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Introduction

Preliminaries

The roots of the Greek sciences of harmonics and acoustics go back to the fifth century B.C., perhaps even the sixth. No treatises survive from this period, and only one or two short quotations: even these are of questionable authenticity. Later writers, reflecting on the past, offer tantalising hints about pioneering efforts in the field. Some of these, referring to just one of several traditions of enquiry, are collected in my first chapter, and others appear in texts translated elsewhere in the book. It seems likely that these beginnings were fairly unsystematic, and were usually embedded in writings of wider scope. The classification of sciences into distinct domains and their pursuit as autonomous intellectual enterprises are things that were only beginning in the later fifth century, and came into their own in the fourth.

Even for the first three quarters of the fourth century we have very little from the pens of specialists in the musical sciences: our small collection of quotations and paraphrases of the work of Archytas, important though they are, give a pretty thin representation of the work of seventy-five years. Some of them are also included in chapter 1. But for these years we have another significant source of information in the writings of the philosophers, especially Plato and Aristotle (chapters 2 and 3). Though neither made the scientific study of music a central part of his own investigations, both found that their reflections in other areas required them to pay these subjects careful attention, and each made contributions to them which exercised a powerful influence on later theorists. Both also give us valuable information about the work of their contemporaries and predecessors.

Among the sciences which Aristotelian methods of classification identified as independent enquiries was physical acoustics; and from the years after Aristotle's death there survives a compilation, made within his 'school', the Lyceum, of problems that arise in that field, together with suggested solutions. In the same collection is a comparable set of puzzles relating to music more generally, some of which bear on harmonics. Selections from both are given in chapter 4. Something approaching the status of a complete treatise is translated in chapter 5: parts of it are certainly missing, but even as it stands it is a substantial essay in acoustics, still within the Aristotelian or 'Peripatetic' tradition. A long fragment by another philosopher in this school, Aristotle's successor Theophrastus, occupies chapter 6. It reviews, very critically, the assumptions and procedures of all theorists up to that time who had conceived

the study of music, from any point of view, as a quantitative or mathematical discipline.

So far, all that we know of specialist writings in harmonics comes either indirectly, in the reports and comments of others, or in the form of fragmentary quotations, torn from their context. But from the end of the fourth century we have two impressive works, the *Elementa Harmonica* of Aristoxenus (chapter 7), and the *Sectio Canonis* attributed to Euclid (chapter 8). The former is incomplete, and as we have it is probably the remains of more than one treatise, edited into a continuous piece at a later date, but its importance for the history of the subject can hardly be exaggerated. Subsequent theorists treated Aristoxenus' writings both as the foundation stone of one major tradition in Greek harmonics, and as the pinnacle of its achievements. The *Sectio Canonis* is dated only uncertainly to this period, but it represents neatly and systematically, and perhaps in almost its complete and original form, a rounded exposition of harmonic theory in a quite different style from that of Aristoxenus, one based in mathematics and physics rather than in musical experience. It is an illuminating specimen of the tradition that may roughly be called 'Pythagorean' or 'Platonist', in the more scientific and rigorous of its various guises.

The enterprises of theoretical harmonics, under the banners of two principal schools of thought (loose confederations of interests and approaches, each internally divided in significant ways), were now well under way. Disappointingly, very little remains of writings in either tradition over the next three hundred years. From the time when our direct evidence begins to resurface, in the first century A.D., we can put together a tolerable amount of material representing contemporary ideas, and giving some notion of their relation to their fourth-century precursors: relevant passages from two major sources are assembled in chapter 9. My remaining chapters contain three complete treatises, those of Nicomachus, from about the end of the first century (chapter 10), Ptolemy in the second century (chapter 11), and Aristides Quintilianus, whose date is uncertain, but perhaps belongs to the third century or fourth (chapter 12). Of the three, those of Ptolemy and Aristides are of great intrinsic value: Ptolemy's for his intellectual rigour, his compelling and original method, his detailed critiques of previous theories and his independent development of new ones; Aristides' for its impressive (if only fitfully successful) attempt at the extraordinary project to which its author set himself with such enthusiasm, that of bringing everything that could be said about music into a compendious scheme embracing all human life, the cosmic order and God. Both convey a mass of information about musical practice, and about the ideas of earlier writers, much of it unknown elsewhere. The work of Nicomachus, though slighter than the others, is significant as our earliest complete specimen of the genre from which Aristides' much wider vision developed, the 'Pythagorean' essay in musical metaphysics. Like the other two, it must also be treated as a landmark in musicological history for the influence it came to exert on theorists of post-classical times.

In selecting the texts to translate I have been guided by the conviction that certain major works should be presented complete, or in as complete a form as the surviving documents permit. By applying this policy to the *De Audibilibus*, Aristoxenus, Euclid, Nicomachus, Ptolemy and Aristides Quintilianus, I have inevitably squeezed out many writings with a good claim on our attention. Those who know the field will no doubt deplore the absence, in particular, of the 'Aristoxenian handbooks', essays put together in the first few centuries A.D. as compilations of the doctrines of that school. (They include the treatises of Cleonides, Bacchius and Gaudentius, and those collectively known as 'Bellermann's Anonymous'.) In mitigation I can plead that the bulk of the information they offer can be found either in Aristoxenus' *El. Harm.* itself, or in the first book of Aristides Quintilianus. On points of detail, admittedly, these authors often differ among themselves, and I have tried to make comparisons where they are relevant, especially in the notes to chapter 12. The most important of them, Cleonides, is translated in Strunk (1952). I might add that they make dreary reading: Aristides Quintilianus, for all his faults, at least has some fire in his belly.

Two other kinds of material are obtrusively missing from this volume. One relates to notation, which I have dealt with only by brief comments on the sources who mention or use it (principally Aristoxenus and Aristides Quintilianus). The treatise on which we must rely for most of our knowledge of Greek notation, that of Alypius (printed in MSG), is a work whose translation would serve little purpose: it is principally valuable for its tables of notational signs. These and the systems of ideas underlying them have been ably discussed elsewhere: see especially Gombosi (1939), chapters 3 and 5, Winnington-Ingram (1956), Henderson (1957), pp. 358ff., Barbour (1960), Pöhlmann (1970), pp. 141ff. Similarly, I have not reproduced any of the surviving scores of Greek melodies, though I have referred to them occasionally in the commentary. Most of them are printed and discussed in Pöhlmann (1970), and there is also a useful brief analysis in Chailley (1979). They are not documents in the 'literature' of music, and would therefore be out of place in this book, since no ancient theorist discusses them. The exercise of applying the theorists' constructions to the analysis of the scores is one that I must leave to the reader.

Traditions of enquiry in harmonic and acoustic science

Greek harmonics, broadly conceived, is the study of the elements out of which melody is built, of the relations in which they can legitimately stand to one another, of the organised structures (e.g., scalar systems) formed by complexes of these relations and of the ways in which different structures are generated by combinations or transformations of others. As I have already hinted, there was no single, homogeneous Greek approach to this study. For most of its history, Greek harmonic writing can be classified under two fairly distinct traditions, the 'Aristoxenian' and the 'Pythagorean', each with its own

characteristic presuppositions, methods and goals. It must be said at once that neither school is monolithic – there are important internal distinctions to be drawn – and that the work of each did not flow onwards quite independently of the other. There were occasional attempts to bring the two approaches together in a coherent synthesis, as well as more frequent polemical interactions across the doctrinal divide.

As Aristoxenus conceived it, harmonics is a science whose data and explanatory principles are independent of those in any other domain of enquiry. Its subject is music as we hear it, the perceptual data offered to the discerning musical ear. Its task is to exhibit the order that lies within the perceived phenomena; to analyse the systematic patterns into which it is organised; to show how the requirement that notes must fall into certain patterns of organisation, if they are to be grasped as melodic, explains why some possible sequences of pitches form a melody while others do not; and ultimately to display all the rules governing melodic form as flowing from a coordinated group of principles that describe a single, determinate essence, that of ‘the melodic’ or ‘the well-attuned’ itself.

