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~~Ch III~~ Pythagoras and Pythagoreanism. p 49-84.

A. The Myth: Chaos and Premordial Germ.

B. Mathematics: The Infinite as the Indeterminate.

acknowledged a positive infinity was that of the Megarians, who declared infinity to coincide with perfection and the Good.

The Pythagoreans started in their philosophy from pairs of contrasting principles. In the process of evolution, the developing beings were separated from each other by the void, inhaled as a breath from the surrounding universe. Their theories left room for individual things to exist separately as distinguished from other things. This way of thinking was not shared by Parmenides. With all his energy he stresses the unity and homogeneity of Being: ἐὼν γὰρ ἐόντι πελάζει, being borders on being, there is no void to be inhaled. Seen historically, Parmenides is the last defender of the divine unity of all things. After him the universe, as it were, explodes into a multitude of individual things, even, in Democritus' theory, into atoms.

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CHAPTER III

PYTHAGORAS AND PYTHAGOREANISM

A. THE MYTH: CHAOS AND PRIMORDIAL GERM.

One of the central points in the so-called Orphic theologies was the myth of the world-egg. In an account by Damascius on the central themes of Orphic myth the story runs thus (Kern 54 = DK 1 B 13). In the beginning there was earth and water, or, according to a different reading, earth and slime. From these two came forth never aging Time (Χρόνος ἀγήραος), represented as a winged monster with three heads: a man's, a bull's and a lion's. Time, accordingly, had three names: Χρόνος, Ἡρακλῆς and Ἀνάγκη, i.e. Time, Heracles and Fate, obviously only loosely corresponding to the three heads. Time not only had three names and three heads, but was also flanked by the powers of Ἀνάγκη and Ἀδράστεια, Fate and Ineluctability. These powers serve as expressions of the qualities of the Time-deity himself. It is said that Adrasteia embraced the whole universe, keeping hold of its boundaries (διωργυιωμένην ἐν παντὶ τῷ κόσμῳ, τῶν περάτων αὐτοῦ ἐφαπτομένην). We recognize in this description the qualities of the ancient Time-deity, whose offspring in the history of philosophy was Anaximander's ἄπειρον.

Damascius next says that the Time-deity generated a new triad: moist air (Αἰθῆρ), endless chaos (Χάος ἄπειρον), and dark nether-world (Ἐρεβος ὁμιχλῶδες). In the middle of this Triad the world-egg comes into existence, generated by Χρόνος. This world-egg is hermaprodite (ἀρρενόδηλος). It contains the male and female germs of all things that were developed from the primeval Egg. The further generation of the universe, from the primeval Egg onwards, is again described as a triadic process. We are left with the impression that the account, as Damascius gives it, is to a high degree a mixture of mainly synonymous elements, arranged according to a triadic principle. The triad Ether, Chaos and Erebus e.g. is a triple variation on the same idea of yawning dark Void, filled with unformed matter. In

many other Orphic fragments Chaos and Erebus appear as synonyms, and Ether has a similar function: that of an indefinitely extended space, in which all things are developed. The name of Ether has been introduced, as is clear from other sources, in order to have the necessary scenery for explaining how light came into existence. At all events, the Orphic myth contains the idea of a widely extended cosmic space, in the middle of which a cosmic egg is generated, containing the germs of all future beings.

This idea of a cosmic egg, or variations of it in more philosophical language, is frequently found in our sources for the Orphic theology. The account by Damascius is followed, in Kern's collection, by a parallel passage from Apion (Kern 55). Here the texture of the story is less mythological, as there are no gods having symbolic forms. The formulas are those of a philosophical cosmology. There is an unformed primordial matter moving disorderly in the depths of a yawning abyss. The unformed mass of primordial matter already possesses the animated germs of the four elements (τετραγενής ὕλη). At a certain time the most fertile elements of the mixture were blown by a kind of whirlwind into a central region, where they condensed into a 'critical mixture' (κριτική σύστασις), capable of generating life. When concentrating in this central region, the fertile seeds had drawn with them the surrounding wind (τὸ περικείμενον πνεῦμα), which encompassed it like a divine breath (θειώδης πνεῦμα). The rotating mixture developed a sort of skin (κύτος), and was made pregnant by the encompassing divine breath. The pregnant egg sprang open, and in the greatest light gave birth to a godhead called Phanes.

The divine breath as factor in Orphic cosmogony is also found in a remark of Aristotle in *De anima* (410 b 28 - 29; this passage is quoted by Philoponus and Iamblichus, see Kern 27). Possibly the wording of Aristotle's remark was influenced in some degree by what he knew about the Pythagorean theory, but there can be no doubt as to its contents: "this is also the case with the story told by the Orphic poetry; it says that the soul enters from the universe by a process of inhaling, borne by the winds."

The testimonies, quoted so far, differ in the way they represent the myth. The account by Damascius seems to be a late mixture, incorporating many superfluous elements. The terminology in which Apion presents the theory has been adapted to philosophical use. What Aristotle tells us is very short and essential. The image itself has an archaic character and must have been a current one because

many sources repeat it. There can be no reasonable doubt about its authenticity.

The oldest text in which we hear of the myth of the world-egg is the well-known chorus in Aristophanes' *Birds* (vs. 693 - 702). Here, too, the story is about how the cosmos came into existence, starting from Chaos, Night, dark Hades and broad Abyss. In the endless bosom of dark Abyss, dark-winged Night generated the first pregnant germ, an egg, from which sprang Eros, life-giver:

Ἐρέβους δ' ἐν ἀπείροσι κόλποις
τίκτει πρότιστον ὑπηνέμιον Νύξ ἡ μελανόπτερος ὦν,
ἔξ οὗ . . . ἔβλασεν Ἔρως. (694 - 6)

The elements of the myth are the same: there is the unbounded primordial void, in which the first pregnant germ came into existence; there is the divine breath of life, represented by the adjective ὑπηνέμιον: the primordial egg is said to have been surrounded by wind. The wording of the verse seems to imply that Night generated this primordial egg by wrapping it up in a whirlwind. It is not impossible that Aristophanes added a second meaning, if ὑπηνέμιον ὠϊόν is to be understood as 'wind-egg'. The expression existed in Greek as well as in English, as is proved by *Frogs* 186. In the *Clouds* Aristophanes makes fun of the whole theory by representing the whirlwind in the primordial chaos as a 'God Whirlwind', and playing upon the many vulgar connotations of this idea (*Clouds* 379 - 393). This need not exclude that the fun took its starting-point from expressions in use with the Orphics. In the same work even the orthodox Orphic term ἀήρ ἀπέραντος 'unbounded air', is found. In the same way we can recognize an element of Orphic cosmogony in the adjective ὑπηνέμιον: the primeval germ was surrounded by divine breath and took its vital impulse from it. This is in line with the ancient idea that the soul, as the principle of life, consisted of breath. As has been remarked, no clear distinction existed between air and void.

It must be surmised that the Orphics themselves did not distinguish in their myths the principles which were to be considered essential in contrast to less important elements. This distinction is, in general, not peculiar to mythical thought. The exuberant way in which many synonymous powers and gods have been incorporated into the myths bears witness to this mythical rhetoric. In the course of centuries, moreover, a number of additions have been introduced, which, in our late sources, have been more or less brought into a system. From the standpoint of the more rationalized theories of Greek philosophy, we

may focus our attention on three important elements of Orphic cosmogony: (1) a primordial chaos, in which an unformed mass of material elements moves disorderly; (2) a pregnant nucleus, having the shape of an egg, brought forth by a whirling blast of life-giving breath; (3) life inhaled from the outer void and filling the pregnant nucleus, which, bursting open, gives birth to a winged divinity, called Phanes (Kern 56, 58, 60, 61 and passim), and sometimes Mêtis (Kern 56, p. 135) or Ericapaeus (Kern 60, 65), less often Eros (Kern 28, 74).

The pregnant nucleus or primeval egg draws its vitality from its begetter; who probably for this reason is called by the name of Heracles, the giant of vital power. The dynamism he had infused into his offspring was such as to cause the enormous egg to explode. Its two halves grew into heaven and earth. In Aristophanes' chorus, quoted above, (*Birds*, 690 - 702 = DK 1 A 12 = Kern 1) it is not heaven and earth, but winged Eros who is born from the egg, and who, by making a mixture of all things, brings forth living beings and man. In one of the Orphic hymns (Quandt 6 = Kern 87), the god, born from the egg, called Phanes here, is addressed as *γένεσις μακάρων θνητῶν τ' ἀνθρώπων*, 'begetter of the blissful and of mortal man'. All this means that Phanes or Eros concentrates in himself all generative power. The origin of this vital power is the breath inhaled from the universe. The same idea is hidden in a verse in Aristophanes' *Clouds* (627), where Socrates exclaims:

Μὰ τὴν Ἀναπνοήν, μὰ τὸ Χάος, μὰ τὸν Ἀέρα,
 "by the Deep Breath, by Chaos, by the Airy Space."

The picture we gather from these Orphic sources is completely different from the myth which could be reconstructed from the proverbial sayings, discussed in our first chapter. This myth was the ancestor of the first apeiron-theory, and contained, in essence, a unifying cosmological theory. The evolution of all things and birth and death of all beings were ruled in this theory by one supreme divinity. The philosophical theories which are descended from this myth are all characterized by their tendency to maintain the unity of all things and to describe the unifying principle as supreme and divine. Anaximander declared it to be a law of cosmic and reciprocal justice that all things should return to that from which they originated. In Xenophanes, the ancient conception caused a strong emphasis to be laid on the unity and divinity of the universe, to such an extent that the description of it borders on pantheism. To Parmenides the unity

of Being is a sacrosanct truth, preached with divine authority, and venerated with religious awe.

The Orphic myth represents quite a different way of thinking. To the later rationalizing development the fact that, in Orphic myth, two or three different working powers are described, is of foremost importance. These primordial powers generate the cosmos by their interaction. They are called primordial chaos, pregnant germ, and divine vital power or Eros. It is interesting to find Eros described, in Plato's *Symposium*, as the godhead of poverty, that is as the vital desire for self-expansion, acting in the sphere between nothingness and fulfilment. Even in Aristotle's doctrine of form and matter there is a third principle called *στέρησις* or privation which suggests that ὕλη so to say "desires" for the *εἶδος* which is to give it the state of actual being.

Giving the name of primordial matter to the unordered mixture of the Orphic chaos may cause the impression of anticipating later philosophic development. In many late sources on Orphism this primeval chaos is interpreted as representing the ὕλη of the philosophers. Though there may be a historical connection, the identification of the two is an anachronism. When describing Presocratic theories, the term ὕλη or matter in the technical sense of Aristotelean philosophy must be avoided altogether. There is, however, in modern linguistic use a non-technical sense in which the word matter may be used to designate the substance of which things are made. In this sense the term 'matter' can be used without implicitly introducing Aristotelean metaphysics. It is, at the same time, the sense in which the concept had existed for many centuries before the dawn of Greek philosophy.

