

**Abstract.** Michele Sbacchi examines the impact of the discipline of Euclidean geometry upon architecture and, more specifically, upon theory of architecture. Special attention is given to the work of Guarino Guarini, the 17th century Italian architect and mathematician who, more than any other architect, was involved in Euclidean geometry. Furthermore, the analysis shows how, within the realm of architecture, a complementary opposition can be traced between what is called "Pythagorean numerology" and "Euclidean geometry." These two disciplines epitomized two overlapping ways of conceiving architectural design.



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## Euclidism and Theory of Architecture

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### INTRODUCTION

It is well known that one of the basic branches of geometry which, almost unchanged, we still use today was codified by Euclid at Alexandria during the time of Ptolemy I Soter (323-285/83 BC) in thirteen books called *Stoicheia* (*Elements*). This overwhelmingly influential text deals with planar geometry and contains the basic definitions of the geometric elements such as the very famous ones of point, line and surface: "A point is that which has no part;" "Line is breathless length;" "A surface is that which has length and breadth only" [Euclid 1956, I:153]. It also contains a whole range of propositions where the features of increasingly complex geometric figures are defined. Furthermore, Euclid provides procedures to generate planar shapes and solids and, generally speaking, to solve geometrical problems. Familiarity with the *Elements* allows virtually anyone to master the majority of geometrical topics. Although all this is well known I nevertheless find it necessary, given our misleading post-Euclidean standpoint, to underline that 'Euclidean Geometry' was 'Geometry' *tout court* until the 17th century. For it was only from the second half of 17th century that other branches of geometry were developed—notably analytic and projective geometry and, much later, topology. Yet these disciplines, rather than challenging the validity of Euclidean geometry, opened up complementary understandings; therefore they flanked Euclid's doctrine, thus confirming its effectiveness. In fact, Euclidean geometry is still an essential part of the curriculum in high schools worldwide, as it was in the *quadrivium* during the Middle Ages. That is not to say that Euclid's teaching has never been questioned. In fact a long-standing tradition does not necessarily imply a positive reverence: some Euclidean topics have, indeed, undergone violent attacks and have fostered huge debates. The ever-rising polemic about the postulate of the parallels is just one notorious example of the many controversies scattered throughout its somewhat disquieted existence.

Euclid was far from being an original writer. Although conventionally referred to as the inventor of the discipline, he was hardly an isolated genius. Historians of mathematics have clarified how he drew from other sources—mainly Theaetetus and Eudoxus.[1] Hence, rather than inventing, he mostly systematized a corpus of knowledge that circulated among Greek scholars in somewhat rough forms. Therefore, Euclid's great merit lies in the exceptional ability to illustrate and synthesize.. Although marred by contradictions and gaps, the *Elements*, in its

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time, represented a gigantic step forward, especially compared to the fragmentary way in which geometry was known and transmitted. It soon became an immensely useful text for all the fields where geometry was applied. Optics, mensuration, surveying, navigation, astronomy, agriculture and architecture all benefitted in various ways from a newly comprehensive set of rules able to overcome geometrical problems. As its popularity grew, the *Elements* went through several translations. Following the destiny of most Greek scientific texts, it was soon translated into Arabic and was known through this language for almost fifteen centuries. A well-known Latin translation was made by Adelard of Bath in the 12th century but at least another translation existed earlier.[2] Campano's Latin translation of 1482 was the first to be published. Nevertheless a translation directly from Greek into Latin was made by Bartolomeo Zamberti in 1505. Federico Commandino's Latin edition of 1572 was to become the standard one. The first English translation is due to Henry Billingsley in 1570, with a preface by John Dee [Wittkower 1974:98; Rykwert 1980:123]. No less significant are the commentaries upon the *Stoicheia*, if only because they witness the continuous debates that scholars engaged in about the text. Certainly the most renowned commentary is the one made in the 5th century A.D. by Proclus on the First Book. Because of this vast and lasting tradition, the *Elements* may be appropriately compared to the Bible or to the *Timaeus* as a cornerstone of Western culture [Field 1984:291].

