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ON SOME ASPECTS OF EARLY GREEK ASTRONOMY

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(Read November 11, 1971)

SOME EIGHT YEARS ago a "Symposium on Cuneiform Studies and the History of Civilization" was held at a meeting of this Society. As one of the speakers on that occasion I presented a paper on "The Survival of Babylonian Methods in the Exact Sciences of Antiquity and Middle Ages"¹ in which I tried to distinguish as far as possible between those areas of ancient astronomy in which Babylonian influence was decisive and those which represent an independent development. In the present paper some sections of this earlier study will be amplified. Its main purpose, however, is methodological. The progress of modern astronomy since Brahe and Kepler is inextricably connected with the ever-increasing accuracy and range of observational techniques and it therefore has seemed plausible to assume that a similar trend existed also in the first phase of astronomical development in the Greek world, that is, in the period from the beginnings in the fifth century B.C. to the crowning achievement, Ptolemy's "Almagest" in the second century A.D.

I think that this retrojection of conditions prevailing during the last five centuries into a fundamentally different milieu, two millennia earlier, has resulted in a severe distortion of the actual situation and has deprived us of a better insight into the origin of scientific methods that are difficult enough to reconstruct from our fragmentary sources. Furthermore, since most of the sources in question were made accessible through the industry and philological competence of the classical scholars of the nineteenth century we also have inherited much of their basic attitudes. Classicists during this period were still undisturbed by fields concerned with "Ueberresten von gemischter Art"² (e.g., Archaeology or Papyrology—not to mention oriental material), and so they could act *sicut Deus, scientes bonum et malum*. Thus it was simply taken for granted that "progress" from Eudoxus and Aristotle to Aristarchus and Hipparchus

could be measured by the increasing agreement with modern data and methods. Wilamowitz (in 1897) did not hesitate to declare that around 240 B.C. (in the reign of Ptolemy III Euergetes) "man arbeitete auf der Sternwarte Alexandrias an einem Fixsternkataloge"—although there exists no trace of organized observational activity before the Abbasid period. There is no need to multiply such examples of baseless anachronisms; they would easily fill another paper.

Instead, I shall make an attempt to describe a drastically different aspect that emerges from fragments of early Greek astronomy, i.e., from the period from Eudoxus (early fourth century B.C.) to Archimedes and Apollonius (i.e., to about 200 B.C.). I think it is essential for our understanding of this early period to realize, first that its approach to fundamental problems of astronomy is in many respects totally different from what we customarily consider to be "Greek" astronomy, and, secondly, that Greek mathematics and Greek astronomy progressed in quite distinct levels, a distinction which left its effects until deep into the Renaissance.

Two more introductory remarks. I shall abstain from giving the bibliographical references and the discussions of details which would be necessary to support statements made in this paper; I hope to do this elsewhere within a wider framework. Secondly, it is not my intention to present a complete picture of what I think we do know about early Greek astronomy. I have only selected certain topics which seem to me particularly revealing for the situation in the formative period of Greek astronomy. But I shall far transcend the traditional chronological and geographical framework of early Greek science simply because I am convinced that many a medieval and non-Greek source gives us important information about hellenistic origins.

1. MEASUREMENT OF TIME

The division of the day into 24 hours—itsself the outcome of a complicated mixture of Egyptian, Babylonian, and Greek components—presents

¹ *Proc. Amer. Philos. Soc.* **107** (1963): p. 528-535.

² Fr. Aug. Wolf, *Museum der Alterthums-Wissenschaft* **1** (1807): p. 77.

TABLE 1

Month	hora		
	3	6	9
I et XII	17	11	17
II XI	15	9	15
III X	13	7	13
IV IX	11	5	11
V VIII	9	3	9
VI VII	7	2	7

itself in two different forms: one, of greater popular appeal in the Mediterranean world, is the division of the time of daylight and night separately into 12 ("seasonal") hours each; the other operates with 24 hours of constant length which agree with the seasonal hours at the equinoxes, therefore called "equinoctial" hours and commonly used in astronomical contexts, though the separation seasonal/equinoctial is by no means equivalent to popular versus scientific usage. Even the same text may have both types simultaneously. The "shadow tables" to be discussed presently concern 12 seasonal hours for each month but they also give the length of daylight for the same months in equinoctial hours—without any distinction in the terminology.

