

45 Pythagoras at the forge: tuning in early music

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Preliminary considerations

Much work has been done in rediscovering the forces, timbres and styles appropriate to the performance of pre-Classical musical repertory. Integral to this has been the readoption of documented tuning systems for instruments of fixed tuning such as the organ, clavichord, harpsichord and lute. Perhaps the most recent development in this field lies with the viol consort, where the best groups have rediscovered the possibility of achieving a proportion of Just or Perfect major thirds. (Throughout this article the words Just and Perfect are synonymous.)

For players of instruments that permit flexibility of tuning, and for unaccompanied singers, intonation is far harder to define than other parameters of performance since it is at best a blend of good aural training, sound technique and a knowledge of temperament, and, at worst, a total lack of blend, resulting from poor technique and ignorance of tuning systems. In practice a keyboard temperament can only be a reference standard since no performer can possibly temper intervals in the precise way demanded by fixed tuning. Indeed, any attempt at tempering will interfere with rather than help a good performance. The instincts of the best performers will produce tuning that is closer to Just or to Mean Tone tuning than to modern Equal Temperament. However, all performers need special guidance in achieving satisfactory tuning in what I will label 'Pythagorean' repertory.

Solo performers have the opportunity to be self-critical through the medium of commercial or studio recording. They can monitor their own level of achievement and take or reject the advice of their producer, and with luck the tuning will approximate to what feels right for that particular repertory. For a group of singers or instrumentalists the question of tuning becomes much more complex as each line may be in or out of tune with itself or the other lines.

The perception and taste of the performer is usually determined by early experience or by professional training. Players whose experience has been primarily in nineteenth- or twentieth-century chamber music or in orchestras are aware and may be suspicious of the narrow, flat Just major third used by period instrumentalists. After the physical differences of their instruments, the most obvious feature of historically-aware performance is the much reduced use of vibrato and a tuning closer to Just or Mean Tone than to Equal Temperament. String players are likely to produce Just major thirds even if a continuo keyboard is in a Classical or Baroque temperament such as Vallotti in which the third is a little wider than Just.

If tuning could be isolated from all the other parameters, what would tell the performer or listener that it was good or bad? Octaves, fifths and fourths are very basic to the structure of musical texture. If they sound sour, then there is little hope for any of the smaller intervals. Defective technique often results in inconsistency of intonation, but an added hazard for singers is vowel colour, and in any case

singers do not always hear accurately the pitch that they think they are producing. The listener can perceive that some vowels flatten the pitch while others sharpen it. Take the word 'Alleluia'. A cruel test is to ask a singer to sing the word slowly on one note without any vibrato, dividing it into five syllables. It is likely that 'Al-' and '-le-' will sound flat on the note given; '-lu-' will probably sound in tune, '-i-' will sound sharper; and the final '-a-' flat. *A cappella* singing is particularly vulnerable to bad vowels. A trained singer can find the right placing of the vowel to correct such pitch deviations, but one badly placed vowel can sour the effect of an otherwise justly tuned triad.

Since vocal music is more important than instrumental music before 1600, I shall concentrate on the issues of tuning in vocal performance where, in theory at least, greater subtlety is possible.

Fine shadings of tuning can best be heard between paired voices or instruments that share a very similar harmonic or overtone spectrum, because our ears identify the quality of intervals between voices by the interaction of the overtones. The beauty of an unaccompanied melodic line is determined by the consistency of pitch so that each note of the melody has precisely one 'slot'. In polyphony some notes of the scale or mode will have two or even three 'slots' depending on the harmonic context. The addition of a third and fourth line to a two-part texture gives greater responsibility to everyone since at any one time one note of the basic triad is likely to be doubled. Good tuning in polyphony demands accurate melodic movement as a starting point, but the vertical relationship of harmony must be accommodated simultaneously. Trained choirs and vocal consorts tend towards Mean Tone tuning, but the best performers, when thoroughly rehearsed, are capable of Just or Pythagorean tuning.

So, in what directions must pitch be 'bent' to achieve good tuning, and what tuning is 'right' for a particular period and a particular European country? We shall find that geography is indeed important, and that the 'right' tuning varies with the repertory: it is not, unfortunately, carved in stone for each century.

