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ARITHMETIC AND CHAUCER

Geoffrey Chaucer was an avid student of every aspect of the world, human, material and celestial. He was not only a poet. One of the great loves of his life was astronomy, and so of necessity he was a calculator. He was that also by inclination, I believe, and not only in astronomy. In his daily life he worked with money, a subject that makes a useful starting point for any account of medieval arithmetic.

It is not by chance that many leading scientists have concerned themselves with monetary theory – Copernicus and Newton, for example. In Chaucer's century, Nicole Oresme gave much thought to the subject. Boethius had done so long before¹. You can hardly use money at all without a basic skill in the practice of arithmetic, but the regulation of an economy requires much more. In economic life after the twelfth century Europe was beginning to learn some of the subtler points in the workings and customs of exchange. There came a marked increase in the use of money as an alternative to barter. Waged labour made for greater specialization, which in turn led to a growth in expertise of many different kinds, especially in the towns. Great enterprises – from the building of great churches to the waging of great wars – could be carried through more efficiently, without recourse to slavery or serfdom, or their many near equivalents. Money provided an alternative to the more ancient and often more violent means of ranking human society, and it loosened many of the old social bonds. It made wealth more mobile. It allowed for greater efficiency in giving and lending, taking and borrowing, charity, robbery, taxation. In most of these respects it called for arithmetic. But arithmetic aside, it made life

¹ It will be recalled that Theodoric, king of Italy and of the Goths, gave Boethius the task of reforming the coinage.

more colourful, and the poet Chaucer was able to cash in on the colour of the age.

Then as now, certain number-words were used for impressionistic exaggeration rather than as serious indicators of the truth. Armies were often said to be multiples of a hundred thousand strong. When writing of knights and towns, Chaucer has a strong preference for « an hundred », reserving « an hundred thousand » for the purest hyperbole. (In the Franklin's tale, the rocks have slain « An hundred thousand bodyes of mankynde ».) This use of number has nothing to do with the precise ordering of life. For many ordinary purposes an ability to count, to record, to add and subtract accurately was needed, but even for those with administrative responsibilities, the help of an abacus or counting board would usually have been enough². The recording of simple results could be done using tally sticks, sticks on which debts or payments were recorded by notches. The sticks were then split across the notches, one half of the stick going to the debtor and the other to the creditor. The grain of the wood and the line of the split allowed reliable matching – a safeguard that was probably harder to fake than the magnetic code on the average credit card. Of course in the more literate ranks of society numerical records made use of roman numerals, or occasionally of an alphabetical system similar to that used by the ancient Greeks. All of these things survived in England into the nineteenth century, and indeed roman numerals and even lettered numerals are still with us, for such purposes as the numbering of prefaces.

² The word « abacus » (ultimately from the Greek) was not originally used of a frame with beads on wires, as seems to be popularly believed in English-speaking countries. It referred to a board. This might be ruled with counters, for example shells, or might even be a board with a thin dusting of sand on which marks could be made with a stick and intermediate results rubbed out. When Martianus Capella speaks of an abacus he means only a dusted board on which *geometrical* figures could be drawn. His character representing Arithmetic uses finger-reckoning. The western Arabic forms of the numerals are known by the name for dust (*ghubâr*).

Chaucer knew the exhausting culture of the counting house from his experience as controller of customs, and later as controller of building works for the king. There is ample internal evidence from his astronomical allegories that he was a skilled reckoner, for instance with the most advanced astronomical tables then available. Not surprisingly, references to « rekenyng » are frequently made in his treatise on the astrolabe, although this was only written for his ten-year-old son. More in keeping with the commercial practices he might have known from his father's dealings in the wine trade is the vivid picture the Shipman paints in his tale of the merchant who goes « into his countour-hous... / To rekene with hymself » whether his wealth had increased or not, that year³. « His books and his bagges many oon / He leith bifrom hym on his countyng-bord. » The merchant's wife, the equivalent of a modern computer-widow, later shows some impatience:

Quod she ; "What, sire, how longe wol ye faste?
How longe tyme wol ye rekene and caste
Youre sommes, and youre bookes, and youre thynges?
The devel have part on alle swiche rekenynges!" Sh 215-8

Her reward is an announcement that he plans to leave for Flanders on business.