Three points are of special importance in this programme. First, the science begins from the data presented to perception and grasped by it as musical. It is these that must be brought into a comprehensible order, in precisely that guise under which they strike the ear as melodic, or as exhibiting specific kinds of melodic relation: they must not be redescribed, for scientific purposes, as (for instance) physical movements of the air. The order Aristoxenus seeks is a set of relations between items grasped in their character as notes, and there is no need, and no reason, to suppose that the same relations hold between the physical movements that are their material causes. (To suggest a modern analogy, the note A is the dominant of D major, but 440 cycles per second cannot be the dominant of anything.) Secondly, and as a consequence, harmonics must *describe* the phenomena in terms that reflect the way in which they are grasped by the ear. Various special conclusions flow from this, as well as a general tendency to write in a language developed out of the terminology of practising musicians: the most important is that notes should be treated as located at points lying on a continuum of pitch, and the relations between them as ‘distances’ or ‘intervals’, *diastēmata*. Intervals must themselves be described in autonomously musical terms, as distances of various sizes in the dimension of pitch (as tones, half-tones, and the like). An interval is not to be defined by reference to something non-musical, something that the musical ear does not grasp as such (for instance, as the ratio between the speed of the movements by which the relevant pitches are produced). Thirdly, the coordinating principles of the science must themselves be found by abstraction from the perceived musical data. We explain why this sequence is melodic while that is not, by identifying a general pattern of order, essential to all melody, to which the former, and not the latter, conforms. The principle stating that this order is essential is extracted, along with all other principles in the domain, from the experience of the trained musical ear. They are not sought outside it.

for instance in mathematics: it is not because a sequence can be described by a neat mathematical formula that it is melodically coherent, nor is it because two structures differ in ways significant for mathematics that they fall into aesthetically distinct harmonic categories. (Boundaries between structures that differ significantly from a mathematical point of view may not coincide with ones that are musically important.)

Aristoxenus’ own approach involves a number of subtleties that this sketch ignores. It is also true that he had predecessors and successors, discernibly of the same musicological tendency, whose methods do not tally with his in all respects. Those whom he acknowledges as his precursors seem to have been ‘empiricists’ of a cruder sort, content to tabulate the perceptual data without seeking to discover their coordinating principles. By contrast, many later ‘Aristoxenian’ writers sought only to give a scholastic exposition of the master’s ‘doctrines’, and to reduce them to an academic system, neglecting the need for harmonic understanding to be grounded in real musical experience, and ironing out many of the penetrating ideas that Aristoxenus had derived from that source himself. Where that experience was lacking, or not applied in the proper way, no suitable yardstick was left by which the relative importance of different aspects of Aristoxenus’ analyses could be assessed. Throughout the tradition, however, two central features persist. The data are described in autonomously musical terms (conceived as representing the phenomena in the way that perception grasps them, and developed in large part from the vocabulary of musicians themselves): they are neither described nor explained by reference to concepts drawn from mathematics and physics. Secondly, intervals are consistently treated as linear distances on a continuum of pitch, measured only by the units through which musical experience is articulated, and not transformed to fit into some extra-musical metric: notes are located in (sometimes, identified with) the ‘breadthless’ points that are these intervals’ boundaries.

On the other side of the fence are those theorists who can be bundled together, for convenience, under the description ‘Pythagorean’. The word is a slippery one. Not all of those embraced by it here would have professed any allegiance to Pythagoras himself. Few were genuine members of the Pythagorean brotherhood, dedicated to its rituals and its way of life as well as to its intellectual ideas: in its original form the brotherhood had evaporated by the early fourth century. Some, in particular, were specialist mathematicians whose wider philosophical commitments, if any, had little bearing on their work in this field. Even if we restricted the term to people who would have given it to themselves, there is a world of difference between the immediate followers of Pythagoras in the sixth and fifth centuries B.C., the Pythagorising successors of Plato in the fourth and third, and the enthusiasts of the neo-Pythagorean revival in the first centuries A.D. But in harmonics they had certain views and attitudes in common, and if their work is not a single enterprise it is at least a network of interconnected strands; the crude classification is not altogether arbitrary.

We have an enormous amount of information about Pythagoreans, but so far as it bears on the sixth and fifth centuries B.C., the bulk of it is misinformation, a mixture of speculative reconstruction, anachronistic interpretation and plain forgery. From Plato onwards the ideas allegedly developed by early Pythagoreans were refurbished, embroidered and newly tailored to suit each passing intellectual or religious fashion; and such was the veneration inspired by the long-dead founder of the sect that these novel, often bizarre and gaudy philosophical garments were seldom labelled with their own makers' names, but with that of Pythagoras himself, or those of his direct disciples. As a result, our evidence is in confusion. I can do little to untangle it here: the reader should consult especially the pioneering work of Burkert (1962, translated 1972), along with that of Thesleff (1961, 1965) and Philip (1966).

From the beginning, Pythagoreans were not typically interested in the study of music for its own sake. Their researches in harmonics arose out of a conviction that the universe is orderly, that the perfection of a human soul depends on its grasping, and assimilating itself to that order, and that the key to an understanding of its nature lies in number. Music enters the matter with the discovery that the relations between notes framing an organised melodic structure can themselves be expressed in very simple and neat numerical formulae. The lengths of two sections of a string giving notes an octave apart are in the ratio 2:1, while the ratio 3:2 gives a fifth and 4:3 a fourth. These fundamental harmonic relations thus correspond to what are evidently elegant and fundamental mathematical relations, and encourage the idea that all properly harmonic intervals gain their musical status *because* of their mathematical properties. So we arrive at a position that conflicts sharply with that of the Aristoxenians. The order found in music is a mathematical order; the principles of the coherence of a coordinated harmonic system are mathematical principles. And since these are principles that generate a perceptibly beautiful and satisfying system of organisation, perhaps it is these same mathematical relations, or some extension of them, that underlie the admirable order of the cosmos, and the order to which the human soul can aspire.

These ideas fuelled enthusiasm for investigations in mathematics, together with less rationalistic speculations about the symbolic import of individual numbers. Particular attention was focussed on the mathematics of ratio and proportion, with constant reference back to the paradigmatic ratios governing basic musical structures. By the end of the fifth century, an analysis in terms of ratios had been extended to at least one pattern of attunement spanning a complete octave, where all the important relations between its notes are described as ratios of numbers; others, representing different systems of attunement, appear during the early fourth century in the work of Archytas. Questions of a more abstract order also come into sharp focus in the writings of Archytas, and insistently in Plato. Most importantly, given that such and such a set of ratios combines to form a coherent scalar system, why? What are

the mathematical principles to which a set of ratios must conform if its structure is to be harmoniously coordinated? What is the special feature of those principles that gives them their role as the source of order and coherence, whether in musical, psychic or cosmic? Again, what is it about some mathematical relations that is reflected in the concordance of their musical counterparts, while other equally intelligible ratios correspond to discords?

Such questions looked for answers in mathematics, but not only there. Pythagorean ideas about music probably began, as I have said, from observations about lengths of string. But musical sounds can be produced by other means, and in any case the audible sounds are not themselves lengths. Then when two notes stand in the relation of the octave, what are the items that stand to one another in the ratio 2:1? Here mathematical harmonics needs a supplement from speculations in physics, enquiries into the nature and causes of sound as a physical phenomenon. We know little about their origins: the earliest substantial account is in the first fragment of Archytas, which most later researches in physical acoustics took as their point of departure. A few of the details will be mentioned below: more will appear in the translations. The crucial fact is that these theorists were able to identify a quantitative physical variable (most commonly, the speed of a sound's transmission through the air) as that which determines its pitch. Ratios between different values of this variable could then be identified as the ratios with which harmonics concerns itself, and it became possible to offer physical, as well as mathematical explications of such phenomena as concordance.

In some writings, notably the Euclidean *Sectio Canonis*, the task undertaken by the author is to show how the propositions of harmonics can be demonstrated as theorems within mathematics itself, given certain assumptions about the physical nature of the musical phenomena. In others, the metaphysical and mystical sides of Pythagoreanism come to the fore, in attempts to relate the order of music to the harmony of the heavens and the intelligible organisation of the universe at large. Sometimes, as in Plato and the writers who follow him, harmonic analysis focusses only on those forms of attunement which exhibit the mathematical and metaphysical principles of coordination in their purest form – other kinds of system are dismissed as mere human aberrations. Elsewhere in the tradition, however, a mismatch between theory and practice will more probably be taken as a sign of defective theory. Archytas, for instance, and most notably Ptolemy (though his concepts and methods derive as much from his original genius as from a Pythagorean tradition), direct their analyses at least in part to musical systems in actual use, in an attempt to show that they too exhibit coherent patterns of mathematical order. But in all 'Pythagorean' harmonics there are common features that distinguish their enterprises from those of the Aristoxenians. The most important are these.

First, notes are treated as entities one of whose attributes, that of pitch, varies quantitatively, and can be expressed in numbers. Intervals between notes

are to be expressed as ratios of numbers. Notes, then, are items possessing magnitudes of some sort. They are not points on a line, with intervals as the linear 'distances' between them.

Secondly, the principles on which the structure of harmonic systems is to be analysed and by which their coherence is to be explained are mathematical. More generally, the proper language for the rigorous discussion of harmonic issues is that of mathematics, of which harmonics is a branch: it is not an autonomous discipline to be discussed in an independent terminology developed out of the professional patois of practising musicians.