A brief historical survey

In addition to the performing instinct that seems to get the tuning 'right', I believe that the approach of the performer should be determined by a desire to appreciate the attitude of medieval and Renaissance composers and performers to their craft. Broadly speaking, Western harmony developed from the parallel fourths and fifths of organum, through the discovery of the consonant version of the major third to the concept of ways of tuning that would allow the development of chromatic harmony towards enharmonicity. Total enharmonic freedom occurs only with Equal Temperament. In all other temperaments each accidental has its own discrete identity so that an enharmonic shift involves a tuning adjustment on a held note. At this point I should mention that Just tuning makes similar demands, if and when it is attempted, in order to achieve the smoothest possible tuning of two adjacent harmonies.

For many centuries the ideas of Greek philosophers, in particular those of Pythagoras, permeated the teaching of the sciences, of which music was one. For all practical purposes octaves, fifths and fourths were considered perfect consonant

intervals; thirds, sixths and sevenths dissonances. (Seconds, whether major or minor, are best left out of this discussion for the moment.) On the European continent, the development of two- and three-part polyphony up to and including Machaut and his contemporaries shows increasingly complex linear elaboration, but it also reveals a distinct harmonic hierarchy. The most common static intervals, as opposed to passing harmonies, are octave, fifth and fourth. If a major third occurs on a long note, then it almost without exception demands outward resolution to an open fifth. Composers knew that the thirds resulting from tuning instruments of fixed pitches like organ and harp with pure octaves, fifths and fourths were the wide, rough Pythagorean major thirds, and the correspondingly dull Pythagorean minor third. Lutes, with several strings and, usually, adjustable frets, seem to have been tuned from quite early times in something approximating to Equal Temperament. The mellow harmonic content and the inability to sustain allowed a degree of tolerance in tuning matters that was not granted on the organ nor later the harmonically bright-toned harpsichord. (Indeed, much evidence of late medieval tuning has been adduced in recent years from research into the clavichord.)

When one examines the stylistic innovations of the fifteenth century, the most obvious harmonic development is the increasing use of the major third and a corresponding shift towards its function as a consonant interval: fifths begin to resolve inwards to major thirds, and final cadences with the fifths filled in by a major third become more common. The documentation of keyboard tuning seems to be in step with this aspect of stylistic change. Early keyboard anthologies such as the Buxheim Organ Book eschew the third, but when thirds do occur they do so predominantly as Schismatic thirds. (A Schismatic third occurs on a keyboard when three thirds are nearly pure but the remainder are wide, rough Pythagorean thirds. An octave, for example, is the sum of two Pythagorean thirds and a Schismatic or nearly pure third.) It was realized that appropriate transposition of the tuning would place the Schismatic thirds in 'usable' keys. If the Wolf fifth is placed between B and F# (or technically Gb), then these nearly pure thirds occur between A and C# (Db), D and F# (Gb) and E and G# (Ab). These approximations to pure thirds are a mathematical coincidence. The sequence of tuning involved is actually: C upwards to G, G downwards to D, D up to A, A down to E, E up to B; then (starting from the C above the first C) C down to F, F up to Bb, Bb down to Eb, Eb up to Ab, Ab down to Db and finally Db up to Gb in which every fifth and fourth is tuned perfect. The point is that the Db, Gb and the Ab sound like pure-tuned C#, F# and G#. (If you are puzzled, but sufficiently motivated to explore, then you might try tuning the octave between middle and treble C on a harpsichord.)

The next and more far-reaching development was Mean Tone tuning. The essence of this system is to accommodate as many Just major thirds as possible on a standard keyboard. Europe became obsessed with thirds. This may sound like an exaggeration, but it is abundantly clear from a study of fifteenth-century harmonic procedures and of keyboard tuning evidence. It is not fanciful to suppose that this change was triggered by the work of English composers and performers. A prominent element in the 'Contenance angloise' observed by French medieval writers must have been the way in which the English singers tuned their music (questions of tone colour and expressivity can only be conjectural). A study of the English repertory demonstrates an early awareness of the possibilities of harmonic

expression allowed by consonant, pure thirds compared to the more arid climate of fifths and fourths. Dufay, Josquin and succeeding continental composers appear to have taken note of the work of John Dunstable and probably of the English carol and its increasingly modern harmonic feel. A very apt and well-known example of this genre is 'There is no rose of such virtue', with its definite 'major key' feel; consistently perfect, Just tuning of its component harmonies can be achieved without undue difficulty.