An obsession with arithmetical reckoning was not the only consequence of an obsession with accumulating wealth. A little dishonesty here and there was not unusual. The Host hints broadly of this in his words to the Manciple, in the Prologue to his tale. Beware, the Host warns, lest the Cook take his revenge and carp at small items of dishonesty in your calculations, that is

(...) speke wole of smale thynges,
As for to pynchen⁴ at thy rekenynges,
That were nat honest, if it cam to preef. Mcp 73-5

The word « preef » (proof) here does not have the logical force of the word as now commonly used in deductive sciences such as

³ Sh 75-81. All quotations from Chaucer will be taken from *The Riverside Chaucer*, ed. L. D. BENSON, Boston, 1987.

⁴ Compare « Ther koude no wight pynchen at his writyng », Prolog 326.

geometry – for which the medieval scholar used the word « demonstration ». It means only a check, a reworking, to confirm an earlier calculation. The Reeve, as we learn elsewhere, was expert in this kind of proof. A reeve was in this sense of the word a bailiff, and here it was a question of keeping a tally of his lord's estate, in which matter « Ther was noon auditour koude on him wyne »⁵. This one line of verse gives us an insight into fourteenth-century commercial life. An auditor was in the first place a listener, but, since accounts were then checked orally, the word was beginning to take on a fresh meaning that is of course still current. Just as today, so in Chaucer's time an auditor examined accounts with the help of written documents or such material evidence as tallies, and allowed or disallowed items in them. The Reeve was a match for the auditor just as he would now be for an Inspector of Taxes.

Skill in reckoning at the simple levels thus far mentioned here could have been obtained in many ways. The Venerable Bede (d. 735) contributed a text *On [calendar] computation, or a language of the fingers*⁶ in which numbers up to 9999 could be represented by the arrangement of one's fingers in relation to the palm of the hand, and calculation done accordingly. (Martianus Capella – Chaucer's « Marcius » – had previously made fun of the contortions that might be called for in this type of manual representation, something familiar to racegoers even today.) There were few similar tracts, and what survives is of no great mathematical moment, but the fact is of little consequence since personal tuition must have played – as it does now – the most important role in elementary education. As for the abacus, there seem to have been very few medieval texts devoted to its use, despite its having been commonly used in ancient Rome, but one would again expect the oral tradition to have been far more important than written instruction.

⁵ Prol 594.

⁶ *De computo vel loquela digitorum*. See J.-P. MIGNÉ, *Patrologiae cursus completus, series latina*, vol. 90, Paris, 1850, col. 295-8.

The most common elementary text for the teaching of arithmetic, one that was used throughout the middle ages, was written in late antiquity by the Roman patrician scholar Boethius (fl. 480-524). I refer of course to his *Institutions of Arithmetic*⁷. It is very likely that Chaucer knew it: he made a translation of Boethius' most widely read work, the *Consolations of Philosophy*, and would surely have taken more than a passing interest in the Roman scholar's oeuvre as a whole. The arithmetic, being little more than a translation or paraphrase of another, written in Greek by Nicomachus of Gerasa about four centuries earlier, had a more theoretical than practical and computational character⁸. As regards arithmetical theory this work certainly could not compare in intellectual depth with the arithmetical books of Euclid's *Elements of Geometry*, a fact that had to be discovered painfully and slowly by successive generations, beginning in the middle ages but continuing even into the nineteenth century⁹.

⁷ *Arithmeticae Institutiones*. A complete edition of the Boethian and pseudo-Boethian works (Basel, 1546, and in a fuller version 1570) was reprinted in Migne's *Patrologia Latina*, vol. LVIII and LXIV (Paris, 1847), apart from some passages where later editorial substitutions were made. The mathematical and musical works were reedited by G. FRIEDLEIN, *Anicii Manlii Torquati Severini Boetii De institutione arithmetica libri duo, De institutione musica libri quinque. Accedit geometria quae fertur Boetii*, Leipzig, 1867. A useful modern edition with French translation is J.-Y. GUILLAUMIN, *Boèce, Institution arithmétique*, Paris, 1995.

⁸ After Boethius' political disgrace, his kinsman Cassiodorus replaced him (523) as « master of offices ». Cassiodorus, once regarded as the saviour of classical culture by virtue of a series of translations he brought together at a monastery he founded (Vivarium, on his estate at Squillace/Scylacium, around 550), had two translations of Nicomachus available to him. The better of them had been checked by Boethius himself as early as 510, the year he first became consul.

⁹ All references to this key text will be made tacitly to the translation (from the text of J. L. HEIBERG) and commentary by Sir T. L. HEATH, *The Thirteen Books of Euclid's Elements*, Cambridge, 1925, 2nd ed., 3 vol. (since reprinted). It is now generally supposed that the arithmetical books (V and VII, and XII if we include the method of exhaustion) were due largely to Eudoxus, who anticipated some of the ideas of R. Dedekind and K. T. W. Weierstrass. See T. L. HEATH, *A History of Greek Mathematics*, vol. 1, Oxford, 1921, p. 326-327.

