the term appears. For instance, in *Propositio VIII* itself, Newton wrote: “*reciproce ut quadratum distantiae inter centra*”

(Newton 1972, p. 577). In the *scholium* he wrote: “*reciproce ut quadrata longitudinum chordarum*” (Casini 1981, p. 41; Casini 1984, p. 32). The first “*reciproce*” certainly meant the inverse of the square of the distance; might the second mean anything else? I believe it improbable. Moreover, the mistake in the musical model allowed Newton to conclude his argument as he hoped; that is, in the case of unison, the weight decreases with the square of the distance just like the gravitational attraction g of the sun on planets:

$$w \propto \frac{1}{r^2} ; g \propto \frac{1}{r^2} .$$

So, according to Newton, the Pythagoreans would really have “foreshadowed” his gravitational law. It seems to me that his mistake betrays (in the sense of revealing a secret) the deep convictions he had.

This error is so blatant, and so curious for a person of Newton’s capabilities, that it can only be explained by a wish to arrive at the conclusion of his argument at any cost. Does Newton himself therefore belong to the Pythagorean tradition?

3. *The scholium in the secondary literature*

Right after the passage in the *scholium* that contains the “*reciproce*” here discussed, Newton wrote: “*Subtilis quidem est hæc argumentatio, sed veteribus tamen innotuit* (Actually this argument is subtle, but it was known by the ancients as well)”. Scholars who have in recent decades considered this *scholium* have had the intention of reevaluating the Neo-Platonic and alchemist Newton. However, their analyses had singular shortcomings. I here comment on some of the articles on Newton and music. It is noteworthy that other papers, though they set out to cover the topic of science and music in its generality, do not consider Newton.

According to McGuire and Rattansi all the *scholium* was a “tortuous interpretation of the Lyre of Apollo” (McGuire Rattansi 1966, p. 136). On the contrary, Newton’s argument was not “tortuous” at all but so coherent that he was forced to a mistake in order to maintain his desired thesis. Here the famous philosopher of nature seemed to be led by a belief in the music of the spheres he has scarcely been suspected of in the current literature.

In his introduction to the first complete edition of the classical *scholia*, Casini was able to identify many sources of them. On this basis, criticizing McGuire and Rattansi's interpretation, he concluded that the "tortuous interpretation of the Lyre of Apollo, the Pipes of Pan, and the Harmony of the Spheres" was not "an authentic invention of Newton" but "a commonplace of writers of handbooks of mythology" (Casini 1981, p. 23; Casini 1984, p. 13). Neither McGuire and Rattansi nor Casini noticed Newton's convenient error. Perhaps, these scholars were more concerned with establishing a position in the contest between a positivist and an esotericist interpretation of Newton than with a full understanding of the text. Anyway, even after three centuries, Newton was able to hide his secret thoughts. At least in that sense he could be called a Pythagorean.

Sigalia Dostrovsky, in her considerations of the early vibration theory, uncritically followed McGuire and Rattansi and so concluded that "Mersenne's law $v \propto \frac{\sqrt{F}}{l}$, which Newton assumed to have been known to the Pythagoreans,

does hold for a given planet in a given orbit provided that one take l to be the planet's distance from the sun, F to be the gravitational attraction of it, and furthermore that one takes v to be proportional to the planet's angular velocity, as Kepler did (section 2.2)" (Dostrovsky 1974/75, p. 211). One can easily see from the text appended hereto that Newton's argument was quite different and therefore that such a reconstruction is too rational (or too irrational). What is worse is that it obscures a problem Newton faced regarding Kepler in preparing this *scholium*, as I will explain later.

In Susi Jeans' article on Newton in *The New Grove Dictionary of Music and Musicians* the *scholium* is not mentioned. However a book by Jeans, *Sir Isaac Newton and the Science of Music*, is cited as "in preparation" (Jeans 1980). As far as I have been able to determine, it has not yet appeared. It is to be hoped that in it the *scholium* will be considered and properly reported.

In her paper "Newton and Music", Penelope Gouk judged the *scholium* important for its author. "The idea that the Pythagorean myth of the harmony of the spheres contains the essential truth about God's geometrical construction of the heavens seems to have occurred to Newton at the same time as he developed his own inverse square law theory, if not before", (Gouk 1986, p. 52) she writes. Further, "Newton believed that Pythagoras was in this way showing that he was explaining the inverse square law theory of gravity rather than simply the characteristics of 'real' musical ratios. He as-

sumed that Pythagoras knew that the weights of the planets towards the sun vary reciprocally as the squares of their distance from the sun”. And, she concludes, “Such a belief made Newton, like Kepler before him, a true Pythagorean” (Gouk 1986, p. 53). It is unfortunate that Gouk, uncritically following McGuire and Rattansi and Casini, did not notice Newton’s error.

