

Aristoxenus and the Intervals of Greek Music



R. P. Winnington-Ingram

The Classical Quarterly. Vol. 26, No. 3/4. (Jul. - Oct., 1932), pp. 195-208.

Stable URL:

<http://links.jstor.org/sici?sici=0009-8388%28193207%2F10%291%3A26%3A3%2F4%3C195%3AAATIOG%3E2.0.CO%3B2-U>

The Classical Quarterly is currently published by The Classical Association.

ab rlf

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/classical.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.

3424
WINNINGTON, R.P.
1932

ARISTOXENUS AND THE INTERVALS OF GREEK MUSIC.

ANCIENT Greek music was purely or predominantly melodic; and in such music subtleties of intonation count for much. If our sources of information about the intervals used in Greek music are not always easy to interpret, they are at any rate fairly voluminous. On the one hand we have Aristoxenus, by whom musical intervals were regarded spatially and combined and subdivided by the processes of addition and subtraction; for him the octave consisted of six tones, and the tone was exactly divisible into fractions such as the half and quarter, so that the fourth was equal to two tones and a half, the fifth to three tones and a half, and so on. On the other hand we have preserved for us in Ptolemy's *Harmonics* the computations of a number of mathematicians, who realized correctly that intervals could only be expressed as ratios (e.g. of string-lengths), that the octave was less than the sum of six whole tones and that this tone could not be divided into equal parts. These authorities are Archytas, the Pythagorean of the early fourth century, Eratosthenes (third century), Didymus (first century) and Ptolemy himself (second century A.D.). To these we must add the scale of Plato's *Timaeus* (35B) and, closely related to it, the computations of the pseudo-Philolaus (*ap.* Boethius, *Mus.* III, 8) and of Boethius himself (IV, 6).¹ Aristoxenus is less easy to understand than the mathematicians because of the unscientific nature of his postulates. His importance, however, is very great, not only from his comparatively early date but because he claims to champion the direct musical consciousness against the scientific approach of some of his predecessors and contemporaries. But if they are under suspicion of letting irrelevant factors intrude into their calculations, he must equally be suspected of yielding to the attractions of symmetry and convenience. Only their mutual agreement, perhaps, can establish any point strongly. This article is an attempt, first of all, to state what precisely Aristoxenus says directly or by implication about the intervals of Greek music, secondly, to compare his evaluations with the ratios of the mathematicians and so consider what his rough-and-ready mathematics may conceal in the way of real musical intervals.

I. In the fragments of Aristoxenus' writings known to us as the *Harmonic Elements* are preserved two separate but similar accounts of the genera or γένη τῆς μελωδίας (I, pp. 21-27; II, pp. 46-52, Meibom). Like theorists before and after him he considers a typical tetrachord or group of four notes composing the interval of the fourth, of which the extremes (μέση, ὑπάτη) remaining fixed the means (λιχανός, παρυπάτη) move in each of the three genera (diatonic, chromatic, enharmonic) within a certain locus. The genera are characterized by these loci; the nuances (χρόαι) are special cases of each genus, selected by Aristoxenus on a principle that will be discussed later. These loci he sets out to discover (22, 24), taking first the higher note, λιχανός. The upper limit he finds in the distance of a tone from μέση, appealing to the practice of the diatonic genus.² The lower limit is a matter of controversy, in which Aristoxenus' emotions are closely engaged. In his view the λιχανός that is separated by a ditone from μέση was not only a real fact of music but characterized

¹ Gevaert (*Histoire et Théorie I.*, pp. 304-327) was the first to give these formulae the importance that is their due. This article owes much also to the work of P. Tannery (*Mémoires*

Scientifiques III, 97-115, reprinted from *Revue des Et. Gr.* XV, 336-352).

² 22, 33 οὐχ before ὁμολογεῖται is rightly bracketed by Macran.

its noblest and most beautiful type, as employed in certain ancient styles to which he refers. To his grief this true enharmonic is being banished by contemporary performers, who prefer the higher lichanos of the chromatic, and even when they employ the enharmonic approximate it to the chromatic with an inevitable alteration of character. The implications of this passage (23, 4-22) are of the utmost importance.

The locus of lichanos is thus a tone. That of *παρυπάτη* is the smallest diesis, i.e. a quarter-tone, since it is never less than a quarter-tone or more than a semitone above *ὑπάτη*. The two loci, then, meet at a point a semitone above *ὑπάτη*. Aristoxenus now turns to consider the genera and nuances individually. He defines a *πυκνόν* as a combination of two intervals which together are less than the remaining interval that makes up the fourth. The smallest *pycnon* consists of two smallest enharmonic dieses (he has already—p. 21—defined the smallest enharmonic diesis as a quarter of a tone, the smallest chromatic diesis as a third); next to it comes that which consists of two smallest chromatic dieses. And so on. Aristoxenus establishes the position of lichanos in each of his six nuances. We need not follow the details here, but we may notice certain points. His use of the imperative of definition suggests perhaps something arbitrary. And indeed on his own assumptions it must be so. If lichanos moves within a certain locus, then theoretically it may occupy an infinity of positions (26, 14 *νοητέον γὰρ ἀπείρους τὸν ἀριθμὸν τὰς λιχάνους*). There is not one enharmonic lichanos solely. Aristoxenus' argument is not that the *δίτονος λιχανός* gives the only enharmonic intonation, but that it gives the best.

Secondly, note the great importance given to the note lichanos in establishing the nuance and the relatively small stress laid upon *parhypate*, to which he turns in 26, 29. There are two loci of *parhypate*, one peculiar to the enharmonic, the other shared by the chromatic and diatonic. This for the first time brings us to the question of the combination of lichanos and *parhypate* in a single nuance and the relative size of the three intervals of the typical tetrachord. So far we have only been told that the smallest (enharmonic) *pycnon* consists of two smallest enharmonic dieses, the next smallest (soft chromatic) of two smallest chromatic dieses. Does this imply that the constituent intervals of a *pycnon* are always equal? On p. 27, 2 it is laid down that (1) the lowest interval (*ὑπάτη—παρυπάτη*) is either equal to or less than the middle one (*παρυπάτη—λιχανός*); (2) the middle interval is either equal to, less or greater than the highest (*λιχανός—μέση*). To illustrate this last possibility he appeals to the sharing of *παρυπάται* by chromatic and diatonic; that tetrachordal division also is legitimate that has the lowest chromatic *parhypate* and the highest diatonic lichanos;¹ or, to express it in figures: $\frac{1}{3} + 1\frac{1}{6} + 1$. We shall see the importance of this later. This still leaves untouched the question of the division of a *pycnon* proper, except, apparently, in the cases of the enharmonic and lowest chromatic. We must now turn to the parallel passage in Book II.

On the way thither, however, we may note a passage on p. 44, where there is a reference to mixture of the genera. This mention of mixed types (cf. *μιγνυμένων τῶν γενῶν* 7, 3; also Cleonides 9, 30) is yet more evidence that Aristoxenus was prepared to admit considerable variety within the framework of his scheme.

In Book II, 46, 19 Aristoxenus begins with a brief description of the loci of lichanos and *parhypate* in slightly different terms. He then answers the objection that notes bounding different magnitudes should be called by different names. Among other arguments he points out that this would make necessary an infinity of names. It would also be impossible (and this is the point of interest) to decide between rival claimants for the title of e.g. lichanos. *τί μᾶλλον τὴν δίτονον λιχανὸν*

¹ P. 27, 9: the text is corrupt, but Macran's *χρωματικῆς τῆς βαρυτάτης* for *χρωματικῆς παρυπάτης* is almost certainly right. Marquard imported the more general phrase used in the correspond-

ing passage in Book II. There is no reason why Aristoxenus should not have been more precise here than there and in III, 73 (*τῆς παρυπάτης ἐπὶ βαρὺ κινήσεως*).