This last point leads directly to a consideration of the instincts of late-twentieth-century professional singers when confronted by repertory from between 1150 and 1600. Some years of experience in this field have convinced me of a number of shortcomings in British vocal training, and ignorance of the basic anatomy of tuning is one. Even the most experienced ensemble singers have received little or no tuition. Only those singers who graduated in music or who happen to have a knowledge of keyboard temperaments are likely to have received any guidance in the theory of tuning. Even so, it is difficult to relate the theory to the practice: the simple truth is that good intonation is largely done by feel. In my experience, the hardest tuning to achieve in unaccompanied vocal music is that demanded by the Pythagorean régime, that is, medieval repertory up to and including Machaut. During the initial stages of rehearsal or with singers less experienced in the early repertory, they usually fail to stretch the major third and to make the minor second (diatonic semitone) sufficiently small. Also, they fail to give the major second its full, Just width. As rehearsal proceeds, the tuning will tend towards Pythagorean as singers progressively perfect their fifths and fourths: in the early stages the pitch can suffer badly.

With post-Machaut and most English fifteenth-century repertory, the 'right' tuning is something between the keyboard compromise of Mean Tone and the perfection of Just Tuning. Music students may read about Just Intonation and understand the theory of simple whole number ratios of pitch frequencies, or they may have approached the subject via the harmonic series, but they often do not know that there is a sizeable body of repertory that can be sung in true Just intonation.

Renaissance polyphony achieves its aural perfection through many parameters. Blend, balance and intonation are probably the main considerations, and they link up to each other in an indivisible way. The blend of an individual voice part within a choir is determined principally by uniformity of vowel colour: one singer producing the wrong vowel upsets the unison and 'sticks out'. Good tuning between two or more vocal parts is achieved by good balance of dynamic and, again, uniformity of vowel colour. On this topic there are many instances of modern editions where the editor has failed to rationalize word underlay and presents melismas with interacting lines on different vowels when there is no definite evidence for such a discrepancy. Discrepancies of this kind can be a hindrance to a satisfactory performance. Then there is the question of vocal vibrato which was used, according to historical evidence, only as an expressive device. Before modern times it was not considered desirable as an ever-present quality. Good tuning is audible only when vibrato is virtually absent or perfectly matched between the voices, both in width and in speed, and this applies to all repertoires. The best players and singers are capable of hitting notes in the middle, and the best performers instinctively clarify lush polyphony or highlight poignant harmony by passages or brief moments of vibratoless Just intonation.

Some practical advice

If my analysis of performance practice is correct, and my conscious application of tuning theory appears to fit in with the predominantly unconscious application of the principles by fellow performers, how, then, does theory intermingle with practice?

A superficial examination of the repertory to be performed will reveal the function of the major third. If few thirds are present, and those that are occur as passing harmonies and not as the result of a harmonic resolution, then the composer clearly had the Pythagorean aesthetic as his starting point. Where thirds are abundant and perceptible points of repose, then it is likely that the composer expected to hear Just thirds. The compositional style of English composers demonstrates the latter as a distinct shift away from the harmonic language of Landini, Machaut and the Mannerists, although there is a large repertory of Anglo-French conductus that pose the possibility of alternative or mixed tunings of thirds without compromising the trueness of octaves, fifths and fourths. The progressive erosion of the Pythagorean tuning system by the adoption of Mean Tone tuning demonstrates the increasing importance that was becoming attached to thirds at the expense of pure fifths and fourths. It is arguable that singers always tended towards Just, pure thirds and naturally sang perfect fifths and fourths anyway. Machaut and the Mannerists were not conservative in the notes that they wrote, but their harmonies feel most convincing if some attempt is made to tune to the Pythagorean rather than a Just or Mean Tone scale. A distinct feature of Dunstable and his contemporaries is that the music works in predominantly Just intonation. Let us look at these tuning systems in a little more detail.

