

3914
NETZ, R.
2002

II

Greek Mathematicians: A Group Picture

R. NETZ

As my starting-point for the study of Greek mathematics I take a recent book dealing with modern mathematics. I have in front of me Dauben's fascinating biography of Abraham Robinson,¹ and I turn to pages 142, 331, and 399. The first has a picture with the caption 'Faculty and staff, Royal College of Aeronautics, Cranfield', apparently taken in 1946. Thirty-eight men, mostly middle-aged, sit and stand in three rows; all wearing suits and ties. The second is captioned 'Robinson at the International Congress for Logic, Methodology, and Philosophy of Science, Jerusalem, 1964'. Forty-five men of greatly varying ages (some obviously students) sit and stand in about three or four rows—the arrangement is very informal, and best described as a huddling together rather than a strict arrangement by rows, with the first 'row', in fact, sitting on the floor, Bedouin-style. This does not appear incongruous, since the dress is very informal as well: there are no suits, a single tie, and a variety of shirts, more or less unbuttoned. (Besides the obvious implications for social *mores*, this picture seems to have been taken at the height of an Israeli summer.) The final picture is 'Participants at the UCLA Summer Institute on Axiomatic Set Theory, July 1967'. Eighty-three persons stand in four rows—'persons', for the first time: there are now at least three women in the group. The uniformity is broken ethnically, as well: by contrast with the first two pictures, a considerable minority here do not seem to be of European descent. Dress is as informal and varied as in the second picture (six suits, five ties; and I think I see a clerical collar, and a Scottish kilt—not on the same person—but I admit I might be dreaming both). In short, the three pictures, however unrepresentative and heterogeneous, seem to have a clear moral in terms of the social history of twentieth-century mathematics.

This paper is a close relative—no more—of R. Netz, *The Shaping of Deduction in Greek Mathematics* (Cambridge, 1999), 271–312. I have benefited from the comments of many people but I wish to thank in particular R. Duncan-Jones, R. Sharples, and especially S. Cuomo—none of whom bears any responsibility for the mistakes contained herein or for the views offered.

¹ J. W. Dauben, *Abraham Robinson: The Creation of Nonstandard Analysis. A Personal and Mathematical Odyssey* (Princeton, 1995). Robinson (1918–74) had a rich and variegated mathematical career, and he is chiefly remembered as the author of nonstandard analysis.

If only we could do the same for the Greeks! This is my ambition: to try to think who the Greek mathematicians were—to try to take a group picture. This paper describes and discusses an imaginary picture of this sort, taken at a conference of ancient Greek mathematicians—a conference held in heaven, so that everyone is present, from classical times down to late antiquity. Of course, we shall have to make up our minds who is to receive invitations: the definition of a mathematician is a problem to which I shall return later. To make a start, I shall say that a mathematician is anyone who has written down an original mathematical demonstration, no matter in what context: what he or she may have done besides writing down mathematical demonstrations is a separate issue. Here, then, is the imaginary picture, and I move on to its description.²

To begin with, I have used the expression 'he or she' when referring to Greek mathematicians. Do we actually see any female faces in the group? Surprisingly, perhaps, we do. Of course, there is Hypatia, the well-known mathematician and philosopher of the fifth century AD from Alexandria.³ But, about a century earlier in the same city, we hear from Pappus about another female mathematician,⁴ Pandrosion. Pappus is very critical towards her, but then he is just as critical towards Apollonius and indeed towards almost everyone except (to a large extent) Euclid and Archimedes.⁵ So there are two well-documented women in our group, which is not a little given the obvious obstacles in the way of women in antiquity. It is probably relevant that both examples are from late Alexandria, a place and a time where many old barriers were brought down,⁶ but in general ancient women did not live strictly according to the expectation of either classical society or modern scholarship, and they were not always 'silent': for instance, there are twenty-nine women poets known from antiquity, important qualitatively and not only quantitatively.⁷ But there were, of course, many more poets than mathematicians: I shall return to such numbers below.

An issue comparable to that of gender is that of age. How old are the faces

² For a 'catalogue' of ancient mathematicians see R. Netz, 'Classical Mathematics in the Classical Mediterranean', *Mediterranean Historical Review*, 12 (1997), 1–24. My criteria for identifying mathematicians are quite different from those used by S. Cuomo, *Pappus of Alexandria and the Mathematics of Late Antiquity* (Cambridge, 2000), 9–56, when discussing the place (even 'public profile': 56) of mathematics and mathematicians in late antiquity.