λεκτέον ἢ τὴν μικρῶ συντονωτέραν; ἁρμονία μὲν γὰρ εἶναι τῇ αἰσθήσει κατ' ἀμφοτέρας τὰς διαιρέσεις φαίνεται, τὰ δὲ μεγέθη τῶν διαστημάτων δῆλον ὅτι οὐ ταυτὰ ἐν ἑκατέρᾳ τῶν διαιρέσεων. Again we have admission of a type of enharmonic whose lichanos was slightly higher than the *δίτονος λιχανός* that Aristoxenus approved, but still unmistakably enharmonic.

At 50, 15 he redefines *πυκνόν*. He then proceeds to give those tetrachordal divisions 'which stand out from the rest as familiar (*ἐξαίρετοί τε καὶ γνώριμοι*), because the magnitudes of the intervals in them are familiar' [Macran]. Thus he gives one enharmonic ($\frac{1}{4} + \frac{1}{4} + 2$) and three chromatic, *μαλακόν* ($\frac{1}{3} + \frac{1}{3} + 1\frac{5}{8}$), *ἡμιόλιον* ($\frac{2}{3} + \frac{2}{3} + 1\frac{3}{4}$), *τονιαῖον* ($\frac{1}{2} + \frac{1}{2} + 1\frac{1}{2}$). In the description the emphasis is on lichanos again (that is, on the total size of the pycnon), but here it is definitely stated that these pycna are divided in half by parhypate. Up to this point, he says (51, 11), both notes move; after it parhypate stands still, having run through its locus, while lichanos rises by a quarter-tone; the division ceases to contain a pycnon, and we reach the first diatonic variety, *μαλακόν* ($\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4}$). One more diatonic remains, *σύντονον* ($\frac{1}{2} + 1 + 1$). There are then six different *λιχανοί*, one for each nuance, but only four *παρυπάται*, because the last three varieties have the same. Not only so (and this is the same significant addition as before), but all the three higher *παρυπάται* are common to diatonic and chromatic alike. Again we have the rules for the relative size of intervals in the tetrachord. But here the possibility that the lowest interval may be smaller than the middle one is illustrated not only from the diatonic but from the following particular case of the chromatic: $\frac{1}{3} + \frac{2}{3} + 1\frac{1}{2}$. καὶ γὰρ αἱ τοιαῦται διαιρέσεις τῶν πυκνῶν ἐμμελεῖς φαίνονται.¹ The intervals are still what Aristoxenus would call *γνώριμα*, but he has acknowledged that his standard types in which parhypate moves proportionately to lichanos are simplifications which do not cover all the possibilities of genuine melody. He goes on to deny that the lowest interval can be greater than the middle one, giving two divisions which he stigmatizes as *ἀνάρμοστοι*: $\frac{1}{2} + \frac{1}{4} + 1\frac{3}{4}$ and $\frac{2}{3} + \frac{7}{24} + 1\frac{5}{8}$.² As an instance of a middle interval greater than the highest Aristoxenus does not this time give the specific case of $\frac{1}{3} + 1\frac{1}{6} + 1$, but speaks generally of the combination of the highest diatonic lichanos with a parhypate lower than that which is a semitone above hypate. He excludes presumably the enharmonic parhypate, thus leaving two possibilities: $\frac{1}{3} + 1\frac{1}{6} + 1$ (as in Book I) and $\frac{2}{3} + 1\frac{1}{8} + 1$.

To sum up, we may say that Aristoxenus' primary object is to delimit the spheres of enharmonic, chromatic and diatonic by defining the loci of the movable notes in each (a task, he says, never before attempted in theory: 35, 4); then within each of these to enumerate certain simple and intelligible types. He himself reveals that they do not represent all the genuinely melodious divisions, and in particular that equal division of the pycnon is not obligatory. Far less do they represent all conceivable divisions, which are infinite; and in particular he refers to an enharmonic lichanos lying between his lowest enharmonic and his lowest chromatic as being popular in his day but not accepted by him. Was it partly because he could not reduce the intervals it gave to *γνώριμα μεγέθη*?

To facilitate comparison with tetrachordal divisions expressed in ratios, I here give the nuances that occur in Aristoxenus' account with the value of their intervals in logarithmic cents (1,200 to the octave) in brackets:³

¹ The same possibility of equal or unequal division of a pycnon is expressed in III, 73, 20. In I, 29, 16 he says that the small intervals are equal *ὡς ἐπὶ τὸ πολὺ*.

² The latter is close to the enharmonic of Archytas, as we shall see, and the former not far from the chromatic of Didymus, both of which break this rule laid down by Aristoxenus and repeated by Ptolemy (*Harm.* II, 14). Was this

chromatic in actual use at an early date? And is there a polemical purpose behind the selection of these two instances?

³ In order to give whole numbers I have allowed inconsistencies involving one cent in the figures for one and a half tones and between the three-quarter tone in the soft diatonic and the three-quarter tone pycnon of the *ἡμιόλιον χρῶμα*.

Enharmonic : $\frac{1}{4} + \frac{1}{4} + 2$ (50 + 50 + 398).

Chromatic :

(μαλακόν) $\frac{1}{8} + \frac{1}{8} + 1\frac{5}{8}$ (66 + 66 + 366).

(ἡμιόλιον) $\frac{3}{8} + \frac{3}{8} + 1\frac{3}{4}$ (75 + 75 + 348).

(τονιαῖον) $\frac{1}{2} + \frac{1}{2} + 1\frac{1}{2}$ (100 + 100 + 298).

(mixed, p. 52) $\frac{1}{3} + \frac{2}{3} + 1\frac{1}{2}$ (66 + 133 + 299)

Diatonic :

(μαλακόν) $\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4}$ (100 + 149 + 249).

(σύντονον) $\frac{1}{2} + 1 + 1$ (100 + 199 + 199).

Diatonics with chromatic parhypate :

$\frac{1}{3} + 1\frac{1}{6} + 1$ (66 + 233 + 199).

$\frac{3}{8} + 1\frac{1}{8} + 1$ (75 + 224 + 199).

II. *Enharmonic*.—Eratosthenes apart, the evaluations of the enharmonic fall into two groups according as they make the pycnon consist of a major semitone ($\frac{1}{2}$) or a leimma ($\frac{2}{3}$). To the former class belong: *Archytas* $\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}$ (63 + 49 + 386); *Didymus* $\frac{3}{4} \times \frac{3}{4} \times \frac{5}{6}$ (55 + 57 + 386); *Ptolemy* $\frac{4}{5} \times \frac{2}{3} \times \frac{5}{6}$ (38 + 74 + 386). The lichanos here is a major third from μέση, less by a comma ($\frac{8}{9} = 22$ c.) than the full ditone ($\frac{9}{8} \times \frac{9}{8} = \frac{81}{64} = 408$ c.). The latter class is, in effect, based upon the diatonic sequence ($\frac{2}{3} \times \frac{5}{6} \times \frac{9}{8}$) which we find in Plato's *Timaeus*. It is the product of λήψις διὰ συμφωνίας, and so to be found in those authors who give us a 'Sectio Canonis.' It has been held to be characteristic of the Pythagorean school, but on inadequate grounds. It certainly owed its later theoretical importance to the influence of Plato; and it is very doubtful if the Pythagoreans of the sixth and fifth centuries did more than establish the harmonic framework of standing-notes with the series of numbers 6, 8, 9, 12.¹ There is reason to believe that the pseudo-Euclidean *Sectio Canonis* is an Academic document, while the enharmonic and chromatic tetrachords of the pseudo-Philolaus (*l.c.*) cannot possibly be Pythagorean, since they ignore the impossibility of equal division of tone and semitone, which must have been recognized by the early Pythagoreans. We need only note in passing that pseudo-Philolaus divides his enharmonic pycnon of a leimma into two equal diaschismata without attempting to evaluate the latter. Boethius (IV, 6) divides the $\frac{2}{3}$ pycnon into $\frac{5}{6} \times \frac{4}{5}$ by the naïve formula $\frac{2x}{x+y} \times \frac{x+y}{2y} = \frac{x}{y}$. The tetrachord of Eratosthenes is close to the ditonal scale with a slight variation : $\frac{4}{5} \times \frac{3}{4} \times \frac{1}{2}$.²