As is well known, the arithmetical books going under the name of Euclid deal with number and proportion in an axiomatic manner, with commendable generality. Books I to VI were on the university lists for students in arts in the middle ages, and of these, Book V contains a general theory of proportion that is applicable to arithmetic, geometry, music, and indeed all mathematical sciences. Numbers are represented in Euclid by lines and referred to by letters. Nicomachus, by contrast, tries to convince us of the truth of his statements by illustrating them with simple concrete numerical examples. He tries to make his subject enjoyable to his audience, at times by making the results seem miraculous, and carries some slight responsibility for later mathematical mysticism, albeit mostly indirect. In his *Arithmetic*, the staple text of the medieval university, Boethius corrects some of these stylistic shortcomings, although he has a weakness for wordy definitions, as when he introduces « unevenly even numbers » and « evenly even numbers », « perfect », « abundant », « deficient », and « figurate » numbers, and so forth. It is not that he invented all this. This sort of material points back to a time before Book VII of Euclid's *Elements*, and stems from Pythagoras or his followers¹⁰. Boethius has some useful introductory material on prime numbers and on arithmetical, geometrical and harmonic proportion – the last being of interest to him through its implications for musical theory¹¹. Again his immediate source was Nicomachus, and so ultimately Euclid and earlier writers.

It was not until the *Arithmetic* of Jordanus de Nemore – an early thirteenth-century scholar of whose life virtually nothing is known – that the nascent universities had an arithmetical treatise available to them with anything approaching the deductive style of Euclid. Jordanus

¹⁰ Note the definition of perfect number in Euclid, Book VII, def. 22, a point to which I return. Evenly even numbers are integral powers of 2 (such as 64, for example). This replaces Euclid, Book VII, def. 8, and follows instead Nicomachus.

¹¹ The problems that interested them were those of finding arithmetic, geometric and harmonic means (*medietates*), the latter being of importance, for instance, in identifying a semitone. Boethius lists ten means altogether

provides the reader with about 400 propositions, arranged in ten books, and shows much subtlety in what we would now regard as algebraic proofs. For the common clerk of the time, however, and still in Chaucer's century, this was tough going, and the book was certainly not suitable for use in grammar and cathedral schools, business schools, or the like. What all such institutions needed was an arithmetic that could rise above the primitive calculational methods then in use. First and foremost, arithmetic was in need of a « place-value » notation to replace the ubiquitous roman numerals. This it obtained by two different routes, the astronomical and the more purely arithmetical, and it is paradoxical that of these the astronomical was the more influential.

The sexagesimal system was regularly used in western academic circles, for time-reckoning and astronomical purposes generally, long before the arrival of the new way of « Indian » decimal reckoning, and it must have prepared the way to some extent. The social groups using both systems overlapped considerably, but all who used the sexagesimal system in the west did so at first in a doubly mixed way. We need to remind ourselves that theirs was not a thoroughly consistent system, any more than is ours, when we mix seconds, minutes, and hours of time with days of 24 (not 60) hours; and that their way of writing down the number's elements was until late in the thirteenth century often in the form of *roman* numerals. A third obstacle to understanding was the fact that all systems of money, weights and measures made use of equivalents peculiar to themselves – twelve pence in a shilling and twenty shillings in a pound, four bushels in a peck, six feet in a fathom, and so forth. Chaucer alludes to many such units: « for peny ne for pound », « half a pekke », « fyve fadme at the leeste », and so on. The best thing that can be said about such systems is that they preserve a healthy scepticism about the sanctity of the number ten.

The new system of arithmetic, when it arrived, came in two forms, one relating to the abacus, the other of more general application. The first of these is easily overlooked, but it helped familiarize scholars with the new forms of expression.

A traditional abacus, such as that used in the much-reproduced illustration from Gregorius Reisch's *Margarita Philosophica* (1503), has a series of horizontal lines marked on a board (the « counter ») divided down the middle. This serves to represent two numbers, one on the left of the division, and one on the right. The top horizontal was

typically for the thousands, the second for the 500s (assuming the Roman system), the next for the hundreds, fifties, tens, fives and units. Four pebbles (counters, jetons, or whatever we are to call them) on the hundreds line and seven on the units line would represent our number 407; and so on. Simple procedures for adding two numbers, one on the left and one on the right, by shuffling all counters across to one side and carrying upwards when a line is full (as is the 500 line when it has two counters, for which a single counter is moved to the thousands line). More complex procedures were needed for subtraction, multiplication and division, but for our purposes they may be left to the imagination.