Thirdly, the application of mathematical concepts to musical phenomena is mediated by a physical theory that re-identifies the entities under discussion, perceived in the guise of notes, as movements in a material medium. It is to these movements that the quantitative characteristics can be attached directly.

Finally, in the majority of 'Pythagorean' writers, the study of harmonics is part of a much larger enterprise, designed to show how the same principles govern 'harmonious' relations between the elements of all significant structures in the cosmos. The universe and its parts are all subject to the same perfect patterns of intelligible mathematical order. This programme was pursued in many different ways and with different preconceptions about the nature and source of that order, but the projects undertaken in their various ways by Philolaus, Archytas, Plato, Theon, Nicomachus, Ptolemy and Aristides Quintilianus all stem from a similar aspiration. In mathematics, and especially in mathematical harmonics, lies the key to the rational organisation of the universe.

Some writers, particularly during and after the first century A.D., made heroic attempts to combine elements drawn from both harmonic traditions. From the point of view of musical analysis, the Aristoxenians had a far richer and more flexible repertoire of concepts to draw on, and were able to give systematic accounts of a much greater variety of musical structures. The Pythagoreans offered nothing comparable, but their procedures appealed to a demand for intellectual rigour and demonstrative argument, and fed, as we have seen, into scientific and metaphysical enquiries of far wider scope. Few theorists found persuasive ways of reconciling the two approaches, which were indeed often held to conflict in their concrete conclusions as well as their conceptual apparatus. In Theon, Nicomachus and Aristides Quintilianus, Pythagorean and Aristoxenian analyses sit uneasily side by side. Only Ptolemy, whose methodological and mathematical ingenuity far exceeded theirs, offered an intellectually convincing way of coordinating a mathematically rigorous form of analysis, close to that of the Pythagoreans, with a realistic sensitivity to the complexity and variability of actual musical structures, preserving some of the musical richness of Aristoxenian accounts while wholly rejecting their framework of concepts and methods.

Ideas about the physical nature and attributes of sound were first developed in detail in the context sketched above, as part of the explanatory paraphernalia

of mathematical harmonics. Certainly there were speculations in this area before Archytas, some within the Pythagorean school, others offered by Presocratic cosmologists in the course of their researches into the constitution of the material world, and the way its perceptible phenomena arise. But Archytas frag. 1 is our first surviving sustained essay on the subject, and its enquiries set the agenda for most later researches, which refined and modified its hypotheses many times over. After Archytas and Plato, it was the school of Aristotle that conducted the most important work in physical acoustics. Here it was to some extent pursued as a science in its own right, detached from harmonics, though inheriting some of the problems and most of the presuppositions bequeathed by Pythagorean students of music and mathematics. The two disciplines are recombined in some of the Aristotelian *Problems* and in the *Sectio Canonis*, and later in the neo-Pythagorean writings of the first centuries A.D. Their authors were able to draw on physical theories from a wide range of Pythagorean, Platonist, Aristotelian and Stoic sources to underpin their programmes for harmonics.

All Greek acoustic theories begin from the proposition that sound is a form of movement in the air, caused by an impact made on the air by a solid body, or by an emission of breath. One major question for debate was how it is transmitted from place to place. The earliest authorities suppose that portions or currents of air actually travel from the source to the hearer's ear. Others (particularly in some of the *Problems*), reflect on differences between the behaviour of a sound and that of a solid missile, and hint at a different view, expressed most clearly in the *De Audibilibus*. No parcel of air travels when a sound moves: its transmission is rather a spreading pulsation in a stationary elastic medium. The second issue of prime importance concerned pitch. What is it that distinguishes a movement perceived as a high-pitched sound from that perceived as a deep one? The question is obviously crucial to mathematical harmonics, and to the interpretation of the ratios by which musical intervals are described. It is essential to identify the variable whose 'magnitudes' the terms of the ratios quantify. Several approaches can be disentangled. The earliest (that of Archytas and Plato) and much the most popular in later writings identified this variable with the speed of a sound's transmission from place to place: theories of this sort continually resurface throughout antiquity. But they posed obvious problems, most notably that of explaining how two sounds of different pitches, simultaneously produced, could be heard as simultaneous at some distance from their origin. A second theory is suggested in a number of sources, but explicitly stated only in one, the *Sectio Canonis*. It begins from the observation that a plucked string, in generating an apparently continuous sound, oscillates back and forth. The string is therefore conceived as making not one impact on the air, but a sequence of detached blows, though these follow one another so rapidly that our hearing grasps the resulting movements as a single, sustained and uninterrupted sound. This conception is next generalised to cover all forms of sound production. It is allied to a second observation, that a string generating a higher pitch oscillates

more rapidly. The conclusion of the writer of the *Sectio Canonis* is that it is precisely this relative frequency of impacts striking the ear that is responsible for or constitutes the perceived phenomenon of higher or lower pitch. Other writers (for example the author of the *De Audibilibus*) subscribed to the theory of discrete impacts, and remarked on the greater frequency of those of higher pitched sounds, but saw this as a secondary characteristic, associated with higher pitch, but not its cause; they continued to identify that cause with the velocity of a sound's transmission. There were other theories too, not all of which treated a sound's pitch as rooted in something directly quantifiable: some linked it, for example, with the 'shape' of the sound's movement. But the two I have sketched were much the most influential.

A third fundamental question, straddling the divide between acoustics and mathematical harmonics, concerned the phenomenon of concordance. Pairs of sounds heard as concordant were held to 'blend' in a characteristic way, whereas discordant pairs did not. In mathematics the question was: 'What is it that marks out the classes of ratio corresponding to concords as ones that are peculiarly well coordinated and unified?' In physical theory it was rather: 'What sort of physical interaction is there between movements with certain relative velocities, or with certain relative frequencies of impact, which causes or constitutes this blending, and which is lacking in other cases?' 'Velocity' theorists found the problem particularly troublesome: Plato offered a solution, but its details are barely comprehensible. Under the aegis of 'relative frequency' theories an ingenious hypothesis was developed of which there are traces in reports of early Pythagorean speculations, in the *Problems*, and in the *De Audibilibus*, but the issue cannot be said to have been satisfactorily resolved.

These three questions attracted the most persistent and painstaking attention, though many others were discussed, especially by Aristotle and in the *De Audibilibus*. The science never advanced to a very high level of theoretical rigour or methodological sophistication. Speculations about the quantitative determination of pitch gained support from observations of the properties of sounding strings or pipes, and we have reports of 'experiments' with other sound-generating devices too, but few inspire any confidence, and many are plainly mere 'thought-experiments' which could not have worked in practice. None, in any case, was adequate to decide between the rival theories. They were supported, if at all, by abstract arguments of modest persuasiveness: satisfactory empirical tests were never devised. Hypotheses about the physical causes of other qualifications of sound, such as volume, clarity, harshness and so forth, were similarly mounted on a basis of plausible argument combined with informal observation: in the *De Audibilibus* such hypotheses are loosely unified under an impressionistic theory about the qualitative resemblance of physical cause to acoustic effect. The central conclusions of the science, though problematic and inadequately explored, were sufficient to encourage the programmes of Pythagorean musical metaphysics, and they could stand as presuppositions that underpinned, for example, Ptolemy's detailed pre-

scriptions for the construction of instruments through whose use his theoretically based conclusions in harmonics could be assessed by the ear. In other parts of the field, there are isolated examples of acute observation and inspired guess-work. But while the harmonic sciences exhibit some of the Greeks' most impressive intellectual achievements, it would be disingenuous to make a similar claim for their acoustics.

The organisation of harmonic space

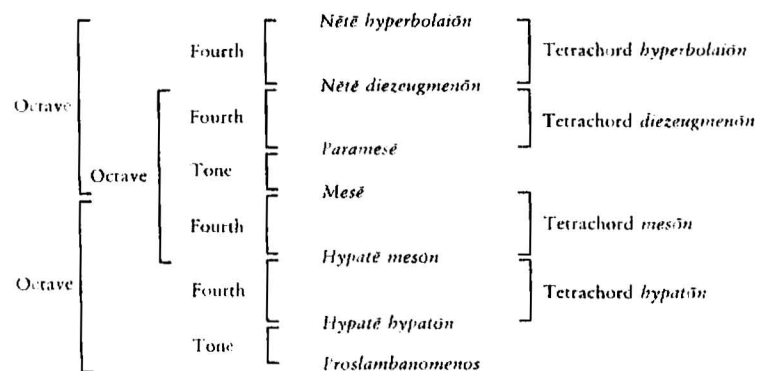
A complete account of the concepts and structures analysed by the harmonic theorists would be out of place here. A schematic sketch of some of the most important of them may be helpful as a preliminary, but I must emphasise in advance that my account will be oversimplified both conceptually and historically, particularly in its first two sections. It attempts to set out what is broadly common ground between a variety of writers and periods. It will say little about their areas of disagreement except in the section on *tonoi* below (p. 17), and less about the ways in which musical structures changed over time. Those of the period before about 400 B.C., in particular, raise problems which I shall not try to address.