Before we can consider which of these types Aristoxenus intended to represent by his enharmonic, we must of course be clear that he intended to represent one of them. The term 'equal temperament' is often used in connection with Aristoxenus; and in a sense by dividing the octave into six and the tone into two he has produced 'equal temperament.' But the difference between his procedure and the temperament of modern theory and practice is more important than their resemblance. Our equal temperament is dictated by practical convenience in the matter of modulation. The modern theorist knows that the intervals are distorted upon a tempered instrument and by how much. But Aristoxenus did not live in an age when temperament in the modern sense was either necessary or desirable. The very existence of such variant intonations as he describes would have reduced it to futility. He believed that his semitone was equal to half a tone; he did not say 'I will make a semitone which shall be half a tone, and that tone slightly less than a true tone.' He did not set out to distort slightly his fourths and fifths; but when in the course of experiment difficulties arose he said 'The consonances vary within a minute locus.' Four

¹ It may be, as Tannery suggests, that confronted with the incommensurability of the tone and varying musical practice they relegated the movable notes to the realm of ἀπειρον and refused to speculate upon them, till Archytas tackled the problem on more realistic lines. Aristoxenus himself, who in Books I and II selected certain magnitudes as γνώριμα from the infinite possibilities, in Book III, 69, 6 affirms that in these matters of pitch and magnitude scientific treatment is impossible, since the possibilities are

unlimited; the functions and mutual arrangements of notes in the scale only are πεπερασμένα.

² The ratios of Eratosthenes' enharmonic are dictated by his choice of the minor third ($\frac{4}{5}$) for the upper interval of his chromatic and by the assumption, made also by Didymus and Boethius, that the pycnon of the enharmonic should be equal to the lowest interval of the chromatic. Dividing his chromatic pycnon $\frac{3}{4}$ into $\frac{2}{3} \times \frac{1}{2}$, he then takes $\frac{2}{3}$ as his enharmonic pycnon, leaving $\frac{1}{2}$ for the highest interval.

passages throw light on his point of view. Two are from the first book, two from the second; and they seem to show a development. On p. 24 he shelves the question of the commensurability of the intervals making up the fourth, but decides to assume that it consists of two and a half tones (*ὡς φαινόμενον ἐκείνου δύο τόνων καὶ ἡμίσεος*). The guilty conscience appears again on p. 28. The ditone, he says, is either eight times the smallest diesis or very slightly less. Between the writing of the two versions represented by Books I and II he seems to have devised an experiment, if it can be dignified by such a name, proving to his satisfaction that the fourth consists of two and a half tones. It is described on p. 56. We need not examine it in detail. Its success depends on a slight distortion of the fifths or fourths or both by means of which it is conducted; if these are all true the interval finally obtained will be in error by 24 cents. Its success, that is to say, in theory; for in practice a strictly accurate result was unlikely. Is it not a sufficient commentary upon it that, apart from the initial fourth and the supposed resulting fifth, it involves the judging by ear of ten successive consonances? If he had once after an accumulation of small errors obtained a recognizable fifth, would not that have satisfied him? How often did he conduct this operation? If it did not always come out right, it may have been this that led him in the fourth of our passages (p. 55) to say that even the magnitudes of the consonant intervals perhaps vary within an extremely minute locus (*τόπον . . . παντελῶς ἀκαριαῖόν τινα*). This is a very vague kind of 'temperament'!

Which, then, of the enharmonic types does Aristoxenus intend to represent by his enharmonic? His ditone of approximately 398 cents falls between the major third (386 c.) and the true ditone (408 c.). It is slightly closer to the latter, which I believe it to represent. Since Aristoxenus believed that his tone, ditone and semitone could be obtained by means of the consonances, there is surely a presumption that they are respectively $\frac{9}{8}$, $\frac{8}{6}\frac{1}{4}$ and $\frac{2}{2}\frac{5}{4}\frac{6}{8}$. It needs a distortion of consonances to obtain 'equal temperament' by this method, but a still greater one to obtain the major third and semitone. There is a further reason. As we have seen in the earlier section, Aristoxenus more than once mentions a certain type of enharmonic as popular in his day. It had a slightly higher lichanos than the type he preferred, but was still genuinely enharmonic; that is to say, its highest interval was greater than that of his own lowest chromatic, greater than 366 cents. It is hard not to believe that his own *δίτονος λιχανός* is that which gives an upper interval of a strict ditone, while that which is *μικρῷ συντονωτέρα* gives the major third of Archytas' enharmonic. If that is so, Aristoxenus recognizes here, at least, the difference of a comma between the ditone and the major third. And this is the difference between a major and a minor tone. The further implications of this must wait till we consider the diatonic.

But if the ditone is really a ditone, then the semitone is really a leimma (90 c.), and less than an equal semitone, though nearer to it than is the major semitone (112 c.) on the other side. As a rough approximation this is as good as we can expect from our author. But we must here face a piece of evidence that tells against the interpretation that has been adopted. In a passage already noted (p. 28) Aristoxenus grudgingly admits that the ditone may be slightly less than eight times the enharmonic quarter-tone. That is, the semitone is slightly more, not slightly less, than the equal semitone. The passage would seem to point to the lichanos of Archytas. In that case, it is hard to conceive what the *μικρῷ συντονωτέρα λιχανός* can be. I think we can understand the working of Aristoxenus' mind which produced this statement without abandoning our first hypothesis. How did he arrive at the conclusion that the ditone might be less than eight quarter-tones? Not by adding quarter-tones together till he found out, certainly. It is a concession to mathematical doctrine in some form. It might, it is true, be an admission that the $\frac{5}{4}$ interval of Archytas was less than four times $\frac{1}{16}$, if that fact had ever been

brought to Aristoxenus' notice. But it might also be due to a much simpler form of objection. It had certainly been established by mathematicians that the fourth was less than two whole tones and a half. Now Aristoxenus believed, first on faith, later on the evidence of a bad experiment, that his semitone was a correct half-tone. If any concession was to be made it should be made in the other direction: the ditone should be less than two full tones rather than the semitone less than half a tone.