³ See M. Dzielska, *Hypatia of Alexandria* (diss. Cambridge, Mass., 1995).

⁴ Papp. *Coll.* 3. 1, 30. 2.

⁵ On Pappus' polemical attitude towards contemporary and earlier mathematicians, see now Cuomo (n. 2), esp. 55, 71, 73–4, 84, 86, 89–90, 108, 128–34, 186–7, 194–9.

⁶ P. Brown, *The Body and Society* (New York, 1988), 145 ff.

⁷ Quality is important, historically, because it ensures visibility: in the group picture, as it were, it is important to stand in the first row. Sappho—simply one of the greatest poets of all time—could serve as a model for later poets, both female and male. See J. Balmer, *Classical Women Poets* (Newcastle upon Tyne, 1996), for an informal introduction to ancient female poetry, with translations. Among the scholarly studies, I mention J. M. Snyder, *The Woman and the Lyre: Women Writers in Classical Greece and Rome* (Bristol, 1989).

we see? Here, of course, the evidence is even thinner, but some remarks can be made. First of all, to speak in very general terms, Greek culture tended to appreciate old age more, and young age less, than at least some strands of contemporary culture.⁸ Against this background, the small evidence there is for mathematical achievement in youth becomes more noteworthy. Theaetetus, of course, is relevant. He appears first in the Platonic dialogue called after him as a mere boy who already proved a certain result (143 E, 147 E–148 A), and in later dialogues he is a vividly characterized figure, the bright youth, a charming contrast to the *gravitas* of the Stranger from Elea in the *Sophist* and the *Statesman*. He becomes the embodiment of young brilliance. A literary topos is thus struck, and in the pseudo-Platonic *Amatores* boys discuss mathematics (132 A–B). This popular image exists independently of Plato. Isocrates associates an interest in mathematics with young age.⁹ Aristotle explicitly asserts that the young fare better at mathematics (*Nicomachean Ethics* 6. 8. 5–6, 1142^a3–21).

Popular impressions aside, at what age did mathematicians produce their works? Very little is known for certain, of course. Pythocles, known to us especially from the letter Epicurus sent to him, was known in antiquity for his achievements when not yet 18, and he studied especially mathematics (or astronomy—to the extent that a distinction between the two can be made).¹⁰ The known astronomical observations of the great Hipparchus date from 161 to 126 BC. This makes at least thirty-five years of activity, probably considerably more. So how young was he when he started?¹¹ Or another example: Diogenes Laertius gives the age of death for many of his subjects. The youngest is Eudoxus, who died aged 52; is this young, given his achievement?¹² On the other hand, we now know that Apollonius must have been quite old when producing the *Conics*; but this is just the tip of his mathematical iceberg.¹³ Also, when we look at other sets of dated obser-

⁸ M. Kleijwegt, *Ancient Youth* (Amsterdam, 1991), argues for the absence of adolescence culture in the ancient world. Less controversially, he points out the value of adulthood in antiquity (e.g. 58 ff., 188 ff.).

⁹ 15. 261 ff., where, unfortunately, the issue is complicated by the conjunction of mathematics with philosophy. See also Isoc. 11. 23; 12. 26–7; Plut. *Mor.* 43 A–B, 52 C. The alleged attraction of the young to mathematics has recently been discussed by R. Wallace, 'What was Greek about Greek Mathematics?', *SCI* 15 (1996), 82–9 at 87–8.

¹⁰ Phld. *De morte* (P. Herc. 1050) XII. 30–1; Plut. *Mor.* 1124 C = Epicur. fr. 118. In Epicurus' view 'Pythocles was a sort of Alcibiades' (Alciph. *Epist.* 2. 3 = Epicur. fr. 162), the model of precocious *jeunesse dorée*. See also Epicur. fr. 81; and D. N. Sedley, 'Epicurus and the Mathematicians of Cyzicus', *Cronache ercolanesi*, 6 (1976), 23–54 at 43–6.

¹¹ In general for observations known through the *Almagest*, see O. Pedersen, *A Survey of the Almagest* (Odense, 1974), appendix A.

¹² Eudoxus' chronology is, however, very difficult: P. Merlan, *Studies in Epicurus and Aristotle* (Wiesbaden, 1960), 98–104; H. J. Waschkies, *Von Eudoxus zu Aristoteles: Das Fortwirken der Eudoxischen Proportionentheorie in der Aristotelischen Lehre vom Kontinuum* (Amsterdam, 1977), 34–58, esp. 50.

