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Incommensurability Then and Now

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Summary

The incommensurability of scientific theories is not the only famous incommensurability issue in the history of western philosophy. The commensurability of all magnitudes (things) by means of ratios of integers (arithmetical ratios) was *the thesis* of Pythagoreanism. The diagonal and side of a square, however, are not commensurable, thus the Pythagorean thesis is refuted. Most philosophers ancient and contemporary would agree that Pythagoreanism was refuted by the counter-example and the concomitant argument or proof. The incommensurabilists were victorious. The present paper examines the prospects of the contemporary thesis of the incommensurability of scientific theories in the light of the history of the Pythagorean thesis. What factors were responsible for the rather clear-cut victory of the *incommensurability* side? How were they able to carry through a refutation? How likely is it that the contemporary dispute over the commensurability of scientific theories will be resolved in such a sharp manner? The paper concludes that it is not at all likely.

De Paradigms Non Disputandum Est is the rallying cry of the Kuhn-Feyerabend axis. Scientific theories under different paradigms are incommensurable with one another. There is no objective yardstick for measuring the scientific superiority of one theory over another. Different theories carry their own yardsticks with them, and the respective yardsticks do not share a common unit of measurement; the two yardsticks cannot be traded in for one yardstick applicable to both theories since such a yardstick is simply unavailable.

The incommensurability of scientific theories is not the only famous incommensurability issue in the history of western philosophy. The commensurability of all magnitudes (things) by means of ratios of integers (arithmetical ratios) was *the thesis* of Pythagoreanism. The diagonal and side of a square, however, are not commensurable and unless this monster be barred, the Pythagorean thesis is refuted. Most philosophers ancient and contemporary would agree that Pythagoreanism was refuted by the counter-example and the concomitant argument or proof. This famous incommensurability argument was then a victory for the incommensurability side. The Pythagorean thesis was refuted. But these were the good old days and perhaps things were a lot simpler then. What factors were responsible for the rather clear-cut victory of the *incommensurability* side? How were they able to carry through a refutation? How likely is it that the contemporary dispute over the commensurability of scientific

theories will be resolved in such a sharp manner? The general question then can be put as follows: What factors go into an incommensurability proof [What are the necessary and sufficient conditions for a successful incommensurability proof?]

First of all, a little logico-grammatical spadework. *Incommensurability* is a relational state. Nothing is simply incommensurable; it is incommensurable with something else. (Every x is commensurable with itself). If x is incommensurable with y then y is incommensurable with x . This can simply be read off from the literal meaning of *incommensurable*. (Incommensurable-having no common unit of measure.) So the incommensurability relation is irreflexive and symmetrical. Furthermore, the relation is not transitive. One side of a square is incommensurable with its diagonal. The diagonal is incommensurable with any other side of the square. But all sides of the square, being equal, are commensurable. Here we have a case of the relation holding between x and y , y and z , but not between x and z . [In fact, I cannot think of any cases where transitivity does hold. The incommensurability relation appears to be *intransitive*.]

The reader has probably noted that treating incommensurability as a two-place relation definitely represents an over-simplification. To say of any two things that they have no common measure, is not usually to make a claim about *all conceivable* systems of measurement. Such claims would be extremely difficult to defend. Usually the claim is much less ambitious. The diagonal and side of a square have no common *rational* (integral) (Pythagorean-arithmetical) unit in common. For all integers x and y it is not the case that $D = (\frac{x}{y}) S$. If $D = (\frac{x}{y}) S$ then my making our units small enough [making our '1' small enough] we could find a common unit. [If $A = 1/2B$ we can set our unit ($1 = 1/2$, $2 = 1$ etc.) equal to $1/2$ and have $2A = B$ - A is two units and B one]. But if we permit x and y to range over the real numbers (rational numbers supplemented with irrational numbers) then there is an x and a y such that $D = (\frac{x}{y}) S$. $D = \frac{\sqrt{2}}{1} S$. [The irrational number $\sqrt{2}$ makes the difference, of course]. The point of all this is that incommensurability statements are most appropriately construed as ranging over certain systems of measurement, e.g. the system of rational numbers. A is incommensurable with B now because A is incommensurable with B in respect to system of measurement C . In the Pythagorean case it was quite clear what the system C was. It was the system comprising positive integers and their ratios (they had an *implicit rational* number framework, since rationals can be defined as pairs--ratios--of integers. Explicitly, the Pythagoreans appear to have limited numbers to the positive integers). Put baldly, in the Pythagorean case the commensurability claim was clear. No matter what things you choose, a number is in principle assignable to it, and each pair of things is in some ratio x to y where x and y are positive integers. Alternatively, each pair of things has a common unit of measurement--perhaps more accurately--everything has a common unit of measurement with everything else).