Apart from this passage there is an objection of a more general kind that might be made to the view advanced. Aristoxenus claims to trust his ear and to represent the facts of practical music. If the ditonal scale was a theoretical elaboration of the Academy, it is unlikely that Aristoxenus had its intervals in mind for his enharmonic. Further, to our ears the scale of Archytas with its major third would appear far more melodious than the Platonic scale with its harsh third.¹ The answer is suggested by various considerations. Aristoxenus' *δίτονος λιχανός* is clearly a lost cause. The enharmonic in any form is rarely attempted and when it is the *δίτονος λιχανός* is slightly raised. The process is described as one of 'sweetening' (*γλυκαίνειν*). What is this but the employment of the major third instead of the harsher ditone? Secondly, though the ditonal scale may owe its theoretical importance to Plato's *Timaeus*, it is absurd to suppose it was invented by him. Immediately the ratio expressing the tone ($\frac{9}{8}$) had been discovered it was surely likely that theorists, not necessarily Pythagoreans, would try to evaluate the tetrachord by subtracting the tone twice from the interval of the fourth. By the remainder they were baffled, and it is quite possible that the speculations (and terminology) found in the pseudo-Philolaus are pre-Platonic. But, more important still, a scale of this sort is the direct result of tuning a stringed instrument by means of the consonances of the fifth and fourth. Starting from μέση A a fourth and a fifth give the two E's, a fourth upward D; a fifth downward from D gives G, a fourth upward from G C; thence a fifth downward F, and we have a diatonic scale consisting of major tones and leimmata. The effect must have been familiar, even if the tuning was subsequently adjusted to produce sweeter thirds. Indeed, Aristoxenus may be right in holding that this was the actual intonation of the older and severer style of music. That it did not entirely disappear is testified by Ptolemy, where he describes (*Harm.* I, 16; II, 16) the intonations most in use in his day for the lyre and cithara.²

As to the division of the pycnon, practice, at least in his day, may have approximated to Aristoxenus' version by aiming at strict equality. Eratosthenes and Didymus both give approximate equality in the pycnon, and the slight difference between the two ratios is really a mathematical fiction. Ptolemy's lower interval is little more than half the upper one and less than a fifth of a tone.³ Both he and Aristoxenus declare that the lower should not be the greater; and, in so far as the parhypate was a leading-note subject to attraction by hypate (as the leading-note of our major scale is subject to attraction *upwards* towards the tonic), the principle seems reasonable. But we know so little of the real nature of Greek melody that we cannot sum-

¹ The tempered thirds we tolerate are actually nearer to Plato than to Archytas.

² Note that it is the practical musicians, not any theoretical *κανονικοί*, to whom it is ascribed. They tended, he says, to substitute it for the intonation $\frac{1}{2} \times \frac{3}{4} \times \frac{1}{2}$. And, as it was used in combination with a tetrachord of $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2}$, we can see clearly one reason why they did so: $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4}$ in the E mode gives two false fourths and two false fifths; the substitution of $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4}$ in the upper tetrachord gives all true fourths and only one false fifth [F-C]. It is also interesting to note that these intona-

tions of the practical musicians of Ptolemy's time show a complete avoidance of both major and minor thirds, except in the tetrachord for which the ditonal type was substituted!

³ Tannery suggests that this is the legacy of some earlier theorist who wished to deny that Aristoxenus' quarter-tone was the smallest melodious interval. It seems to me more likely that he adopts here for the sake of uniformity the principle of division by tripling the terms ($\frac{1}{2} = \frac{1}{3} = \frac{1}{4} = \frac{1}{6} = \frac{1}{8} = \frac{1}{12}$) which gave him most satisfactory results for his chromatics. The enharmonic was extinct in his day.

marily dismiss the $\frac{28}{27}$ interval that Archytas places in the lower position, and we shall later see reason to relate it to other information we possess.

Diatonic.—As the chromatic nuances of Aristoxenus present points of particular difficulty we will pass straight to the diatonic. If our interpretation of the enharmonic is correct it carries important implications for the diatonic. For us this enharmonic has in the strict sense the *δίτονος λιχανός*. A comma higher comes the lichanos of Archytas' enharmonic. To this Aristoxenus refers, but does not take it as one of his types, partly perhaps because he disliked the school of Archytas (he was, by all accounts, a maliciously-minded person), more probably because he could not reduce it to such fractions as seemed to him intelligible. The smaller intervals of his system are the quarter- and third- tones; the interval of three-eighths of a tone only enters it *quâ* one and a half times the quarter-tone, and even in that nuance the lichanos, the important note, is three-quarters of a tone above *ὑπάτη*. He might have represented the pycnon of Archytas by seven-twelfths of a tone, but he would not have considered it *γνώριμον*. However, that he did distinguish his own enharmonic from that of Archytas shows that he was aware of the comma difference, and makes it therefore unlikely that he would fail to distinguish a major from a minor tone if he wished to represent a diatonic scale which combined the two. Actually the varieties of diatonic in Aristoxenus are as follows:

- (1) $\frac{1}{2} + 1 + 1$ (100 + 199 + 199). (2) $\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4}$ (100 + 149 + 249).
 (3) $\frac{1}{3} + 1\frac{1}{6} + 1$ (66 + 233 + 199). [And perhaps (4) $\frac{2}{3} + 1\frac{1}{3} + 1$ (75 + 224 + 199).]

Let us set against them the evaluations of the mathematical theorists. (a) First we have the scale of the *Timaeus* and the *κανονικοί*, which is also that of Eratosthenes and Boethius and employed under certain circumstances by the professional string-players of Ptolemy's day: $\frac{2}{3} \times \frac{5}{4} \times \frac{9}{8} \times \frac{9}{8}$ (90 + 204 + 204). (b) Didymus and Ptolemy (*σύντονον*) both give us a diatonic consisting of major and minor tones and a major semitone. Ptolemy's is as follows: $\frac{1}{1} \times \frac{9}{8} \times \frac{5}{4} \times \frac{10}{9}$ (112 + 204 + 182); Didymus has the tones in reverse order. (c) Archytas and Ptolemy (*τονιαίον*) both have a diatonic containing a septimal tone: $\frac{2}{3} \times \frac{9}{7} \times \frac{9}{8}$ (63 + 231 + 204). (d), (e) Ptolemy gives us two further varieties, a soft diatonic: $\frac{2}{3} \times \frac{10}{9} \times \frac{9}{8}$ (85 + 182 + 231) and the curious *δμαλόν*: $\frac{1}{1} \times \frac{1}{10} \times \frac{10}{9}$ (151 + 165 + 182). Now if we take Aristoxenus' enharmonic as equivalent to a pycnon of $\frac{2}{3} \times \frac{5}{4}$ plus a strict ditone, we are bound also to take his diatonic (1) as the equivalent of the scale of the *Timaeus*; for he expressly states that the lichanos of the one is the parhypate of the other. Where then is that variety in the size of the tones that we find in the other computations? Above all, where is the minor tone ($\frac{10}{9}$) that we find in our own 'just' intonation? If nowhere else, do we not find here a kind of 'temperament' in Aristoxenus, by which a perfect fourth consists of a major semitone and two equal 'tempered' tones? This is the view of Tannery (*l.c.*), and it is superficially attractive. But it is inconsistent even with such evidence as we have in favour of 'temperament' in Aristoxenus. It is inconsistent with his view that the tone is a magnitude that can be obtained by subtracting the fourth from the fifth. Tone, ditone and semitone are all intervals which he believed could be found by the process of *λῆψις διὰ συμφωνίας*, so that any tempering that was contemplated must also apply to the consonances. The most that could be said is that he failed to distinguish, or turned a blind eye upon, the comma difference between major and minor tones. The likelihood of this depends upon the general degree of accuracy shown in all his computations; and judgement should thus be suspended until we have them all under review. But at least the absence of variety of tones cannot be adduced as evidence, unless the nature of Aristoxenus' system is misunderstood. His nuances are not exclusive but representative types. Quite apart from the soft diatonic (2), of which consideration must be postponed, he does in effect admit a variety of diatonic containing unequal tones.

The intervals of his diatonic (3) are, expressed in cents, $66 + 233 + 199$. Those of Archytas' diatonic (c) are $63 + 231 + 204$. The discrepancy is minute. It is surely impossible to doubt that the diatonic Aristoxenus had in mind was that of Archytas, and that it was in general practical use. Any doubt of its practicality would be set at rest by the discovery that Ptolemy not only records it but regards it as the most fundamental type of diatonic, and declares that the artists of his day only used the other intonations in combination with it, a tetrachord of each. This, then, is a most interesting fact about Greek music that alike in the fourth century B.C. and the second century A.D. the Greeks used a diatonic scale containing septimal tones ($\frac{8}{7}$).