¹³ See P. M. Fraser, *Ptolemaic Alexandria* (Oxford, 1972), i. 415–16, or G. Toomer, 'Apollonius of Perga', *DSB* i (1970), 179–93.

vations, they give us much less than the thirty-five years for Hipparchus. The picture is not easy to read, and, it seems, not only because of our limited evidence: apparently there were no simple rules for age, no necessary 'burning out' at a certain age. Such 'burning out' is a modern myth: the ancient myth was different—Archimedes carried away from his diagram by a Roman soldier, at the age of 75.

This confrontation—between the Roman and the Greek—reminds us of another dimension, that of ethnicity. There is in fact a great divide between east and west: almost all our figures come from the eastern Mediterranean.¹⁴ Are they all 'Greek'? The terms themselves are difficult to apply in the ancient world,¹⁵ but it must be stressed immediately that not all ancient mathematicians were Greek—in some senses of the word. Dositheus, to whom Archimedes addressed his works, was probably Jewish.¹⁶ Marinus, a Neoplatonist and a late commentator on Euclid's *Data*, was born a Samaritan.¹⁷ Toomer discusses the possibility that Zenodorus, a mathematician from the early second century BC, may have been of Semitic origin, and notes that Basilides of Tyre, later in the same century, was probably another mathematician from that background.¹⁸ None of these is from the first rank, but Pappus is the most important mathematician from late antiquity—and he may well have been Jewish.¹⁹ The most important thing is not the evidence itself but the fact that the evidence for Marinus is exceptional (as a pagan philosopher, he chose to tell us about his religious progress): for Dositheus, Zenodorus, and Pappus the argument is based purely on their names. From the writings, ethnicity cannot be judged: perhaps this should make mathematics an ideal arena for cross-ethnic achievement? Once again, we see that the distinctions we impose are irrelevant: mathematics would cross the borders of ethnic identities, just as it crossed the borders of gender and age. (I shall go on using the term 'Greek mathematics', meaning 'mathematics written in Greek': just as we would use the term 'Arabic mathematics', later on, to describe mathematics written in Arabic by writers from similarly mixed ethnic backgrounds.)

¹⁴ See Netz (n. 2) for a discussion of the geographical distribution of ancient mathematicians.

¹⁵ F. G. B. Millar, *The Roman Near East, 31 BC–AD 337* (Cambridge, Mass., 1993), for instance, studies the notions of identity in a central period and area, concluding with the absence of clear-cut identities.

¹⁶ R. Netz, 'The First Jewish Scientist?', *Scripta Classica Israelitica*, 17 (1998), 27–33.

¹⁷ See O. Schissel von Fleschenburg, 'Marinos (1)', *RE* xiv (1930), 1759–67 at 1759.

¹⁸ G. J. Toomer, 'The Mathematician Zenodorus', *GRBS* 13 (1972), 177–92. In the end he prefers to identify the mathematician Zenodorus with an Athenian (there is an inscription mentioning an Athenian Zenodorus), but the balance of evidence seems to me in favour of a Semitic Zenodorus.

¹⁹ Jewish inscriptions from Graeco-Roman Egypt can be followed through W. Horbury and D. Noy, *Jewish Inscriptions of Graeco-Roman Egypt* (Cambridge, 1992). There we find several Pappi and related names (with differing spellings): one dedicates a synagogue, another dies at the age of 3 while his brother (sharing the same tomb, dead at the age of 4) was called 'Joseph'. Briefly, we see that among Egyptian Jews Pappus was a common name.

Another border, more difficult to cross, is that of class. Of course, precise definition is again difficult, but the question is clear: what sort of people were the Greek mathematicians? Were they involved in politics, for instance? Naturally, the evidence is skewed. Greek mathematicians would appear to us in fuller personal colours to the extent that they are exceptional, which in itself implies a high status. A few do, which is noteworthy. Archytas was the leading citizen of Tarentum, the most important Italian Greek city of his time.²⁰ Eudoxus may have been a (if not the) leading citizen at Cnidus.²¹ Theaetetus' father was a well-reputed, and especially a very rich, citizen.²² Meton may have been expected to furnish a trireme: an important citizen, therefore.²³ Hippias was a trusted ambassador of Elis, and also very rich (86 A1 DK). In the Hellenistic world and later, political roles are defined more often in relation to courts, and Eratosthenes, for instance, the Alexandrian librarian, should be thought of as an important courtier.²⁴ Vitruvius addressed Augustus;²⁵ Thrasyllus served Tiberius.²⁶ Philonides' role in the Seleucid court should be compared.²⁷ Finally, the earliest important Greek mathematician, Hippocrates of Chios, is said on the authority of Aristotle to have been foolish enough to lose a huge sum of money: a sign of some wealth.²⁸