It is obviously equinoctial hours that are meant when the calendar page for the month of June in the "Tres belles heures de Notre Dame" says, "*Les heures de la nuit 6 et du jour 18.*" This "Book of Hours" of the Duke of Berry (about 1400) presents us with a simple pattern for the variation of length of daylight from month to month during the year: the maximum $M = 18^h$ in June, the minimum $m = 6^h$ in December and a fixed increase or decrease of 2^h per month, i.e., a strictly linear variation between m and M . However, not only does the abrupt change from increase to decrease stand in obvious contradiction to the most elementary experience, but a maximum of 18^h corresponding to a geographical latitude of almost 58° (as tabulated, e.g., in the *Almagest*), correct about half-way between Copenhagen and Stockholm, but surely not in Paris or Bourges.

How these data got into a Book of Hours I cannot say. Only the antiquity of the pattern is evident: "linear zig-zag functions" are a characteristic feature in cuneiform astronomical texts of which we have a great variety from the centuries between Artaxerxes and Caesar. But this is by no means the earliest evidence for linear calendaric schemes: a hieratic papyrus, known as

the "Cairo Calendar," written in the Ramesside period (twelfth century B.C.) shows exactly the same scheme as the Book of Hours of 1400 A.D., i.e. the same extrema of 18^h end 6^h and a linear variation with 2^h each month. I can only once more admit my inability to explain the origin of the basic parameters.

Fortunately arithmetical schemes constitute the leading principle also in a group of texts, the so-called "shadow tables," where we can reach a fairly clear historical understanding. To begin with the European medieval tradition, we have, e.g., a "*Horologium Horarum*" of the ninth or tenth century which gives in six pairs information of the following type: "*Januarius et december, hora 3 et 9 pedes 17, hora 6 pedes 11.*" Here "hora" must mean seasonal hour, the 6th always representing noon. The "pedes" measure the length of the shadow of a man standing upright and using his own feet as units of length. The resulting scheme is very simple (table 1). The last noon shadow of 2 feet is an obvious arithmetical error, both with respect to the sequence of the noon shadows and the shadows 3 hours before or after noon that are always 6 feet ($= 1 + 2 + 3$) longer than the noon shadow.

Many more shadow tables are preserved in Greek from the Byzantine period (thirteenth and fourteenth centuries). They lead us to distinguish two major types and to recognize a systematic error in many tables, e.g., in the above-given Latin example (table 1). The arrangement in six pairs of months is wrong: there should be seven entries, one for each extremum alone (e.g., December and June) and five pairs of equidistant months (e.g., I and XI or V and VII). This holds for both types of tables: the one which uses names of months for the entries, the other which does not depend on calendaric conventions but uses the solar positions in the signs of the zodiac. For this second type one has one single entry for Capricorn and Cancer each and five pairs of signs symmetric to the solstices.

Another solid group of shadow tables is preserved in Ethiopic codices, all of very recent date (e.g., seventeenth and eighteenth centuries) but undoubtedly copied from much older (presumably Coptic) sources. Except for the adaptation to the Ethiopic calendar and many scribal errors, these tables are closely parallel to the Byzantine ones. This parallelism is further emphasized by two peculiarities. Several tables mention for each month the length of daylight, and these numbers again form a linear zigzag function, now always

with $M = 15^h$, $m = 9^h$. Since according to ancient geography $M = 15^h$ is characteristic for the "clima" of the Hellespont, and since Byzantine relations with Ethiopia are well attested for the early Middle Ages (e.g., sixth century), a transmission of Byzantine astronomy to Ethiopia seems evident here. A second element of parallelism is found in the textual preambles to the tables, which address a "King Philip." In astronomical context one will think, of course, of Philip Arrhidæus whose regnal years constitute the basis for the "Era Philip," used, e.g., in the famous "Handy Tables" of Ptolemy and Theon.