There remains unmentioned by Aristoxenus the type (b), our 'just' diatonic, the σύντονον of Ptolemy, with the variation of it that occurs in Didymus. If Aristoxenus could speak of an enharmonic lichanos which, compared with the δίτονος λιχανός, was μικρῶ συντονωτέρα, he could equally well have spoken of a diatonic παρυπάτη that was similarly slightly raised. He does not, and various causes may have led him not to do so. As in the case of the enharmonic, the resulting intervals cannot be expressed in the fractions he favours; and there was no necessity for him to mention varieties to him anomalous, unless he had either a point to make as in the case of diatonic (3) or a grievance to air as in the case of the enharmonic. But perhaps it weighed more with him that such an admission would have disorganized his theory of the loci of lichanos and parhypate. To admit a lichanos higher than the lowest lichanos was simple, but to have a parhypate wandering by however small an interval into the locus reserved for λιχανοί would never do. The loci might meet at a point but not overlap. Thus, if he was aware of a kind of diatonic in which this took place, he has suppressed it—or else it is impossible to extract any consistent sense out of his doctrine. But it is not certain that our normal diatonic (or that of Didymus) was in common use at that time, and it may well have been that when the citharodes had turned their instruments to the ditonal scale the intonation to which they adjusted them, if at all, was that which was obtained by slackening the παρυπατοειδῆ only to obtain a septimal tone from the λιχανοειδῆ. The evidence of Archytas is strong. We must now see what bearing it has upon the even harder problems of the chromatic nuances.

Chromatic.—Aristoxenus gives three types: (a) μαλακόν $\frac{1}{3} + \frac{1}{3} + 1\frac{5}{8}$ ($66 + 66 + 366$), (b) ἡμιόλιον $\frac{3}{8} + \frac{3}{8} + 1\frac{1}{4}$ ($75 + 75 + 348$), (c) τονιαῖον $\frac{1}{2} + \frac{1}{2} + 1\frac{1}{2}$ ($100 + 100 + 298$); and we may here consider also (d) the soft diatonic: $\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4}$ ($100 + 149 + 249$). The mathematical theorists give us two types in which the pycnon is a minor and a major tone respectively. Into the first class fall: (i) Eratosthenes— $\frac{2}{19} \times \frac{1}{18} \times \frac{6}{5}$ ($89 + 93 + 316$); (ii) Didymus— $\frac{1}{15} \times \frac{2}{24} \times \frac{6}{5}$ ($112 + 70 + 316$); (iii) Ptolemy (μαλακόν)— $\frac{2}{27} \times \frac{1}{14} \times \frac{6}{5}$ ($63 + 119 + 316$). Into the second fall those κανονικοί who made the pycnon consist of leimma + apotome, the latter being the difference between the tone and the leimma (cf. Gaudentius, p. 343, Jan), the pseudo-Philolaus, who split the difference between leimma and apotome without attempting a mathematical evaluation, and (iv) Archytas: $\frac{2}{27} \times \frac{2}{24} \times \frac{3}{27}$ ($63 + 141 + 294$).

Let us consider Aristoxenus' (c) first. Does its pycnon of a tone represent the 182 cents of Eratosthenes or the 204 cents of Archytas? The difference is less than a comma in either case, but it is definitely nearer to that of Archytas. Further, the fact that Archytas prefers the above formula with clumsy ratios that are not of the favoured $\frac{n+1}{n}$ type to the simpler $\frac{2}{27} \times \frac{1}{14} \times \frac{6}{5}$ would seem to indicate that the major-tonal pycnon was favoured in contemporary practice. But Aristoxenus differs from Archytas in the division of the pycnon by giving it equal intervals.¹ His parhypate is that

¹ The nearest approach to these equal semi-tones in mathematical dress is to be found in

Aristides Quintilianus, p. 117, Meibom, who divides the tone as follows: $\frac{8}{9} = \frac{1}{17} \times \frac{1}{16}$.

of the diatonic, so that, if that gave a leimma to hypate, it gave an apotome to lichanos here. The difference is the 'Pythagorean comma' of 24 cents. It seems likely that this difference between two consecutive small intervals was neglected by Aristoxenus, while he was aware of a slightly smaller difference in two rival tunings of the same string, comparison being obviously more difficult in the former case. This is the nearest to equal division of the major tone that makes musical sense.¹ But we must also call to mind that Aristoxenus regarded all the *παρνάται* above the enharmonic as common to all chromatic and diatonic *λιχανοί*, and gave as an example of a melodically satisfactory combination: (e) $\frac{1}{3} + \frac{2}{3} + 1\frac{1}{2}$ (66 + 133 + 299). Compare this with Archytas' chromatic: 63 + 141 + 294. The maximum difference is one of 8 cents, that is, practically negligible. Just, then, as we discovered Archytas' diatonic in Aristoxenus' scheme, so now we have also found his chromatic. This agreement is surely very significant. We can hardly avoid the conclusion that in the *τονιαῖον χρώμα* also Aristoxenus' tone is the major tone. Before we pass on to the chromatics of lower lichanos, we may note that none of Aristoxenus' evaluations corresponds closely to the chromatics of Eratosthenes, Didymus and Ptolemy (soft). I find it easier to believe that this reflects the practice of his time than that he either failed to recognize the distinction between major and minor tones or deliberately adopted a compromise between them.

There remain the two lower chromatics, (a) *μαλακόν* and (b) *ἡμιόλιον*, and the soft diatonic. I cannot pretend to solve the problems that arise when we try to interpret them in terms of musical practice. I have, however, various suggestions to make. The first point that is remarkable is the extraordinary closeness between the two chromatics, which Aristoxenus yet thought it worth while to record as distinct types. Even between the respective *λιχανοί* of (a) and (b) there is only a difference of 18 cents, less than a comma; that between the *παρνάται* is only half as much. We must, however, keep in the forefront of our minds that Aristoxenus was not wedded to equal division of the pycnon in the chromatic but regarded the various *παρνάται* as common; we must then examine all the values involved independently as well as in combination.

First we should consider a theory advanced by Tannery in an article published posthumously ('Sur le spondiasme dans l'ancienne musique grecque,' *Rev. archéol.*, 1911, Vol. I, pp. 41-50 = *Mém. Scient.* III, 299-309). There he attempts to identify the soft chromatic lichanos with that of Archytas' enharmonic and the hemiolic lichanos with that of the chromatic of Eratosthenes. Thus two-thirds of a tone represents $\frac{1}{3}\frac{6}{5}$, three-quarters of a tone represents the minor tone $\frac{1}{9}\frac{6}{5}$; and it is pointed out that the minor-tonal pycnon of the hemiolic type is very nearly one and a half times the major semitone $\frac{1}{4}\frac{6}{5}$. There are then, in effect, only four upper intervals in enharmonic and chromatic nuances ($\frac{8}{8}\frac{1}{4}$, $\frac{5}{4}$, $\frac{6}{5}$, $\frac{3}{2}\frac{2}{7}$), and the four types of Aristoxenus are inaccurate interpretations of them. The conclusion would be a satisfying one if we could feel it was honestly come by. But there are objections. In the first place the hemiolic lichanos of Aristoxenus is one and a half times not his soft chromatic but his enharmonic lichanos; 182 cents is near enough to being one and a half times 112 cents; but in the case of 182 and 90 the error is too great. That is to say, Tannery's interpretations of the two chromatics are mutually inconsistent. There are also difficulties about each taken separately. The equation of two-thirds of a tone with $\frac{1}{3}\frac{6}{5}$ involves an error of about a comma; it therefore just doubles the true difference between $\frac{2}{3}\frac{6}{5}$ and $\frac{1}{3}\frac{6}{5}$. The equation of three-quarters of a tone with $\frac{1}{9}\frac{6}{5}$ involves the still larger error of 32 cents. Further, whatever three-quarters of a tone means here, it presumably means the same also in the soft diatonic, where it occurs as the middle