What about Archimedes? The greatest mathematician of antiquity merits a brief detour: he may also prove an exception to the evidence above. The literary evidence for Archimedes' social standing is contradictory,²⁹ and may not represent any specific knowledge on the part of the ancient authors. Neither Cicero (who makes him come from humble origins) nor Plutarch (who makes him a Syracusan aristocrat) is necessarily of any special value as a witness: they had his writings, just as we do. But perhaps both were right? His father may have been called Pheidias,³⁰ a suggestive name: the *Realencyclopädie* knows of five persons named 'Pheidias': the famous artist, his son, two other artists—and Archimedes' father. The published volumes of Fraser and Matthews' *Lexicon of Greek Personal Names* (i–iiiB) have between them 81 additional individuals, most of whom are too obscure to have anything like their occupation identified. One is a mercenary in Samos, another is a vice-headmaster in Teuthrone, another a *hieraulēs* (sacred flute-player) in Athens, a fourth a chorus-trainer again in Athens; there is also

²⁰ 47 A 1 DK.

²¹ Diog. Laert. 8. 88, on the authority of Hermippus (1026 F 9): doubtful, therefore, but not inherently implausible. ²² Pl. *Thl.* 144 c 5–8.

²³ N. Dunbar, *Aristophanes: Birds* (Oxford, 1995), 551; *APF* 391.

²⁴ See Fraser (n. 13), 322–3.

²⁵ *Tac. Ann.* 6. 20–1.

²⁶ See e.g. Fraser (n. 13), i. 416 and n. 322, ii. 602.

²⁷ The evidence is in 42 A 2 DK.

²⁸ See E. J. Dijksterhuis, *Archimedes* (Princeton, 1938), 10.

²⁹ *Aren.* 1. 3, p. 220. 21–2 Heiberg. This Pheidias is mentioned in the context of an astronomical hypothesis, and so he may be assumed to be, at least among other things, an astronomer.

a doctor from Rhodes, living in Athens, and a contractor in Delos:³¹ a variety of individuals, but I find it striking that all are artisans. In a famous passage Plutarch noted that, however impressed by him, no aristocrat would wish to become like the banausic Pheidias;³² and so, to the extent that Plutarch was representative, no such aristocrat would call his son 'Pheidias'. The name became marked—Pheidias after all was the most famous artist of antiquity. I know of a young man whose first name is 'Leon Battista Curbosiero' and, surprisingly enough, his father is an architect. (He himself is a mathematician.) A remarkable result, then: I do not know what was Archimedes' social standing, but I guess that his grandfather was an artisan, perhaps an artist. It must have been an extraordinary family: the artist, his son the astronomer, and the grandson Archimedes, the friend of King Gelon—but this becomes fictional biography and the hypothetical social mobility of this family is the exception, not the rule.

The truth is, of course, that ancient society was much more polarized than modern society is or at least used to be. It is doubtful whether the concept of a 'middle class' has much relevance for antiquity. The rich could be more or less rich, the poor could be more or less poor, but the rich were rich and the poor were poor.³³ Thus Greek mathematicians, by and large, should be assumed to have led a privileged life. However, money cannot buy you mathematics, and here is the one negative observation I wish to make on gender, age, ethnicity, and class. Greek mathematicians had many different faces, and there is no need to look far to explain this. Mathematics, perhaps more than other disciplines, calls for specialized cognitive skills. They may be culturally developed, but a residue remains, of highly variable individual capacities. It is such capacities and inclinations, not wealth, status, age, or gender, which determine whether an individual could become a mathematician. Of course upper-class male middle-aged Greeks would have more chance of having their talents developed, or of being exposed to mathematics at all (an important consideration to which I shall return later): but the force of 'mathematical skills' cannot be dismissed altogether—and it is an egalitarian force.

So this is my first observation, which is mainly negative: our picture contains all sorts of people. On the next point I intend to be much more positive. This is what I see as the most important question: not *whom* do we see in the group, but *how many* people we see there. I ask how many Greek mathematicians there were.