Towards the end of the tenth century, Gerbert of Aurillac, later to become pope as Sylvester II (d. 1003), wrote a treatise on a new form of abacus of which he had learned from Muslim scholars in Spain. This type of abacus, rigorously using a scale of ten, had counters (*apices*: sing. *apex*), each bearing a numeral from 1 to 9 (written in the western Arabic form peculiar to north Africa and Spain), and each therefore capable of standing for the corresponding number of stones. To take an example: where the old type of abacus (or counter) with unmarked stones would have required as many as twelve stones to represent the number 7603 (that is, $7 \times 1000 + 1 \times 500 + 1 \times 100 + 3 \times 1$), only three apices would have been needed for the same purpose, or four if there was an apex for the zero. There was only a small penalty to be paid for using this new sort of abacus, for one needed to memorize (or use fingers for counting out) sums and products of numbers between 1 and 9.

The apices might just possibly have been what Chaucer had in mind by the « augrym stones » mentioned in the inventory of the clerk Nicholas's possessions: « His augrym stones layen faire apart... » (Mil 3210). Most editors who gloss the word « augrym » note correctly that it is an alternative to « algorism », and derives from the name of the eastern scholar al-Khwārizmī, but they do not go any deeper into the two possible forms of abacus. The question is: were the clerk's augrym stones numbered like apices or not? They were most probably unnumbered. I am not basing this suspicion on the well known hostility to Hindu-Arabic numerals – such as in the 1299 Florentine edict forbidding their commercial use – but on the relative paucity of references to abaci with apices. The word « apex » originally referred to the conical shapes of the stones used as counters. It eventually became applied to the way of writing down the numerals. The same

was true of the word « augrym ». When in his *Treatise on the Astrolabe* Chaucer speaks of inscribing the instrument with « nombres of augrym »¹², this plainly has nothing to do with augrym stones but refers merely to the modified Hindu-Arabic script, more or less that we use today. « Augrym stones » that had no such numbers on them might seem like a contradiction in terms, but at the end I shall give another reason for thinking that they indeed had none, and were not apices.

As a footnote to the use of moveable pieces with numbers on them, let me mention very briefly that arithmetical game known as *rhythmomachia*¹³. I mention it simply to try to convey something of the arithmetical ambience of the middle ages. It was played on a double chess-board (8 by 16). The pieces were triangles, squares, circles and pyramids, and each had a numerical value – but at first roman numerals seem to have been used. The game, which seems to date from the eleventh century, had a complex set of rules and required computational skill of quite a high order. (The climax was when a player had managed to get four pieces in a row representing all three progressions simultaneously, namely arithmetic, geometric and harmonic. There were only six possibilities, with the pieces provided.) The very fact that it survived, and even flourished, over three or four centuries, shows that we must not underestimate medieval proficiency in mental arithmetic within an educated élite.

To return to Chaucer's « augrym », and Hindu-Arabic numerals: as is well known, al-Khwārizmī's name was rendered « Alchorismi » or « Algorismi » by his Latin translators (beginning with Adelard of Bath, Robert of Chester, and Gerard of Cremona in the twelfth century)¹⁴. His

¹² *Astr* 1.7.4/6, etc.

¹³ Often spelt « (a)rithmomachia », as if deriving from the word « arithmetic ». It seems rather to have been formed from the Greek words *rhythmos* (proportion) and *machê* (contest).

¹⁴ Abū Ja'far Muhammad ibn Mūsā al-Khwārizmī (c. 780–c. 850), mathematician, astronomer, and geographer, was a member of an important scientific academy at Baghdad (« the House of Wisdom ») in the time of the caliph al-Ma'mūn. There is no extant Arabic version of his arithmetic. Written after another notable work of his on

Treatise on calculation with the Hindu numerals spawned numerous treatises similar works, and so it was that the entire subject took its generic name *algorismus* from his personal name, giving us our words «algorithm» and «algorism». The two most widely diffused algorithmic treatises in the West were by authors well known to Chaucer. The first was a *Song of algorithm* (*Carmen de algorismo*) by Alexander de Villa Dei¹⁵. Composed in hexameters in the first two or three years of the thirteenth century, it became enormously popular as an aid to calculating the movable feasts of the calendar. It is significant that what chiefly recommended the theory, even to scholars, was the need to perform a common task. This was a pragmatic science for a pragmatic age.