The Pythagorean thesis was clear. What about the contemporary *incommensurability* thesis *re* scientific theories?¹

T_1 is commensurable with T_2 IFF _____. First of all 'commensurable' is being used in a somewhat extended sense. We are not speaking literally of units of measurement. Whereas the Pythagoreans were concerned with *mathematical commensurability*, philosophers of science are concerned with what we might term *logical commensurability*. We need some logical relations which can serve to order (compare) theories. Strictly speaking, if it can be shown that there is at least one such logical relation, the commensurability thesis is saved. Theories can always be compared in respect to consistency, but consistency is not a relation holding between theories. An inconsistent theory is one which permits the derivation all statements (expressible in the language of the theory). We can compare theories as to theorems. We would want to say that if T_1 is inconsistent and T_2 not, there are no sentences of T_1 which are not theorems of T_1 , whereas T_2 does not possess this property. We do however have the following logical (or epistemological) relation which does contain some basis for comparing theories. X is known to contain contradictory pairs of sentences which y is not known to contain, and it is not the case that y is known to contain a contradictory pair of sentences which x is not known to contain. In any event, theories can be compared in respect to internal consistency.

But theories are *presumed* to be at the very least consistent. For the most part there is no question anyway of *proving* a theory consistent (except in the sense of one theory being consistent *if* some other theory is). What does the commensurability thesis boil down to for consistent theories? First of all, the claim would be simply that *for the most part or for the important cases* two theories (belonging to different paradigms) are commensurable. [Every two theories needn't be commensurable. And of course, there are degrees of commensurability]. In order to compare two things we needn't assign a precise measure to the two things. The Pythagoreans knew that the diagonal of a square was longer than any of its sides even though they could not assign a precise measure to both side and diagonal. They knew *approximately* how long the diagonal was in comparison with its sides. *Commensurability* needs to be distinguished from *comparability*. [By loosening a *commensurability* standard we get comparability.]

T_1 is commensurable with T_2 iff T_1 can conflict with T_2 if $\Diamond^*$ (there is a sentence Φ of T_1 and a sentence Ψ of T_2 such that Φ iff $\sim \Psi$). What *appears* to make two theories commensurable in the relevant sense is that they share a common language, or, more precisely, that there is

sufficient overlap between the languages of T_1 and T_2 to permit commensurability. A necessary condition for *sufficient* overlap is that there is some overlap, and if there is some overlap then there is at least one sentence which belongs to each language. But when are we prepared to say that a given sentence is shared by two languages? This is a ticklish question which fortunately we need not answer. All we really need is a sentence Φ in T_1 which is true iff a sentence Ψ in T_2 is false, or iff $\sim \Psi$. The *iff* here needn't be (indeed, is best not) interpreted as an analytic relation; Φ and Ψ needn't have the same *shape*, or the same *meaning* (sense). They needn't even be about the same thing (have the same subject-matter). For suppose we can 'establish' the following scientific law. It snows in Bermuda (Φ) iff $\sim$ Smith becomes a great tennis player ($\sim \Psi$). This law belongs neither to T_1 nor T_2 , yet it suffices to underline the conflict between T_1 and T_2 . If we are permitted the logico-semantic relation '____ is true iff ____ is true', the commensurability thesis stands. When the thesis is put sharply, the very possibility of an incommensurability proof seems to evaporate. What would such a proof look like? The incommensurabilists seem to have an extravagant *super-scientific* thesis to defend - a proof of the impossibility of our finding suitable principles which employ the above mentioned logico-semantic relation.

The story goes that Hippasos of Metapontion was drowned at sea for revealing the incommensurability of the diagonal and side of a square. Fortunately, today's incommensurability situation hardly requires anyone even get wet.

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¹ What has become the *locus classicus* for this question is Thomas Kuhn's, *The Structure of Scientific Revolutions*, Phoenix books 1962. The literature on the subject is already voluminous. The present paper essays a simple-minded approach which is hopefully not too simple-minded. * = it is possible that.