I have made a catalogue of known Greek mathematicians (cf. n. 2), which contains 144 individuals. Of course this is to some extent tentative, but I

³¹ The references are, respectively, L. Robert, *Études épigraphiques et philologiques* (Paris, 1938), 114; *SEG* xxii. 304. 3; *SEG* xxviii. 170. 46; *SEG* xxvii. 19; *IG* ii². 483. 12–13, 22; *ID* 446 A 3.

³² *Per.* 2. 1. ³³ G. E. M. de Ste Croix, *The Class Struggle in Antiquity* (London, 1981), *passim*.

believe the number is representative. This is roughly the number of individuals about whom we have some scrap of evidence suggesting that they were mathematicians. Of these, a large number are known through references, often very slight, from late antiquity. Pappus, Proclus, and Eutocius each refer to about 30 individuals. Another 30 or so are preserved in the manuscript tradition. These four sources, Pappus, Proclus, Eutocius, and the manuscript tradition, can be conceived as four selections out of the unknown group of ancient mathematicians. The interesting fact is that these four selections are quite similar. This offers a first ground for suspicion that not many mathematicians were known at all, at least in late antiquity.

I pause to explain a methodological point. We are all used to the question of the transmission of works in manuscripts, and we know that only a small proportion of works survived from antiquity. But what I am concerned with now is a very different question: the survival not of a *work*, but of a *name*. To survive as a name, all you need is that your name will be mentioned in some surviving manuscript. This is far easier than to have your *work* survive. To have your *name* survive, all you need to do is to become parasitic, as it were, upon some other person's work which did survive. And many 'hosts' were not averse to such 'parasites'—on the contrary, throughout antiquity there were always authors who were interested in mentioning names from the past. So the names we hear about from an antiquarian-minded author form a large proportion of the names *he* heard about and, to a lesser extent, of the names *he could* hear about. And this can be checked: by comparing the names mentioned by late commentators, it is possible to extrapolate a probable number of names the late commentators could be aware of. When Proclus writes a commentary and mentions some mathematical names, this is not completely unlike choosing balls out of a box. If four different selections out of a box are made, it is possible to make a guess at the total number of balls in the box. This is based on an obvious, crude probabilistic method. And the number reached in this way is consistently less than 300, even after we add some natural modifications to the probabilistic model. So I believe the number of mathematicians whose names were at all known in late antiquity was not more than 300.³⁴ Extrapolating from this number, I guess that the number of mathematicians throughout antiquity was around one thousand.

A. E. Housman estimated that literary critics are rarer than the appearance of Halley's comet.³⁵ The heavenly body set up to measure the appearances of Greek mathematicians is the sun. Year by year, the sun returned and a mathematician was born. Of course, nothing as regular as this: but

³⁴ The data for this calculation are given in R. Netz, *The Shaping of Deduction in Greek Mathematics* (Cambridge, 1999), 282 ff.

³⁵ Cambridge Inaugural Lecture (1911), repr. in C. Ricks (ed.), *A. E. Housman: Collected Poems and Selected Prose* (Harmondsworth, 1988), 302. Orators, poets, sages, saints, and heroes, by contrast, appear more frequently.

I believe that this is the right order of magnitude, and I now move on to discuss some evidence.