¹ Similarly, the minor tone can be divided with approximate equality, as by Eratosthenes; but it is the division of Didymus that gives the more convincing intervals.

interval. The mathematicians give us two evaluations with a minor tone in this position—Didymus' diatonic: $\frac{1}{8} \times \frac{1}{9} \times \frac{8}{9}$ (112 + 182 + 204) and Ptolemy's soft diatonic: $\frac{2}{3} \times \frac{1}{9} \times \frac{8}{7}$ (85 + 182 + 231). Aristoxenus' 100 + 149 + 249 can scarcely be intended for the former, since 100 cents absurdly misrepresents the difference between the two tones.¹ The latter is more plausible, granted the error of 33 cents; but we can find a closer parallel than this to Aristoxenus' soft diatonic in Ptolemy.

Let us return to the soft chromatic of Aristoxenus. Its parhypate is a third-tone above $\acute{\upsilon}\pi\acute{\alpha}\tau\eta$, its lichanos two-thirds of a tone. But with the interval of a third-tone we are already familiar and have seen reason to equate it with the ratio of $\frac{2}{3} \times \frac{8}{7}$ occurring in Archytas' chromatic and diatonic. This would seem to be a fixed point. But if we have found parhypate, we are far from finding lichanos. The nearest interval to two-thirds of a tone that we can find among the mathematicians is Archytas' $\frac{2}{3} \times \frac{4}{5}$. But this gives us an incredible top interval for the tetrachord, consisting of the combination of a minor tone, a major semitone and Archytas' third-tone. The major third is 20 cents larger, but must belong either to Aristoxenus' enharmonic or to that variant of it that is expressly distinguished from the chromatic. There is in fact no musically probable interval that can be held to be represented by the top interval of this tetrachord and yet distinguished from that of the hemiolic chromatic. I suggest that Aristoxenus, favouring the equal division of pycna, and knowing his third-tone ($\frac{2}{3} \times \frac{8}{7}$) to be a true musical interval, assumed that by doubling it he could obtain a satisfactory lichanos, and so produced a completely factitious nuance.

The hemiolic chromatic and soft diatonic alike contain the interval of three-quarters of a tone, the former as pycnon, the latter as middle interval. The interval is also known to us as *σπονδειασμός*, both from pseudo-Plutarch *de mus.* § 112 (Weil and Reinach) and from Aristides Quintilianus, p. 28 (Meibom). It was employed apparently in the old Spondeion scale for the undivided 'pycnon' in the upper (and presumably also in the lower) tetrachord, instead of the semitone of the enharmonic pycnon or lowest diatonic interval. Tannery, as we have seen, interprets this three-quarter tone as the minor tone. But there is a musical interval occurring in Ptolemy's tetrachords that is far closer in value. Three-quarters of a tone is worth 148 cents, and the interval of $\frac{1}{11} \times \frac{2}{3}$ is worth 151. The latter occurs as the lowest interval of Ptolemy's *ῥαλλὸν διάτονον*. Whatever may be the truth about this peculiar-looking tetrachord ($\frac{1}{11} \times \frac{2}{3} \times \frac{1}{10} \times \frac{1}{9}$), it is unlikely to have been pure invention of Ptolemy, and it would seem to have relation to the Spondeion. If the 'lichanos' of the Spondeion was $\frac{1}{11} \times \frac{2}{3}$ above the hypate,² did it remain so when in the course of development the pycnon was divided? If so, into what intervals was it divided? These questions cannot be answered with certainty. $\frac{2}{3} \times \frac{4}{5} \times \frac{2}{3}$ would merely give the familiar fiction of equal division. The musical interval nearest in value to Aristoxenus' three-eighths of a tone (75 cents) is our own minor semitone $\frac{2}{3} \times \frac{5}{4}$ (70 cents), which occurs as the middle interval of Didymus' chromatic, but it will not conveniently combine in a tetrachord with an upper interval of $\frac{1}{9}$. Perhaps we must here, as in the soft chromatic, consider the validity of parhypate and lichanos separately, and regard the intervals of a minor semitone and an undecimal three-quarter tone as both familiar to Aristoxenus from actual practice, but their combination in a single nuance as factitious.

¹ If Aristoxenus wished to represent this type of diatonic and was taking $\frac{1}{9}$ as three-quarters of a tone, he was faced by a dilemma. Either he must grotesquely exaggerate the difference of the tones, as above, or, representing the major tone, as usual, by his own whole tone, make the lowest interval also three-quarters of a tone, which is absurd.

² The interval of $\frac{1}{9}$ will not really square

with the account in Plutarch, which demands the possibility of confusion between the upper interval of the lower tetrachord (F-A) and the combination of disjunctive tone and *σπονδειασμός* (A-C). $\frac{1}{11} \times \frac{2}{3}$ as *σπονδειασμός* makes these intervals practically identical. On this point, and on the Spondeion in general, I would refer the reader to my article on 'The Spondeion Scale' in *C.Q.*, Vol. XXII, 1928.

This same interval of the three-quarter tone occurs also in the soft diatonic of Aristoxenus. This tetrachordal division assumes the possibility of dividing the fourth into two equal parts, each $1\frac{1}{4}$ tones in size. The division of the fourth ($\frac{4}{3}$) into simple ratios which comes nearest to this is $\frac{5}{7} \times \frac{7}{8}$ ($231 + 267$). Aristoxenus' $1\frac{1}{4}$ tones (249) lie exactly half-way between these two ratios. But it would be somewhat in favour of equating the upper interval of his soft diatonic with $\frac{7}{8}$ that elsewhere he appears to represent Archytas' $\frac{8}{7}$ by the interval of $1\frac{1}{8}$ tones. On the basis of the above division Ptolemy constructs two tetrachords, his soft diatonic $\frac{21}{10} \times \frac{10}{9} \times \frac{8}{7}$ ($85 + 182 + 231$) and his *σύντονον χρώμα* $\frac{22}{11} \times \frac{12}{11} \times \frac{7}{8}$ ($80 + 151 + 267$). If we compare Aristoxenus' soft diatonic with these we find that in the former case, which sets three-quarters of a tone against the minor tone (as on Tannery's hypothesis), the maximum error is the large one of 33 cents, in the latter it is less than a comma; the middle intervals are practically identical, the semitone, which in the enharmonic represented 90 cents, is here 80, the upper interval of $1\frac{1}{4}$ tones represents the septimal third $\frac{7}{5}$. This last interval shows a divergence of 18 cents, yet I hope to show that it is really the strongest possible reason for equating Aristoxenus' soft diatonic with Ptolemy's *σύντονον χρώμα*.¹ For the part played by this septimal minor third in Greek music has not yet been fully recognized.

The soft diatonic nuance of Aristoxenus is not the only place in Greek musical theory where the interval of $1\frac{1}{4}$ tones is mentioned. Two theorists (both perhaps depending ultimately on Aristoxenus) mention the term *ἐκβολή* and define it as a rising interval of five dieses, or quarter-tones. These are Aristides Quintilianus and Bacchius. It is also mentioned, without definition, in pseudo-Plutarch, *de musica*.