In Hesiod already we find chaos as the origin of men and gods. (*Theog.* 116, Cornford 1912, 66, 70; Cornford 1952, II, 194). The pedigree of Chaos, however, is much older, for it goes back to the Babylonian notion of the primeval ocean, called Tiamat, from which all beings originate. Akin to this Babylonian idea is the Biblical description of *tehôm*, the waters over which hovered the Spirit of God. The Ἑρεβος ὁμιχλῶδες of the Orphics, Hesiod's Chaos, water as the first principle of all things, as Thales took it —, all probably have a common origin with the Babylonian and Biblical myths. (See: Heidel 1942; Solmsen 1950, II; Cornford 1952, 248; Eisfeldt 1952; Hölscher 1953; Guthrie 1957, 17 - 18; Dornseiff 1937 and 1956; Laemmli 1962; Brandon 1963.)

An interesting parallel of these myths is found in certain Phoeni-

cian conceptions, an account of which is found in Eusebius (Eusebii Pamphyli *Evangelicae Praeparationis libri XV*, rec. E. H. Gifford, Oxonii 1903, I 33 c - d = ed. Mras, Berlin 1954, vol. I p. 42-43. The text also in Hölischer 1953, 394).

Τὴν τῶν ὅλων ἀρχὴν ὑποτίθεται ἀέρα ζοφώδη καὶ πνευματώδη, ἣ πνοὴν ἀέρος ζοφώδους, καὶ χάος θολερὸν ἐρεβώδες. Ταῦτα δὲ εἶναι ἀπειρα, καὶ διὰ πολλὸν αἰῶνα μὴ ἔχειν πέρας. "Ὅτε δέ, φησὶν, ἡράσθη τὸ πνεῦμα τῶν ἰδίων ἀρχῶν καὶ ἐγένετο σύγκρασις, ἣ πλοκὴ ἐκείνη ἐκλήθη πόθος. Αὕτη δὲ ἀρχὴ κτίσεως ἀπάντων. Αὐτὸ δὲ οὐκ ἐγίνωσκε τὴν αὐτοῦ κτίσιν· καὶ ἐκ τῆς αὐτοῦ συμπλοκῆς τοῦ πνεύματος ἐγένετο Μῶτ. Τοῦτ' οὖν τινὲς φασὶν ἰλύν, οἱ δὲ ὑδατώδους μίξεως σῆψιν. Καὶ ἐκ ταύτης ἐγένετο πᾶσα σπορά κτίσεως, καὶ γένεσις τῶν ὅλων. Ἦν δὲ τινα ζῶα οὐκ ἔχοντα αἰσθησιν, ἐξ ὧν ἐγένετο ζῶα νοερά, καὶ ἐκλήθη Ζωφασημίν, τοῦτ' ἔστιν οὐρανοῦ κατόπτται. Καὶ ἀνεπλάσθη ὁμοίως ῥοῦ σχήματι. Καὶ ἐξέλαμψε Μῶτ, ἥλιός τε καὶ σελήνη, ἀστέρες τε καὶ ἀστρα μεγάλα.

"As the first principle of all things he posits a misty air moved by a breath, or also a breath of misty air, and chaos as a turbid and dark abyss. He says all this is unbounded and has no end throughout eternity. When this breath, as he says, fell in love with its own first principles and a mixture came about, the intertwinement received the name of love's desire. This was the starting-point for the building-up (or: the creation) of all things. It did not know its own building-up. From this intertwinement of breath arose Moot. Some call this slime, some a fermentation in a watery mixture. From this fermentation originated every germ of the building-up of the universe, and the origin of the whole. There existed a number of living beings, without any perception, from which spiritual beings originated, and these were called Zophasemin, that is contemplators of the heavens. It was moulded just like the shape of an egg. Then from it emerged shining Moot, sun and moon, stars and great heavenly bodies."

As the source from which he got his information, Eusebius mentions a certain Philo of Byblos. Under the name of this author no other writings have been preserved, which means that it is extremely hard to assess his reliability. The account as quoted above seems somewhat muddled and mixed-up, as if it were a compilation from various sources. The Greek expressions Philo used have caused a considerable distrust of its authenticity. One of the earliest critical studies on the subject of this Phoenician mythology was published by Ernest Renan in 1858. It was not until the excavations at Ras Shamra that Eusebius' account regained a good deal of its reliability.

The Ugaritic texts, found at Ras Shamra proved the existence of very old mythological texts in this same style, and also their parallelism with the Phoenician mythology given by Eusebius. Due to these discoveries, an account given by Eudemus and preserved in a passage of Damascius (Eudemus ed. Wehrli fr. 150) has gained a background. In this text of Eudemus we find a comparison between the mythology of the Greeks, the Babylonians and the Phoenicians. In what Eudemus says about the 'mythology of the Sidonians' there are very close parallels with the extracts from Philo of Byblos given by Eusebius. Among other things we find here three principles of Being: Χρόνος, Πόθος and Ὀμίχλη, Time, Desire and Mist. When Desire and Mist combined, airy Breath originated. To this account by Eudemus Damascius adds his own information, taken directly from Phoenician sources: from airy Breath new beings came forth, among which there was a heavenly egg. When this egg burst open, heaven and earth came into existence.

However confused and complicated these genealogies may be, this much is clear that many elements of the so-called Orphic theology are akin to elements of eastern, and more especially Babylonian, resp. Phoenician mythology. The most important are: the dark abyss of unbounded chaos, the egg as the first germ of vital power, the breath inhaled as a generating principle, and the part played by Eros in the development of the universe. It is worth noticing that in Eusebius' account the primeval mixture is indicated as a 'watery mixture' (ὑδατώδης μίξις), or 'slime' (ἰλύς), probably the same kneadable material out of which, in the story of *Genesis*, man was moulded and called to life by the Spirit of God.

The Pythagorean theories are in many respects a duplicate of the Orphic traditions. Cornford (1912, 198) even regards Pythagoreanism as a reformation of Orphism. Their doctrines coincide practically on two points: the immortality of the soul, conceived as a rebirth after death, and the way they represent the cosmic evolution. Mathematics was probably a specially Pythagorean branch of activity.

As early as in antiquity the affinity of the Pythagorean to the Orphic doctrines has been noted. Herodotus mentions both of them, when describing an Egyptian taboo on clothing (II 81). The soul's migration, sacred to the Orphics, is one of the Pythagorean doctrines of which we have early evidence, viz. in Xenophanes' verses (DK 21 B 7) where the story is told of the wailing dog, in whose voice Pytha-

goras recognized the voice of a deceased friend. Plato fairly often quotes Orphic and Pythagorean theories, though the theories are not always indicated as Orphic or Pythagorean. He seldom says anything about Pythagoras himself. The name of Pythagoras occurs only once (*Rep.* 600 B). When talking about the Pythagorean doctrine, Plato generally says that the 'Pythagorean way of Life' (ὁ Πυθαγόρειος τρόπος τοῦ βίου) implied this or that consequence, or he indicates the adherents of this doctrine by the name of 'the Pythagoreans', οἱ Πυθαγόρειοι (e.g. *Rep.* 530 D). Sometimes he even restricts himself to the indication 'this is a theory handed down as a secret doctrine' (ὁ ἐν ἀπορρήτοις λεγόμενος λόγος), e.g. when Socrates in the *Phaedo*, describes the Pythagorean convictions of the Thebans Cebes and Simmias (62 B). From the way Plato quotes Pythagorean theories it is obvious that he is speaking about the Pythagoreans he knew in his own days, not about Pythagoras himself or the first school of his adepts. Nevertheless, we may safely assume that the theories mentioned, especially the theory of the soul's migration, and the purity-rites and taboo-precepts connected with it, were part of the early doctrine of Pythagoras' school. It is mainly their affinity with the Orphic doctrines which assures their being ancient.

Plato's statements on the Orphic doctrines can generally be identified more easily than his accounts on Pythagorean theories, because more often he mentions the Orphics by name. Plato does so, when he is quoting verses from Orphic sources, by saying that οἱ ἀμφὶ Ὀρφέα 'the adepts of Orpheus', or οἱ Ὀρφικοί, 'the Orphics', were the authors of the verses or the proverb (e.g. *Crat.* 400 C, *Laws* 782 C). Many times, however, the Orphics are also hinted at indirectly. In these cases Plato introduces his account with formulas such as 'according to an ancient theory' (παλαιὸς λόγος e.g. *Phaed.* 70 C, *Ep.* VII 335 A), or 'in the initiation-rites' (οἱ τὰς τελετὰς κατὰστήσαντες *Phaed.* 69 C). A passage such as *Crat.* 402 B shows that Plato had a fairly specified knowledge of these rites and doctrines. In the *Apology* Socrates professes his belief in immortality in formulas of the Orphic creed (cf. de Vogel 1936, 61). Of all the numerous passages in which Plato quotes Orphic doctrines, a good many are devoted to the belief in a life hereafter, and, in that connection, to the doctrine that the soul is to be purified by practices of abstinence and purificatory rites. It is this class of doctrines of which we have the most conclusive evidence that the very earliest Pythagoreans had also accepted them (cf. Guthrie 1962, 157-169).

There is a special problem which, for the moment, must be left out here; that is the problem of how far Plato's mathematical doctrines, and also some of his scientific views in the *Timaios*, were his own or had been borrowed by him from the Pythagoreans. This is a very important problem, because in Plato's later philosophy the mathematical method of thinking is to such a high degree interwoven with his cosmological theories and sometimes with his metaphysical theories as well. It is difficult to draw a line between the historical development leading up to Plato's theories, and Plato's own special way of making use of the material that was handed down to him by tradition. The most extreme hypotheses on this point have been worked out by Burnet (G.P.), Taylor (1928) and Frank (1923) (cf. de Vogel 1936, 23 - 31, 116 - 118, 213 - 216).

It is, however, the cosmological myth which interests us at this moment. The oldest forms of Pythagorean and Orphic cosmology are based on a small number of first principles which, in their later rationalized form, grew into the principles of a dualistic cosmology, in which form was opposed to matter. This dualistic theory stands in contrast to the monistic doctrines of the oldest Milesian tradition. The roots of the dualistic cosmology go back to Phoenician and Babylonian mythology.

The Pythagorean and Orphic movements must have been manifestations of a broad religious current spreading over the whole Mediterranean in the sixth century B.C. Evidence of the Orphic movement is found in almost every region where Greek was spoken. One of the more active centres of Orphism must have been South-Italy. This is suggested by the numerous allusions to Orphism in authors living there, such as Empedocles (cf. Jaeger *Theol.* 143 and Rostagni 1924, 183-247) and also by the fact that several times Plato makes allusion to Sicilian or Italian sources when talking about Orphic theories. The most eloquent evidence are small golden plates with Orphic inscriptions, found in towns of *Magna Graecia*, which is in the country where Pythagoras lived (DK 1 B 17 from Petelia, DK 1 B 18, 19, 20 from Thurii). As we have found convincing parallels between Pythagorean and Orphic myths, these findings by themselves confirm the view that both movements were branches of that large complex of religious convictions which emerged in so many Greek towns in the sixth century. Orphism provides us with the link between Pythagorean cosmology and Babylonian myth. A number of elements in the Orphic theology presents so striking a resemblance to these

eastern myths that it is hard not to believe in an eastern origin. An example is the history of *χάος* discussed above, and also the many heterogeneous divinities, described in Orphic sources, and reminiscent of Babylonian monsters (e.g. DK 1 B 13).