#### ARCHITECTURE THEORY, GEOMETRY AND NUMBER

Architecture, a discipline concerned with the making of forms, perhaps profitted most from this knowledge. I find it unnecessary to dwell here upon such a vast and overstudied issue as the relationship between architecture and geometry. Instead, it suffices to stress that the geometrical understanding of, say, Vitruvius, Viollet Le Duc and Le Corbusier was basically the Euclidean one — that of the *Elements*. It is nevertheless true that the other branches of geometry, which arose from the 17th century on, affected architecture, but this can be considered a comparatively minor phenomenon. In fact, the influence exerted by projective geometry or by topology on architecture is by no means comparable to the overwhelming use of Euclidean geometry within architectural design throughout history.

The relevance of Euclidean methods for the making of architecture has been recently underlined by scholars, especially as against the predominance of the Vitruvian theory. According to these studies [Rykwert 1985; Shelby 1977], among masons and carpenters Euclidean procedures and, indeed, sleights of hand were quite widespread. Although this building culture went through an oral transmission, documents do exist from which it can be understood that it was surely a conscious knowledge. 'Clerke Euclide' is explicitly referred to in the few remaining manuscripts.[3] Probably the phenomenon was much wider than what has been thought so far, for the lack of traces has considerably belittled it. We can believe that during the Middle Ages, to make architecture, the Euclidean lines, easily drawn and visualized, were most often a good alternative to more complicated numerological calculations. Hence we can assume that an 'Euclidean culture associated with architecture,' existed for a long time and that it was probably the preeminent one among the masses and the workers.

Yet among the refined circles of patrons and architects the rather different Vitruvian tradition was also in effect at the same time [Rykwert 1985:26]. This tradition was based on the Pythagorean-Platonic idea that proportions and numerical ratios regulated the harmony of the world. The memorandum of

Francesco Giorgi for the church of S. Francesco della Vigna in Venice, is probably the most eloquent example illustrating how substantial this idea was considered to be for architecture [Moschini 1815, I:55-56; Wittkower 1949:136ff]. This document reflects Giorgi's Neoplatonic theories, developed broadly in his *De Harmonia mundi totius*, published in Venice in 1525, which, together with Marsilio Ficino's work, can be taken as a milestone of Neoplatonic cabalistic mysticism. The whole theory, whose realm is of course much wider than the mere architectural application, was built around the notion of proportion, as Plato understood it in the *Timaeus*. Furthermore, it was grounded on the analogy between musical and visual ratios, established by Pythagoras: he maintained that numerical ratios existed between pitches of sounds, obtained with certain strings, and the lengths of these strings. Hence, the belief that an underlying harmony of numbers was acting in both music and architecture, the domain respectively of the noble senses of hearing and of sight. In architecture numbers operated for two different purposes: the determination of overall proportions in buildings and the modular construction of architectural orders. The first regarded the reciprocal dimensions of height, width and length in rooms as well as in the building as a whole. The second was what Vitruvius called *commodulatio*. [4] According to this procedure, a module was established — generally half the diameter of the column — from which all the dimensions of the orders could be derived. The order determined the numerical system to adopt and, thus, every element of the architectural order was determined by a ratio related to the module. Indeed it was possible to express architecture by an algorithm [Hersey 1976:24]. Simply by mentioning the style a numerical formula was implied and the dimensions of the order could be constructed. These two design procedures are both clearly governed by numerical ratios — series of numbers whose reciprocal relationships embodied the rules of universal harmony.

If we now compare again these procedures with the Euclidean ones, it appears more clearly that the difference between the two systems is a significant one: according to the Vitruvian, multiplications and subdivisions of numbers regulated architectural shapes and dimensions; adopting Euclidean constructions, instead, architecture and its elements were made out of lines, by means of compass and straightedge. The 'Pythagorean theory of numbers' and the 'Euclidean geometry of lines' established thus a polarity within the theory of architecture. [5] Both disciplines were backed up and, in a way, symbolized by two great texts of antiquity: the *Timaeus* and the *Elements*. [6] Although in architecture the dichotomy was brought about substantially by the issue of proportion, the difference is, in fact, a more general one. Every shape and not only proportional elements can be determined either by the tracing of a line or by a numerical calculation. This twofold design option is somehow implied in the epistemological difference between geometry and arithmetic. Socrates' remark, in Plato's *Meno*, to his slave who hesitated to calculate the diagonal of the square, epitomizes the two alternatives: "If you do not want to work out a number for it, trace it" [Plato *Meno* 84].