(a) Aristides Quintilianus inserts between his accounts of Modulation and Melodic Composition (p. 28) a brief description of three intervals, namely: *ἐκλυσis*, a fall of three dieses; *σπονδειασμός*, a rise of three dieses; and *ἐκβολή*, a rise of five dieses. These intervals had to be employed, he says, by the ancients *πρὸς τὰς διαφορὰς τῶν ἀρμονιῶν*. They were called *πάθη τῶν διαστημάτων* (whatever that may mean) owing to the rarity of their employment. They are, then, connected in some way with the old *ἀρμονίαι*.

(b) Bacchius §§ 41, 42 defines *ἐκλυσis* and *ἐκβολή* similarly, using the phrase *ἀπὸ τινος φθόγγου ἀρμονίας*, where *ἀρμονία* might perhaps mean 'scale,' though in the only other place where the word occurs in Bacchius it means 'enharmonic.' (That he illustrates the intervals from that part of the scale in which the tetrachords *συννημμένων* and *διεξευγμένων* overlap is probably because this offered the only opportunity of illustrating the interval of three dieses.) In an earlier passage, however (§§ 36, 37), he definitely associates them with the enharmonic. After distinguishing between 'standing' and 'movable' notes, he adds to the definition of the latter class the remark 'δι' ὧν τὰ διαστήματα πάντα ἀνίσταται καὶ ἐπιτείνεται πλὴν δύο. The catechism continues. 'Which are these?' '*ἐκλυσis* and *ἐκβολή*.' 'How is this?' '*ἐκλυσis* is a fall in pitch, *ἐκβολή* a rise in pitch.' 'In what genus?' 'In the enharmonic and in no other.'

(c) The pseudo-Plutarch passage (§ 287) associates *ἐκβολή* and *ἐκλυσis* with Polymnestus and may ascribe to him their 'invention' (the reading is doubtful).²

¹ If this equation is correct, we have the interesting fact that the only type of chromatic that Ptolemy found in practical use in his time was actually regarded by Aristoxenus as diatonic!

² καὶ τὴν ἐκλυσιν καὶ τὴν ἐκβολὴν πολὺ μείζω πεποιηκέναι φασὶν αὐτόν. πολὺ μείζω certainly makes bad sense, and Reinach brackets it as concealing a marginal *Πολύμνηστον* or *Πολύμνιστον* and translates *πεποιηκέναι* by 'il créa.' But *εὐρίσκω* (or some compound) is usual in

pseudo-Plutarch of musical 'inventions,' and I prefer Westphal's hypothesis of a lacuna after *ἐκβολήν*; these accusatives then are constructed with *Πολυμνήστω* . . . ἀνατιθέασιν in the preceding phrase. It is conceivable that, just as Terpander is associated with the employment in melody of Dorian *νήτη*, so Polymnestus popularized the addition of D below an enharmonic E octave. See below.

It must be confessed that our authorities are unsatisfactory. Bacchius is very incoherent. Can any intervals except the fourths and fifths of the harmonic framework (E-A-B-E) and the disjunctive tone be said to be independent of the movements of the 'movable' notes? But it seems that he is trying to distinguish these intervals from the normal mutations within the tetrachord. I believe that the clue to *ἐκβολή* at least is to be found in the disjunctive tone and also in the tone between *ὑπάτη* and the note a tone below *ὑπάτη* (Aristides and Theo Smyrnaeus give us the convenient term *ὑπερυπάτη*). This interval is found combined with an enharmonic pycnon in the old *ἀρμονίαι* described by Aristides (p. 21) and also in the Orestes fragment.¹ There we twice find an instance of the progression P Φ, namely from enharmonic *παρυπάτη* to diatonic *λιχανὸς ὑπατῶν* (or *ὑπερυπάτη*)—an interval, that is, of five dieses. These tones, both, it will be noted, outside the tetrachordal variation, offer no clue to intervals of three dieses; and *σπονδειασμός* must certainly be associated, on the strength of pseudo-Plutarch's evidence, with the lower interval of the Spondeion trichord. It will have been remarked that the intervals quoted from the Orestes fragment are falls, not rises, of five dieses. Why is the term *σπονδειασμός* limited to a *rise* of three dieses? If *ἔκλυσις* is to be taken as its corresponding fall, it is thereby divorced from *ἐκβολή*. Was there no name for a fall of five dieses? It is hard to believe that the account of these intervals has come down to us aright. It seems to me at least credible that there has been a confusion here and that *σπονδειασμός* meant a rise or fall of three dieses and was associated with the Spondeion scale, while *ἔκλυσις* and *ἐκβολή* were respectively a fall and rise of five dieses.

However this may be, there is evidence (including that of an actual musical document) for an interval of five dieses, called into being through the relation of the enharmonic pycnon and a tone (unaffected by the genera) lying immediately below it. Now, the employment of such an interval may or may not have been rare, as Aristides says; but in any case it is likely that it was some comparatively simple musical interval and not the result of a haphazard approximate splitting of a semitone. Indeed such an interpretation of the enharmonic pycnon is only possible if we imagine the *μεσόπυκνον* to have been employed merely in relation to the extremes as a 'Durchschleifen durch das Intervall,' as Westphal puts it. It is one of the great values of the Orestes fragment for us that it shows us that this was not so. Now, there are only two intervals smaller than the major semitone ($\frac{1}{16}$) which will make with the tone below a satisfactory interval. One is $\frac{6}{63}$, which makes with the major tone ($\frac{9}{8}$) the septimal tone ($\frac{8}{7}$). But this is smaller (27 cents) than any interval which we can postulate for Greek music.² The other is $\frac{2}{27}$, which with the tone makes the septimal third $\frac{7}{6}$.

Perhaps we can now see why Archytas selected $\frac{2}{27}$ to give the common parhypate of his three genera. As Tannery points out, to understand his ratios it is necessary to consider not only the typical tetrachord (E-A) but also the tone below (D-E). For he obtained his enharmonic *lichanos* by dividing the fifth (D-A) into

¹ It is found also in the second Delphic Hymn, but there the pycnon is more probably chromatic.

² This interval is the difference between the septimal tone ($\frac{8}{7}$) and the major tone ($\frac{9}{8}$). It plays an important part in the theories of Dr. W. Perrett (*Some Questions of Musical Theory*, 1926 and 1928). Though I find difficulty in accepting them in detail, I believe with him that the Greeks used intervals strange to us with precision. They can scarcely, however, have used so small an interval as 27 cents. It is possible that the aulos-player, who could control

the intonation at will, varied the pitch of the enharmonic parhypate according as it was employed in relation to hypate or to hyperhypate. If then in the course of the same piece he made the interval between hyperhypate and parhypate that of $\frac{7}{6}$ and also employed the *lichanos* of Aristoxenus, a leimma above hypate, he would in effect be employing two notes distant only by $\frac{6}{63}$. Procedure of this sort might also have led Aristoxenus to evaluate $\frac{7}{6}$ ($= \frac{9}{8} \times \frac{2}{27}$) as $1\frac{1}{3}$ tones (see p. 207), which an interval of $\frac{8}{7}$ ($= \frac{9}{8} \times \frac{6}{63}$) between hyperhypate and parhypate would not do, as that he elsewhere equates with $1\frac{1}{3}$ tones.