The essence of the probabilistic argument I mentioned above is the close prosopographic repetition between our various sources. No matter from which angle we look at Greek mathematics, we always see the same faces. The trouble with this sort of argument is that we do not actually get to look at Greek mathematics from radically different angles, if what we use is Pappus, Proclus, and Eutocius: the different commentators, to a certain extent, have similar interests, and it may be the similar interests (rather than the limited pool of names they could draw upon) which explain the similar sets of names they mention. It is thus vital to get a completely different hold, to try and view Greek mathematics from as independent a viewpoint as possible. To begin with, the perspective from which the mathematicians themselves viewed mathematics. Both Archimedes and Apollonius carry with them a group of mathematicians for whose existence they alone testify (at least as far as the literary evidence goes): Dositheus, Pheidias, and Zeuxippus in the case of Archimedes, and Attalus, Eudemus, Philonides, Thrasydaeus, and Naucrates in the case of Apollonius. These are contemporaries. As for predecessors, Apollonius refers to Euclid alone, Archimedes to Democritus, Eudoxus, Aristarchus, and (perhaps) Euclid. Furthermore, one of the contemporaries to whom Archimedes writes is Eratosthenes, who is well known as a mathematician from other sources, and another, Conon, is referred to by Apollonius as well. In other words: of the five contemporaries referred to by Archimedes, two are known independently. This is the heyday of Greek mathematics, a period where I am willing to imagine the birth of up to three mathematicians a year, i.e. the total number of active mathematicians might be as large as 100. The brief glance Apollonius and Archimedes afford us is not compatible with anything more than 100—in fact is hardly compatible even with 100 active mathematicians. When Archimedes first approaches Dositheus, following the death of Conon, a note of desperation is detectable—as if he cannot find anyone to whom he can communicate his results;³⁶ Apollonius, approaching Attalus in the introduction to book 4 of the *Conics*, is in a similar situation. No 'school of mathematics' is ever hinted at by the mathematicians themselves, and the death of a single person seriously affects the network. In the introduction to the *Method* Archimedes hoped that his method would be picked up by 'either present or future' investigators—a meaningful qualification.³⁷ And in the introduction to *Spiral Lines* he said that 'though many years have

³⁶ *Quadr.* 262. 2–8 Heiberg = 164. 3–9 Mugler: 'Archimedes to Dositheus greeting. When I heard that Conon, who was my friend in his life-time, was dead, but that you were acquainted with Conon and withal versed in geometry, while I grieved for the loss not only of a friend but of an admirable mathematician, I set myself the task of communicating to you, as I had intended to send to Conon' (trans. 'T. Heath).

³⁷ *Method* 430. 15–18 Heiberg = 84 Mugler.

elapsed . . . I do not find that any one of the problems has been stirred by a single person'.³⁸ That is, Archimedes complained that for many years he had not heard about a single person even *attempting* to answer the open problems he had announced to the world through Conon. No wonder he was so desperate following Conon's death.

Another set of evidence—another angle on the mathematical world—is offered by the documentary evidence from antiquity: inscriptions and papyri. This is especially valuable, since this is the closest we come anywhere in classical studies to a random sample. Of course, we no longer deal now with the size of the active community. The data reflect rather the passive audience—an important issue. We get a glimpse of the 'market': and it did not go for mathematics. The papyrological evidence for mathematics is almost non-existent. I ignore here metrology, astrology, cosmological astronomy, and very elementary arithmetic and measurements for schoolchildren. Beyond these, there is almost nothing. The few bits which do exist repeat on the whole material known from Euclid. It is symptomatic of the arbitrary, patchy nature of the mathematical papyri that the most extensive and serious piece of 'papyrus' mathematics is on a series of ostraka.³⁹ This is while the documentary evidence, both on papyri and on inscriptions, amply testifies to the existence of philosophers, not to mention grammarians, rhetoricians, and, of course, doctors. Very often (relatively speaking), such professions are mentioned in decrees, funerary stelae, and everyday correspondence.⁴⁰ On the other hand, the noun *μαθηματικός* occurs only once in the Duke papyrological collection. Invariably, the use of *γεωμετρία* and its cognate forms (of which there are over a thousand occurrences in papyri) refers to land measurement for tax purposes. This *γεωμετρία* is the equivalent of the Latin *agrimensura*, not a reference to the 'geometry' we know from the literary evidence.⁴¹

A similar comparison can be made through Diogenes Laertius' list, based on Demetrius, of persons having the same name. This list is obviously not a

³⁸ *Spir.* pref. (2. 18–21) = 8 Mugler. (I use Heath's translation).

³⁹ For a full discussion of mathematical papyri see D. H. Fowler, *The Mathematics of Plato's Academy*, 2nd edn. (Oxford, 1999), § 6.2. (Users of the first edition of Fowler's work should note the addition of a new Euclid fragment in W. Brashear, 'Vier neue Texte zum antiken Bildungswesen', *Archiv für Papyrusforschung*, 40 (1994), 29–35.) There are six pieces of literary papyri relating to Euclidean material. Five of these, it should be pointed out, relate to book 1. I do not know of any other Euclidean-style mathematical papyri.