The second work, *Common algorithm* (*Algorismus vulgaris*) by John of Sacrobosco, was a rather more academic study, originally meant for students in arts at the university of Paris¹⁶. Written at a time of vigorous university expansion, it incorporated material from Boethius' *Arithmetic*, but it was essentially a by-product of the al-Khwārizmī treatise, and a sentence from its opening paragraph is revealing for its use of the shortened name «Algus»:

A certain philosopher named Algus wrote this brief science of numbering, for which reason it is called *Algorismus*, which is understood to be the art of numbering or the art introductory to number.

algebra, it shows Babylonian and Indian influences, and is in fact the oldest text of Arabic origin on Indian arithmetic of which we have detailed knowledge. The best edition and general account of it is now: M. FOLKERTS, P. KUNITZSCH, *Die älteste lateinische Schrift über das indische Rechnen nach al-Hwārizmī*, Munich, 1997 (with German translation, English summary, an invaluable Latin glossary, and a manuscript facsimile).

¹⁵ Alexander de Villa Dei, *Algorismus*, ed. J. O. HALLIWELL, *Rara mathematica*, London, 1841, p. 73–83.

¹⁶ Included in Maximilian Curtze's edition of the commentary on it by P. NIGHTINGALE, *Petri Philomeni de Dacia Algorismus vulgarem Johannis de Sacrobosco Commentarius una cum Algorismo ipso edidit*, Copenhagen, 1897. Much of it is translated and annotated by E. GRANT in his *A Source Book in Medieval Science*, Cambridge, Mass, 1974, p. 94–101.

In other places we find «Algo», «Argo Philosophus» or the like, a fact that explains the «Argus» who is pictured in Chaucer's *Book of the Duchess*, as reckoning to a scale of ten. So many beasts were there, we are told.

That thogh Argus, the noble countour,
Sete to rekene in hys countour,
And rekene with his figures ten—
For by tho figures mowe al ken,
Yf they be crafty, rekene and noumbre,
And telle of every thing the noumbre—
Yet shoulde he fayle to rekene even
The wondres me mette in my sweven.

BD 435-42

I ask the reader to remember this reference, for I shall come back to the *Book of the Duchess* shortly.

Like Sacrobosco's astronomical text-book, this arithmetic enjoyed three centuries and more of almost universal European use, appearing under various different titles, of course, but also in a Middle English translation¹⁷. It would have qualified as an «*Algorismus integrorum*» that was required reading for determination at Oxford. When given in this form the title is in contrast to an «*Algorismus de minutis*», that is, a work on vulgar fractions. (Al-Khwārizmī's work had included both.) By Chaucer's time a notation for these had been evolved that more or less corresponds with our own ($\frac{3}{4}$, $\frac{1}{2}$, etc.), but even then, manipulating all but the simplest fractions was the cause of many a medieval headache. It is tempting in a superficial survey to speak chiefly of forms of numerical representation and to evade questions of the degree of difficulty of the subject matter. Suffice it to say that al-Khwārizmī's treatise dealt with such topics as the multiplication and division of sexagesimal as well as decimal fractions, and procedures for extracting the square roots of integers as well as fractions, not to mention some attempts at justifying the procedures adopted.

¹⁷ *The Art of Nombryng*, in R. S. STEELE (ed.), *The Earliest Arithmetics in English*, London, 1922.

With such materials to hand, it is hard to understand why Europe produced so few truly new arithmetical studies at this time. It is a curious fact that the most original arithmetician of the middle ages, Leonardo of Pisa, alias Fibonacci, was not a scholar sitting in an ivory tower but a man trained in algorithmic methods for purposes of business. His good fortune was to have been born the son of a merchant, who was also secretary to the Pisan Republic. I do not have much of an answer to the wider problem of western failure, although I believe it has much to do with the question of notation, and the ease – or lack of it – with which number problems can be manipulated. I am not simply suggesting that number theory came into its own when the hideous Roman notation was replaced, although that is a large part of the story. Number theory flourishes as long as representational methods are adequate. Pythagoras and company found many interesting properties of number while representing number by pebbles and lines, for example. Scholars of the middle ages remained for the most part stuck in similarly simple modes of representation. Did they do nothing of value, working within these modes?