4. The sense of the text in the context of other works by Newton and other natural philosophers

Now we must pose some questions. Why was Newton so intent on obtaining the musical model at which he arrived? Would it have been difficult to develop a musical model of gravitation coherent with the rules of sound, where tension decreases with the inverse of the square of the distance? It would not: changing the music of the spheres, instead of the rules of sound, as Newton did, could have sufficed. In fact at the beginning of the *scholium* the planets were not considered to sound in unison, but Saturn, the farthest one, sang in the Dorian mode – that is, low – and the others, which are nearer, in higher tones. If Newton had chosen that way, fixing the pitch of planetary music as inverse to the square of the distance from the sun, he would have obtained exactly his own gravitational law:

$$p \propto \frac{1}{l^2} \propto \frac{\sqrt{w}}{l}$$

implying

$$g \propto w \propto \frac{1}{l^2}.$$

But in that case he might have been even more troubled. For he would have reiterated an argument developed by a forerunner who had certainly thought about the music of the spheres much more than he had. In *Harmonices Mundi Libri V*, published in 1619, Johannes Kepler had made distant planets, like Saturn and Jupiter, sing lower tones and the nearer ones higher tones. The planets in the *Harmonices* formed a complex polyphony, certainly not unison, just because their orbits were elliptical (Kepler 1619) (Walker 1967) (Warrain 1942) (Dickreiter 1973) (Koyré 1973) (Stephenson 1994). Could Newton have taken

the risk of letting the law of gravitation be “foreshadowed” not only by the remote, perhaps unreal, Pythagoras but also by that closer presence Kepler, to whom already owed so much? The mistake he made saved Newton's faith in the music of the spheres without requiring him to credit the insight to Kepler².

Against over enthusiasm regarding the significance of this error on Newton's part it could quite reasonably be argued that, as he did not publish the classical *scholia*, they must have been relatively unimportant to him. Yet it should be recalled that Newton was generally reluctant to publish, and the exclusion of the classical *scholia* from the 1713 and 1726 editions of the *Principia* and their replacement with the general *scholium* may be ascribed to something else. What made Newton abandon the classical *scholia* might not have been lack of conviction and interest, for he had after all written them. Rather the reason might be found in his well-known resentment toward rivals and his assiduousness at avoiding direct engagement in polemics. Both these tendencies led him to forms of self-censorship, including avoidance of calling attention to his rivals (Westfall 1980) (Casini 1981, p. 7 ff; Casini 1984, p. 11 ff).

In any case the *scholium* on *Propositio VIII* of the *Principia* is by no means the only place where Newton dealt with music. In other papers he considered musical problems, such as the division of the octave to get the most suitable tones for music. In some notes written in 1665, he divided the octave into twelve equal parts (semitones), that is using the proportion $1: \sqrt[12]{2}$ (Jeans 1980) (Gouk 1986, p. 43).

Especially in the *Opticks* Newton developed a synesthesia between light and sound so far as to set a strict analogy between the colours spreading out from the prism and the tones of the musical scale. First of all, colours had to be seven like the tones: violet, indigo, blue, green, yellow, orange, and red. In Book I, part II, Proposition III, problem I, colours were distributed along a line divided through the harmonic proportions of a musical scale (Newton 1952, p. 128):

$$1, \frac{8}{9}, \frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{9}{16}, \frac{1}{2}.$$

The intervals corresponded to the sequence from violet to red. The synesthesia came out again at Proposition VI, problem II of the *Opticks*. This time colours were distributed along a circumference divided into seven parts “proportional to the seven Musical Tones” (Newton 1952, pp. 154–155). The representation in the circle of the musical monochord recalled René Descartes'

Compendium Musicae (Descartes 1908, pp. 104, 118, 120) as well as Kepler's *Harmonices* (Kepler 1619, chap. I book III). The analogy between colours and tones was again proposed by Newton in Book II of the *Opticks* at the beginning of part II through a more complex model and calculations (Newton 1952, pp. 225–229). In the end, in Query 14 of Book III, part I, the English natural philosopher was wondering: “May not the harmony and discord of Colours arise from the proportions of the Vibrations propagated through the Fibres of the optick Nerves into the Brain, as the harmony and discord of Sounds arise from the proportions of the Vibrations of the Air? For some Colours, if they be viewed together, are agreeable to one another, as those of Gold and Indigo, and others disagree” (Newton 1952, p. 346) (Gouk 1986, p. 48).

It is to be noted that when the string is shortened the sound increases in pitch. But in Newton's picture the synesthesia was represented in such a way that, when the sound increased in frequency, the light turned from violet to red. In modern language, the frequency of light decreased whereas the frequency of sound increased. However Newton should not be charged with this incoherence, and this should not surprise us for, as is known, he maintained a corpuscular theory of light where the frequency had no place.