Tetrachords and fixed notes

The Greeks conceived their scalar systems and patterns of attunement as expressions of the divisions and organisations imposed by melody on the tonal 'space' or range which it inhabits. The range analysed did not normally exceed two octaves, and it was of 'abstract' pitch: that is, it was not identified as any particular pitch-range, but simply as the range occupied by any melody, or by the structure implied by a melody as its foundation.

The structure was standardly envisaged as being framed by a set of notes forming the boundaries of tetrachords. In the most basic form of organisation, these tetrachords were themselves ordered in a regular way, and the notes bounding them were regarded as 'fixed', invariable in their relations to one another. Thus the central octave of the most fundamental system was divided into two principal parts, each spanning a fourth, and separated ('disjoined') by a tone. Above this octave, and sharing its highest note, lay a further tetrachord: below it was another, also in 'conjunction' with the lower of the two central tetrachords. The double octave range was completed by the addition of one more note at the bottom, at the interval of a tone below the lowest note of the lowest tetrachord. Names were attached to these fixed notes, and to the tetrachords, as in the table below.

An alternative structure, sometimes treated as a variant of the first, sometimes as an independent parallel system, retained the two lowest tetrachords, but proceeded upwards from *mesē* to a third tetrachord in conjunction with the second. This tetrachord was called *synēmmenōn* ('of conjoined notes'), as its counterpart was *diezeugmenōn* ('of disjoined notes').



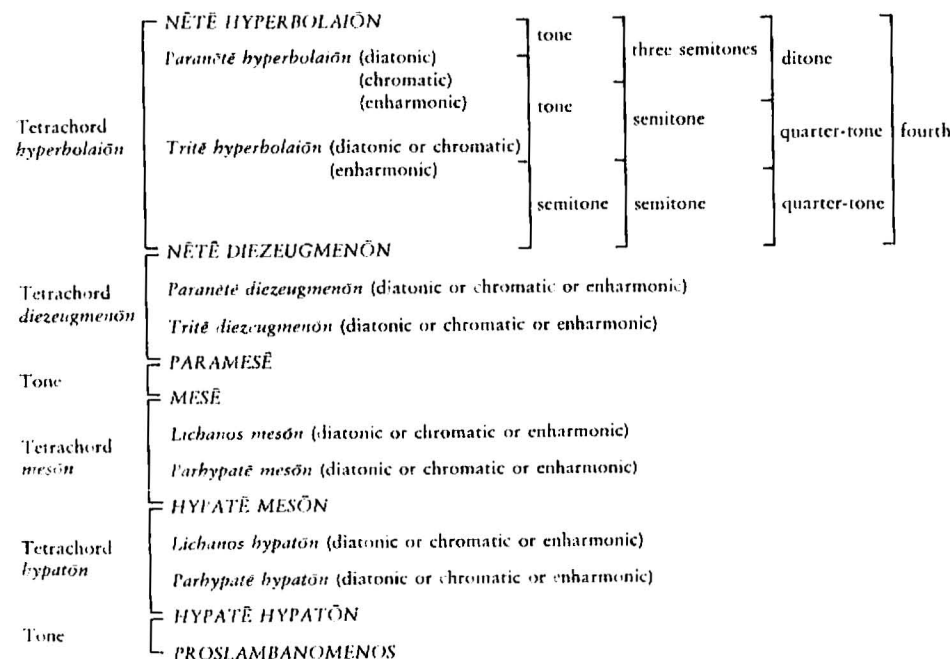
The structure was not usually conceived as extending to a further tetrachord above the highest note of this one, *nētē synēmmenōn*.

Moveable notes and the genera

Each tetrachord spans a fourth: between the boundaries of each two further notes remain to be inserted. These, however, were not invariable in their relations to their neighbours. Different systems were available in which these internal notes might be higher or lower with respect to the tetrachord's boundaries. According to Aristoxenus, for example, the higher 'moveable' note in a tetrachord might lie at any distance from a tone to a ditone below the upper boundary, and the lower moveable note at any distance from a quarter-tone to a half-tone above the lower boundary. Though an indefinite number of variations of position within these ranges were in principle permissible, certain sets of tetrachordal divisions were the most familiar, and these fell into three aesthetically distinguishable groups or 'genera', the enharmonic, the chromatic and the diatonic. Theorists differed considerably in their quantifications of these divisions (especially those theorists who described the intervals as ratios of numbers, rather than as 'quarter-tones', 'semitones', etc.). They differed also over the question of how many distinct and legitimate kinds of division there were in each genus. They typically agreed, however, that what determined difference of genus was primarily the distance between the higher moveable note and the upper boundary of the tetrachord. This distance was greatest in the enharmonic genus, smallest in diatonic, intermediate in chromatic. Let us pass over the complications, and record only the simplest and commonest of the tetrachordal divisions, as expressed in the terms used by Aristoxenus and his successors. Here an enharmonic tetrachord divides the span of a fourth between fixed notes, from the bottom upwards, into quarter-tone, quarter-tone and ditone; a chromatic tetrachord into semitone, semitone, tone-and-a-half; and a diatonic tetrachord into semitone, tone, tone.

The notes lying between fixed notes were therefore given names which

qualified them as enharmonic, chromatic or diatonic. We can now fill in the two-octave framework with the moveable notes placed between the boundaries of each tetrachord, to complete what is known as the Greater Perfect System (hereafter GPS). I have shown the details of the commonest tetrachordal divisions only in the highest tetrachord; the others are identically formed. Fixed notes are capitalised.



The two lower tetrachords, if conjoined at *mesē* with the tetrachord *synēmmenōn*, formed the Lesser Perfect System (LPS). The moveable notes between *nētē synēmmenōn* and *mesē* were named as *paranētē synēmmenōn* and *tritē synēmmenōn*. Notes in this tetrachord will sometimes stand in the same pitch-relations to *mesē* as do some of those in the parallel system between *paranētē diezeugmenōn* and *paramesē* (just which coincidences occur will depend on genus): the analyst has to inspect the whole structural context, not merely the size of the interval between, for example, *mesē* and some other note, to determine that other note's identity. Something similar is true of other coincidences of pitch in the system, for instance that between an enharmonic *lichanos* and a diatonic *parhypatē*.

The *harmoniai*

The systems so far described provide the foundation for all others, but they are not the end of the story. Nor indeed, are they properly speaking its beginning, but represent a convenient intermediate stage in the history of harmonic investigations. It will be appropriate to move next to a form of analysis that is relatively early, predating Aristoxenus, one that concerns itself with structures spanning only an octave.

From the seventh century, if not before, the Greeks were familiar with a number of distinct melodic styles, associated with different regions or peoples of the Aegean area. Although one such style, called 'Dorian' after the Dorian race of Greeks, came to be thought of as peculiarly and nobly Greek, interaction between Greeks from different places, and contact with non-Hellenic cultures, led to the adoption of several other such styles into the music of the major centres of civilisation. By the sixth century this process was well advanced, and the literature of the sixth and fifth centuries gives hints of the ways in which the styles were distinguished and employed. They were not assimilated into a single, undifferentiated cosmopolitan mélange: Ionian, Phrygian, Lydian and Dorian music seem to have retained distinct characters, credited with distinct emotional, aesthetic and moral effects, and found their places in different religious or cultural niches. Large-scale works by sophisticated fifth-century composers might shift from one style to another in the course of a single piece, but this was a way of generating changes of feeling and mood, not merely an exhibition of complicated technique. Poets like Aristophanes in the late fifth century could still indicate distinguishable musical characters simply by means of the regional names, and the differences were still sufficiently marked in the fourth century for philosophers, notably Plato, to use them as the foundation for their theories about the distinct moral characters and influences of music of different sorts.

Even if our authorities are right in attributing theoretical musical writings to authors as early as Lasus of Hermione (late sixth century), all such works are lost, and we have little solid information about the ways in which distinctions between regional styles might, at that period, have been described from a technical point of view. By the later fifth century, however, a fairly clear general conception begins to emerge: the main differences between regional types are identified as differences of what is called *harmonia*. The word has many uses, but here its primary significance is 'attunement', specifically 'pattern of attunement over the span of an octave'. Its principal application is to the organisation of intervals between notes sounded by the strings of a *lyra* or a *kithara*. Whatever may have been the case earlier, it seems that at this stage differences of pitch-range had little to do with the matter. One *kithara* probably differed little from another in usable range of pitch, but by retuning the intervals between the strings a performer could prepare his instrument for a piece in a different *harmonia*, Dorian, Phrygian or whatever.