A perfectly-tuned triad demands a root, a fifth and a third. The frequency of vibration of the fifth must be precisely half as fast again as that of the root. Expressed more scientifically they are in the ratio 3:2. The major third in the triad has a vibration frequency with the ratio 5:4 to the root, while a minor third has the frequency ratio 6:5 with respect to the root. Thus, the notes of the major triad are in the relationship 4:5:6. They are in fact adjacent notes in the harmonic or overtone series counting the fundamental as 1 (one). These simple ratios define Just or perfect tuning. The actual ratio of a major third in Equal Temperament is the mathematically irrational (twelfth root of two to the power four):1 which comes to 1.259921:1. The Just third is 1.25:1 precisely. Why do perfectly tuned triads not automatically produce perfectly tuned harmony in the course of a harmonic progression or indeed the course of a piece? The answer is that the Just third is not 'structural' to the vast majority of repertory since the Just Intonation scale allows of only three major and two minor triads. Additional triads require additional tuning slots.

The essence of the Pythagorean system of tuning is that all fifths and fourths are perfect except for one of each. All major seconds are the pure major tone with a ratio of 9:8. All minor seconds (diatonic semitones) are the very small Limma (256:243), and the major third is the discordant irrational interval of 81:64.

The essence of the Just Intonation scale is that the three degrees of the scale that form major thirds above a root are flattened from their Pythagorean slots to form Just thirds. The remaining 'white notes' fall into their Pythagorean slots. Similarly, the Just minor scale requires the raising of those degrees that form minor thirds above a root.

How much of a compromise is the Mean Tone system? The fifth is narrowed by a twentieth of a semitone in Equal Temperament; the fourth is stretched by the same amount. The major second is narrowed by nearly a tenth of a semitone from its Just/Pythagorean dimension. The minor second and the chromatic semitone are wider than Just. These two varieties of semitone are wider than their Just counterparts, but all four are distinctly different from their single counterpart in Equal Temperament. In the context of a major scale or mode, the most noticeable deviation is in the fourth and the seventh degrees, or in modern parlance the subdominant and the leading note. Melodically, the fourth sounds sharp and the seventh sounds flat. The receptive ear soon accommodates to these slightly strange scales because the many Just thirds more than compensate. (Indeed, the use of chromaticism in late sixteenth-century keyboard repertory often exploits the two sizes of semitone to great effect.) Gesualdo's apparently tortuous chromaticism actually conforms to a Mean Tone structure. He will appear to modulate

Ex. 1

Carlo Gesualdo, Tenebrae Responsory for Holy Saturday

Versus (Cantus et Sextus tacent)

clau - stra in - fer - ni, et sub - ver -

De - stru - xit qui - dem clau - stra in - fer - ni, et sub - ver -

clau - stra in - fer - ni, et sub - ver -

De - stru - xit qui - dem clau - stra in - fer - ni, et sub - ver -

- tit po - ten - ti - as di - a - bo - li.

- tit po - ten - ti - as di - a - bo - li.

- tit po - ten - ti - as, po - ten - ti - as di - a - bo - li.

- tit po - ten - ti - as di - a - bo - li.

Nam et ille.

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sharpwards or flatwards, but rarely in both directions within the same piece. As an example of this in the Versus section of the Fourth Responsory for the Tenebrae for Holy Saturday, he employs an E $\sharp$ in the final chord, the last F $\sharp$ occurring five bars earlier. This separation allows ample time for singers to accommodate the E $\sharp$ (see Ex. 1). The actual triadic sequence of this passage, set to the words '... et subvertit potentias diaboli', is B $\flat$ major $\rightarrow$ F major, 1st inversion $\rightarrow$ A minor, 1st inversion $\rightarrow$ E major $\rightarrow$ minor $\rightarrow$ B major, 1st inversion $\rightarrow$ E major $\rightarrow$ C $\sharp$ minor, 1st inversion $\rightarrow$ C $\sharp$ major. The reprise ('Nam et ille captus est...') takes the key back to an F major triad at the end of the second bar. True modulation occurs since the pitch of the E $\sharp$ is unlikely to coincide with that of the F $\sharp$ s cited.