I have outlined how, during the Middle Ages, Euclidean and Vitruvian procedures empirically coexisted within building practice. This situation would undergo an important change in the 17th century. During the Renaissance the advent of an established written architectural theory, based as it was on the dialogue with Vitruvius' text, fostered the neo-Pythagorean numerological aspect of architecture. Leon Battista Alberti, the most important Renaissance architectural theorist, was well aware of Euclidean geometry, [7] a discipline

which he dealt with in one of his minor works, the *Ludi Mathematici*. Yet Alberti's orthodox position within the Classical tradition could not allow him to challenge the primacy of numerical ratios for the making of architecture. Therefore, not surprisingly, Euclidean methods are left out of his *De Re Aedificatoria*, where he quite decidedly states that: "... the three principal components of that whole theory [of beauty] into which we inquire are number (*numerus*), what we might call outline (*finitio*) and position (*collocatio*)" [Alberti 1485:164v-165]. For him numbers were still the basic source. Accordingly, his seventh and eighth books, fundamental ones of *De Re Aedificatoria*, are devoted to numerical topics. Yet it might be speculated that his emphasis on *lineamenta* (lineaments) and lines, never fully understood, could be an acknowledgement of a building practice leaning more toward geometry than toward numerology. With Francesco di Giorgio Martini's *Trattato di Architettura Civile e Militare*, the Euclidean definitions of line, point and parallels make their first appearance within an architectural treatise, although in a rather unsystematic way. Serlio, later, goes a step further: his first two books include the standard Euclidean definitions and constructions; yet they are intended to be the grounds more for Perspective than for Architecture. Traces of Euclidean studies can be found also in Leonardo: the M and I manuscripts, the Foster, Madrid II and Atlantic codices contain Euclidean constructions and even the literal transcription of the first page of the *Elements* [Lorber 1985:114; Veltman 1986].

#### GUARINO GUARINI AND EUCLIDISM

It is only with Guarino Guarini, in the second half of the 17th century, however, that Euclidean geometry abandons the oral realm and makes its open appearance within a treatise. His posthumously published *Architettura Civile*, written presumably between 1670 and his death, marks a fundamental moment of the relationship between Euclidism and theory of architecture. But first, a reflection on Guarini's activity allows us to understand that his being the first to include Euclidean geometry extensively within an architectural treatise was no accident. I do not want to dwell upon his general involvement with geometry and the vast use of geometrical schemes for his buildings, two issues doubtlessly but loosely related to this fact. I would rather point out more circumstantial events. Firstly, being a professor of mathematics, Guarini was almost unavoidably obliged to consider Euclidean geometry. His Euclidean interests probably arose during his early teaching of Mathematics at Messina where distinguished Euclidean scholars such as Francesco Maurolico and his pupil Giovanni Alfonso Borelli had taught previously. There Guarini found himself in one of the most stimulating scientific centers of the time where a long-standing Euclidean tradition existed.[8] Maurolico wrote a commentary of the *Elements*, [9] while Borelli was author of the *Euclides Restituitus*. Yet it was more likely in Paris, where Guarini taught mathematics between 1662 and 1666, that his concern with Euclidean geometry expanded. For there he encountered a lively scientific milieu and particularly Francois Millet de Chales. A most distinguished mathematician, this latter was the author of *Cursus seu mundus mathematicus*, an encyclopedic work on mathematics that also dealt with architecture.[10] More relevant to the present discussion are Millet's two commentaries on the *Elements*, *Les Huit Livres d'Euclide* and *Les éléments d'Euclide expliqués d'une maniere nouvelle et très facile*. Guarini was deeply influenced by Millet [Guarini 1968:5, note 1]; he is referred to frequently in Guarini's books, not just for geometrical or mathematical matters. Out of this background developed Guarini's *magnum opus* on geometry, the *Euclides Adauctus et methodicus mathematicaque universalis*