I have no doubt that both conclusions are wrong. The preserved material amply suffices to restore the original pattern of the shadow tables (*cf.* table 2). Both the lengths of the noon shadows and the lengths of daylight form arithmetical sequences with difference 1. Since a daylight of 12^h is necessary for the equinoxes, such a sequence leads automatically to $M = 15^h$ and $m = 9^h$. In other words, these extrema are the consequence of a primitive arithmetical pattern, not the result of observations which we could then utilize to determine the underlying geographical location, e.g., Hellespont or Byzantium. In fact we have good evidence from Hipparchus and from Geminus that the ratio 15:9 was considered by Eudoxus, Aratus, and Attalus as representative for Greece in general. Thus our pattern does not belong to the Byzantine period but originated in early Greek astronomy in Greece, presumably at Athens. This conclusion is supported by another consideration. Shadow tables have nothing to do with a chronological era but they naturally belong to the "parapegmata," i.e. texts which associate the risings and settings of fixed stars in the course of the year with weather conditions—much in the way that our Farmer's Almanacs still do. Ptolemy, among others, wrote a whole treatise on these "Phaseis," and he cites his authorities for all predictions, e.g., "unwholesome air and turbulence according to Callippus, Euctemon and Philip; rain and thunder according to Eudoxus. . . ." Finally, Ptolemy gives the list of his authorities from Eudoxus to Caesar and mentions the regions where they obtained their climatic experiences. Here "Philip" is associated with the Peloponnesus, Locris, and Phocis, and there is little doubt that we are dealing with Philip of Opus, who flourished in the first half of the fourth century B.C. (well known because of his connection with Plato's "Epinomis").

TABLE 2

LENGTH OF SHADOW IN FEET FOR THE HOURS OF DAYLIGHT BEGINNING AT SUNRISE (NOON = 6^h), DEPENDING ON THE SOLAR POSITIONS BETWEEN WINTER SOLSTICE (♄) AND SUMMER SOLSTICE (♊)

hour	♄	♊	♈	♉	♊	♈	♉
1	28	27	26	25	24	23	22
2	18	17	16	15	14	13	12
3	14	13	12	11	10	9	8
4	11	10	9	8	7	6	5
5	9	8	7	6	5	4	3
6	8	7	6	5	4	3	2
7	9	8	7	6	5	4	3
8	11	10	9	8	7	6	5
9	14	13	12	11	10	9	8
10	18	17	16	15	14	13	12
11	28	27	26	25	24	23	22

length of daylight:	15	14	13	12	11	10	9	hours
noon shadow:	2	3	4	5	6	7	8	feet

Thus it seems fairly certain that the arithmetical patterns of table 2 for the shadow lengths and lengths of daylight originated in early Greek astronomy, and this conclusion is supported by the remark that the zodiacal patterns (which, in view of the Greek lunar calendars, must be the original form) presuppose the Eudoxan norm which places solstices and equinoxes in the middle of the zodiacal signs. Again, Athens seems to be the plausible center for this development. A lucky accident allows us to show that shadow tables of the type discussed here appear already in the Ptolemaic period in Egypt. A papyrus fragment of an astronomical treatise (now in Vienna) has preserved a little corner of such a table, just enough to demonstrate the identity with the pattern of our table 2.

Having once established the basic structure of these tables it is no longer difficult to recognize their survival in more or less significant variations all around the Mediterranean medieval world: in an Armenian treatise, in Syriac, in Coptic, in Nubia (a Greek inscription in a temple at Taphis), in North Africa and Spain (in "Anwa" "tables"), and in monastic manuscripts of France and England. Incidentally we can now say that the apparent arithmetical error 2 in the last line of our table 1 is the only correct residue of the original pattern, whereas all other numbers are adapted to the faulty six-pair pattern that replaced the original $1 + 2 \cdot 5 + 1$ scheme. It should be noted, however, that this modification of the ancient Greek scheme is only a misguided arithmetical re-

other above-mentioned works to realize that their real goal is to demonstrate the power of the mathematical approach, not the solution of some specific astronomical problems.