⁴⁰ I have made a CD-ROM survey of the following four sequences: *ιατρ*, *φιλοσοφ*, *ρητωρ*, *γραμματικ*, in the Attica inscriptions and in *P. Oxy.* I have ascertained that the usages are relevant. Attica: *ιατρ*—103, *φιλοσοφ*—35, *ρητωρ*—36, *γραμματικ*—8. *P. Oxy.*: *ιατρ*—67, *φιλοσοφ*—12, *ρητωρ*—10, *γραμματικ*—8. We already saw one of those doctors: the Pheidias from Rhodes, mentioned in a decree from 304/3. No decree ever mentions anyone as a mathematician. None of these two sources had any *γεωμετρ*, *αστρονομ*, or *αστρολογ*, and the single *μαθημ* in *P. Oxy.* (x. 1206. r. 6) is irrelevant.

⁴¹ See Cuomo (n. 2), 16–25, 31, 38–40, on the different ancient meanings of *γεωμετρία*, and their relations to *agrimensura*.

random sample: it was edited with a view to intellectual pursuits. Of the 188 persons in the list, only three to five at most may be mathematicians, and one of these is known from elsewhere. Or the *Digest*: various exemptions from civic duties are accorded to doctors, rhetors, and grammarians. The numbers allowed for exemption are between 3 and 10, depending on the subject and the size of the city. The *Digest* goes on to explain that 'the number of philosophers has not been laid down, since there are so few philosophers'.⁴² The numbers in the *Digest* are minima, grudgingly allowed by a tax-hungry empire, and they give little indication of absolute numbers. However, a sense of the relative numbers is made clear, and is consistent with searches on the documentary material. Perhaps as much as half of the professional intellectuals are doctors, the rest being mainly teachers of skills related to language. Mathematicians are so few as to be unquantifiable.

Catalogues have been provided for other areas, similar to the one offered (tentatively) in the article cited in n. 2 (which gives the number of mathematicians as 144). Runia, for instance, reckons that there were 316 pagan writers of philosophy in antiquity.⁴³ Felix Jacoby's still unfinished *Fragmente der griechischen Historiker* catalogues all ancient Greek historians (again, roughly until the Christianization of the empire): Jacoby himself assembled 856 authors, his successors have already published fragments of a further 81, and the eventual total will be well over a thousand. So more than two philosophers for every mathematician, about ten historians for every mathematician. Add doctors and rhetors—probably the two largest groups—and we can say that no more than about 2 or 3 per cent at most of prose writers were mathematicians. But it must be realized that, where a group is small, the proportion of its membership represented by surviving references may be exaggerated because the few famous individuals form a larger proportion of the whole. So 2–3 per cent may well be an over-estimate and, at any rate, it is clear that it is somewhat misleading to say that 'the ratio of mathematics to philosophy in antiquity was 144:316'. Any acquaintance with the sources shows that the ratio was more heavily towards philosophy: people did not read mathematics that much, as I have already made clear when discussing papyrological evidence. (I shall return to this below.)

But first we must get a sense of the total size of the audience for high culture in antiquity. I am not an expert on demography—and this is an advantage which I shall now try to exploit. Professional demographers have

⁴² 27. 1. 6. 7. For the significance of this evidence see R. Duncan-Jones, *Structure and Scale in the Roman Economy* (Cambridge, 1990), 161. I owe the reference to Dr Duncan-Jones. The *Digest* contains a fair amount of evidence about mathematicians in the broad sense (cf. n. 2), which is discussed in Cuomo (n. 2), 30–46.

⁴³ D. T. Runia, 'Aristotle and Theophrastus Conjoined in the Writings of Cicero', in W. F. Fortenbaugh and P. Steinmetz (eds.), *Cicero's Knowledge of the Peripatos* (New Brunswick, 1989), 23–38. The limitation to pagan writers means that e.g. both Socrates (who did not write) and Augustine (a Christian) are excluded.

a professional reputation to defend, and therefore tend to be cautious in their guesses. But what would be really useful is to have not a prudent guess, but a *precise* guess—a set of numbers. I begin from what still is—remarkably—the starting-point for all such discussions, namely K. J. Beloch, *Die Bevölkerung der griechisch-römischen Welt* (Leipzig, 1886). In that work Beloch estimated the number of inhabitants of the 'Greek east' (i.e. the eastern Mediterranean under Roman rule), at around the death of Augustus, at 28 million (p. 507). Of course not all of these were Greek, and some allowance must be made for this. It is true, as I have argued above, that persons who were ethnically non-Greek could participate in Greek culture, but they had to belong to Greek culture—to begin with, to be literate in Greek: and so the people we are looking for had to belong not only to the Greek geographical sphere, but also to the Greek *cultural* sphere, which must have been more limited. Not everywhere in the 'Greek east' was really Greek. Now I shall soon introduce further constraints, such as 'living in a city', and these to some extent would make the geographical sphere coincide more with the cultural sphere. To live in an eastern city would have been almost the same as being (at least potentially) exposed to Greek culture. But not completely: Greek culture stood in competition with other cultures in the eastern Mediterranean, and highly literate Jews in Palestine, for instance, would not necessarily be primarily literate in *Greek*.⁴⁴ Add this to the fact that the death of Augustus is a relatively high point in ancient Greek demography and that we are averaging over the classical, Hellenistic, and Roman eras (a period during which the eastern Mediterranean was *gradually* Hellenized) and I think it is, if anything, optimistic to estimate the population of the Greek cultural sphere at about 20 million persons. I shall use this convenient number.