published in 1671. As the title makes clear it, was both a commentary on the *Elements* and an attempt to summarize the mathematical knowledge of the time, much in the manner of his beloved Millet. It turned out to be a rather successful book for it was republished five years later. Guarini, therefore, falls well within the tradition of Euclidean commentators. His interest for the discipline went beyond the mere content, however, as Euclidean geometry was for him a sort of universal key for human knowledge. The extent to which Guarini considered Euclidean norms as the basis of every scientific work is also clear from another work of his, the *Trattato di Fortificazione*, where the Euclidean basic definitions of point, line, etc. are provided at the very beginning as a kind of conditional entry to the topic.[11] The same approach occurs with his *Del modo di Misurare le fabbriche*, a booklet on surveying.

*Architettura Civile* came later; it was definitely written after the *Euclides* since the latter is mentioned in it. As I have suggested, the Euclidean intrusions in *Architettura Civile* are far too many to justify them only on the grounds of a mere unconscious professional bias. The argument that the geometer prevailed over the architect misses the importance of the issue. In the first treatise of the five constituting the book, Guarini early on states his geometrical interests: "And since Architecture, as a discipline that uses measures in every one of its operations, depends on Geometry, and at least wants to know its primary elements, therefore in the following chapters we will set out those geometrical principles that are most necessary".[12] Consequently the following chapter explores the "Principles of Geometry necessary to Architecture." It contains the nine definitions of point, line, surface, angle, right angle, acute angle and parallel lines. Chapters dedicated to surfaces, rectilinear shapes, circular shapes follow and the whole first treatise continues basically in this way with postulates, other principles and several typical Euclidean transformations such as "To draw a line from a given point in order to make it touch the circle" [Guarini 1968:41]. The Euclidean discipline of Geodesia fills the Fifth Treatise — the way of dividing and transforming planar shapes into other equivalents.[13] Some of these parts are literally transported from his own *Euclides*, some are slightly elaborated on in light of their architectural application. Guarini's Euclidean purism—as opposed to arithmetics—is remarkably evidenced, when, in the Geodesia treatise, he considers progressions as purely geometrical and not numerical [Capo 8]. The dismissal of numerical progression, an attitude taken also by Francois Derand, was shared by those who wanted to reestablish the foundation of logarithms from a geometrical basis rather than from exponential equations.[14] Thus the issue proposed is once again the opposition between the two disciplines. In *Architettura Civile*, however, the most significant fact for the purpose of my argument is that even the theory of the orders, the very core of Vitruvian numerology, is overshadowed by the alternative geometrical approach. Remarkably the modular commodulatio procedure, rooted in numbers, is replaced by a mixed system where the dimensions of the architectural elements are determined by geometrical constructions and only in some cases by numerical operations. Therefore, Guarini breaks away from a long-standing tradition where the only possible way of making the orders had to be numerical.

#### THE REVIVAL OF EUCLIDISM

In this revival of Euclidean culture Guarini was not alone. His acknowledged source was the treatise of the Milanese architect Carlo Cesare Osio. Osio's treatise, which also bears the title *Architettura Civile*, sets forth a system for the orders that is, even more geometrical than Guarini's. Of course Osio's ideas, probably regarded as unorthodox or extravagant by others, strongly appealed