The conceptual beauty and mathematical elegance of the Eudoxan homocentric spheres are undeniable, but equally evident is their inability to explain obvious details in the observable planetary motions. I think it is in vain that modern scholars have tried to reconstruct numerical parameters to be substituted in order to make the Eudoxan model represent the planetary motions with some semblance of truth. I think such numerical data never existed. And I confess that I consider it quite possible that exactly the same thing could still be said with respect to the sophisticated investigations of epicyclic and eccentric motions by Apollonius some 150 years later.

The same general attitude explains the much discussed disrespect of Aristarchus for observational data. We know from Archimedes that Aristarchus was aware of the fact that the apparent diameter of the sun and moon is about $1/2^\circ$; nevertheless, in his treatise he computes with a value of 2° , thus avoiding trigonometric difficulties with small angles. The basic mathematical idea to determine the ratio of lunar to solar distance is very neat (cf. fig. 3): if one knows the angle η of elongation between sun and moon when the latter appears exactly half illuminated, then one has only to solve one right triangle to obtain the desired ratio. Astronomically the method is totally impracticable: the moment of dichotomy cannot be determined with any accuracy, and the value of η is so near 90° that the difference falls below the limit of accuracy of visual observation—Aristarchus's value of 87° is purely fictitious. What really interests him in this treatise is, on the one hand, the trigonometry of the problem (at his time an undeveloped topic) and, secondly, an accurate mathematical investigation of a question which is again without any practical importance: how far is the terminator between light and darkness removed from the great circle which contains the radius moon-earth

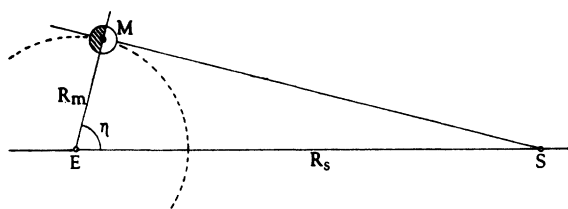


FIG. 3.

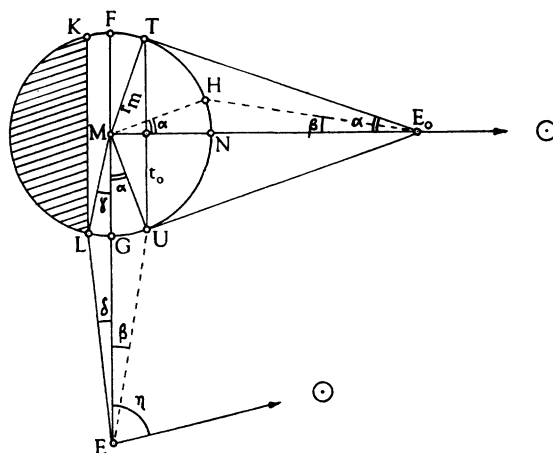


FIG. 4.

(*ME* in fig. 3). The answer that a minute eccentricity of the terminator cannot materially affect the ratio of the distance is, of course, obvious from the very beginning. But it is the rigorous mathematical aspects of the problem and not observational techniques which concern Aristarchus.³ And his treatise ends without giving actual distances and sizes.

To determine the distance of sun and moon Hipparchus eventually introduced a method which is based on geometric conditions that prevail at a lunar eclipse and which are well suited for observational refinements. None of these methods were applicable to planetary distances, and ancient astronomers remained entangled in philosophical doctrines which eventually led Ptolemy to the model of nested spheres that dominated Islamic and western astronomy during the Middle Ages. In a completely different direction, however, lies a system of planetary distances, proposed by Archimedes, that escapes our understanding so far as the underlying principle is concerned.

It is a very peculiar accident which has preserved for us a record of this Archimedean theory. Hippolytus, in the first half of the third century,

³ The figure which belongs to the discussion of the terminator had been garbled already in antiquity, a fact not realized by Pappus (early fourth century A.D.) or by the modern editor, Sir Thomas Heath (1913). Actually two cases were condensed into one figure: one, the earth *E* at quadrature (cf. fig. 4), the other *E*₀ in the axis moon-sun in order to find the smallest terminator *TU*, assuming equal apparent diameter of sun and moon. The problem is to find how big *LG* < *GU* appears from *E*. Pappus did not realize that *EM* must be perpendicular to *E*₀*M* and he therefore discusses the apparent size of *GU* as seen from *E*₀, which is absurd because this arc is invisible from *E*₀.