Of mathematical value, relatively little. But what of aesthetic values? Here it is useful to put oneself in the frame of mind of a person who has either fortuitously discovered some unexpected property of simple numbers, or had it pointed out to them, but who does not naturally look for explanations, formal links with other properties of number. They are just there, exciting, beautiful, or whatever. Perhaps there is some explanation of another sort, a religious one, for instance. This is not the place to review the long history of the Pythagorean tradition, or the unifying theme of harmony – a theme that added a moral dimension to mathematics and music, for example – but we must not forget what every medieval clerk well knew. He knew that the Pythagoreans began to investigate the properties of numbers, prompted by the discovery of musical harmonies: he knew of the geometrical representations that led up to figurate numbers – triangular, square, pentagonal, pyramidal, and so on. He was aware of the Platonic view of numbers as significant (even sacred) entities in themselves, of which the things numbered are no more than a pale reflection. In his *Timaeus*, Plato describes the world-soul as built up out of numerical harmonies, and all who accepted this captivating idea expected all works of beauty, for instance of art and literature, to carry the imprint of numerical harmony. Scriptural support was easily found. Words from the *Wisdom of Solomon*, for example, « Thou has ordered all things by measure and

number and weight » (11:20)¹⁸, were turned by St Augustine into a thesis of Christian Neoplatonism that would become a cornerstone of medieval aesthetic theory.

The Christian God was evidently a mathematician. When Christian architects planned the shapes and proportions of their churches, it was natural for them to turn to those parts of Euclidean geometry with the most obvious bearing on proportion and harmony. When Geoffrey of Vinsauf (c. 1200) wrote his *Poetria nova* – a treatise on poetic theory from which Chaucer drew – he set out an analogy between the techniques of the poet and the geometrical procedures of the architect. It should not be surprising to find Chaucer making use of geometry for his own aesthetic ends. Arithmetic is merely the other side of the coin, a part of the larger theme of harmony – as would have been manifest at the time in discussions of figurate numbers, integer harmonies and integer geometry, for example. I will show very briefly how this might have been worked into poetry.

It has been argued, for example by Thomas Hart, that Chaucer's finest poem, *Troilus*, was entirely laid out according to geometrical patterns, and that the poet was calculating these to a high degree of accuracy. Hart thinks that the reader is pointed towards this¹⁹. I cannot go into the argument in detail, but I will try to give something of its flavour. Hart finds 3782 lines from the beginning of the poem to a place where Chaucer inserts a reference to a theorem of Euclid, and 4457 lines from there to the end of the poem. These numbers are as close to the ratio of 3 to $5\sqrt{2}$ as it is possible to get, in breaking up a poem of length 8239 lines into two parts with no fractions of lines. The theorem reference comes at mid-stanza. Taking the end of the stanza to be the break-point, the division would be into lengths of 3787 and 4452 lines. This is now an ideal representation of the ratio of the radius of a circle circumscribing a regular pentagon and the length of its side. It is

¹⁸ Or 11:21 in the Vulgate: « omnia in mensura et numero et pondere disposuisti ».

¹⁹ T. E. HART, *Ibid.*

a ratio geometrically related to golden section, but not to be confused with it²⁰.

Now how one responds to this sort of reading of a fourteenth century poem will depend very largely on one's familiarity with other literature, where such tricks were quite openly played. Dante, for example, tells us something of his own schemes in his famous letter to Can Grande²¹. Boccaccio does the same in his *Chiose*. Looked at in the light of internal evidence, what of Chaucer? There are several texts from which he might have obtained accurate enough numerical values for the square roots in question to have allowed him to calculate the accurate placement of such markers had he so wished²². Only with a much longer poem could he have made *both* breaks fall at the end of a stanza, and the fact that one fails to do so is not an argument against it. The hardest point to accept is that Chaucer would have introduced two

²⁰ When a line is cut so that the product of a part and the whole is equal to the square on the other part, it is said to be cut in « extreme and mean proportion » or in « golden section ». The ratio of the whole to the first part is then $(\sqrt{5} - 1)$ to 2, which can be shown equal to the ratio of the length of the side of a regular decagon (a ten-sided polygon) to the radius of the circumscribing circle. If one can inscribe a regular decagon in a circle one can obviously inscribe a regular pentagon, by joining up alternate vertices.