Guarini.[15] Hence, it is hardly surprising that Osio, despite being a rather obscure architect, is taken by Guarini as a primary authority, second only to Vitruvius, and is continuously quoted throughout his *Architettura Civile*. With Guarini and Osio, therefore, the Euclidean heritage is consciously acknowledged within the learned realm of theory and no longer belongs to an oral and empirical culture. Osio's Euclidean opposition to numerology is clearly self-confessed: in the preface of his book he describes the difficulties of the traditional modular systems: ".....such those that (perhaps in order to avoid subdivisions that are intricate in themselves) follow the fashion of the more modern with the establishment of the modules, in which, relying on the discreet property of the numbers.....".[16] And he then states that his method will avoid the modules used by architects before him: "Thus henceforth it always appeared that these were the possible ways, and the only ones capable of putting in proportion the quantities of the same order, both in themselves and amongst themselves. And still in any case, through divine favour, I hope in this work of mine to enrich Architecture to more certain and more perfect effect. With Geometrical rules, which have for their basis and support the Euclidean Demonstrations, I hope to aid...".[17] His new attitude is also emphasized by a symbolic representation: in the frontispiece he is significantly portrayed with two books bearing the names of Vitruvius and Euclid, alluding unambiguously to the double tradition I have outlined so far. Just as conscious and deliberate is Guarini's Euclidism. Indeed *Architettura Civile* turns out to be a rather peculiar trattato where Euclid and Millet de Chales—two geometers—are advocated as architectural authorities, even in the most quintessentially architectural parts.[18] The Euclidean leaning is revealed by a number of other circumstances. In *Architettura Civile* quite often the elements of geometry become the elements of architecture *tout court*. For Guarini, for example, a wall is a 'surface' and a dome a 'semisphere.' Consequently, 'architectural design' most often seems to be identified with 'architectural drawing': as a true geometer Guarini describes the production of the project rather than the production of the building. In contrast to the two treatises of his pupil Vittone, where technical problems are preeminent, Guarini's *Architettura Civile* completely disregards the constructional aspect of architecture in favor of detailed descriptions of drawing techniques. This is striking, especially if we think of the technological emphasis often displayed in Guarini's buildings. In this regard it is curious that drawing tools are in fact grouped under the title "Architectural Instruments". The problem, for him, was not 'how to build' but 'how to draw.' Therefore, not only Euclidean geometry has become a part of architectural theory but it has also carried with it its implied linearis essentia (linear-like essence) which in Guarini and Osio pervades the all matter.

The expression linearis essentia is Francesco Barozzi's. An outstanding mathematician and friend of Daniele Barbaro, Barozzi was the leader of a movement of general reappraisal of Euclidean geometry, which centered around Barozzi in Venice and Padua and around Federico Commandino in Urbino.[19] The achievements of this group of scholars are essential to understanding how Euclidean geometry passed from Serlio's timid acknowledgement to Guarini's broad inclusion within architecture.[20] Barozzi, Barbaro, Commandino and their circles contributed to the recognition of geometry as a modern science. Consequently they took the rigorous rereading of the Euclidean text as a conditional starting point. Commandino dedicated all his life to retranslating and clarifying Greek texts on science, among them the *Elements*. Francesco Barozzi edited a renowned edition of Proclus's commentary, in which, as already noted, he acutely observed and stressed the fundamental linear-like essence of

geometry. But Barozzi and Barbaro's epistemological interest dwelled upon another important notion, that of "demonstration" (*demonstrazione*), not coincidentally a basic requisite of the Euclidean axiomatic-deductive procedure. For them, but also for other mathematicians of the Paduan circle such as Giuseppe Moletto as well, the theory (*teorica*) would have been valid only in conjunction with demonstrations [Tafuri 1985:202]. [21] Barozzi also polemized with Alessandro Piccolomini and Pietro Catena, who argued for the separation of Aristotelian syllogism from mathematical logic, thereby putting the latter on an inferior level. On the other hand, Barozzi in his *Opusculum: in quo una Oratio e duo Questiones, altera de Certitudine et altera de Medietate Mathematicarum continentur*, dedicated to Daniele Barbaro, stressed that "the certitude of mathematics is contained in the syntactic rigor of demonstrations" [Tafuri 1985:206]. To carry this idea into architectural theory was, as is well known, Barbaro's task in his Vitruvian commentary, where syllogism (for Barbaro, *discorso*) and demonstration are key elements. Therefore not only was geometry at that time compellingly reevaluated but the epistemological value of the geometrical demonstration was appreciated as well, with an interesting architectural twist.