TABLE 4

A		B	
From earth to	Stadia	From earth to	Stadia
Moon	5544130 = 1a + 2d	Venus	50815160
Sun	55816195 = 11a + 3d	Mercury	52688256
Venus	76088260 = 15a + 4d	Sun	121604451
Mercury	126904455 = 25a + 7d		
Mars	167448585 = 33a + 9d	Mars	132418581
Jupiter	187720650 = 37a + 10d	Jupiter	202770646
Saturn	227992715 = 45a + 11d	Saturn	222692711
Zodiac	248264780 = 49a + 12d		
a = 5 · 10 ⁶ d = 272065			
Moon to Mercury = Mercury to Zodiac = 121360325 = 24a + 5d			
Mercury to Saturn = Sun to Saturn = 101088260			

was perhaps the last Greek-educated bishop of Rome. He was of an uncompromising fighting spirit, not only directed against his more worldly competitors for the episcopal throne but also against heresies about which he composed a still extant treatise. Though not a heresy by itself, astronomy is full of speculations about the structure of the world and is thus potentially a preparation for heretical ideas. As an example of such useless speculations he gives the list of two sets of distances assumed by Archimedes (*cf.* table 4), unfortunately without any further details.

All that we can say is derived from the bare numbers. For no apparent reason we find two different arrangements: in A the sun is nearer to the earth than Venus and Mercury, in B it is located beyond these two planets. In the first scheme, Mercury is exactly at the midpoint between moon and zodiac (*cf.* fig. 5) and the distance Mercury-Saturn in A is exactly the same as the distance sun-Saturn in B. But the most surprising feature is the structure of the numbers in A. Each one of them is a linear combination of a multiple of $a = 5000000$ and of $d = 272065$ (*cf.* table 4). Why these numbers were chosen and why they were combined in this peculiar fashion is a complete mystery. All that the pious bishop has to say is that Archimedes should have taken numbers that satisfy harmonies postulated by

Plato. We must, alas, admit that we do not understand how Archimedes, the greatest scientist of antiquity, came to these numbers.

3. TRIGONOMETRY

Aristarchus's discussion of the solution of one right triangle demonstrates the absence of a systematic trigonometry in the time before Hipparchus. It is obviously nonsense that Pliny ascribes to Hipparchus a work on chords in 12 books. The total of ancient trigonometry, plane and spherical, with proofs, tables, and applications, requires only two of the 13 books of the whole *Almagest*. How Pliny's error originated I do not know; at any rate it is not worth the effort to rescue this passage in Pliny by emendations. It is important, however, not to be misled by Pliny into the assumption of a very developed stage of trigonometric tables in the time of Hipparchus, comparable, say, to the table of chords in the *Almagest* computed in steps of $1/2^\circ$. On the contrary, I think we have good evidence that suggests a rather crude set of tables as the core of Hipparchus's trigonometry.

That Hipparchus had no spherical trigonometry in the proper sense is certain, simply because it did not become clear before Menelaus (in the first century A.D.) that spherical triangles must be made up exclusively of great circle arcs. This does not exclude, however, the possibility of solving certain problems of spherical astronomy by means of the so-called "Analemma" methods, to which also Ptolemy devoted an elegant treatise. The basic idea can best be described as using "descriptive geometry" for the transformation of three-dimensional configurations to two-dimensional problems which then only require plane trigonometry for their solution. These methods fortunately do not depend on the distinction between great circles and parallels which have such a natural place in spherical astronomy. The theory of sundials (as preserved, e.g., by Vitruvius) also makes use of the analemma methods. Though direct proof is lacking, it seems very likely that Hipparchus knew (if indeed he did not invent) these methods; it is certainly not accidental that analemma methods play a vital role also in Indian astronomy.

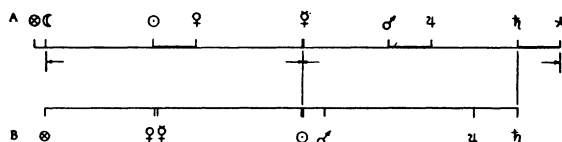


FIG. 5.