The pedigree of this mythological cosmology is of special importance when we ask for the origin of certain cosmological theories, which are positively ascribed to the Pythagoreans by Aristotle. They most clearly show the old ground-pattern of a Chaos and a primordial vital nucleus. In terms of later date this nucleus is indicated by Aristotle as "the One, or the faces of the solids, or the Sperm, or principles even they themselves cannot account for". Aristotle's obvious irritation in his description of their wild theories once more guarantees the authenticity of the description (see Ross' commentary ad *Met.* 1091 a 13 - 18).

Οἱ μὲν οὖν Πυθαγόρειοι πότερον οὐ ποιοῦσιν ἢ ποιοῦσι γένεσιν οὐδὲν δεῖ διστάζειν· φανερώς γὰρ λέγουσιν ὡς τοῦ ἐνὸς συσταθέντος, εἴτ' ἐξ ἐπιπέδων εἴτ' ἐκ χοιῶς εἴτ' ἐκ σπέρματος εἴτ' ἐξ ὧν ἀποροῦσιν εἰπεῖν, εὐθύς τὸ ἐγγιστα τοῦ ἀπείρου [δτι] εἴλκετο καὶ ἐπεραίνετο ὑπὸ τοῦ πέρατος (*Met.* N 3, 1091 a 13 - 18).

"Now there is no need to be ambiguous about the question whether the Pythagoreans did or did not introduce becoming; for, as everyone may convince himself, they say that when the One had been formed, whether from planes or surfaces or sperm or from principles even they themselves cannot account for, instantly the nearest parts of the Infinite were drawn in [as a breath] and defined by the definite."

Εἶναι δ' ἔφασαν καὶ οἱ Πυθαγόρειοι κενόν, καὶ ἐπεισιέναι αὐτὸ τῷ οὐρανῷ ἐκ τοῦ ἀπείρου πνεύματος ὡς ἀναπνέοντι καὶ τὸ κενόν, ὃ διορίζει τὰς φύσεις, ὡς ὄντος τοῦ κενοῦ χωρισμοῦ τινὸς τῶν ἐφεξῆς καὶ διορίσεως (*Phys.* VI, 213 b 22 - 26).

"The Pythagoreans also said there was a void, and that it entered heaven from the unbounded breath, because the heaven inhaled this void, which sunders the several natures. In this they started from the supposition that the void is a kind of separation of and a boundary between things that come next to each other."

As always, Aristotle does not speak of Pythagoras, but of 'the Pythagoreans', and these must have been either the Pythagoreans that were his contemporaries or the Pythagoreans whose writings Aristotle had read. By itself, Aristotle's account gives us no clue as regards the antiquity of the doctrine. Once we know, however, the

contents of the theory, we can see from a remark by Xenophanes that it goes back to the earliest generations of Pythagoreans. Diogenes Laertius (IX 19 = DK 21 A 1) states that, according to Xenophanes, "the Godhead is spherical and sees and perceives with all of itself, but that it does not inhale." The spherical figure belongs to the tradition of the all-embracing heavenly deity, which was discussed in our first two chapters. The addition 'that it did not inhale' calls for discussion. The best interpretation seems to be, that these words were a reaction of Xenophanes to the current Pythagorean cosmology. If this interpretation is sound, the words must be more or less authentic, because in later centuries they would have sounded superfluous. They only acquire their full sense if Xenophanes had opponents who held a theory of a breathing universe. This seemingly strange theory was found with the Pythagoreans only. It was a point that was likely to arouse the criticism of thinkers of the type of Xenophanes and Parmenides, who saw a divine unity in the universe and absolutely rejected the view that this divine world had developed from a first germ like a living being.

In this theory of the breathing universe three points are of importance. (1) The evolution of the cosmos starts with a primordial unity, which is opposed to an undetermined Void. (2) The first living nucleus in the Pythagorean theory developed by breathing. The breath it inhaled is called, in Aristotle's account, *ἄπειρον πνεῦμα*, 'boundless breath', or simply *τὸ ἄπειρον*, 'the Unbounded'. As we saw above in the discussion on Parmenides (p. 40-41), a very ancient conception implied that breath, air and void were identical. When speaking of Anaxagoras, Aristotle informs us of an experiment intended to prove that air was a thing. A wine-skin was filled with air and closed. The air in the skin then offered resistance (*Phys.* 213 a 26). Aristotle adds the remark that "people stick to the view that where no bodies can be perceived there is empty space; supposing all existing things to be bodies, they say that void is that in which there is absolutely nothing; this amounts to saying that what is full of air is empty." In two other places (*De anima* 419 b 34, *De part. an.* 656 b 15) Aristotle remarks that 'according to popular thinking' (*δοκεῖ, τὸ καλούμενον*) the air is a void (cf. Burnet EGP 74 n. 2, 109 n. 1).

(3) The air or void which is inhaled does not only serve as the principle of growth for the living being, but at the same time it has the function of separating individual things, thus making the universe discontinuous. This is expressed in the words *διορίζει τὰς φύσεις* (*Phys.* 213 b 24). In this expression the subject of the clause is *τὸ κενόν*: the

void brings separation between beings. This may give rise to the question whether the void is to be considered the delimiting, and, consequently, form-giving principle. Probably the early Pythagoreans did not make this sharp distinction, formulated in concepts of Aristotelean metaphysics. First of all, the fragment taken from the *Metaphysics* seems to convey the opposite meaning: the void inhaled was itself delimited by the *πέρας*. The quotation is not to be interpreted in terms of the Aristotelean distinctions. Burnet (EGP 108) pointed out that the question is unnecessary, because *διορίζει* simply means: 'keeps the units apart from each other'. This is also made clear by the expression *χωρισμοῦ τινος*: the void represents the interval between things. A further confirmation of this is found in a fragment from Aristotle's work on the Pythagorean philosophy (DK 58 B 30), where we read that "the void causes separation between the fields, occupied by things", *διορίζει τὰς χώρας*, more or less in the sense that the void is the principle of discontinuity. The term *διόρισις* must be understood in the sense that the void is the cause of the universe being discontinuous. The void is the principle by virtue of which the Pythagoreans, though in a somewhat primitive way, tried to ensure the well-delimited individuality of things.

Summing up, we must say that, as far as we can discern, the earliest Pythagoreans admitted two different kinds of oppositions in the construction of their cosmology: the opposition of Chaos and primordial germ, and the opposition of *ἄπειρον* and *πέρας*. The two oppositions have a certain parallelism, though a rather vague one. Chaos, identified with infinite space or void, surrounding the universe, is described as awakening the primordial germ to life. Thus infinite void or breath, or, in the Greek word, *πνεῦμα* = spirit, becomes the determining factor, inhaled by the developing germ. The parts of determining and determined factors are not as clearly distributed as might be desired. Though standing in contrast to the infinite void, the developing germ also has in itself a determining factor. As a germ it contains a principle of form which will work itself out into a developed constitution from the moment when the life-giving breath or spirit intervenes. It is as if, in terms of later philosophy, the germ bears within itself the innate idea, which is going to express itself in the special characteristics of the fully developed being. The distribution of the parts becomes more clearly marked when the opposition of *ἄπειρον* and *πέρας* is added as a parallelism. As we shall see in the

eighth chapter, the theory of *ἄπειρον* and *πέρας* in Plato's *Philebus* was probably the nearest preparation to Aristotle's hylemorphism. In this dialogue, Plato uses the term *ἄπειρον* not in the Anaximandrian sense, but strictly as the opposite of *πέρας*, thereby introducing the Pythagorean pair of opposites as a starting-point for his ontological theory. The *ἄπειρον* is the indeterminate principle, *πέρας* is the delimiting factor. It seems not unlikely that, in the early Pythagorean theory, the two sets of opposites, on the one hand Chaos and primordial germ, and on the other hand *ἄπειρον* and *πέρας*, may have been viewed as embodying the same principle of explanation. The interaction of indeterminate void and living nucleus produced the evolution of the universe, just as the interaction of *ἄπειρον* and *πέρας* produced the special characteristics of every individual thing.

The great importance of Pythagorean philosophy to the history of Greek thought lies in the fact that it introduced dualistic principles in order to explain the qualities of things and the evolution of the cosmos. The starting-point of this philosophy is in direct opposition to the Ionian way of explaining the universe. In the Ionian view, an all-embracing godhead ruled all cosmic life. The Ionian tradition implies a monistic tendency, because the whole universe forms a unity of interplaying forces, in which the first principle of life and being is omnipresent and omnipotent. This unity of all things, therefore, was invested with divine majesty. The philosophers who followed this tradition have emphasized different aspects of this one principle of unity, but they all show the same religious veneration in describing and defending it. It is not saying too much that the introduction of an undetermined void as an antagonistic principle in the production of individual beings must have horrified the philosophers whose thoughts were penetrated by the consciousness of the divine unity of the cosmos. In the verses of Parmenides we are, as it were, eye-witnesses to Parmenides' reaction to this horror: the new theory is banished with all force as an impiety. Perhaps we may add: as a philosophic impiety, because the rejection has its ground in the argument of salvation for human reason, rather than in religious awe of a highest divinity: *ἀλλὰ σὺ τῇσδ' ἀφ' ὅδου διζήσιος εἶργε νόημα*, "but thou, keep thy thought away from this course of investigation" (fr. 7, verse 2). Parmenides speaks as a philosopher, but nevertheless he preaches, and he does so with the fervour of an apostle. The old awareness of divine omnipresence and cosmic unity has maintained its sway over his thinking.

The dualism described here is a capital point in the history of the concept of ἀπειρον. From the moment the Pythagorean influence comes into play, this concept carries with it an inner contradiction. This contradiction can be traced to its historic origins. In the Ionian philosophy the ἀπειρον is the very positive infinity of an all-embracing divine power, uniting all beings, and principle of their existence and unity. Due to the Pythagorean influence the term acquired the negative meaning of cosmic void and indeterminate principle, cause of the discontinuity of things and of their being a multitude. It is this latter signification which develops widest in the following centuries. In a mathematical sense it is present in the concept of 'measurable field', in cosmology it appears as the empty space offering room (χώρα) for coming into existence to all things, metaphysically it is the undetermined substrate (ὑποδοχή) for all things, necessary to, though not participating in, their existence. These three significations can be observed in Plato's work. The measurable field plays its part in the construction of the groundpatterns of the elements as regular solids. Empty space and undetermined substrate are deeper and more general foundations for the theory by which Plato attempts to explain the nature and existence of matter.

As in the case of the development which led from the ancient Time-deity to the philosophical concept of ἀπειρον in its Ionian sense, so in the development of the Pythagorean ideas a certain stage was reached at which the concepts lost their mythical contents and grew into metaphysical theories. Aristotle describes the Pythagorean theories (*Met.* 986 b 2 - 3 and 987 a 13 - 19) as if from the very start the concepts of ἔν, ἀπειρον and πέρας had been conceived as ontological principles. This may have been influenced by Plato's theories on these points. In the case of the Pythagoreans, the actual border-line of the transition from myth to metaphysics cannot be identified with all clearness. It seems not impossible that it was Plato himself who rationalized the ground-patterns of Pythagorean thought for the first time to such a degree that they could form the starting-point of a coherent metaphysics.