Performers of pre-1600 music can quite easily accustom themselves to lower leading notes, but the raising of the subdominant in Mean Tone is quite at odds with the function of that note as a dominant seventh. A Just-tuned seventh above the dominant, possible in unaccompanied singing, is pitched distinctly lower than a Just subdominant and a lot lower than a Mean Tone subdominant, and this partly explains the late arrival of the seventh chord as a usable entity. Significantly, it appears to be the English madrigal composers who first exploited sevenths and even ninths to artistic effect in the late-sixteenth century.

Professor Easley Blackwood's remarkable account of tuning systems ought to be required reading for all professional musicians with any interest in tuning. The most significant aspect of his book is the introduction of a simple method of notating tuning by suffixing simple fractional numbers to notes printed on staves or simply giving their names by upper-case letters. Blackwood's starting point is the 'white note' Pythagorean major scale:

C D E F G A B (C) which he notates as C $_0$ D $_0$ E $_0$ F $_0$ G $_0$ A $_0$ B $_0$ (C $_0$).

The tiny interval between a Pythagorean E above C and a Just E above C is the Syntonic Comma (ratio 81:80). Blackwood simply defines the Just E as E $_{-1}$, that is to say, one comma less (lower or flatter) than E $_0$. Since a Just major second added to a Just minor third make a perfect fifth, it follows that a Just minor third is a comma wider than its Pythagorean counterpart, so that a Just minor third above D $_0$ is F $_{+1}$. Alternatively, a Just minor third below F $_0$ is D $_{-1}$. Thus it is clear that small adjustments can produce Just intervals as required. Blackwood notates the Natural or Just Intonation scale as C $_0$ D $_0$ E $_{-1}$ F $_0$ G $_0$ A $_{-1}$ B $_{-1}$ (C $_0$). To avoid confusion, it is worth noting that all notes sharing the same subscript, if used in combination as in a chord, stand in a Pythagorean relationship. If that combination produces octaves, fifths, fourths and major seconds, then these intervals are identical with Just intervals; Pythagorean sevenths, thirds and minor seconds are most definitely not Just intervals. The bugbear of the Just scale is that it contains a Pythagorean minor third between the second and the fourth degrees (D $_0$ F $_0$) and a sour, narrow fifth between the second and the sixth degrees (D $_0$ A $_{-1}$). This presents a serious problem in the Dorian mode, and it would seem clear, therefore, that medieval theorists did not recognize the possibility that performers might tend towards Just intonation – or did they? Marchetto da Padova insisted upon sharpened major thirds and very narrow leading notes to tonic intervals as in the standard double leading note cadence, but it is reasonable to presume that singers in the fourteenth century sang closer to Pythagorean since all instruments of fixed tuning, particularly organs, were in that tuning. These technicalities serve to hint at some of the problems which performers ought to be

aware of: if intractable problems arise in the preparation of a performance, they often involve tuning difficulties. Some of the repertory works only if someone in a group knows something about the ambient tuning system. A work as late as the Requiem Mass of Lassus has a harmonic structure that can catch out the unwary. It starts innocently in C major, but soon switches to a Dorian D minor and then oscillates between the two. The result of this oscillation is usually a fall in pitch by the end of the first Kyrie. The clue here is that the ambient system for Lassus was Mean Tone. Whether a keyboard is present or not, the knowledge that the second degree is a little dull and the fourth degree distinctly high should help the tuning and the pitch stability.