TABLE 5

M	Hipparchus			Ptolemy		
	a	Δa	a	φ	$a = \bar{\varphi} - \epsilon$	Δa
16 ^h	9 cubits	3 cub. = 6°	18°	48;32°	17;37°	5;30°
17	6	2 cub. = 4°	12	54;1	12;8	4
18	4	1 cub. = 2°	8	58	8;9	3
19	3		6	61	5;9	

In dealing with problems of spherical astronomy Hipparchus was not restricted to the Analemma; we know from data transmitted by Strabo that he also made use of arithmetical patterns that are characteristic for Babylonian astronomy. The problem in question concerns the altitude the sun can reach at the winter solstice at given geographical latitudes. The correct solution has been given by Ptolemy and the results are tabulated in the *Almagest* (cf. table 5, right half). Hipparchus also answered the question for latitudes where the longest daylight is 16^h to 19^h. The corresponding solar altitudes are given in "cubits," an angular measurement of 2° well attested as a Babylonian norm (cf. table 5, left). His answer agrees fairly well with the correct data, but the basis is obviously a simple sequence of second order with 1, 2, 3 as differences.

What Hipparchus's plane trigonometry looked like is not to be judged from Pliny but from Indian astronomy whose dependence on Hellenistic (and thus Babylonian) prototypes has long been recognized. Our insight into this process of transmission has moved one important step forward with the proper interpretation of two Greek papyri concerned with the motion of the moon: P. Lund Inv. 35a from the time of Nero-Domitian (A.D. 60 to 84) and P. Ryl. 27 written around A.D. 250. The methods displayed in these papyri are based on Babylonian parameters and procedures; their exact counterparts had been found two centuries ago by LeGentil in South India where he attempted to observe the Venus transits of 1761 and 1769 from the French colony at Pondicherry. He missed the transits (first because of war and then because of clouds), but he succeeded in getting native Tamil scholars to compute for him the circumstances of a lunar eclipse using rules which Cassini eventually was able to explain and which are, as we now know, identical with the

methods in the above-mentioned papyri. It is certainly no accident that a site near Pondicherry, Arikamedu, was a center of Roman trade in the early imperial period. No doubt the interest in astrology, a discipline brought to its highest development in hellenistic Egypt, was the vehicle that transmitted also the strictly astronomical techniques to India.

It is not within the scope of this paper to describe how much we owe to Indian astronomy (e.g. through Varahamihira's *Pancasiddhāntikā*) for our knowledge of Babylonian and Greek astronomy of the hellenistic period. I shall only draw attention to some features of Indian trigonometry which, I think, reflect very accurately the type of Hipparchus's trigonometry.

In the only work of Hipparchus that has come down to us, his *Commentary on Aratus* (more a sharp critique than a "Commentary"), he uses peculiar spherical coordinates—arcs on the ecliptic in combination with arcs on circles of declination,⁴ a system also used in Indian astronomy. As units he repeatedly uses "signs," i.e., twelfths of the circumference regardless of whether these arcs lie on the ecliptic or not. Again the same terminology is found in India.

Another peculiar terminology is revealed to us by Theon's *Commentary to the Handy Tables* (fourth century A.D.) according to which 15°-sections, i.e., 24ths of the circumference, are called "steps" (*βαθμοί*), in particular in relation to lunar latitudes and solar declinations. Once alerted to this concept, one finds many instances of its application from Roman Egypt to the late Byzantine period. It is particularly significant in the present context that the above-mentioned P. Ryl. 27 uses this concept in relation to the lunar argument of latitude. These "steps" also appear in the astrological literature, e.g., in Vettius Valens (second century A.D.) and in relation to planetary latitudes. An apparently very old connection exists with meteorology (i.e., to the same background from which the *paraepgmata* and shadow tables come) since the steps of the four quadrants are related to wind directions.

Finally, we find in the geographical literature, e.g., in fragments from Eratosthenes, in Posidonius, and in Geminus, a 48-division of the circle, resulting in "parts" (*μέρη*) of 7;30°. All these units are more or less interwoven in our sources

⁴ This peculiar system could well have been suggested by its convenience for stereographic projection which is fundamental for the "astrolabe"; but explicit evidence for Hipparchus is still missing.

and represent together a simple sequence of arcs: 30° , 15° , and $7;30^\circ$.