There is no reason to suppose that these patterns of tuning originally stood

in any straightforward and organised relations to one another. Our information is sparse: just one late source sets out what purport to be a collection of scales 'which the ancients used for their *harmoniai*', that is, to prepare the attunements of their instruments (12 Arist. Quint. *De Mus.* 18.5ff.), and even that does not claim to refer to anything before the time of Plato. If the scales it describes are genuine (and it is a big 'if'), they reveal attunements that differ in the sizes of intervals used, in their order, in the number of notes brought into play, and in their overall ranges, and they cannot all be easily treated as modifications or transformations of one another.

Attempts to reduce such *harmoniai* to a system, and in particular to express them as orderly transformations of a single structure, probably originated in the later fifth century, partly as a consequence of the growing use of modulations between *harmoniai* in practical music, partly under the influence of detached, abstract musical theory, which was just beginning to appear as a serious technical discipline. We cannot assign a definite date to Eratocles, but somewhere within a decade or two of the year 400 is a reasonable guess, and it is to him and his school of harmonic theorists that Aristoxenus attributes a representation of 'the seven octachords which they call *harmoniai*', as cyclic reorderings of a given series of intervals within the octave. That is, if we begin from a series of intervals spanning an octave, constituting some one *harmonia*, we can generate another by removing the extreme interval at one end and replacing it at the other end of the series, shifting the pitches of the inner strings so that the whole always remains within the same overall compass. In this way we produce the seven 'species' of the octave, each constituting one *harmonia*.

We also hear from Aristoxenus that his predecessors' analyses were confined to the enharmonic genus, whose tetrachords were divided into two steps of a quarter-tone each, plus one of a ditone. Drawing on later sources (especially Cleonides and Aristides Quintilianus), as well as on Aristoxenus, we can ascribe to the school of Eratocles the following system of *harmoniai*, represented on a diagram that divided the space of an octave into twenty-four quarter-tones.

Mixolydian $\frac{1}{4}, \frac{1}{4}, 2; \frac{1}{4}, \frac{1}{4}, 2; 1$

Lydian $\frac{1}{4}, 2; \frac{1}{4}, \frac{1}{4}, 2; 1; \frac{1}{4}$

Phrygian $2; \frac{1}{4}, \frac{1}{4}, 2; 1; \frac{1}{4}, \frac{1}{4}$

Dorian $\frac{1}{4}, \frac{1}{4}, 2; 1; \frac{1}{4}, \frac{1}{4}, 2$

Hypolydian $\frac{1}{4}, 2; 1; \frac{1}{4}, \frac{1}{4}, 2; \frac{1}{4}$

Hypophrygian $2; 1; \frac{1}{4}, \frac{1}{4}, 2; \frac{1}{4}, \frac{1}{4}$

Hypodorian $1; \frac{1}{4}, \frac{1}{4}, 2; \frac{1}{4}, \frac{1}{4}, 2$

The list retains a number of the old names. Others, such as 'Aeolian' and 'Aeolian' have disappeared, and instead we find what are obviously specialists' terms, Hypolydian, Hypophrygian, Hypodorian. This is already evidence of a shift away from traditional practice towards systematic theorising: never-

theless, it is unlikely that the process of tidying up the *harmoniai* was completely divorced from the realities of performance. Comparison with the allegedly ancient and relatively disorganised scales mentioned by Aristides Quintilianus suggests that the Eratoclean *harmoniai* might fairly be construed as rationalised but recognisable versions of their older counterparts. Nor is there good reason to doubt that the rationalised versions represented systems of attunement which practical musicians did in fact adopt, and to which, perhaps, they had already begun to approximate before the theorists got to work.

If we now consider the table of *harmoniai* in relation to the notes and intervals of the GPS, it will be clear, first, that the central, Dorian species of the octave corresponds to the sequence, in the GPS, from *hypatē mesōn* to *nētē diezeugmenōn*, two tetrachords between fixed notes separated by a tone. This remains the case whether the analysis is set out in the enharmonic genus, as above, or in any version of the chromatic (e.g., $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{4}$; 1; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{4}$) or diatonic (e.g., $\frac{1}{2}$, 1, 1; 1; $\frac{1}{2}$, 1, 1, as on the white notes from e to e' on a modern keyboard). Each of the others is also represented in its own range of the GPS. Thus the Mixolydian structure is that which runs upwards for an octave from *hypatē hypatōn*; Lydian begins from *parhypatē hypatōn*; Phrygian from *lichanos hypatōn*; Dorian, as we have seen, from *hypatē mesōn*; Hypolydian from *parhypatē mesōn*; Hypophrygian from *lichanos mesōn*; Hypodorian from *mesē*. This sort of account is also found in our sources, for instance at 12 Arist. Quint. *De Mus.* 15.10ff.

One might therefore suppose that if these structures were to be described as standing to one another at various relative pitches, against the background of the GPS, Mixolydian would be treated as lowest, Hypodorian as highest. In fact the reverse is the case, consistently in all our sources, and it is important to see why. First, a given melody remains the same melody just so long as its pattern of movements through intervals is preserved. The range of pitch within which it is performed is not relevant. Correspondingly, the notes of the melody are identified and named by reference to the organisation of the series of intervals surrounding them, not by their absolute pitches: the note below the higher disjunction of the GPS, for instance, is always *mesē*, no matter whether, in absolute terms, it is performed at a high or low pitch. Then, if we think of an instrument with eight strings spanning an octave, two different melodies, each of the same genus and each within an octave range, may require the strings to be attuned in different arrangements of intervals. (Thus, both might be playable on the white notes of a keyboard, but one might demand an arrangement of intervals like that between e and e', while the other required one like that between a and a'.) Each will thus reflect a different *harmonia*; and given that all *harmoniai* can be found, in one location or another, in octave sequences in the GPS, each will project onto the range used a different slice of that system.

The absolute pitch of the range used is irrelevant: *harmoniai* are of 'abstract pitch', and may be described as if the pitch-range inhabited by the melody was

in each case the same. (Concretely, we may imagine a performer retuning his instrument for each *harmonia* without altering the pitches of the highest and lowest of his eight strings, those bounding the octave.) Then Mixolydian, for instance, projects onto that range the interval series, and the corresponding notes, between *hypatē hypatōn* and *paramesē*, while Hypodorian projects onto it the series from *mesē* to *nētē hyperbolaion*. In that case any given note or interval of the GPS appears in Mixolydian higher in the range of the octave employed than it does in any other *harmonia*, and in Hypodorian it appears lower. In Mixolydian, the disjunction between *mesē* and *paramesē* is the highest interval of the octave; in Hypodorian it is the lowest.

This, then, is the sense in which Mixolydian is 'higher' than Lydian, Lydian than Phrygian, and so on. It follows that relations between the *harmoniai* were not conceived as displaying the relative locations of the octave-species on a diagram, an instrument or a group of instruments whose notes covered the whole span of the GPS. If they had been, the pitch-relations would have come in precisely the reverse order. The relations are those between the levels of the octave range onto which a particular note or segment of the GPS is in each case projected: it is as though the octave range were held constant, and the GPS moved up or down to bring different parts of it within the range. When it is moved up (bringing 'lower' notes into the range) the *harmonia* is 'higher', because *mesē*, for example, has travelled into a higher part of the octave. In that case the *harmoniai* are not distinguished by their location in different regions of pitch, and their differences have nothing to do with relative pitch of performance. Though the terminology is risky, we may say that a *harmonia* is a good deal more like a 'mode' than a 'key'.

The *tonoi*

Questions about the 'relative pitches' of different *harmoniai*, conceived in the way discussed above, become musically important in connection with issues concerning the possibility of modulation between them. Thus if a melody were to shift in mid-course, for instance, from a Dorian arrangement of the intervals in its octave range to a Hypodorian arrangement, this would require that a sequence of intervals corresponding to a certain stretch of the GPS should be available in two positions, the latter at the interval of a fourth below the former. Such a melody could obviously not be played on an instrument with just eight strings, unless their pitches were altered by some expedient during the performance. (This may sometimes have been done, and we also know that composers of the later fifth and early fourth centuries, who were notoriously addicted to the practice of modulation, often added extra strings to their instruments to make a total of eleven or twelve. See, for example, the passage of Pherecrates quoted at ps.-Plut. *De Mus.* 1.141d ff., GMW vol. 1, pp. 236–8.) If we number the strings from 1 to 8 (from highest to lowest), we can assign note-names to them in each *harmonia*, and identify the intervals between them (see table below): I have specified the relations according to

Aristoxenus' quantification of the enharmonic genus, with their diatonic counterparts in brackets.