The importance of the Pythagorean theories to our investigation is that they provide us with the missing link between the development leading up to the Aristotelean hylemorphism and its mythical ancestors. Aristotle's theory was prepared by Plato's theory on ἀπειρον and πέρας, and probably also the μὴ ὄν-theory of the *Sophistes*, as well as the theory of χώρα of the *Timaeus*, played their part in suggesting to

Aristotle his doctrine of hylemorphism. As it seems, the most direct preparation lay in the πέρας - ἀπειρον doctrine of the *Philebus*, which has its roots in Pythagoreanism. Thus, we may assume that, with a certain number of intermediary stages, the Aristotelean hylemorphism was a descendant of the Pythagorean doctrines. These, in their turn, go back to the Eastern myth, in which the infinite Void or Chaos was contrasted to the life-bearing primordial nucleus. In the pedigree of the theory the earliest ancestor of Aristotelean hylemorphism was Babylonian chaos.

B. MATHEMATICS: THE INFINITE AS THE INDETERMINATE

In the first volume of a recent work on the history of Greek philosophy Guthrie gives a comprehensive account of the history and the doctrines of the earliest Pythagoreans. He starts his discussions by noticing that the literature on this subject is 'a bottomless pit' (p. 146). The range of this literature is inversely proportional to the scarcity of our sources on the earliest form of Pythagoreanism, that is to say, the scarcity of sources antedating the year 400 B.C.

There are plenty of later authorities on early Pythagoreanism, because during the Hellenistic period there was an important revival of Pythagorean doctrines. Since Zeller, the tendency has been to reject these later sources as mainly pious legends devised by preachers of Pythagorean morals to meet the needs of the period. It was not until recent times that a new evaluation of the evidence has been attempted. In 1961 Thesleff published his investigation on '*the Pythagorean writings of the Hellenistic period*'. The traditional view persists in the works of Burkert (1962) and Philip (1966), who, in the wake of Frank (1923), tend to regard the tradition on the early Pythagorean theories as mainly an invention of later, 'pythagorizing' philosophers. As such they consider Plato's successors in the Academy, Speusippus and Xenocrates, and, of course, the writers of the Hellenistic period. An unprejudiced account of the evidence was given by Kurt von Fritz in RE (± 1962). In 1966 a work was published by C. J. de Vogel, in which the evidence from the later sources is taken into full consideration.

The abundance of modern investigations makes the problem even more complex. The outlines of the earliest Pythagorean doctrines have to be built up from materials coming to us from widely dispersed sources. Each separate source often calls for a special critical treat-

ment. Uniting the scattered evidence is in itself a complex task but sometimes it produces remarkable results. In such cases the modern investigations often come to have the status of source-material by their own standards. A highly ingenious method of criticism had to be developed in order to deduce a coherent picture of Pythagorean doctrine from this scattered material. It was F. M. Cornford and J. E. Raven who showed great intelligence as well as patience in performing this painstaking work of reconstruction. They built on the foundations laid by John Burnet (EGP and GP). Their work was continued by W. K. C. Guthrie.

One of the special problems concerning early Pythagoreanism is that of Pythagorean mathematics. Direct information on the mathematical doctrines of early Pythagoreanism must, for the greater part, be derived from Hellenistic sources. Certain theories are mentioned by Aristotle whose work on the Pythagoreans is unfortunately lost. It is mostly the archaic character of the theories which determines their being assigned to the early stages of Pythagoreanism. There are no contemporary sources for these early stages. Both Plato and Aristotle when speaking on the subject are, as a rule, discussing theories of the Pythagoreans who were their contemporaries. Nevertheless, the authority of the later sources may be relied on whenever the mathematical theorems they attribute to the earliest Pythagoreans confirm this attribution by their archaic character.

The decipherment of Babylonian texts has brought to light to what extent the earliest Greek geometers borrowed their techniques from Babylonian tradition (cf. van der Waerden 1948, 1966, Neugebauer 1936, 1952). It is no longer possible to think the Greeks were original in developing mathematics. The so-called theorem of Pythagoras had existed in Mesopotamia for more than a thousand years before Pythagoras. Not even the attempt at developing a general form for the demonstration of a theorem can be regarded as properly Greek. A mathematic tradition was transmitted to the Greeks from the East, probably along the same caravan-routes by which trade went east- and westward. The general direction in this migration of techniques and science was from Mesopotamia to Ionia. It seems very probable therefore that Thales in Miletus and Pythagoras on Samos acquired rather extensive mathematical and astronomical knowledge from this eastern tradition.

Little information about Pythagoras' life has come down to us from the first century after his death, but what we know is sufficient

to confirm the later, more detailed accounts. There are four lines by Xenophanes (DK 21 B 7), in which Pythagoras' doctrine of metempsychosis is ridiculed. Heraclitus criticizes his ambitious way of accumulating knowledge without making the proper use of it in two passages, transmitted to us by Diogenes Laertius. In one of these two passages (DK 22 B 129) he says that "Pythagoras, son of Mnesarchus, surpassed all people in collecting information, and by making a choice from these collections came into possession of wisdom, much learning, and evil practices." The other passage (DK 22 B 40) reads: "Much learning does not impart knowledge, for else Hesiod and Pythagoras would have learnt from it, as well as Xenophanes and Hecataeus."

Xenophanes and Heraclitus may have been late contemporaries of Pythagoras. Their remarks make it clear that Pythagoras advocated the doctrine of metempsychosis, and that he occupied himself very extensively with the then available science. The way he did so was vulgar in Heraclitus' eyes. There is a fragment of Empedocles (DK 31 B 129), probably referring to Pythagoras and showing the same tendency.

After Xenophanes and Heraclitus comes Herodotus (II 81, II 123, IV 95 - 96). Herodotus may have taken his information from sources contemporary to Pythagoras, but at the same time his way of telling the stories suggests that Pythagoras must have become the subject of myths soon after his death. Of the passages quoted II 123 does not mention the Pythagoreans. It contains, however, the same theory about the transmigration of souls which we know to have formed part of the Pythagorean doctrine, and even has the detail that there is a return of the soul in 3000 years' time, just as Plato has it in his myth (*Phaedrus* 249 a). Herodotus says he does know the names of those Greeks who are adherents of this doctrine, but he does not want to write them down. This may be due to the fact that, in his later years, Herodotus lived in South-Italy, where the Pythagorean sect was eager to keep its traditions secret. In II 81 Herodotus mentions a taboo on wearing woollen clothes, observed by the Egyptians, adding that "in this respect they have the same traditions as the followers of Orpheus and Dionysus, who, in fact, are Egyptians and Pythagoreans." Leaving aside the question whether some Pythagorean convictions really originated in Egypt, we may gather from this account by Herodotus that the Pythagoreans were known to Herodotus as a sect observing certain religious taboos. The story in IV 95-96

is about a slave of Pythagoras' named Salmoxis, who migrated from Samos to the barbarian country of the Thracians, taking with him the knowledge and the skill he had learnt from 'Pythagoras the sophist'. By playing tricks on the credulous Thracians he convinced them both of his immortality and of their own. Probably Salmoxis was a Thracian hero or divinity, wrongly associated in the myth with the great name of Pythagoras. The conclusion, if any, to be drawn from the account, is that to the Greeks the name of Pythagoras stood for something between a wisdom-monger playing sophisticated tricks and the founder of a religious sect with rather curious customs. This latter feature may have been the more dominant one in the current tradition about Pythagoras. The irritation of Heraclitus possibly had a real motive if Pythagoras used his knowledge in the manner of a self-confident preacher, head of a mysterious sect and guardian of a number of very exclusive theories. Xenophanes' story about the dog whose wailing voice reminded Pythagoras of a dead friend whose soul had become a dog's soul and now greeted him, may well give the impression that Pythagoras rather liked to show off his peculiar convictions. In later biographies Pythagoras is depicted as the leader of a closed group of initiates much devoted to their master. Outsiders may have gathered the impression that Pythagoras really enjoyed being revered as a master of wisdom.

When we come to Plato there is a special problem in interpreting his statements about Pythagoreans or Pythagorean doctrine. It is difficult to find statements in the texts of the dialogues, which may unequivocally be explained as matter-of-fact information in this respect. The reason for this is the large extent to which Plato found inspiration in Pythagorean doctrines. In general we may assume that they were the doctrines held by the Pythagoreans of Plato's own days. Plato incorporated these doctrines into his own views and, although the general tendencies can easily be identified, it is hardly possible to state where the borrowing from Pythagorean sources ended and Plato's own thinking comes in. Aristotle (*Met.* A 6, 987 b 10 - 26) tries to lay down a criterion for distinguishing Pythagorean from Platonic thinking. Its core is a misapprehension of Aristotle as to the principle of the ἄπειρον which Plato defined as a more-and-less. Though Aristotle did not rightly understand what Plato meant by it, it is beyond any doubt that the theory of ἄπειρον and πέρας of the *Philebus*, and also that of the ἄπειρον as a more-and-less, which is the later form of the same theory, were developed by Plato from Pythagorean principles.

This means, to the present problem, that there was a Pythagorean as well as a Platonic theory on the dualistic principles of ἄπειρον and πέρας. As far as our sources go, it is not possible to draw a clear dividing line between the two.

Plato once mentions Pythagoras' name (*Rep.* X, 600 B), in a passage where he says that Pythagoras, like Homer, showed others the way to culture (ἡγεμὼν παιδείας), and was the founder of a certain 'way of life' by which the Pythagoreans distinguished themselves. In *Rep.* VII, 530 D he expresses his sympathy to the view held by the Pythagoreans (οἱ Πυθαγόρειοι), that astronomy and music are sister-sciences because both are founded on the theory of numbers and proportions. This makes it clear that the Pythagoreans of Plato's days already had a well-developed arithmetical theory, on which they founded their speculations about astronomy as well as music. This, in turn, confirms the view that the Pythagoreans of the first days, and even Pythagoras himself must have occupied themselves with speculations of this order, though we have no contemporary sources to confirm this (cf. v. d. Waerden 1943).

A very important passage is *Philebus* 16 C. It is the page where Plato starts his exposition of the theory of πέρας and ἄπειρον. He does so by saying that this theory must have been 'handed down from the region of the gods through some Prometheus'. This has since antiquity been understood as pointing to Pythagoras. Burkert (1962, 76-81), who is very critical about attributing later theories to early Pythagoreans, has a penetrating discussion of this passage. He analyses the discussion of 16 C together with the one in 23 C where, after a digression, it is resumed with the words: "We said that it was the god who made us aware of the existence of ἄπειρον and πέρας." Burkert distinguishes this as Pythagorean from the two concepts which Plato, according to his view, added to the theory: the 'mixture' and the 'cause'. He then expounds the further development of the theory, marked by the change in terminology from ἄπειρον - πέρας, via μᾶλλον καὶ ἥττον to the ἀόριστος δυνάς of the 'unwritten doctrine', an exposition which may by now be regarded as representing a *communis opinio* of Platonists. For the moment we may leave this further development aside, and notice that Burkert, critical as he is about early Pythagorean theories, must confess that πέρας and ἄπειρον were taken by Plato from Pythagorean sources. We may assume that they belonged to the common stock of Pythagorean doctrines, and that they went back to the first generations of Pythagoreans, probably to the founder of the school himself. More-

over, the Pythagorean character of the theory is confirmed by what Aristotle says about it. It is a curious fact that in the fragments of Aristotle's extensive work on the Pythagoreans the mythical and mystic features, not to say the mystifications, once more seem to dominate. (See the texts in Ross, *Fragmenta Selecta*, p. 129-143). There are, however, a number of passages in the extant works of Aristotle, which confirm beyond doubt that the Pythagoreans of Aristotle's days held certain well-defined mathematical and metaphysical theories. One of these is the theory of ἄπειρον and πέρας. The most remarkable characteristic of this theory is that it was exemplified in mathematical form. The theory of ἄπειρον and πέρας in its oldest form must, in fact, have been linked up intimately with the arithmetical theory of odd and even, περιττόν and ἄρτιον. With the aid of what we know about early Pythagoreanism from later sources (see Ross in his commentaries on the *Physics* and *Metaphysics*) we can fairly adequately reconstruct the Pythagorean reasonings as follows.