Technically there is only one Mean Tone system: it divides the Just major third into two equal tones. This is achieved by narrowing eleven fifths by a quarter of a Syntonic Comma. The results are easily visible in Blackwood's notation: $C_0 D_{-\frac{1}{2}} E_{-1} F_{+\frac{1}{4}} G_{-\frac{1}{4}} A_{-\frac{3}{4}} B_{-\frac{1}{4}} C_0$ (remembering that the + and - are the degree of bending away from Pythagorean tuning slots). C major is the most convenient scale for the purposes of this account, but the principles apply to any key or mode or to any 'harmonic cell' – the tonality or chord structure of a single moment in a piece. This last point is of vital importance to a singer since tuning is an instinctive response to each chord or harmony as it occurs. In the Just scale, A_{-1} is the pure tuning for a third above F_0 , but it is not the right pitch for A in relation to a G triad. A_0 is the A that relates to $G_0 B_{-1} D_0$ just as D_0 relates to $C_0 E_{-1} G_0$. A_0 becomes the local supertonic to G_0 . Singers readily take account of the two different versions of A, and that is what good tuning is about. (I have talked of the 'tendency' towards a particular tuning because it is clear that no group of musicians can ever achieve total allegiance to any of the tuning systems outlined. Some thirds in a performance of Landini or Machaut will – and probably should – come closer to Just than to Pythagorean. For those seeking to recreate a medieval performance style, whatever that might be, it is likely that little has changed in this particular area.)

I will conclude this account by mentioning some of the practical problems encountered generally by performers, but particularly by unaccompanied singers when preparing unfamiliar repertory. Singers accustomed to the feel of English and continental polyphony of the late fifteenth and early to mid-sixteenth centuries tend towards a tuning that is somewhere between Just and Mean Tone. Such singers will encounter problems if they have to cope with any essentially Pythagorean textures since they are unlikely to appreciate the mental adjustment required. Sound advice to them is: stretch the major second to its full Just/Pythagorean width, narrow the minor second and stretch the major third well beyond what is comfortable. This applies to every single interval in the piece. Concentration on these principles should assist in producing really pure fifths and fourths and excitingly abrasive harmonies, a quality that is a distinctive feature of Anglo-French conductus of the twelfth and thirteenth centuries. Singers who are skilled enough to give convincing accounts of the tortuous textures of Gesualdo are in fact closer to Just tuning than to Mean Tone. Should the director of a group feel that the singers will be aided by the presence of a keyboard, then it should not be assumed that Mean Tone will help. Allowing for the fine tuning of thirds that singers can achieve, overall pitch stability may be aided by a very discreet

background of Equal Temperament or even a Baroque temperament such as Werckmeister or Vallotti.

Performance of polyphony from the mid- and later sixteenth century is usually aided by an awareness of the principles of Mean Tone tuning, and, if a keyboard is used in performance, then singers and keyboard will be in far closer agreement

Table 1: Interval sizes

Interval	Frequency ratio	Size in cents	Nearest equivalent Equal Temperament interval
<i>Just intervals</i>			
Octave	2:1	1200.00	1200.00
Fifth	3:2	701.96	700.00
Fourth	4:3	498.04	500.00
Major third	5:4	386.31	400.00
Minor third	6:5	315.64	300.00
Major second or Major tone	9:8	203.91	200.00
Minor tone	10:9	182.40	200.00
Minor second or Diatonic semitone	16:15	111.73	100.00
Chromatic semitone	25:24	70.67	100.00
<i>Pythagorean intervals</i>			
Major third	81:64	407.82	400.00
Minor third	32:27	294.13	300.00
Minor second or Limma	256:243	90.22	100.00
<i>Syntonic Comma</i>	81:80	21.51	–
<i>Mean tone intervals</i>			
Fifth	$(3:2)(80:81)^{\frac{1}{4}}$	696.58	700.00
Fourth	$(4:3)(81:80)^{\frac{1}{4}}$	503.42	500.00
Major second or Mean tone	$(5:4)^{\frac{1}{2}}$	193.16	200.00
Minor second	$(256:243)(81:80)^{\frac{1}{4}}$	117.11	100.00
Chromatic semitone	(Major second): (minor second)	76.05	100.00

than if an anachronistic temperament is employed. This advice applies to the mainstream repertory and not to highly-chromatic works.

Since tuning is mainly about vertical chording and not about solo melodic lines, I have concentrated upon that and said little about what makes good melodic tuning. Earlier I mentioned consistency of interval size: this is a feature of the best western European singers, and that consistency tends towards Pythagorean. However, eastern European folk-singing, for example, is a very different genre, one strongly characterized by Just melodic tuning.

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IN COMPANION TO MEDIEVAL
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