Dorian	Hypodorian
1 <i>Nētē diezeugmenōn</i> Ditone (tone)	<i>Nētē hyperbolaion</i> Ditone (tone)
2 <i>Paranētē diezeugmenōn</i> Quarter-tone (tone)	<i>Paranētē hyperbolaion</i> Quarter-tone (tone)
3 <i>Tritē diezeugmenōn</i> Quarter-tone (semitone)	<i>Tritē hyperbolaion</i> Quarter-tone (semitone)
4 <i>Paramesē</i> Tone (tone)	<i>Nētē diezeugmenōn</i> Ditone (tone)
5 <i>Mesē</i> Ditone (tone)	<i>Paranētē diezeugmenōn</i> Quarter-tone (tone)
6 <i>Lichanos mesōn</i> Quarter-tone (tone)	<i>Tritē diezeugmenōn</i> Quarter-tone (semitone)
7 <i>Parhypatē mesōn</i> Quarter-tone (semitone)	<i>Paramesē</i> Tone (tone)
8 <i>Hypatē mesōn</i>	<i>Mesē</i>

Such a modulation in enharmonic would require adjustments of pitch, or alternative strings, in three cases, numbers 5, 6 and 7. In diatonic only number 7 is altered. The modulation is relatively straightforward (some others would involve more radical adjustments). It is clear that a systematisation of the relations between *harmoniai*, and an analysis of the shifts involved in modulating from one to another, would have been useful to performers as well as interesting for theorists.

No such treatment survives: Aristoxenus implies that none was given, or none of any value for an understanding of modulation. Even in his time, analysis in terms of the *harmoniai* was outmoded. He refers to them as elements in his predecessors' constructions, but makes no direct use of them in his own (though there are associated issues that he does discuss, and treats as relevant to his own articulation of harmonic structures). In their place, connected with similar problems about modulation, we find references to systems called *tonoi* (sometimes *tropoi* in later writers).

Aristoxenus' full-dress account of the *tonoi* is lost, though he mentions them in several surviving passages. We are left with the hints those passages give, the compressed and often confused reports of later Aristoxenians, and a beautifully clear, meticulously detailed exposition by Ptolemy. Unfortunately Ptolemy tells us little about the systems of Aristoxenus and his followers beyond what can be gleaned from other sources. His account describes his own novel construction, and he mentions others only to criticise them, without fully explaining their nature or their authors' intentions.

The incompleteness, internal confusion and mutual contradictions of our sources make the topic of the *tonoi* one of the thorniest in Greek musical

science; many of its details remain obscure, and I cannot pursue all of them here. But some sketch of their character and functions must be offered, and it is important at least to identify the principal source of difficulty. It lies, I think, in the fact that theorists conceived the *tonoi* in two quite different ways and used them for two different purposes, which they themselves did not always distinguish clearly, and indeed they are interconnected. Very broadly, some treatments envisage them in a manner comparable to our notion of 'key', others in a way closer to that of 'mode' (though both terms, as I have suggested above, are to some degree misleading). Perhaps these conceptions appeared at different times, reflecting real changes in musical practice, or perhaps both – or some fusion of the two – were originally worked out by a theorist involved in a period of transition. In the latter case the theorist in question must be Aristoxenus. If he was responsible for only one version of the theory of *tonoi*, the confusions in our sources will be due to an uncritical conflation of his account with those of later writers, who approached the topic from a different angle. Let us first try to clarify, in general terms, the nature of the two main uses to which the idea of *tonos* was put.

I shall simplify my exposition by ignoring chronology, and considering first, fairly briefly, the system of Ptolemy. Though it is late and perhaps idiosyncratic, it brings out clearly a connection between *tonoi* and species or rearrangements of the octave, which other accounts tend to obscure. For Ptolemy the connection is intimate and essential.

He does not directly follow the principle of the cyclic reordering of intervals which is found in the pre-Aristoxenian system of *harmoniai*. Nevertheless, his results are comparable to those that would arise from an application of this principle to the whole two-octave structure of the GPS, taken in a familiar version of the diatonic genus. As intervals are shifted from the top of the structure to the bottom, they carry with them the names of the notes bounding them: hence for these purposes the highest note of the GPS, *nētē hyperbolaion*, is treated as identical with the lowest, *proslambanomenos*. Ptolemy's main intention, like that of the exponents of the *harmoniai*, is that each *tonos* should project onto a specified octave range a different species of the octave: the range in question lies roughly in the centre of the system, running from its fifth degree to its twelfth. In each *tonos* the fifth degree is occupied by the note from which the corresponding *harmonia* was conceived as beginning – *hypatē hypatōn* in Mixolydian, *parhypatē hypatōn* in Lydian, and so on.

This organisation sounds neat and simple, but in practice involves two related sorts of awkwardness. First, if the intervals of the central, characteristic range are to be projected in each case onto an octave between the same pitches, the outer limits of one double-octave *tonos*, the Hypolydian, must be shifted upwards by a semitone. (Talk of 'semitones' is in fact inappropriate in discussions of Ptolemy, whose representation of intervals as ratios of numbers does not admit exact half-tones, but I shall ignore this complication here.) The alternative would be to locate the outer notes of its central octave a semitone below those of the other *tonoi*, but since the differences between *tonoi* are

conceived as those between their organisations of intervals within a single abstract octave of pitch, such a manoeuvre would be out of place. But secondly, as I have said, Ptolemy does not derive the pitch-relations between his *tonoi* (defined by the intervals between their *mesai*) directly from the principle of cyclic reordering. Such a method would obviously generate serious complications when applied to different generic divisions of the tetrachords of the system. The relations between the *mesai* of Ptolemy's *tonoi* do not alter with changes of genus, and correspond in fact to a form of the diatonic series. They are derived, however, from a principle that is independent of considerations of genus: roughly, it is that each *mesē* should be locatable by movements through concords (fourths, fifths, octaves) from every other. But given that the relative positions of the *mesai* are unaffected by genus, it is inevitable that in some genera, the boundaries of the 'central octave' will sometimes be moved upwards or downwards after all, since in some *tonoi* these boundaries are occupied by moveable notes.

The diagram below may be helpful, but in certain respects it simplifies the matter drastically. First, it represents its intervals in the way they would have been treated by an Aristoxenian, as tones, semitones, etc., not in ratios as Ptolemy does. Secondly, it presents them in only one genus, the most familiar form of Aristoxenus' diatonic (it corresponds roughly, but not exactly, to Ptolemy's ditonic diatonic). These expedients blur some of the complexities mentioned above. Thirdly, I have avoided the difficulties of expression introduced by Ptolemy's dual terminology for naming notes, by *thesis* and by *dynamis* (see 11 Ptol. *Harm.* Book II ch. 5). The vertical lines are the boundaries of semitones, numbered from 0 to 25. The notes of each *tonos* are located on this grid, and are also represented by numbers, corresponding to their ordinal positions in the GPS, so that 1 is *proslambanomenos*, 2 is *hypatē hypatōn*, and so on. The numeral 8, which indicates *mesē*, is emphasised in the diagram, since its changing locations determine what Ptolemy thinks of as the

	central octave																									
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Mixolydian	12	13		14	15/1		2	3		4		5	6		7		8		9	10		11		12		
Lydian	13		14	15/1		2	3		4		5	6		7		8		9	10		11		12	13		
Phrygian	14	15/1		2	3		4		5	6		7		8		9	10		11		12	13		14		
Dorian	15/1		2	3		4		5	6		7		8		9	10		11		12	13		14	15/1		
Hypolydian		2	3		4		5	6		7		8		9	10		11		12	13		14	15/1		2	
Hypophrygian		3		4		5	6		7		8		9	10		11		12	13		14	15/1		2	3	
Hypodorian		4		5	6		7		8		9	10		11		12	13		14	15/1		2	3		4	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	central octave																									

pitch-relations between his *tonoi*. Because the systems are cyclic, the top of each *tonos* is conceived as joining on to the bottom: the highest and lowest notes are identical. For the same reason, *proslambanomenos* is always identical with *nētē hyperbolaion*: since the former is the first note in the GPS and the latter the fifteenth, I have represented their joint location by the symbol 15/1. The fifth degree of the scale stands at the same pitch in every *tonos*, seven semitones from the bottom, as does the twelfth, nineteen semitones from the bottom: these mark the boundaries of the central octave, in which the seven species appear.

Where *tonoi* are conceived in this way, the Dorian *tonos* is identical with the GPS in its usual form. All the others take the intervals of the GPS in the same sequence, but rotate them so as to begin from a different starting-point. All of them inhabit the same (abstract) range of pitch, but organise it differently. Relative pitch enters the matter not because the *tonoi* are transpositions of the same structure to a different pitch-level in the manner of keys (for they are not), but because issues to do with modulation require us to consider the size of the interval through which the GPS must be rotated in order to project the intended new organisation onto the same range of pitch. One might seek to define these pitch-relations by reference to the movements of *proslambanomenos*, but its travels between the bottom of the system and the top would confuse the scheme: it is simpler to track the peregrinations of *mesē*, which always remains within the central octave.