In *Metaphysics* A, 986 a 17 - 21 Aristotle says:

τοῦ δὲ ἀριθμοῦ στοιχεῖα τό τε ἄρτιον καὶ τὸ περιττόν, τούτων δὲ τὸ μὲν πεπερασμένον τὸ δὲ ἄπειρον, τὸ δ' ἐν ἐξ ἀμφοτέρων εἶναι τούτων (καὶ γὰρ ἄρτιον εἶναι καὶ περιττόν), τὸν δ' ἀριθμὸν ἐκ τοῦ ἑνός, ἀριθμοὺς δέ, καθάπερ εἴρηται, τὸν ὅλον οὐρανόν.

"The elements of number are the even and the odd, the latter of which is well-determined whereas the former is undetermined (ἄπειρον); the One consists of both these elements (for it is at the same time even and odd), and from the One proceeds number, just as the whole heaven is also numbers, as has been said."

This text is followed by the account of the well-known 'table of opposites'. Our second chapter (see p. 42-43) already called for a discussion of these ten pairs of opposites. The mere number ten arouses suspicion, because it was a sacred number to the Pythagoreans. Tradition in antiquity is not unanimous (see Ross a.h.l.). Simplicius has a list of seven pairs of opposites, Porphyry has six. Even Aristotle's own account implies that the number was not always ten, for he continues with the words ἕτεροι δὲ τῶν αὐτῶν τούτων "others of this same sect hold that the principles are ten, the so-called principles κατὰ συστοιχίαν, i.e. principles ordered in pairs." We may be justified in assuming that, at any rate, those opposites which formed the core of the Pythagorean doctrine must also have formed part of the earliest tradition. The problem of πέρας and ἄπειρον as well as the problem

of the one and many with its formulas of a primitive theory of numbers, were essential to any Pythagorean doctrine.

We may notice that Aristotle's remark on numbers proceeding from the One runs a serious risk of not being matter-of-fact information as to the earliest form of the theory. In later Platonic theory the One indeed was made the origin from which multiplicity emanated, and though this theory was intended metaphysically, it was modelled on number-theory. This may easily have given occasion to project it back to the early Pythagoreans. Not being subject to seeing perspectives, Aristotle may uncritically have adapted his expression to a Platonic way of formulating the doctrine. If the earliest Pythagoreans were dualists they may not have felt inclined to have all things (i.e. all numbers, constituting the universe) proceed from the One. We must ask then how this latter view came to be predominant over the original dualism. A plausible answer is that this was due to the influence of Parmenides, who took great pains to eliminate the element of dispersion and multiplicity, i.e. the Pythagorean ἄπειρον from his world-view. If things really took this course, a last question must remain obscure: were there Pythagorean thinkers who, preceding Plato, already adopted the criticism of Parmenides, or was it Plato himself who first integrated the Parmenidean vision into Pythagorean tradition?

The text quoted earlier from the *Physics* (our p. 58) is followed by the words (213 b 26 - 27):

καὶ τοῦτ' εἶναι πρῶτον ἐν τοῖς ἀριθμοῖς· τὸ γὰρ κενὸν διορίζει τὴν φύσιν αὐτῶν.

"The void is the first among the numbers; for it separates their natures." This can only be understood if we keep in mind that, to the Pythagoreans, one was not a number. The first of the numbers was the number two. Aristotle's statement, then, means that the 'void' is the opposite of the 'one'. This is exactly what we read in Stobaeus' comment on these lines (quoted by Ross a.h.l.): "The heaven is one, and from the unbounded are introduced into heaven Time, Breath and the Void." - Time, Breath and Void must be seen as synonyms of cosmic empty space, because emptiness, air and breath were the same thing, and because the original Time-deity was identical with the outermost cosmic space. It may also be seen from other texts that the Pythagoreans took the heaven to be composed of numbers, e.g. *De caelo* 300 a 15 - 17 and *Metaphysics* 1080 b 18 - 21 where Aristotle says: "The Pythagoreans construct the whole heaven out of

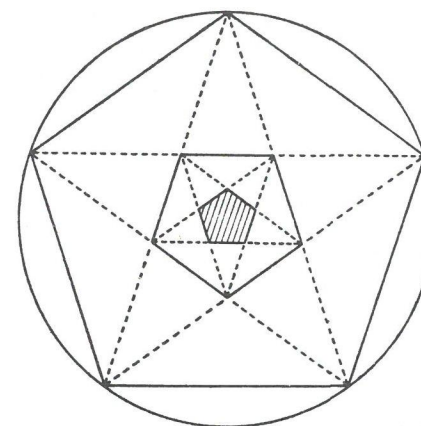
numbers'', adding in his peculiar sneering tone: "When asked how the first One came into being, they seem to be at a loss." This remark runs parallel to that in *Met.* 1091 a 12 - 16. Aristotle disposes of the Pythagorean doctrines as if they were some sort of silly mythology. He fails to notice the capital importance to philosophy of this doctrine of an invisible One as a principle of beings. It was Plato who 'had his eyes open', to use an expression of Aristotle's (*Met.* 986 b 28: $\mu\alpha\lambda\lambda\omicron\nu$ $\beta\lambda\acute{\epsilon}\pi\omega\nu$).

There is another text of Aristotle which tells us how the Pythagoreans treated the concept of $\acute{\alpha}\pi\epsilon\iota\rho\acute{\iota}\alpha$ which in this context clearly means 'indeterminateness.' The text is also important because it shows how the opposition $\pi\acute{\epsilon}\rho\alpha\varsigma$ - $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ and that of $\pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu$ - $\acute{\alpha}\rho\tau\iota\omicron\nu$ were identified.

$\kappa\alpha\iota$ οἱ μὲν τὸ $\acute{\alpha}\pi\epsilon\iota\rho\omicron\nu$ εἶναι τὸ $\acute{\alpha}\rho\tau\iota\omicron\nu$ · τοῦτο γὰρ $\acute{\epsilon}\nu\alpha\pi\omicron\lambda\alpha\mu\beta\alpha\nu\acute{\omicron}\mu\epsilon\nu\omicron\nu$ $\kappa\alpha\iota$ ὑπὸ τοῦ $\pi\epsilon\rho\iota\tau\tau\acute{\omicron}\nu$ $\pi\epsilon\rho\alpha\iota\nu\acute{\omicron}\mu\epsilon\nu\omicron\nu$ $\pi\alpha\rho\acute{\epsilon}\chi\epsilon\iota\nu$ τοῖς οὖσι τὴν $\acute{\alpha}\pi\epsilon\iota\rho\acute{\iota}\alpha$ · σημεῖον δ' εἶναι τούτου τὸ $\sigma\upsilon\mu\beta\alpha\acute{\iota}\nu\omicron\nu$ ἐπὶ τῶν ἀριθμῶν· $\pi\epsilon\rho\iota\tau\iota\theta\epsilon\mu\acute{\epsilon}\nu\omega\nu$ γὰρ τῶν $\gamma\omega\mu\acute{\omicron}\nu\omega\nu$ $\pi\epsilon\rho\iota$ τὸ $\acute{\epsilon}\nu$ $\kappa\alpha\iota$ $\chi\omega\rho\acute{\iota}\varsigma$ ὅτ' $\mu\acute{\epsilon}\nu$ ἄλλο $\acute{\alpha}\epsilon\iota$ γίγνεσθαι τὸ $\epsilon\acute{\iota}\delta\omicron\varsigma$, ὅτ' $\delta\grave{\epsilon}$ $\acute{\epsilon}\nu$ (*Physics*, III 4, 203 a 10 - 15).

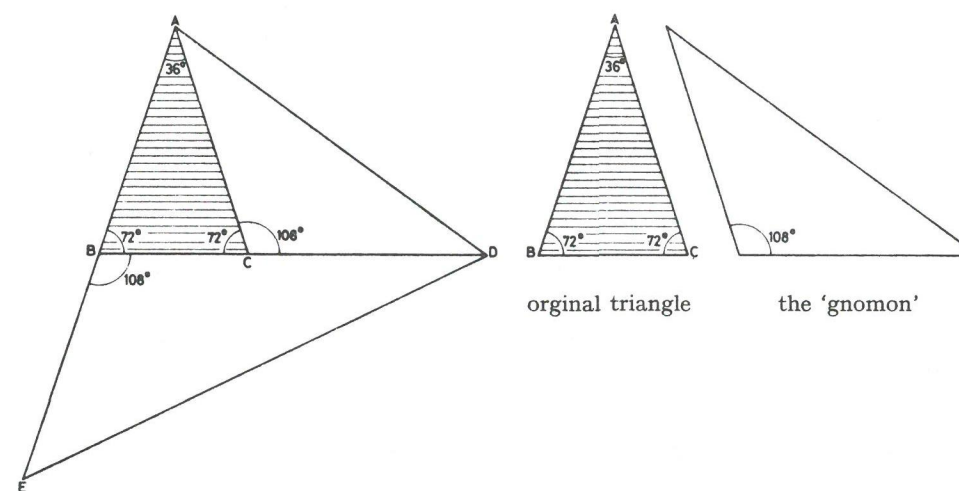
"The Pythagoreans identify the indeterminate and the even; for, they say, when this is taken up ($\acute{\epsilon}\nu\alpha\pi\omicron\lambda\alpha\mu\beta\alpha\nu\acute{\omicron}\mu\epsilon\nu\omicron\nu$) into things and is limited by the odd, it brings indeterminateness to the beings; a proof of this is what happens to numbers; for when the *gnomons* are being laid around the one, or in the other way ($\kappa\alpha\iota$ $\chi\omega\rho\acute{\iota}\varsigma$), in the latter case the figure is constantly changing, in the former it remains the same."

In this text two expressions call for an explanation: $\gamma\omega\mu\acute{\omicron}\nu\epsilon\varsigma$, and $\kappa\alpha\iota$ $\chi\omega\rho\acute{\iota}\varsigma$. The word $\gamma\omega\mu\acute{\omicron}\nu$ means any instrument by which something is marked or by the aid of which it can be recognized. In carpentry it means a carpenter's square. Etymologically the word is derived from $\gamma\iota\gamma\nu\acute{\omega}\sigma\kappa\omega$; the original meaning is 'discerner' (Cf. Heath, *Euclid's Elements*, vol I, 370-372). In the explanation given below it is taken in the meaning of 'carpenter's square', but it has a much wider sense. The use of $\gamma\omega\mu\acute{\omicron}\nu\epsilon\varsigma$ was a characteristic feature of Pythagorean mathematics. The Pythagoreans devoted much attention to geometric diagrams in which a certain characteristic pattern repeated itself. An example of this is the famous pentagram. When in a regular pentagon the diagonals are drawn, the pentagon repeats itself within the intersections of the diagonals.