### THE DECLINE OF 17TH CENTURY PYTHAGOREAN NUMEROLOGY

If the general rise of geometry can explain Guarini's achievement, another phenomenon must be considered. Guarini's Euclidism can also be rightly inserted in a general decline of Pythagorean numerology in the 17th century. In the fields of astronomy and music, at that time, Kepler made an even more radical dismissal of numerology on the grounds of the Euclidean argument. Astronomy had been saturated with Pythagorean ideas but the Copernican revolution shook the whole field, promoting new interpretations. With the moon no longer considered a planet but a satellite, Copernicus's planets became six instead of the Ptolemaic seven. The astronomer Rheticus tried to confer meaning to this number according to a Pythagorean understanding:

For the number six is honoured above all the others in the sacred prophecies of God and by the Pythagoreans and the other philosophers. What is more agreeable to God's handiwork than this first and most perfect work should be summed up in this first and most perfect number? [Field 1984:273]

To this Kepler replied in the *Mysterium Cosmographicum* on a geometrical basis. For him the orbs were six because they defined the spaces between the five regular solids. To substantiate the fact that the bodies were five Kepler cited the last proposition of Book XIII of Euclid's *Elements*. This should not be considered coincidental for, indeed, Euclid was held in the highest consideration by Kepler: for example, in a letter to Heydon in 1605, he writes that the archetype of the world "lies in Geometry, and specifically in the work of Euclid, the thrice-greatest philosopher [*et nominatim in Euclide philosopho ter maximo*]" [Field 1984:283]. But Kepler's most evident Euclidean concern came out in the field of music, where he tried to fight the Pythagorean conception, exactly in the realm where it was strongest. Kepler's *Harmonices Mundi* is specially devoted to the founding of musical ratios on geometry. The first book, in which Kepler outlines his theory, is entirely devoted to geometry, the second on music. He declares:

Since today, to judge by the books that are published, there is a total neglect of the intellectual distinctions to be made among geometrical entities, I thought fit to state at the outset that it is from the divisions

of the circle into equal aliquot parts, by means of geometrical constructions [i.e., using straight edge and compasses], that is, from the constructible Regular plane figures, that we should seek the causes of Harmonic proportions.[Field 1984:283]

Judith Field has pointed out that "... the weight of the geometrical work in *Harmonices Mundi* ... must be seen as indicating that he took very seriously his endeavor to prove that God was a Platonic geometer rather than a Pythagorean numerologist" [Field 1984:284]. The case of Kepler further proves that the opposition between Pythagorean theories and Euclidism was a vast phenomenon which transcended the realm of architectural theory. Moreover, Kepler's attitude reveals that the issue, far from involving merely practical procedures, had ontological facets in the deepest sense.

#### THE CONFLICT BETWEEN EUCLIDISM AND PYTHAGOREAN NUMEROLOGY

To complete my analysis I shall lastly consider a fundamental antithesis. In fact, the conflict between Euclidism and Pythagorean numerology is mirrored by the analogous dualism between two opposite ways of conceiving quantities, as continuous or as discrete. This topic requires a discussion which is too vast for this essay,[22] yet a short treatment is indispensable for the purpose of my argument. Quantities can be intended either as the summation of infinitesimal parts—hence they are discrete—or as the product of the flow of some primary entities—hence they are continuous. This double conception goes back at least to Aristotle and has been widely discussed over centuries. The root of the different approach towards reality adopted in the two disciplines of geometry and arithmetic must be sought in this very duality. In arithmetic quantity is conceived as discrete; this means that it is represented by entities such as numbers. This conception is grounded on two assumptions: that things are separable and that, consequently, they can be enumerated. The idea of quantity as discrete is therefore an essential one for the very nature of arithmetic. The Pythagoreans' enthusiasm about numbers celebrated mystically this very possibility.