Colin MacLaurin, who was a pupil of Newton's, expounded in 1750 his master's discoveries, among which was the musical model of gravitation. MacLaurin wrote, differently from Newton, that in order to keep the same tone in a string of double length the tension must be quadrupled. Also he wrote that the gravity of a planet was quadruple that of a planet at twice the distance. But then he too made efforts to confirm his master's words. He claimed that it was necessary to increase the force on the farthest planets in direct proportion to the square of their distance from the sun to make all of them undergo the same gravity. MacLaurin wanted to keep the same gravity affecting each planet by analogy with strings sounding the same tone. The proportions according to which tension on strings and gravity on planets must be increased to obtain the relative effects thus became certainly the same (MacLaurin 1750, p. 54) (McGuire Rattansi 1966, footnote 7).

The Newtonian law of gravitational attraction and the law of sound and music in unison remained incompatible. MacLaurin was, like Newton, obliged to modify one of them to restore their coherence. In contrast to Newton, he modified the law of gravitation! However, what would the gravitational attraction have turned out to be in his scheme? What the motion of planets?

No doubt for his pupil the attraction of Newtonian authority was independent of the covered distance and the elapsed time and still very strong.

5. *Science and music*

Here I have been concerned with that Newton who found in the music of the spheres a foreshadowing at the mathematical formulation of natural laws. Certainly the numerical regularities discovered dividing the monochord to get consonant tones were as ancient as the regularities of cosmic motions, so the unique God of creation must have established his unique law in both cases. Hence the need to discover these relations.

I will leave for further future discussion real meanings of the tradition from which Newton derived that belief. In the *Magna Grecia* were born at least two different musical and scientific traditions: that of the Pythagoreans and that of Aristoxenos. But after two thousand years, during which they had traveled from there, had crossed with each other, disguised, transplanted, and hidden in many ways, the belief in the music of the spheres still seemed to possess a maieutic function – even though, cautious as ever, Newton refrained from using it like a flag to wave over his own castle. Unfortunately his musical interests have been almost neglected by historians of science. It is only recently that studies such as those cited here have begun to appear.

Among the famous pages Alexandre Koyré devoted to Newton there is no mention of music. The reason is not hard to find. Koyré maintained that during the 17th century the conception of cosmos as an enclosed, hierarchic system had vanished, being replaced (at least until the time of Einstein) by a universe seen as open, undefined, and infinite – a transition Koyré aptly described as “the bursting of the sphere” (Koyré 1965, p. 7). According to Koyré's view any concern with harmony should have disappeared in the course of the ‘progress’ from Kepler to Newton. And yet scientists did keep on dealing with musical theories as before,³ to the extent that the list of those who did not leave us writings about this argument would be shorter than the complementary one. Perhaps Koyré had not breath enough for inflating and exploding the sphere that dominates the books written by Kepler.

The general lack of consideration of music in the secondary literature concerning history and philosophy of science is regrettable. In addition to Daniel Walker, already cited, Claude Palisca has studied the relationship between

science and music (from the side of music) (Palisca 1961)⁴. Now more papers and books have begun to appear, and in 1997 a complete English translation of Kepler's *Harmonices* ... at long last became available.⁵ I hope that consideration of the many and sometimes strong links between science and music will not still be judged irrelevant for understanding the history of science. There may be some who are reluctant to see the current commonplaces undermined if it is found out how scientific evolution has been deeply marked by discussions of music. But comfortable simplifications are unlikely to advance either culture or historical knowledge.

While I was correcting the proof I received the new book by Penelope Gouk, *Music, Science and Natural Magic in Seventeenth-Century England* (1999, Yale University Press). It was published after I had finished my article. The book reconstructs the context in which Newton wrote his papers on music. However, to these papers Gouk does not add anything which changes her previous interpretations and *a fortiori* which might modify the content of my article. Anyway, now, also by means of her book, Newton's musical models become more easily accessible.

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Appendix

On prop. VIII

Ancient peoples did not sufficiently explain the proportion with which gravity decreases getting away from planets. Anyway they are regarded as having foreshadowed it through the harmony of the celestial spheres naming the Sun and the six planets left Mercury, Venus, Earth, Mars, Jupiter, Saturn using Apollo and his seven-stringed lyre, and measuring intervals between spheres