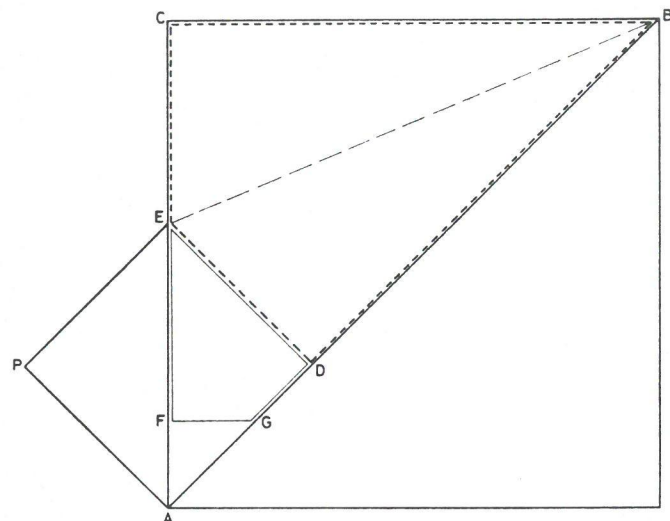
(cf. Becker 1957, 72).

In another way still, this figure offers an example how from a given figure, in this case the triangle with angles 72° , 72° , 36° , a new figure, similar to the original one, can be produced by always adding a similar figure, in this case the triangle 108° , 36° , 36° :



By adding first ACD as a gnomon to the original triangle, we obtain DAB; by adding DBE as a gnomon we obtain EAD. The triangles ABC, DBE and EDA are repetitions of the same ground-pattern. (Naber 1908). The process repeats itself *ad infinitum*.

A third example is given by Becker (1957, 73), and by Heath (1921, I, 208-209; *Transl. Euclid* vol. 3, 19-20). The example is important because, at the same time, it is an instance of the famous method of ἀντανάλξεις or ἀνθυπαίξεις (Eucl. *El.* X 2, cf. v. d. Waerden 1947/9, 689).



In a square the diagonal AB is drawn.
On this diagonal BD is equal to BC
In D a line is drawn at right angles to DB
This line meets CA in E
Then EBC and EBD are congruent, and $\angle AED = 45^\circ$
Therefore $CE = ED = AD$

If we measure AB by BC (= BD = CA), that which is left is AD. Measuring again BC (= CA) by AD, we find that we can subtract AD (= CE = EF) twice from AC. That which is left is AF. This measuring of the larger magnitude by the smaller repeats itself *ad infinitum*.

In the diagram the figure BCED is repeated in the similar figure EDGF. This figure is a kind of gnomon which repeats itself *ad infinitum*. The repetition can be seen more clearly if we draw the square ADEP.

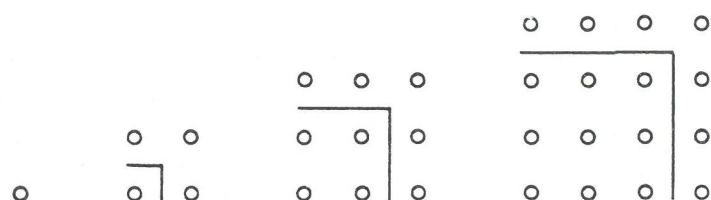
Becker gives this diagram as a possible instance of the method by which the irrationality of $\sqrt{2}$ was proved in its earliest form. At any rate the recurrent figure may have a claim to Pythagorean origin. As regards the method of ἀντανάλξεις, Becker himself seems to imply

in an earlier publication (1936, 550) that this method belongs to a later stage of development, which he calls 'Theodorische Stufe', referring to Theodorus of Cyrene, the mathematician. There is another method, making use of the so-called 'side' and 'diagonal' numbers giving successive approximations to $\sqrt{2}$, which has a better claim to be of an early date (see Heath, *Euclid's Elements*, I 398 - 400, and Taylor in *Mind* 1926, 429-431). Possibly both methods, that of ἀντανάλξεις and the one of the 'side' and 'diagonal' numbers, have had their origins in Pythagorean procedures, as is argued by Heller (1958). The fairest chance of being oldest has the apocryphal Euclidean demonstration of *El.* X 117, which will be discussed at the end of this chapter. Its essential feature is its being founded on the authentically Pythagorean doctrine of even and odd. Heath (*Euclid's Elements* vol. III, p. 2) says about it: "The actual method by which the Pythagoreans proved the incommensurability of $\sqrt{2}$ with unity was no doubt that referred to by Aristotle (*Anal. prior.* I, 23, 41 a 26 - 7), a *reductio ad absurdum* by which it is proved that, if the diagonal is commensurable with the side, it will follow that the same number is both odd and even."

The examples of recurrent figures or gnomons are inserted here because the mathematical method used in these examples is characteristic of the kind of problems with which the Pythagoreans occupied themselves. It is possible that the problem of incommensurability was in its earliest form tackled by the method of ἀντανάλξεις. The fact that this ἀντανάλξεις produced an infinite process in that case may have been taken as proof that the two lines in question were incommensurable. The infinite regressus was visualized in the diagram by the infinite repetition of the same gnomon. The problems caused by the discovery of the incommensurable could at first not be solved adequately, because the necessary theories had as yet not been developed (Heath, 1931, 105). This must have entailed a situation in which the concept of infinity came to be distrusted because problems involving the concept of infinity could not be mastered. The contradictions caused by infinite processes could not be solved by the mathematical methods in use early in the fifth century.

Having seen what the concept of gnomon stood for, we may return to the text quoted above (*Physics* III 4, 203 a 10 - 15). This text indicates two ways of putting gnomons round a given point or set of points. In the first case this method results in a figure which exactly reproduces itself, as should be the case with gnomons. This figure is a square. In the second case the method produces rectangles, but rec-

tangles which at every new step show a changing proportion of the sides. Commentators have variant explanations of the method of the second case, but the fundamental idea seems to be the same. For a discussion of the various comments by Themistius and Simplicius see Ross ad *Phys.* 203 a 13 - 15. In substance the question of the two ways of applying gnomons, distinguished in the text by the indication *καὶ χωρὶς*, amounts to the following arithmetical methods (Cf. Heidel, 1901; Heath, *Euclid's Elements*, vol I, 359; Burnet EGP 103; Raven 1948, 188-194; Michel 1950, 321; Becker 1957, 40; Kucharski 1959).



Starting from the number one and adding the next odd number 3, we obtain a square visualizing the first square number 4. Adding the next odd number 5, we obtain a square visualizing the number 9, and so on *ad infinitum*. This arithmetical method visualizes in geometrical diagrams the principle that every square number is the sum of an uninterrupted sequence of odd numbers starting with 1:

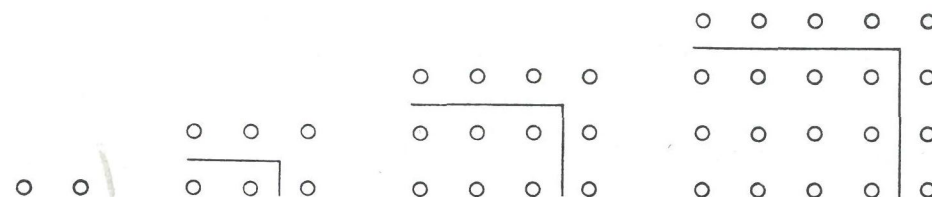
$$\begin{aligned} 1 + 3 &= 4 \\ 1 + 3 + 5 &= 9 \\ 1 + 3 + 5 + 7 &= 16 \text{ etc.} \end{aligned}$$

In the diagram a new square is always produced by adding a gnomon representing the odd number following the odd number of the preceding gnomon. The diagrams visualize the mutual dependence of the sequences:

1	3	5	7	9	11	13
	4	9	16	25	36	49
	2 ²	3 ²	4 ²	5 ²	6 ²	7 ²

The important point is that by adding gnomons the square character of the number is maintained, however far the sequence is made to go on. That is why the Pythagoreans ascribed to the odd numbers the quality of maintaining a figure in its own character. The concepts of 'odd' and 'well-determined' went side by side.

If we take 'the other way' (*καὶ χωρὶς*), starting not from one dot, but from a pair of two dots, we get a surprisingly different result:



The sequence starts from the even number two, and the gnomons added likewise have even values. The diagrams visualize the mutual dependence of the sequences:

2	4	6	8	10	12	14	16
	6	12	20	30	42	56	72
	2 × 3	3 × 4	4 × 5	5 × 6	6 × 7	7 × 8	8 × 9

In this latter form the sequence demonstrates most clearly that the rectangle more and more approximates the form of a square without ever coinciding with it. This observation may have given to the Pythagoreans the first inkling of the properties of infinite sequences. The important thing to them must have been the unstable proportion of the sides of the rectangle. When applying even gnomons to the even number two, the result was an ever-changing diagram, and the changing even went on *ad infinitum*. Infinite processes were evidently linked up with instability, because in their geometrical visualization none of the rectangles was similar to the others. The use of gnomons in this case did not simply lead to infinite repetitions, but to infinite changes of form. The Pythagoreans formulated this by saying that even was in the category of instability, and because this instability went side by side with infinite progression, the result was that evenness, infinity and instability were included in the same category. This is what Aristotle hints at when saying that 'the figure is constantly changing', ἄλλο ἀεὶ γίνεσθαι τὸ εἶδος (*Phys.* 203 a 15, cf. *Simpl. phys.* 456, 16 - 458, 16). Simplicius is more explicit: ἡ δὲ τῶν ἀρτίων περίθσεις οὐ ποιεῖ τὸ σχῆμα ὠρισμένον, "The adding of even gnomons makes the figure unstable" (457, 22). Cf. Taylor 1926.

If our interpretation is right, there can be no doubt that the Pythagoreans occupied themselves with finding methods by which, if one started from one dot or a small number of dots and applied

certain mathematical operations, numbers could be developed which had certain characteristics in common. The statements about this arithmetic come from later sources, but they are confirmed by incidental hints of Aristotle. Moreover, the tradition is practically unanimous on this point. This means that the Pythagorean mathematicians knew the idea of ordering numbers in a sequence obeying to a certain law.

This is an important conclusion to keep in mind when we shall come to Zeno. It is clear that to Zeno the problem of a sequence being continued to infinity and never coming to its end existed. It is present e.g. in the problem of Achilles and the tortoise. Achilles cannot overtake the tortoise unless he first traverses half the distance separating him from the animal, then half the remaining distance and so on, that is, unless he reaches the end of an infinite sequence of halves. Experience showed that this limit was actually reached and even crossed. It is highly probable that these problems were misunderstood by the sophists and later by Aristotle, as if they were meant to show how little our senses can be relied on. In our next chapter we shall discuss in what sense Zeno's paradoxes are to be understood within the context of Zeno's work itself. For the moment we must state that even as early as the Pythagorean arithmetic of the 5th century the existence can be traced of problems about sequences of numbers. These problems demonstrated, among other things, the connection between the concepts of infinity and indeterminateness. In the historical development of philosophy, Pythagorean arithmetic is a junction at which the concept of *ἄπειρον* inextricably becomes mixed up with the concept of instability and indeterminateness. For several generations this latter quality was to dominate every treatment of the concept of infinity. The concept of *ἄπειρον* acquired, metaphysically, a negative ring, only to be replaced by a more positive content when later Platonic thought began speculating about positive infinity.