In geometry the approach is totally different: the entities adopted—line, volume, etc.—are thought of as continuous; they match the continuity of reality in a more comprehensive way than the discrete ones do. For example the geometrical line—not coincidentally taken as the symbol of the "continuous"—represents measurable as well as incommensurable quantities, by means of the infinite series of his points. As a matter of fact the argument about discrete and continuous quantity has historically often been used to distinguish geometry from arithmetic, and sometimes to support the superiority of one over the other.[23] Geometry, in fact, often became synonymous with continuous. Mathematicians such as Barozzi, Tartaglia or Viviani—just to quote those from the period with which I have mainly dealt—were well aware of this distinction, as scientists are today. Architects, instead, only vaguely considered it. The very learned Scamozzi and the rather minor figure Osio are two of the few who included this topic, although very briefly, in their treatises. Guarini, who as a mathematician and philosopher discusses at length *quantitas continua* and *quantitas discreta* in his books, disregards it almost completely in his architectural treatise.[24] This is rather surprising because, as I have tried to demonstrate, the field of architecture was a crucial battleground for the two conceptions. Indeed in the making of architectural forms the choice between a line to trace—i.e. the geometrical approach—or a number to calculate—i.e. the numerological approach—not only implies rather different design methods but also brings about diverse results.

The opposition of the continuous to the discrete enlightens how deep, conceptually, was the opposition of geometry to arithmetic. The change that occurred in architecture at the end of the 17th century, which witnessed a dismissal of Pythagorean numerology in favour of a more explicit adherence to geometry, is therefore a meaningful phenomenon. It consisted in making official rather widespread but disguised procedures. Furthermore, its belonging to a vast cultural phenomenon—of which I have analyzed the revival of Euclidean geometry within Italian scientific circles and Kepler's approach in the fields of astronomy and music—further magnifies its importance.

#### NOTES

[1] In particular the whole theory of proportionals, including the much-debated Definition V was taken from Eudoxus of Cnido (IV c. B. C.) [Euclid 1956, I:1]. See also [Cambiano 1967]. [return to text](#)

[2] Heath has pointed out that a Latin translation, earlier than Adelard's, must have been the common source for at least three documents: Boethius, a passage in the *Gromatici* and the Regius Manuscript in the King's Library of the British Museum [Euclid 1956, I: 91-95]. [return to text](#)

[3] Two manuscripts are located in the King's Library of the British Museum, the Regius manuscript and the Coke manuscript. See [Knoop 1938; Euclid 1956, I: 95; Halliwell: *Rara Mathematica*]. [return to text](#)

[4] "Proportio est ratae partis membrorum in omni opere totiusque commodulatio, ex qua ratio efficitur symmetriarum," [Vitruvius, III, 1, 1]. [return to text](#)

[5] Girolamo Cardano stigmatizes this opposition when in his *De subtilitate* contrapposes an "Euclidis Laus," which praises Euclid's "inconcussa dogmatum firmitas," with a rather critical "Vitruvij Laus," where Vitruvius is accused of being only a compiler. See [Oechslin 1983:23]. [return to text](#)

[6] Mario Vegetti has written, "The tradition of the *Timaeus* remains completely foreign to the theoretic field of the Euclidean-style sciences" ("La tradizione del *Timeo* resta del tutto estranea al campo teorico delle scienze di stile Euclideo") [Vegetti 1983: 156]. [return to text](#)

[7] Alberti owned a copy of the *Elements*. It is now in the Marciana library in Venice. [return to text](#)

[8] Note XVII of Michel Chasles' *Aperçu historique ...* [1875] has the heading "Sur Maurolico and Guarini". See [Baldini 1980-I; Micheli 1980: 489-490]. On Maurolico see [Clagett 1974] and [Dollo 1979]. [return to text](#)

[9] Unpublished manuscript at the Bibliothèque Nationale, Paris. He also translated Euclid's *Phenomena*. [return to text](#)

[10] On Millet de Chales and 17th century encyclopedism see [Vasoli 1978]. [return to text](#)