²¹ The God of the *Divine Comedy* was a Trinity, a fact mirrored in the three divisions of the poem and the nine spheres of the heavens. The numbers ten and a hundred are there as other markers of perfection. Ten is the number of the Empyrean, the heavenly sphere to which the poet makes his way. Each division (*Inferno*, *Purgatorio*, *Paradiso*) has thirty-three cantos, brought up to a hundred with the introductory canto. The terza rima has its eleven-syllable lines always stressed on the tenth. See C. S. SINGLETON, *The Poet's Number at the Center*, in *MLN*, 80 (1965), p. 1-10. Claims to have found an arithmetical structuring of earlier poems have been made, for example for the Old French *La Vie de St Alexis* (1040) and versions of the *Chanson de Roland* from the eleventh and possibly early twelfth centuries. See E. W. BULATKIN, *Structural Arithmetic Metaphor in the Oxford « Roland »*, Ohio, 1972.

²² For instance Archimedean tractates and the arithmetic of al-Khwārizmī. He might have been called upon to start from a sexagesimal notation in some cases, but this would not have troubled him.

alternative geometrical plans so close to one another in position and ratio. (The two ratios differ by only three parts in a thousand.) For the time being, however, let us admit that if only one of the two schemes was truly intentional, then it is likely to have had a geometrical rather than an arithmetical meaning, bearing in mind Chaucer's references to propositions from *Euclid*, a geometrical work as far as the theorem quoted is concerned.

Let me now call upon a completely separate item of evidence as to Chaucer's possible use of a structural arithmetical device. This is from *The Book of the Duchess*, a work from which I have already quoted a couple of the poet's arithmetical allusions. In that work he twice names Pythagoras, explicitly now, once in the middle (at the end of the line that should be numbered 666) and again 500 lines later. There are 1333 lines in the poem altogether²³. There was an ancient tradition, still very much alive in the middle ages and Renaissance, of giving great significance to literary centre-points²⁴. The pattern of the poem, as suggested by the split just mentioned, might thus have been taken as 666+1+666.

I do think that the *Book of the Duchess* is not another piece of geometry in disguise, but that it is plainly arithmetical. I have already set out the broad symmetries that it seems to me to contain, in my book *Chaucer's Universe*, and here I shall only summarize them and add one small additional suggestion which occurred to me recently. The poem seems to have been Chaucer's earliest extensive work – the historical event it seems to record happened in 1368. It opens with words that give a clue to the structural symbolism that is to follow, where the poet speaks of light, and a contrast of day and night. It is a eulogy to the deceased Blanche, former wife of John of Gaunt, and at the same time a

²³ For historical reasons, the line numbering in standard editions is inconsistent. I use actual numbers from the best editions. One might even suspect that the 500-line rule is to be found in *Troilus* too. See my *Chaucer's Universe* for more details.

²⁴ This is touched on in various chapters of A. FOWLER (ed.), *Silent Poetry*. London, 1970, and in his *Spenser and the Numbers of Time*, London, 1964.

consolation to him. (It was through her that John – fourth son of the English king Edward III – acquired the duchy of Lancaster.)

Older accounts of the numerical structure of *The Book of the Duchess* have drawn attention to ones (Blanche was that one), threes, eights and twelves. It is written almost entirely in octosyllabic couplets. It opens with an introduction in which the poet laments his inability to sleep (61 lines), overwhelmed as he is by grief and a haunting fear of death. He retells Ovid's tale of Ceyx and Alcyone (lines 62 to 220) that introduces in a natural way the general theme of bereavement and grief, and tells of the fate of one who, unable to accept the realities of death, grieves to the point of her own death. Returning to the problem of his sleeplessness, he tells two dreams. In one, the dreamer meets the emperor Octovyen's unsuccessful hunting party. The trees are in full leaf, all is green, and the trees themselves, being « Fro other wel ten foot or twelve », are « fourty or fifty fadme lengthe » (420, 422). There is plainly something afoot, as far as number is concerned. Note how 40 or 50 fathoms (each of six feet) introduces yet more tens and twelves. Not far beyond this point, we come across Chaucer's numerological intent, with his reference to Argus, the noble counter, who reckoned with his figures ten.

The second part of the dream concerns an encounter in a deeper region of the wood, when the dreamer meets the Man in Black, of the age of « foure and twenty yer » (455). It is thought that the number was changed with some ulterior motive from the true age of John of Gaunt at the time of his bereavement, twenty-nine. This knight « made of rym ten vers or twelve » (463), a complaint. There are in fact just eleven lines of it, followed by the poet's own observations.