Despite the late date of Ptolemy's system, it seems likely that it reflects some of the preoccupations of much earlier theorists, writing at a time when the old *harmoniai* were still a live element in musical composition. This notion of *tonos* will have continued to be useful just so long as differences between patterns of attunement, within the same range of pitch, remained an important source of aesthetic distinctions in practical music. Music of this sort may fairly be called 'modal', where the modal character of a composition depends crucially on the order in which intervals are taken on a scale spanning its compass. (This of course does not exhaust the notion of 'mode', but we know virtually nothing that would help us to decide whether Greek music of any period incorporated other salient features of modal systems.)

While 'modal' conceptions remained influential, discussions of *tonoi* might be expected to display the following features.

(a) Though the note, *mesē*, by reference to whose position each *tonos* is located, moves up and down in the range (as does every other named note), the two-octave scales belonging to the various *tonoi* do not: their notes and intervals move round in the circle of the same range of pitch. Hence they can all be represented in their proper relations within the same span of two octaves (or two octaves and a semitone), as in our diagram. The *mesē* of each *tonos* is at a different pitch, but every *tonos* occupies the same pitch-range overall. To put it another way, a composer interested in modulations would find the whole compass of every *tonos* available within the boundaries of any single two-octave range he chose.

(b) Correspondingly, in a shift from one *tonos* to another the crucial change is in the arrangement of the intervals in the system, and especially in those of its central octave. It involves no necessary movement by the performer to a new range of pitch, no transposition of 'key', and the scheme is not designed to express relations between scales played on instruments of higher and lower compass.

(c) The number of *tonoi* will correspond to the number of species of the octave, as with the *harmoniai*. As Ptolemy argues, there can be only seven.

In some theoretical writings, however, these features become blurred or disappear. *Tonoi* are said to differ in the levels of pitch they occupy. They are all conceived as identical double-octave scales, following the standard or 'Dorian' form of the GPS, each being a progression in a straight line from low to high, not a cycle, and each beginning and ending above or below its neighbours. Since they form a set of overlapping systems, each spanning the full extent of the GPS, they jointly cover a range that substantially exceeds two octaves. I shall not represent them in a diagram here: the 'wing-shaped' figure printed in the translation of Aristides Quintilianus (see pp. 428–9 below) will serve the purpose. They are commonly described as providing for the needs of higher and lower voices or instruments: again, they are being treated as transpositions of identical interval-sequences to different levels of pitch. Their number is not restricted to seven, for we hear of one system incorporating thirteen, another of fifteen.

These changes in theoretical presentation, found mainly in the Aristoxenian compilers of the first few centuries A.D., must reflect what happened to musical theory in a period when the importance of distinctions between *harmoniai* or 'modes' had waned, leaving one species of the octave, the Dorian, in possession of the field. As a result, the concept of *tonos* came to be treated, by some theorists at least, as something very close to 'key' in its modern sense. To describe a composition as being 'in' a certain *tonos* was no longer an indication of the order of intervals forming its structure: such structures were treated as uniformly 'Dorian'. The *tonos* indicated merely the pitch at which the piece was performed, whether absolutely (in connection with roughly standardised ranges belonging to common types of instrument or voice), or relative to preceding sections of the same composition. There is good evidence of the divorce of *tonoi* from *harmoniai* in some of the surviving fragments of Greek musical scores. Their notation allows us to identify the *tonos* in which they were written: in most cases this bears no relation to the *harmonia* constructible out of the interval-sequence they use, and must indicate only 'key', in a sense related to pitch. Curiously enough, though the Dorian species of the octave retained its primacy, the Dorian *tonos* did not, at least for purposes of notation. At some stage in history (not clearly dateable) it became normal to write melodies of middling pitch in the notation of the Lydian *tonos*, this being conceived, apparently, as representing the two octaves most comfortably fitted to the commonest kind of voice.

Plainly this new conception of 'key', once detached from the modal

harmoniai and used to distinguish only the various transpositions of a single series of intervals, need no longer confine itself to seven *tonoi*. Indeed, it would be odd if it did, since even the seven diatonic versions of the 'modal' *tonoi*, set out as in our diagram, make use at one point or another of every semitonal step in the range of the central octave. Why should not a different *tonos*, in its role as 'key', be associated with each step? That is, why should not the *mesai* of the various possible *tonoi* be arranged at intervals of a semitone over the whole span of an octave?

We might expect a system of *tonoi* conceived in this manner to offer us twelve keys, just as there are in modern 'classical' theory if one ignores the differences between, for example, B flat and A sharp. In fact, however, our sources never speak of a collection of twelve, but typically attribute thirteen to Aristoxenus and fifteen to some later, unspecified authorities. I shall say little about the latter system. It was evidently a purely theoretical construction, three of whose 'keys' are merely repetitions of others at the octave: the additions were made only to yield a certain neatness of nomenclature (see 12 Arist. *Quint. De Mus.* 21.1–4).

The common ascription of thirteen *tonoi* to Aristoxenus raises more interesting issues. There is no doubt that in his time the modal systems were still alive, or at the least vividly remembered, and Aristoxenus' extant writings contain several hints that a discussion of the species of the octave would have substantial importance. The sketchy remarks about *tonoi* and related matters that survive in the *El. Harm.* do not allow us to conclude with certainty whether his treatment linked them to the *harmoniai*, or developed only the newer conception of 'key'. The likeliest hypothesis, I suggest, is that he made some attempt to accommodate both. If so, the confusion of ideas among the compilers would be the more understandable.

Where, then, does a system of thirteen *tonoi* fit into the picture? In particular, how could they have been related to the seven *harmoniai*? Even if they were conceived purely as keys, there should be only twelve. To insist on completing the octave with a thirteenth looks like the act of a theorist more concerned with tidiness than with musical realities, and that is not a description which fits Aristoxenus. Some modern scholars, as a result, have simply refused to believe the sources that ascribe this system to him.

Perhaps some scepticism is warranted, but I think we should avoid it if we can: the unanimity of the sources cannot be disregarded lightly. Let us reconsider some features of the set of 'modal' *tonoi*, each with its own focal note or *mesē*, represented in our previous diagram. Each *mesē* falls on a different degree of the central octave, but there are several semitonal steps in the octave on which no *mesē* falls. If we ignore for the present the repetition of the first note at the octave, there are five steps to which no *mesai* belong; and it is worth noticing that three of them are at pitches on which notes do fall in the fundamental Dorian scale (pitch-numbers 8, 10, 15). One can imagine a practical musician raising the question why he should not use these notes, already present in the Dorian attunement, as *mesai* to which he could

modulate, while generating attunements corresponding to other *harmoniai* as legitimately as he could by the modulations prescribed. Specifically, why should he not use pitch 8 (Dorian *parhypatē mesōn*) instead of pitch 9 as the *mesē* of his Hypophrygian structure; pitch 10 (Dorian *lichanos mesōn*) instead of pitch 11 as the *mesē* of his Hypolydian; and pitch 15 (Dorian *tritē diezeugmenōn*) instead of pitch 16 as the *mesē* of his Lydian? No doubt this involves shifting the boundaries of the central octave down by a semitone, but does that matter? For practical purposes it only means that the outer strings must be adjusted slightly as well as some of the inner ones, and the total number of strings that will need to be shifted from their 'Dorian' pitches is not always greater in these lowered versions of the three *tonoi* than it is when they have their original positions. (The lowered Hypophrygian requires five strings to be altered from their Dorian pitches, including the two outermost ones, by comparison with three in the original version; the lowered Hypolydian requires three alterations by comparison with five; both versions of Lydian require four.) We have already seen that the Hypolydian *harmonia* or species of the octave can only be kept exactly within the central octave by a slightly dubious expedient. The purities of theory are already compromised.

A musicologist who accepted the strength of this argument would find himself acknowledging the legitimacy of two alternative *tonoi*, a higher and a lower, associated with each of these three octave-species. The degree of the octave on which the *mesē* of each *tonos* stands is still the same – the second degree in Hypophrygian, the third in Hypolydian, the sixth in Lydian. What have changed are the distances between these *mesai* and those of the other *tonoi*, and one result of this is to allow more flexibility in modulation. Sometimes it will be more convenient or melodically acceptable to move to the Lydian structure in its higher position, sometimes in its lower one. It depends where we begin from, and what effect the modulation is designed to produce. But now, of course, there is every reason to assign *mesai* to the remaining empty semitonal steps, numbers 13 and 18. They do not, admittedly, appear as notes in the Dorian series, but they do in others, from which one might also wish to modulate. These give us, respectively, a lower *tonos* for the Phrygian octave-structure and a higher one for the Mixolydian.