[11] "The Elements of Euclid are so necessary to every science...and also to whoever would advance themselves in the military arts must believe them to be the basis, principle and fundamental element on which to build, and beyond which to advance, and on which to lay every speculation" ("Gli Elementi di Euclide sono sì necessari ad ogni scienza ... e pertanto qualunque vuole avanzarsi nell'arte militare, deve credere, che questa sia la base, il principio & il primo elemento, di cui si compone, e sopra a cui s'avanza, e cresce ogni sua speculazione") [Guarini 1968: 10]. [return to text](#)

[12] "E perché l'Architettura, come facoltà che in ogni sua operazione adopera le misure, dipende dalla Geometria, e vuol sapere almeno i primi suoi elementi, quindi é che ne' seguenti capitoli porremo que' principi di Geometria che sono più necessari" [Guarini 1968:10]. It is noteworthy that Guarini defines geometry as *ars metendi*. [return to text](#)

[13] There were, in fact, two tradition for Geodesy. The first referred to the lost treatise by Euclid on The Division of Figures, of which existed an Arabic copy by Muhammed ibn Muhammed al Bagdadi, translated into Italian in 1570. The second referred to the Metrics of Hero. See [Guarini 1968: 389, n. 1]. [return to text](#)

[14] "The diffidence of pure geometry with regards to logarithms" ("la diffidenza del puro geometra nei confronti dei logaritmi.") [Guarini 1968: 418, n. 4]. [return to text](#)

[15] The acquaintance between Guarini and Osio is a likely one. Guarini often visited Milan, Osio's town, to meet the publisher of his astronomical work *Caelestis Mathematica*. [return to text](#)

[16] "...come quelli pure li quali (forse per isfuggire le sudette per se stesse intricate subdivisioni) doppo i piú moderni con lo stabilimento dei moduli, ne quali appoggiantesi alla discreta proprietá dei numeri" [Osio 1661: 2]. [return to text](#)

[17] "Laonde parve sempre da qui a dietro che questi fossero i modi possibili, e unici di porporzionare le quantità nei medesimi ordini, tanto in se stesse quanto tra loro. E pure ad ogni modo, mediante il favore divino, io spero in questa mia opera, arricchire l'Architettura a questo effetto piú certa e piú perfetta. Con regole Geometriche, ch'hanno per loro base, e sostegno le Dimostrazioni Euclidean, spero agevolare....." [Osio 1661:2]. [return to text](#)

[18] See [Guarini I,1] where Millet is strikingly quoted together with Vitruvius for the definition of architecture; and I, III, Osservazione 6, where Millet is quoted for the matter of the respect of ancients' rules; see also III, 17, 2, where the topic is the Doric order. [return to text](#)

[19] Daniele Barbaro is quoted together with Vettor Fausto and Nicoló Tartaglia as a restorer of the antique scientific rigor in the dedication of Guidobaldo del Monte, *Mechanicorum Liber* (Pesaro, 1577), quoted in [Tafuri 1985:203]. [return to text](#)

[20] To this might be added John Dee's inclusion of architecture among the mathematical arts. [return to text](#)

[21] The connection between syllogism and geometrical reasoning was known since Socrates' times. See [Mueller: 292ff]. [return to text](#)

[22] A good summary is given by [Evans 1957]. See also [Manin 1982]. [return to text](#)

[23] A position like that of Ramus is to this respect symptomatic. On Ramus and French anti-Euclidism see [Bruyere 1984]. [return to text](#)

[24] Guarini gives this topic primary importance. His Euclides begins with *Tractatus I - De quantitate continua* and *Tractatus II - De quantitate discreta*; these topics are treated also in several other parts of the book. In *Placita Philosophica* one chapter deals with *Quantitas* and another with *De continui compositione*. [return to text](#)

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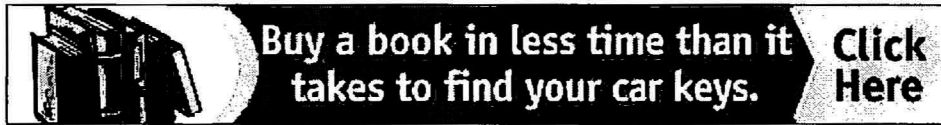
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