Fortune, it seems, has played a game of chess with the Black Knight, has by trickery removed his queen, and has checkmated him. Checkmate is declared in line 659, and the reference to Pythagoras is at 666, as I said earlier. There follow the knight's more extensive complaint, with the poet's interjections, and a series of exchanges concerning mankind's knowledge of, and love of, God. Three times the knight tries to explain his grief, while the dreamer, acting as Christian counsellor to him, conveys the message that God can be loved only by those with self-knowledge. Only as the poem draws to a close, and the dreamer presses the knight harder, does it emerge (1308) that the real cause of the knight's sorrow is that Blanche is dead. The dream is

finally interrupted by twelve strokes of a bell, and the poem as a whole ends eleven lines after that.

Described in this way, the work has a very unsymmetrical arrangement. The encounter with the Man in Black is nearly twice as long as all that precedes it. Some have found threes, and threes within threes, here. Twelves offer themselves, and the other numbers I mentioned earlier, but I do not for a moment believe that the right way of looking at this is as an exercise in number theory. I think that Chaucer was aiming at a series of divisions into units of 60 lines, each marked by one of a handful of key ideas – night and day, death and life, sleep and waking, and the number twelve – applying to the 60 lines after it²⁵. If this is true, then he did not get it quite right, but those who judge life by statistics might care to note that the average divergence from the ideal 60, on my published analysis, is only half a line, while the standard deviation is 4.5 lines.

And what was the reason behind Chaucer's scheme? It seems to point to an allegory of life and time. The twelves in the poem were evidently introduced with great deliberation, for they all have the emphasis that comes of standing at the end of a line. They are surely twelves of the clock and perhaps of the months in the year, and the presence of 60s could well be meant as another time reference connected with either.

I shall here leave the overall pattern of *The Book of the Duchess*, *The House of Fame*, and *The Parliament of Fowls*, while they are not at all clear-cut, also seem to be based on a time cycle of sixty lines. There is one small additional point about *The Book of the Duchess*, however, that brings it closer to my cursory survey of fourteenth-century arithmetic.

At several points in the poem, Chaucer seems to be asking us to open our eyes to the game he is playing. I remind you of the lines about

²⁵ Looking for notions that occur with unexpectedly great frequency, none seems to me more obvious than *hert*, ambiguous as between the animal (hart) and the human heart. There are forty such references, but only very roughly spread over the poem.

Argus, the noble counter. Even Argus, seated to reckon at his counter with his ten figures – by which clever people can number everything – could not have reckoned the wonders Chaucer met with in his dream, he says²⁶. But more significant, perhaps, are the lines near the *middle* of the poem when he tells of how in a game of chess with Fortune, his opponent checks him with a pawn in the *middle* of the chessboard :

Therwith Fortune seyde "Chek her!

And mat in the myd poynt of the chekker,

With a poun errant!" Allas,...

BD 659-61

« The Grek Pictagores », he believes, would have done better than he. But why mention Pythagoras here? Pythagoras was in fact occasionally drawn sitting in front of an abacus. (In Gregorius Reisch's famous woodcut he is paired off with someone working with Hindu-Arabic numerals, namely Boethius – a name Chaucer would have corrected to Argus, but no matter.) The « myd poynt of the chekker » (at line 660) seems to me to be deliberately ambiguous, alluding to the game of chess as well as to the poem, whose mid-point we are approaching (it is seven lines away) and which is being likened to an abacus, a reckoning counter. The word *chekker* is itself ambiguous. The English word *exchequer* comes by a corrupt formation out of the medieval Latin *scaccarium*, a word (with eastern roots) standing both for chess and for a counter. If you can accept that the ambiguity in the poem might have been intentional, consider next how a counter would have looked, arranged to emulate the line numbers of the poem :

The number of lines to each side of the central line, represented by counters for $1+5+10+50+100+500$ is 666, the same as the number of lines to each side of the central line in the poem. The poem was a checker board, an abacus in modern parlance.

²⁶ BD 435-42, quoted earlier.

This is a small point, of no strictly mathematical consequence, but if it is correct then it is further evidence for the artistic use of mathematics by the greatest medieval English poet. Indeed, he was not only that. He was one of the two greatest poets of the European middle ages. He made journeys to France and to Italy, and it is entirely appropriate that he should have played the same sort of numbers game that poets of those two countries had played before him.



Fig. 1. « Typus Arithmeticae », from Gregorius Reisch, *Margarita Philosophica* (1503). Boethius sits on our left, calculating with the new Hindu-Arabic numerals, while Pythagoras, on our right, uses an abacus (« counter » or « checker »).

M
D
C
L
X
V
I

M
D
C
L
X
V
I

Fig. 2. An abacus arranged so as to represent Chaucer's poem *The Book of the Duchess*.

IN. RATIO ET SUPERSTITIO