Our last argument on the Pythagorean treatment of the concept of *ἄπειρον* will start from a discussion of the apocryphal Euclidean theorem of *El. X 117*. It seems possible to show that this theorem can be put in a form in which the *reductio ad absurdum* on which it is based runs parallel to a *reductio ad infinitum*. Before starting on this discussion, we must say something about the date of the first discovery of incommensurability.

It seems that this problem found its origin in the work by Erich

Frank on "*Plato and the so-called Pythagoreans*" (1923). Frank supposed that the whole tradition about early Pythagoreanism had resulted from an endeavour to make the Pythagorean theories seem old in order to confer on them the venerable aura of antiquity. According to Frank, this led to a general tendency of projecting the Pythagorean doctrines back to Presocratic times, a tendency starting with Plato's immediate successors in the Academy. Frank's vision was too rigorous, and now it has practically become an established view that the discovery of irrationality must be put about the middle of the 5th century B.C. Heath thinks that the discovery was made "with reference to the length of the diagonal of a square" (*Euclid's Elements* III 1), which means that the discovery of the irrationality of the division in extreme and mean ratio (which is found in the 'pentagram') did not precede the discovery of the irrationality of $\sqrt{2}$. He decidedly attributes the discovery to the Pythagoreans (*Euclid's Elements* I 351 and 411-414), and thinks that it was made "at a date appreciably earlier than that of Democritus" (1921, I 157). Von Fritz (1945) takes the same line. In *Gnomon* 1958, p. 82 he writes: "Die Entdeckung des Irrationalen wird von Becker ebenso wie von Van der Waerden und anderen mit Recht wieder in die Mitte des 5. Jahrh. gesetzt." The references in this quotation are to the study published by Becker in 1957, of which the article in *Gnomon* was a review, and to Van der Waerden 1948, 153. Van der Waerden thinks that the demonstration of the irrationality of $\sqrt{2}$ must have been found about 450 BC, or at least at a date before 420 BC by a method founded on the Pythagorean theory of odd and even. Burkert, who, at times, is perhaps somewhat hypercritical, thinks it is not certain that the discovery was made by Pythagoreans, and he is of the opinion that the discovery was made gradually and not by way of a shock, as is often supposed (1962, 439). He thinks, however, that the irrationality of $\sqrt{2}$ must have been known before Theodorus of Cyrene proved the irrationality of $\sqrt{3}$ upwards to $\sqrt{17}$, which means not later than about the middle of the 5th century.

An interesting summary of reliable conclusions that can be drawn from the various investigations into this matter is given by Szabó 1956, 136. Szabó mentions several theories which must have existed in full form about the middle of the 5th century. Szabó's discussion takes an unexpected turn when he tries to prove that to the development of abstract mathematical demonstrations the previous development of

an explicit logic was necessary. Szabó points to the rather frequent use in mathematical demonstrations of the *reductio ad absurdum*. He thinks the Eleatics were the first to develop this way of reasoning, and that a fully abstract method for mathematical proofs could not be developed until the Eleatic philosophers had set the example. We are of the opinion that it is not necessary to assume that the mathematicians had to wait for the Eleatics to receive their first training in explicit reasoning (cf. Timpanaro Cardini 1964, 27-36). This is the more unlikely because in Babylonian mathematics already such fine examples of exact reasoning are found (see V. d. Waerden 1966). Moreover, the theories which Szabó himself thinks to be of an early date show a fairly advanced level in the construction of the proofs. This at least leaves us in doubt as regards the supposed Eleatic monopoly of logical thinking.

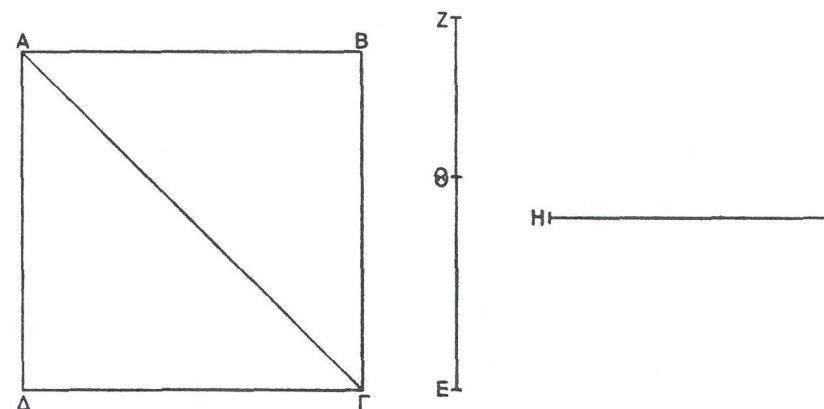
We shall keep to the view that the principle of incommensurability must have been found in the course of the 5th century BC, probably by mathematicians of the generation contemporary to or preceding Socrates. It is possible that the discovery was made by Pythagoreans. If the ascription to the Pythagoreans should seriously be doubted (as Burkert does), then at any rate the Pythagoreans must have taken a keen interest in the discovery. The method of proving the incommensurability of $\sqrt{2}$, as we find it in the addition to Euclid's *Elements* X 117, must, in that case, be regarded as an adaptation to the Pythagorean geometry of numbers. Leaving aside the discussion as to the authorship of the discovery itself, we have, at any rate, a text in which we can see Pythagorean thinking at work. The proof of the theorem seems to presuppose the existence of incommensurable lengths as an established fact. Our point is that the method of demonstration involves a *reductio ad absurdum* in such a form that it coincides with a *reductio ad infinitum*. The text as given by Heiberg is as follows (*Euclidis Elementa* X 117 = ed. Heiberg vol. III p. 408-410, appendix 27). A full translation of it is given by Maria Timpanaro Cardini (1964, 382-387). The proof is summarized in shorter form by Heath, *Euclid's Elements* vol. 3, p. 2, by Ross ad *Anal. Priora* 41 a 26 - 7, and by Becker, 1936, 544 - 5.

Προκείσθω ἡμῖν δεῖξαι, ὅτι ἐπὶ τῶν τετραγώνων σχημάτων ἀσύμμετρός ἐστιν ἡ διάμετρος τῇ πλευρᾷ μήκει.

Ἐστω τετράγωνον τὸ ΑΒΓΔ, διάμετρος δὲ αὐτοῦ ἡ ΑΓ· λέγω, ὅτι ἡ ΓΑ ἀσύμμετρός ἐστι τῇ ΑΒ μήκει.

Εἰ γὰρ δυνατόν, ἔστω σύμμετρος· λέγω, ὅτι συμβήσεται τὸν αὐτὸν

ἀριθμὸν ἄρτιον εἶναι καὶ περισσόν. φανερόν μὲν οὖν, ὅτι τὸ ἀπὸ τῆς ΑΓ διπλάσιον τοῦ ἀπὸ τῆς ΑΒ. καὶ ἐπεὶ σύμμετρός ἐστιν ἡ ΓΑ τῇ ΑΒ, ἡ ΓΑ ἄρα πρὸς τὴν ΑΒ λόγον ἔχει, ὃν ἀριθμὸς πρὸς ἀριθμόν. ἐχέτω, ὃν ὁ ΕΖ πρὸς Η, καὶ ἔστωσαν οἱ ΕΖ, Η ἐλάχιστοι τῶν τὸν αὐτὸν λόγον ἐχόντων αὐτοῖς· οὐκ ἄρα μονὰς ἐστὶν ὁ ΕΖ. εἰ γὰρ ἔσται μονὰς ὁ ΕΖ, ἔχει δὲ λόγον πρὸς τὸν Η, ὃν ἔχει ἡ ΑΓ πρὸς τὴν ΑΒ, καὶ μείζων ἡ ΑΓ τῆς ΑΒ, μείζων ἄρα καὶ ἡ ΕΖ τοῦ Η ἀριθμοῦ· ὅπερ ἄτοπον. οὐκ ἄρα μονὰς ἐστὶν ὁ ΕΖ· ἀριθμὸς ἄρα. καὶ ἐπεὶ ἐστὶν ὡς ἡ ΓΑ πρὸς τὴν ΑΒ, οὕτως ὁ ΕΖ πρὸς τὸν Η, καὶ ὡς ἄρα τὸ ἀπὸ τῆς ΓΑ πρὸς τὸ ἀπὸ τῆς ΑΒ, οὕτως ὁ ἀπὸ τοῦ ΕΖ πρὸς τὸν ἀπὸ τοῦ Η. διπλάσιον δὲ τὸ ἀπὸ τῆς ΓΑ τοῦ ἀπὸ τῆς ΑΒ· διπλασίον ἄρα καὶ ὁ ἀπὸ τοῦ ΕΖ τοῦ ἀπὸ τοῦ Η· ἄρτιος ἄρα ἐστὶν ὁ ἀπὸ τοῦ ΕΖ· ὥστε καὶ αὐτὸς ὁ ΕΖ ἄρτιός ἐστιν. εἰ γὰρ ἦν περισσός, καὶ ὁ ἀπ' αὐτοῦ τετράγωνος περισσός ἦν, ἐπειδήπερ, ἐὰν περισσοὶ ἀριθμοὶ ὁποιοιοῦν συντεθῶσιν, τὸ δὲ πλῆθος αὐτῶν περισσὸν ἦ, ὁ ὅλος περισσός ἐστίν· ὁ ΕΖ ἄρα ἄρτιός ἐστιν. τεμνήσθω δίχῃ κατὰ τὸ Θ. καὶ ἐπεὶ οἱ ΕΖ, Η ἐλάχιστοί εἰσι τῶν τὸν αὐτὸν λόγον ἐχόντων [αὐτοῖς], πρῶτοι πρὸς ἀλλήλους εἰσίν. καὶ ὁ ΕΖ ἄρτιος· περισσός ἄρα ἐστὶν ὁ Η. εἰ γὰρ ἦν ἄρτιος, τοὺς ΕΖ, Η δυὰς ἐμέτρει· πᾶς γὰρ ἄρτιος ἔχει μέρος ἡμισυ· πρῶτους ὄντας πρὸς ἀλλήλους· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα ἄρτιός ἐστιν ὁ Η· περισσός ἄρα. καὶ ἐπεὶ διπλάσιος ὁ ΕΖ τοῦ ΕΘ, τετραπλάσιος ἄρα ὁ ἀπὸ ΕΖ τοῦ ἀπὸ ΕΘ. διπλάσιος δὲ ὁ ἀπὸ τοῦ ΕΖ τοῦ ἀπὸ τοῦ Η· διπλάσιος ἄρα ὁ ἀπὸ τοῦ Η τοῦ ἀπὸ ΕΘ· ἄρτιος ἄρα ἐστὶν ὁ ἀπὸ τοῦ Η. ἄρτιος ἄρα διὰ τὰ εἰρημένα ὁ Η· ἀλλὰ καὶ περισσός· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα σύμμετρός ἐστιν ἡ ΓΑ τῇ ΑΒ μήκει· ὅπερ ἔδει δεῖξαι.



"Let it be required to show that in squares the diagonal is incommensurable in length to the side.

Let a square $AB\Gamma\Delta$ be given, the diagonal being A .

I say that ΓA is incommensurable in length to AB .