This procedure has given us twelve *tonoi*: the thirteenth, the so called Hypermixolydian with its *mesē* on pitch 19, remains to be accounted for. The name means merely 'above Mixolydian', which is apt enough. The problem it raises is why one should posit a distinct *tonos* for a *harmonia* which is merely a repetition of the Hypodorian, and which the new *tonos* does not move into a different pitch-relation with the others. Unlike the cases of the variant Lydians, Phrygians, and so on, calling pitch 19 '*mesē*' instead of pitch 7 will make no difference to the pitch-levels of the strings that form the attunement. The distinction seems merely verbal, or at best 'abstract'. One might perhaps attempt to justify it, granted that *mesē* is in some sense a melodic focus, by a distinction between melodic forms with an impetus upwards and those focussing downwards towards the cadence (we know that the Greeks felt such

distinctions to have definite aesthetic significance). Such a distinction belongs, broadly, to a conception of modally differentiated melody rather than one based on the different keys of a single modal form; and hence, though the argument for the addition of this *tonos* is not strong, and is different in kind from those leading to the adoption of the others, some sort of case can be made. But there is no case for a thirteenth *tonos* at all, beyond a misguided notion of neatness, if the *tonoi* are merely keys.

The reconstruction of these arguments for thirteen *tonoi* in connection with the seven *harmoniai* has been largely hypothetical. According to the hypothesis, there are five pairs of *tonoi* of which each presents two alternative positions for one *harmonia*, a semitone apart, and the fact that these are named as pairs in our sources, as a higher and a lower Phrygian, for instance, may be an indication that the reconstruction is on the right lines. The first and thirteenth *tonoi* are not related in quite the same way, though their *harmoniai* are identical, and the latter could scarcely have been named on the same 'pairing' principle: the expression 'Hypermixolydian' is a pardonable makeshift. Only the Dorian pattern of organisation remains fixed to a single *tonos*, a fact again borne out by the nomenclature. This is only to be expected, since it is by their relation to this *tonos*, the one in which the Dorian structure appears, that the positions of all the others are calculated.

I am inclined to accept, then, that the system of thirteen *tonoi* is correctly attributed to Aristoxenus, and that he may well have developed it initially in connection with the seven *harmoniai*. But there is no doubt that the conception of *tonos* as pure 'key' is also present in what purport to be Aristoxenian sources, though the crucial distinctions are seldom made explicit: they have to be imported in order to make sense of what is otherwise mere confusion. Only in one source is the difference made perfectly plain, and that is in Ptolemy, who insists, as we have seen, on returning to a system of seven *tonoi* corresponding to the seven species of the octave, and who, in developing his ideas, mounts a vigorous attack on those who reduce change of *tonos* to nothing more than the transposition of a fixed sequence of intervals. He also argues that any number of *tonoi* beyond seven must be otiose for their proper purpose of locating the different octave-species in a given range, since additional *tonoi* will only produce duplicate species. If the argument I have offered carries any weight, this need not mean that the thirteen Aristoxenian *tonoi* were developed with no thought of their connection with octave-species or *harmoniai*. The reverse is perhaps more likely, since Ptolemy's counterarguments would otherwise merely miss the point.

Nevertheless, Ptolemy is clearly concerned to reinstate in theoretical analysis the modal conceptions which some accounts of the *tonoi* had obscured. Perhaps, as I suggested earlier, this reflects a renewed attention to distinctions of mode on the part of practical musicians, a 'modal revival' in Ptolemy's own time, at least in the parts of the Greek world with which he was acquainted. Since the mode-related notion of *tonos* and that of key are intertwined in the Aristoxenian writers, there are grounds for supposing that Aristoxenus' own

works included considerations of both, and that the period in which modal distinctions were temporarily eclipsed began – no doubt by gradual stages – at around the time at which he wrote.

There is one fairly well established fact about fourth-century music which may help to make this development understandable. A new focus on key, and an associated loosening of distinctions between *harmoniai*, may have been due, in part, to the increasing importance of the *aulos*, both as a performing instrument and as an adjunct of theory. Aristoxenus' *El. Harm.* mentions no stringed instruments at all, whereas the *aulos* is referred to in several places, and we are even told of a school of theorists who based their whole harmonic system on the properties of this instrument. It is mentioned again, furthermore, in connection with the disposition of the *tonoi*, which some people are said to have set out 'with an eye to the boring of *auloi*'. Now the fourth-century *aulos* was a sophisticated instrument of substantial range: unlike the *lyra* and *kithara* it came in a more or less standardised set of different sizes, corresponding to different ranges of pitch. For performers on such instruments it was natural to regard a shift of *mesē* as involving movement up or down in pitch, rather than as a reorganisation of intervals within a constant range – the conception natural to lyrists and kitharists. Typically, perhaps, an aulete would execute this sort of modulation simply by picking up a pipe pitched in a different key.

This suggestion should not be pressed too far. The predominance of the Dorian *harmonia* was already an established fact in the fifth century. Its foundational status in Aristoxenus' thought is obvious from his analysis of the genera, and of the conjunction and disjunction of tetrachords, where his attention is invariably focussed on the tetrachords lying between the fixed notes of the Dorian perfect systems, not on structures bounded by the outer notes of octave-species belonging to other *harmoniai*. If the modal distinctions between *harmoniai* were losing their importance, for whatever reason, nothing is more probable than that a system of *tonoi* originally designed to articulate relations between modalities might be pressed into service to operate as transposition keys for the triumphant Dorian. Aristoxenus seems to have been by instinct a musical conservative, and despite his emphasis on 'Dorian' structures there are clear indications in the *El. Harm.* of an interest in the other species (or 'forms', or 'arrangements') of systems spanning an octave, a fifth or a fourth. But if he was also a realist, he may well have added an account of the ways in which the mode-related *tonoi* could be adapted, with some differences of presentation, to serve the purposes of the new interest in key. Conceivably he hoped that an account of the relations between mode and key might persuade contemporary composers, seduced by the possibilities of key-variation, to rediscover connections with the music of the past, and to apply their skills to the revitalisation of mode.

There need have been no confusion about these matters in Aristoxenus' mind. It appears, however, that what he wrote was sufficiently obscure or involved to breed nonsense in the minds of some of his later followers. Given the use of the same term, *tonos*, in the two different contexts, and the use of the same names both for *harmoniai* and for *tonoi* in either of their roles, the

ambiguities were always likely to create confusion. Lapses of understanding are found exactly where these considerations would lead us to expect them, in writers more concerned to represent 'Aristoxenian doctrine' in academic summaries than to investigate the application of these ideas to actual musical practice. The fact that both conceptions of *tonos* appear in an author such as Aristides Quintilianus, for instance, may indicate his reliance on different sources, both descended from Aristoxenus, but selecting different parts of his discussions as the basis of their summaries. Aristides himself shows no sign of being aware that he has different ideas in play: the reader must disentangle them for himself. Even Ptolemy, who did at least set out to formulate the relevant distinctions, can hardly be said to have given a straightforward and unbiased account of the approaches and intentions of his rivals. If the subject remained an awkward one for an intellect as impressive and as ruthlessly logical as his, that is some measure of its difficulty, and it is no surprise that the Aristoxenian compilers' grasp on it was less than perfect.

A note on the introductions and commentary

Each chapter in this volume is prefaced by a short introduction. In writing these little essays I have not attempted to follow any fixed plan. I have tried to bring out features of the works translated which it would be helpful for the reader to meet in advance, whether these features are technical, methodological, concerned with the general plan of a treatise or with the biography and other interests of the author. The introductions are also uneven in length and in the amount of detail offered. I have said a good deal less about writers such as Plato, who are the subject of many excellent modern discussions, than about ones like Aristides Quintilianus, who are not. Most of the introductions include a few suggestions for further reading, but they give only the briefest sample of works available. More will be found in the Bibliography.

Something similar is true of the notes. Passages that are technically difficult, or which benefit from full cross-referencing, are heavily annotated, and sometimes the notes have grown quite long. (One or two issues seemed to require fuller treatment than a footnote could stand, and I have moved these into appendices.) Others seemed to call for little explanatory comment, and in some places where thickets of cross-references could have been provided, I have decided that they would serve no useful purpose. As in volume 1, I have entered into the controversies of recent commentators only when it seemed quite unavoidable, restricting myself wherever possible to direct comparisons and interpretations of the ancient texts. Where I know that my own views are heterodox, I have given references to other discussions, but I have used this strategy sparingly. As in the introductions, I have pursued no preconceived plan in deciding which kinds of topic called for discussion and which did not. The scope of my comments has of course been influenced by the direction of my personal interests, but I hope that matters which most readers will think important have not been wholly neglected, and that I am not asking them to ride mere hobby-horses of my own into territory that is not worth exploring.