$\frac{6}{8} \times \frac{5}{4}$; and, having fixed his diatonic lichanos a major tone from A, he divides the fourth (D-G) into $\frac{7}{8} \times \frac{8}{7}$ and so finds his common parhypate a septimal third above D and the interval of $\frac{2}{27}$ above E. This was no mere mathematical trick. If we are to believe the evidence, he was making a genuine attempt to interpret the actual facts of music, and the criticisms of Ptolemy arise in part from Ptolemy's ignorance. Archytas is in a fair way towards being justified in every department. His diatonic and his chromatic are found to square with tetrachordal divisions admitted melodious by Aristoxenus. His enharmonic lichanos appears to be that higher enharmonic lichanos that so roused Aristoxenus' ire, while now his parhypate is seen to give with the *ὑπερπύτη* (or, in the upper tetrachord, with *μέση*), an interval which may have been of great importance in the genuine enharmonic music of ancient Greece (an interval which, it may be added, is produced also by his diatonic, which is also Ptolemy's staple diatonic; it is thus the only type of third which occurs freely in the lyre and cithara scales of Ptolemy). Finally, we may add that what is Archytas' justification is also perhaps the explanation of a peculiarity of the notations that has often been remarked, the fact that parhypate in all genera, including the diatonic, is indicated by the same alphabetic sign.

An obvious inconsistency will have occurred to the reader. *ἑκβολή* is evaluated at five dieses. Similarly the highest interval of the soft diatonic of Aristoxenus is a tone and a quarter. I have been interpreting this as equivalent to the interval of the septimal third ($\frac{7}{8}$), which occurs in Archytas' system as the product of a major tone and the small septimal semitone ($\frac{2}{27}$). But in the discussion of the chromatic and diatonic nuances this same semitone has been taken to be equal to Aristoxenus' third-tone. The distinction is small, whatever Aristoxenus meant by his quarter-tone, but still Aristoxenus makes it. In strict consistency, then, $\frac{7}{8}$ should appear in Aristoxenus' system as $1\frac{1}{3}$ tones. He should, for instance, have evaluated his soft diatonic: $\frac{5}{12} + \frac{3}{4} + 1\frac{1}{3}$, which would then have corresponded exactly with Ptolemy's *σύντονον χρώμα*. To this may be replied that he would not countenance $\frac{5}{12}$ in his scheme any more than $\frac{7}{12}$ for the higher enharmonic pycnon. To the contention that he ought to have evaluated the *ἑκβολή* as $1\frac{1}{3}$ I can only reply that he was determined that his enharmonic should have quarter-tones, that he knew the *ἑκβολή* was an enharmonic interval, that he hoped and believed it was represented by a tone plus his enharmonic diesis, and that he was convinced that the lowest interval of a tetrachord was never larger than the middle one.

If we can, then, equate Aristoxenus' soft diatonic with Ptolemy's *σύντονον χρώμα* with a high degree of probability, does Ptolemy's own soft diatonic, $\frac{21}{10} \times \frac{10}{9} \times \frac{8}{7}$ ($85 + 182 + 231$), find no reference in Aristoxenus? Not, clearly, in his system of nuances. But there is a difficult passage in pseudo-Plutarch *de mus.* (§§ 394-407) which I think provides a mention of it. The speaker (and his authority is almost certainly Aristoxenus), answering the objection that the enharmonic quarter-tone, which he is defending, cannot be obtained by means of the consonances, turns the tables on his opponents by remarking that this objection applies equally to the intervals also which consist of an odd number of dieses. Now, they themselves prefer to employ tetrachordal divisions in which the intervals are for the most part either odd or irrational; for they are continually lowering (*μαλάττουσι*) the *λιχανοί* and *παρανήται*. (The continuation raises a fresh problem of considerable difficulty which is not relevant here.) In what genus did this lowering of lichanos take place? Not in the enharmonic, for it is the raising of lichanos that he cavils at there. More likely it is in the diatonic. The *σύντονον* diatonic for all theorists except Ptolemy has as its upper interval a major tone. The popular tendency was to lower the lichanos and produce a 'soft' diatonic. This could be done in two ways, by substituting for the tone ($\frac{9}{8}$) either the septimal tone ($\frac{7}{6}$) or the septimal third ($\frac{7}{4}$). In the latter case there resulted what Ptolemy called a *σύντονον* chromatic and Aristoxenus

a 'soft' diatonic, of which the two upper intervals were three and five dieses, that is to say *περιττά*. In the former case there resulted Ptolemy's 'soft' diatonic. Now the highest interval of this is in Aristoxenian language $1\frac{1}{2}$ tones, and this is an irrational interval (on any possible theory of that debatable term which is consistent with the evidence of the fragment on Rhythm).¹

This article must have seemed to the reader a mass of hypotheses, some more plausible, some less. No one is more conscious than the writer of the number of loose ends that remain. This was inevitable in dealing with such an unscientific author as Aristoxenus. But it seemed to me worth while to attempt to extract musical sense from his simple arithmetic; and in some cases, notably his implied admission of the diatonic and chromatic of Archytas, the sense appears to be so good that it creates a certain presumption that his other dicta are not wholly nonsensical. The acceptance of Archytas' ratios means in effect the acceptance of the seventh harmonic as an important element in Greek music. What is more likely to provoke opposition is the use of the eleventh harmonic to explain the three-quarter-tone interval (the musical probabilities are hard to estimate); and some may prefer, despite the difficulties I have mentioned, to find in it the minor tone and so equate the *ἡμιόλιον χρώμα* of Aristoxenus with the chromatic of Eratosthenes, his soft diatonic with Ptolemy's soft diatonic. In any case the absence of this latter interval from the above interpretation of Aristoxenus' nuances is a serious matter. Rather, however, than find it in the three-quarter-tone, I would believe that it has been lost by attempting to ascribe too great accuracy to the author. The *τονιαίον χρώμα* is a crucial case. Evaluated in cents it comes closer to the type with a major tone in the pycnon than to that with a minor tone. But it must be noted that twice Aristoxenus' leimma-semitone is almost exactly a minor tone. It is conceivable that the upper interval of this tetrachord may represent not only $\frac{32}{27}$ but $\frac{6}{5}$; that in the latter case its pycnon may, on the principle of common *παρυπάται*, have been divided into a major and a minor semitone (though on Aristoxenus' principles it would have been in the reverse order to Didymus). It is conceivable that he intended his *σύντονον* diatonic to represent either the ditonal diatonic or that with a major and a minor tone (probably in Didymus' rather than Ptolemy's order). He may in either case have disregarded the difference of a comma. But I have already stated why I find it hard to believe that the theorist who was agitated by the raising of the enharmonic lichanos by that interval (and how else can the *συντονωτέρα λιχανός* be interpreted?) disregarded that difference everywhere else. I could believe more easily that he deliberately banished the minor tone from his system along with the major semitone because he could not express them in simple enough fractions, because they were associated with the enharmonic lichanos he deplored, because they disarranged his theory of the loci of parhypate and lichanos. It may even be the case—and this hypothesis would save both the appearances and the reputation of Aristoxenus—that the minor tone was not in fact an interval in common use in this period of Greek music; that the harsh thirds of the ditonal scale, which were sweetened in the enharmonic by substituting the major third, were habitually modified in the diatonic by an alteration in the other direction to give the large septimal third ($\frac{9}{7} = \frac{8}{7} \times \frac{9}{8}$).

However that may be, the chief service of Aristoxenus' account of the genera seems, on examination, to be the confirmation of Archytas.

R. P. WINNINGTON-INGRAM.

TRINITY COLLEGE, CAMBRIDGE.

¹ Aristoxenus' remarks upon the lowering of lichanos here and his polemic against its raising in the enharmonic combine to show the primary importance of this string in determining genus and nuance. As for the two lowest intervals, those of Ptolemy's tetrachord might be expressed

as $\frac{5}{12}$ and $1\frac{1}{2}$ of a tone, and would thus both be irrational. It would be possible to fill up the tetrachord also with $\frac{23}{8} \times \frac{8}{9}$, that is to say $\frac{1}{3} + 1$, intervals more familiar to Aristoxenus. But this has no independent support.