For suppose them, if possible, to be commensurable. I say that, then, the consequence is that one and the same number will be even and odd at the same time. Now it is clear that the square on $A\Gamma$ is double of the square on AB [I 47]. ΓA being commensurable to AB , ΓA has to AB the ratio which a number has to a number. Let it have the ratio which EZ has to H , and let EZ , H be the smallest of the lengths which have the same ratio to one another [VII 33]. In that case EZ is not the unit. For if EZ be the unit, and if it have to H the ratio which $A\Gamma$ has to AB and if $A\Gamma$ be greater than AB , then also EZ will be greater than H , [which is: the unit will be greater than] a number; which is impossible.

Therefore EZ is not the unit. Therefore it is a number.

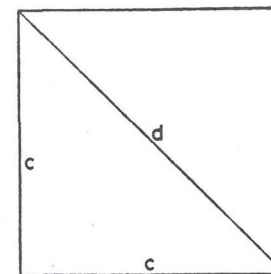
Further, because EZ has to H the same ratio which ΓA has to AB , therefore also the square on EZ has to the square on H the same ratio which the square on ΓA has to the square on AB . But the square on ΓA is double of the square on AB . Therefore also the square on EZ is double of the square on H . Therefore the square on EZ is even; therefore also EZ itself is even. For if it were odd, then also the square on it would be odd, because, if odd numbers be added in whatever multitude and this multitude be odd, the whole is also odd [IX 23]. Therefore EZ is even.

Let it be bisected in Θ . Because EZ , H are the smallest numbers of those which have the same ratio, they are prime to one another [VII 22]. But EZ is even. Therefore H is odd. For if it were even, then the number two would measure EZ and H - for any even number is divisible into two parts - which are prime to one another [VII def. 6 and 12]. Which is impossible. Therefore H is not even. So it must be odd.

And because EZ is double of $E\Theta$, the square on EZ is four times the square on $E\Theta$. But the square on EZ is double of the square on H . Therefore the square on H is double of the square on $E\Theta$. Therefore the square on H is even, and therefore also, for the reasons given above, H is even.

But H is also odd. Which is impossible. Therefore ΓA is incommensurable in length to AB . Q.E.D."

Following the proof step by step we obtain in modern notation:



$$d^2 = 2c^2 \dots \dots \dots (a)$$

d en c are commensurable in whole numbers $\dots \dots \dots (b)$

From (a) and (b) together it follows that d^2 is even, therefore d is even. $\dots \dots \dots (1)$

If a square is even, it is divisible by 4

Therefore also the half of an even square is even

Therefore $\frac{1}{2} d^2$ is even $\dots \dots \dots (c)$

From (a) follows that $\frac{1}{2} d^2 = c^2$

If $\frac{1}{2} d^2$ is even, then also c^2 is even, therefore c is even. $\dots \dots \dots (2)$

If c and d are measurable in whole numbers, their ratio to one another can be reduced to whole numbers which are prime to one another, e.g. $\frac{d}{c} = \frac{p}{q}$ in which p and q are prime to one another. This means that, if p be even, q will be odd and the reverse. The same then holds for $\frac{d}{c}$. Therefore, if we suppose that d is even, c will be odd. $\dots \dots \dots (3)$

(2) and (3) cannot be true at the same time.

The text of this demonstration suggests that its form has been adapted to the structure of Euclid's work, from which several theorems are quoted almost literally (noted in our translation by square brackets). The substance of the proof, however, is clearly Pythagorean, and it must, in its general line, have existed for generations before it found its way as an apocryphal addition to the manuscripts of the *Elements*. Of its early existence we have a reliable testimony in a casual remark of Aristotle (*Anal. Priora* 41 a 26 - 27) who adduces it as an instance of the *reductio ad absurdum*:

Ολον εστι ασύμμετρος ή διάμετρος δια τὸ γίνεσθαι τὰ περιττὰ ἴσα τοῖς ἀρτίοις συμμέτρου τεθείσης.

"E.g. that the diagonal is incommensurate for the reason that odd will be equal to even if it is supposed to be commensurate."

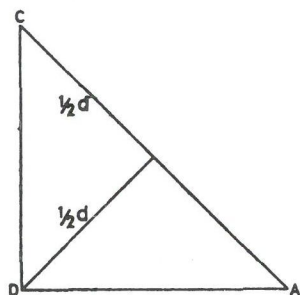
A parallel remark is found 50 a 37 - 38:

οἷον τεθείσης τῆς διαμέτρου συμμετρου τὸ τὰ περιττὰ ἴσα εἶναι τοῖς ἀρτίοις,

"e.g. the conclusion that, if the diagonal is supposed to be commensurable, odd numbers will be equal to even."

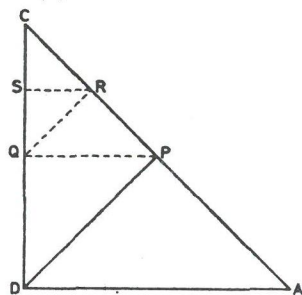
The reasoning in the proof starts from representing length as a certain amount of numbers. This way of representing has an archaic ring, and is completely in line with the way the theory of numbers is treated in *Elements* IX 21 - 34. In fact, Becker has shown these chapters to be authentically Pythagorean. In our text even a theorem is used, which in its explicit form is found in *Elements* IX 23. It is the theorem that numbers of the type $(2n + 1)(2p + 1)$ are always odd.

It is possible to put the proof into a form in which a recurrent figure illustrates the absurdity of the supposition.



$$d^2 = 2c^2 \text{ therefore } d^2 \text{ is even: } d \text{ is even} \dots \dots \dots (1)$$

$$\text{in CPD: } c^2 = 2\left(\frac{d}{2}\right)^2 \text{ } c^2 \text{ is even: } c \text{ is even} \dots \dots \dots (2)$$



$$\text{in PQC: } 2\left(\frac{c}{2}\right)^2 = \left(\frac{d}{2}\right)^2 \quad \left(\frac{2}{d}\right)^2 \text{ is even} \quad \frac{d}{2} \text{ is even} \dots \dots (3)$$

$$\text{in QRC: } 2\left(\frac{d}{4}\right)^2 = \left(\frac{c}{2}\right)^2 \quad \left(\frac{c}{2}\right)^2 \text{ is even} \quad \frac{c}{2} \text{ is even} \dots \dots (4)$$

$$\text{in RSC: } \frac{d}{4} \text{ is even} \dots \dots (5)$$

etc.

This means that by drawing perpendiculars DP, PQ, QR, RS etc. not only d and c turn out to be even at the same time (which is another way of saying: c is even and odd at the same time), but also both d and c are subdivided into halves which will remain even *ad infinitum*. Put into this form the demonstration offers the same characteristics which could be observed in many other Pythagorean problems. By repeating a similar operation a similar figure is obtained. The repetition of the similar figure offers an inexplicable contradiction and this inexplicable contradiction repeats itself *ad infinitum*. It seems plausible to suppose that, if the Pythagoreans were not yet able to master the problem of these infinite repetitions satisfactorily, they may have felt a certain distrust of dealing with infinitesimal problems, and, generally, a distrust of any actual infinity. This distrust of actual infinity is not a supposition *ad hoc*, for we see it still at work in Aristotle's treatment of infinity in *De caelo* I. From the Pythagoreans onwards we observe a fairly uniform tradition in philosophical thinking as regards the concept of *ἄπειρον*. The old lofty conception of an actual divine Infinity is superseded by a way of thinking in which the *ἄπειρον* is seen as the opposite of *πέρας*. Accordingly, it is placed in a negative category, and the signification of indeterminateness prevails. This dyadic way of thinking originated with the Pythagoreans. It is present in Plato's theory of *πέρας* and *ἄπειρον* as expounded in the *Philebus*, and in Aristotle's doctrine of hylemorphism, in which *ὕλη* plays the part of the indeterminate principle.

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| Cornford 1912 | Cornford 1952, II |
| Cornford 1926 | Vlastos 1952 |
| Guthrie 1934 | van der Waerden 1953 |
| Nilsson 1935 | Prümm 1956 |
| Dornseiff 1937 | Dornseiff 1956 |
| Clemen 1939 | Guthrie 1957 |
| Linforth 1941 | Bianchi 1960 |
| Heidel 1942 | Stokes 1962 |
| Barnett 1945 | Laemmli 1962 |
| Frankfort 1949 | West 1963 |
| Solmsen 1950 II | Brandon 1963 |
| Kerényi 1950 | Ziegler RE |

C. History of Mathematics

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|--------------------------|---------------------------------------|
| Tannery 1887 | Brunschvicg 1947 |
| Cantor 1907 | Robin 1948 |
| Milhaud 1900, 1906, 1911 | Reidemeister 1949 |
| Naber 1908 | Michel 1950 |
| Vogt 1909, 1910, 1914 | Neugebauer 1952 |
| Zeuthen 1910, 1913, 1915 | Becker 1957, I and II |
| Loria 1914 | Heller 1958 |
| Heath 1921 | Junge 1958 |
| Taylor 1926 | van Heemert 1963 |
| Hasse-Scholz 1928 | von Fritz 1932, 1945, 1955, 1959, and |
| Dijksterhuis 1930 | RE |
| Heath 1931 | van der Waerden 1943, 1947-9, 1951, |
| Rey 1930, 1933, 1939 | 1966 |
| Dijksterhuis 1935 | Freudenthal 1957, 1966 |
| Becker 1936 | Szabó 1956, 1958, 1960, 1964 |
| Heidel 1940 | |

D. Special topic:

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|--|-------------|
| a discussion on the scientific character | Popper 1958 |
| of Presocratic philosophy: | Kirk 1960 |
| | Lloyd 1967 |

CHAPTER IV

ZENO

In his book on the *Theology of the Early Greek Philosophers* Werner Jaeger ends his chapter on Parmenides with a discussion of an internal discrepancy in Parmenides' ideas (*Theol.* 108). Jaeger states that Parmenides puts great emphasis on determinateness (πεῖρας) as a distinctive quality of being. He thinks Parmenides did so in order to dissociate himself from the Milesian theory of the ἄπειρον. Parmenides, Jaeger says, tries at all costs not to take the view of the Milesians who considered being as unbounded. Likewise he rejects the view of the Pythagoreans, who assumed the first principles to be twofold. The ancient religiously venerated unity of all things dominates his thinking to such an extent that Parmenides wants to defend it with all his might.

In our second chapter we have discussed the meaning of the terms ἀτέλεστον, οὐκ ἀτελεύτητον, τετελεσμένον ἐστί as used by Parmenides. The arguments developed in that chapter suggest that in Jaeger's description of Parmenides' philosophy the shades may be somewhat modified. The contrast to the Milesian theory of the ἄπειρον is not as absolute as Jaeger depicts it, mainly for the reason that as early as Anaximander we find the universe represented as a sphere of being, determined from within (see p. 11). It seems not even excluded that to the Ionians already the first intuition should have presented itself of a theory in which determinateness and infinity were combined (see p. 23-24). Xenophanes may have acted as the intermediary between the Ionian school and the Western, Eleatic tradition. The difference between Parmenidean and Milesian thinking must rather have centered on the word πεῖρας as indicating determinateness. Accordingly the terms οὐκ ἀτελεύτητον and τετελεσμένον are used by Parmenides with the emphasis on the signification of 'well-finished', 'brought to perfection', rather than on the spatial meaning of 'having an end in space'. This Parmenidean emphasis on determinateness must be due to Pythagorean influence.

As was argued in our chapter on Parmenides, the expression ἦδ'