with intervals of tones. This is how they wished the seven tones were generated, whose harmony they named *diapasôn* [octave], Saturn being set in motion by the Dorian mode [from E to E], that is low, while the other planets [being set in motion] by higher modes (as Pliny lib. I c. 22 quotes from the Pythagorean school), and the Sun playing on strings. Therefore Macrobius (lib. I, c. 19) states: “Apollo’s seven-stringed lyre shows many motions of the celestial spheres for which nature is thought to provide the Sun as leader”. And Proclus *In Timaeum* Plat. I. 3 p. 200: “septenarî were sacred to Apollo as the one who owns the songs of the universe. For this reason they named the god *Hebdomageta*, that is the prince of the septenarius”. In a similar way, [he was named] by Apollo’s oracle in Eusebius *Praep. Evangel.* I. 5 c. 14. The Sun is named *της επταφθόγγου βασιλεως* “the king of the harmony of seven tones”. Moreover, using this symbol, they meant that the sun was affecting planets with its force through that harmonically proportion of distances according to which the force of tension affects strings of different length that is inversely proportional to the square of the distance. In fact the force, by which the same tension acts on the same string at different lengths, is inversely proportional to the square of the length of the string. The same tension acts on the string of half-length in a way which is four times more forceful: therefore it generates the octave and the octave is produced [also] by a quadruple force. Therefore, if a string of fixed length and tightened by a fixed weight generates a specific tone, the same string tightened by a quadruple weight generates the octave. And similarly the same tension in a string one third shorter has a nine-fold stronger effect. In fact, it generates the twelfth [the fifth of the higher octave], and also the string, which generates a set tone by a set weight, must be tightened by a nine-fold heavier weight in order to produce the twelfth. And in general, if two strings of the same thickness are tightened by weights fastened to them, these strings will be in unison if the weights are *reciproce* [inversely] proportional to the squares of the lengths of the strings. Actually this argument is subtle, but it was known by the ancients as well. Indeed Pythagoras, as Macrobius confirms, used to stretch sheep’s guts or oxen’s nerves fastening various weights to them in order to work the ratio of celestial harmony out from that. Therefore, from those experiments, he inferred that the weights, with which one can hear all the tones from identical strings, were inversely proportional to the squares of the lengths of the strings, with which the musical instrument utters these notes. As Macrobius testifies, he really applied to the heavens the proportion

discovered through his experiments. And by that, comparing those weights to the weights of planets and the lengths of strings to the distances of planets, he understood by the celestial harmony that the weights of the planets towards the sun were reciprocally as the square of their distances from the sun.

On the other hand, philosophers liked, not to cause mocking, to compose their mystical sayings so as to set forth ordinary things to ordinary people inappositely and under such sayings concealed the truth. In that sense Pythagoras enumerated his own musical tones starting from the Earth, as if from here up to the Moon there were a tone interval, then up to Mercury half a tone interval, and up to the other planets other musical intervals; instead he taught that sounds were generated by motion. And friction of solid spheres, as if the largest sphere uttered the lowest sound, just as it happens beating iron hammers. Here one could have seen the birth of the Ptolemaic system of solid spheres, as meanwhile Pythagoras was concealing under parables of that kind his own system and the true celestial harmony.

[Variant]⁶ As a matter of fact Pythagoras, as Macrobius quotes, while passing by a blacksmith's, realized through experiments that sounds uttered by iron hammers were higher or lower depending on the different weights of the hammers; afterwards, stretching sheep's guts or oxen's nerves, also fastening various weights to them, he deduced that the tones reacted to the fastened weights in a similar way. (It is therefore certain that Pythagoras through those experiments realized the true proportion which is sharing an arcane) secret, also he noticed the numbers generating consonant sounds. Certainly Pythagoras realized through those experiments the true proportion existing between the tones of the strings and the weights fastened to them, that is the weights, by which you can hear all the tones from the same string, are reciprocally as the squares of the lengths of the string with which the musical instrument utters those tones. Pythagoras really applied the proportion, discovered by means of his experiments, to the heavens and then deduced the harmony of the spheres from it. And because of this, making a comparison between those weights and the weights of planets, and between the intervals of sounds and the intervals of the spheres, and between the lengths of the strings and the distances of the planets from the centre of the orbits, he understood by the harmony of the heavens that the weights of planets, towards the sun (whose lyre certainly they dance to), were reciprocally as the square of their distance.

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NOTES

1. I thank Casini for providing me with the photocopy.
2. It is well known that Newton almost never cited Kepler. See the chapter "Newton and Descartes" in Koyré 1965, and Barone 1989.
3. One can find writings on musical theory by famous scientists as late as Jean Le Rond d'Alembert (d'Alembert 1752), and Leonhard Euler (Euler 1926).
4. Thomas Kuhn (Kuhn 1977, pp. 36–37 and 40) is one of the very fews who noticed the deficiency.
5. (Kassler 1982), (Cohen 1984), (Gozza 1989), (Coelho 1992), (Knobloch 1992), (Knobloch 1995), (Settle 1996), (Casini 1994), (Bailhache 1989), (Bailhache 1992), (Bailhache 1993), (Bailhache 1995), (Bailhache 1996), (Toniatti 1997), (Toniatti 1999).
6. The variant was written by Newton on folio 11 verso (Casini 1981, 41).