We can now turn to Indian trigonometry. In India the great step was made of changing the Greek "chords" to "half-chords" i.e., to sines in modern terminology:

$$\frac{1}{2} \text{ crd } \alpha = \sin \frac{\alpha}{2}.$$

Applying this transformation to the previously mentioned units of signs, steps, and parts, we obtain arcs that are multiples of $3;45^\circ$: this is exactly the norm in the Indian tables.⁵ I have very little doubt that these tables are nothing but the transformation of the Hipparchian table of chords to a table of sines.

Through the intermediary of Islamic astronomy these Indian tables did reach medieval western Europe, in particular Spain and England, under the name of "kardaga," a crude rendering of a Sanskrit term meaning "half-chord." In this circuitous way, Hipparchian trigonometry returned to a part of the world from which it had originated well over a millennium before.

A concluding remark must be added concerning the practical use of tables for a trigonometric function, computed only for a few values of the argument, $3;45^\circ$ apart for the sines, $7;30^\circ$ for the chords (24 values in all). Here again the Tamil computers testify to the existence of a rational procedure. Their table of lunar latitudes progressing in steps of integer degrees of the argument of latitude is simply found by linear interpolation between the values computed for the multiples of $3;45^\circ$ as argument. Once more we see at the very end of the "early" period of ancient astronomy the all-pervading convenience of the linear function. And it seems to me of methodological interest that Tamil astronomers of the eighteenth century can provide us indirectly with better information about early Greek astronomy than does Pliny's encyclopedia.

4. EPILOGUE

That we could use sources from the Roman imperial period until deep into the Middle Ages is due to more than a lucky accident of preservation of antiquated material. The extreme simplicity of the arithmetical methods, their entanglement with

a priori speculations of numerological character (politely called "pythagorean"), the lack of observational accuracy, all this makes early Greek astronomy ideally suited to the mental climate of the Middle Ages.

Our results imply a strong warning against periodization of cultural history. Contemporary with the haphazard steps of an elementary astronomy and mathematical geography are the most brilliant achievements of Greek abstract mathematical thought. The theory of irrational quantities by Theaetetus and Eudoxus, Aristotelian logic, Euclid, Archimedes's integrations, and Apollonius's conic sections are examples of mathematical structures whose significance was fully recovered only in modern times. Of the astronomy of the same period nothing remained that could be incorporated into the new kinematic astronomy that emerged from the work of Apollonius and Hipparchus in the second century B.C. While mathematics had clearly passed its peak, astronomy progressed to become an exact science of splendid methodology, including observational techniques, numerical and graphical methods, and theoretical optics. If one had only to rely on methodology, one would date Peurbach, Regiomontanus, Brahe, and Kepler in the century following Ptolemy. In fact it took two centuries to produce the competent mediocrity of Pappus and Theon who turned Ptolemy's work into a segment of higher scientific education, not much less sterile than the tradition followed by astrological practitioners who preserved for us so much of early Greek astronomy.

In our discussion, we have repeatedly referred to the background formed by Babylonian astronomy, whose influence is evident in the use of the sexagesimal system or in the basic procedures of arithmetical methods. Nevertheless, I think that one has to concede to early Greek astronomy a good measure of independence, in particular so far as geometrical considerations are concerned and also with respect to the trend to numerological speculation. How far the Greeks ever reached a detailed understanding of the refined Babylonian methods for the computation of lunar and planetary ephemerides is difficult to say. We only know of the use of basic Babylonian parameters by Hipparchus, and one may also conjecture that the idea of the tabulation of numerical material is an important borrowing from Mesopotamia. But the roads soon parted when geometric models became the basis of Greek astronomy which made possible a physical interpretation some 1,500 years later.

⁵ I know of no proof for the commonly accepted explanation that $3;45^\circ = 225'$ was chosen because it is the largest arc for which $\sin \alpha = \alpha$ numerically. Obviously this implies additional assumptions concerning the length of the radius and the accuracy of the tables.