Of course not everyone in this area was touched by Greek culture. A fair approximation is to assume that only city-dwellers could be touched by high culture: this is the most absolute border in an agrarian society. And the level of urbanization in an agrarian society, again, cannot be high. Fifteen per cent is relatively high, and 3 million city-dwellers in the Greek cultural area is a reasonable number. We know that there were no more than 900 cities in the entire area,⁴⁵ so we may assume—just to have a sense of the possibilities—that the five largest cities had between them a million people,⁴⁶ that the next hundred had between them another million (with

⁴⁴ Jewish Palestine was an extreme case of an inward-looking society (see e.g. M. Goodman, *The Ruling Class of Judaea* (Cambridge, 1987), esp. 97 ff.) but, in general, not all high culture in the eastern Mediterranean became identical with Hellenistic culture.

⁴⁵ A. H. M. Jones, *The Cities of the Eastern Roman Provinces* (Oxford, 1937), appendix IV, names 907 cities in a very inclusive list, covering several overlapping administrative surveys from late antiquity.

⁴⁶ Alexandria at its most populated was said to have 300,000 free inhabitants (R. Duncan-Jones, *The Economy of the Roman Empire* (Cambridge, 1974), 260–1), which makes its entire population something like half a million, comparable to the megalopoleis of early modern times such as Istanbul and Naples (F. Braudel, *The Mediterranean and the Mediterranean World in*

10,000 inhabitants per city, this would leave room for many respectable-sized cities, by ancient standards), and that the remaining cities, somewhat more than 500 in number, had between them the third million (giving rather fewer than 2,000 inhabitants per city—but the ancients would call a city something we would consider to be a large village).

Of these three million city-dwellers, half were female and nearly half were children: this is an iron certainty (a pre-modern society, with an average life expectancy at birth of 25 or less, can have only about half of its members adults). Again, these borders were not impossible to cross—some young people participated in high culture, and so did some women, but for our immediate statistical purposes this may safely be ignored. In other words, there were at any given point no more than about 850,000 adult males living in Greek cities.

Let us assume that 700,000 of them were free.⁴⁷ This then is an important number: it can be said to be the number of 'visible' persons in Greek antiquity. It is a maximum rather than an average: in classical times, for instance, Attica had no more than about 30,000 citizens⁴⁸—and Attica was one of the most populated Greek areas, and certainly one with the highest 'visibility rate'. 'Visible' persons can be defined as the people of whom ancient literature speaks. They are not necessarily yet the people to whom ancient literature spoke. Many of those free urban males were illiterate, or only basically literate: artisans, mercenaries, small-time merchants. Now this is a pure guess, but I suggest that about 300,000 of those 700,000 free urban males were sufficiently literate to be able to read, say, a whole papyrus roll.

Not all of them would do so, at least not on a regular basis. We must still account for sheer philistinism. To be literate is not yet to be interested in written culture; to be able to read a papyrus roll is not yet to have an interest in looking for such rolls and in trying to understand their contents. This then is yet another guess, but again, if anything, an optimistic one: the 'Greek readership' in antiquity comprised no more than about 70,000 people.⁴⁹

the Age of Philip II (London, 1972), 344 ff.); yet bear in mind that 300,000 is a suspiciously convenient number—and that clearly Alexandria was exceptionally populated (though in a later period it might have been equalled by Antioch and, later still, Constantinople; while Rome was always a case apart).

⁴⁷ A conservative estimate of the number of slaves; see Duncan-Jones (n. 46), 273 (and *ibid.* 264 for the percentage of adults).

⁴⁸ M. H. Hansen, *Demography and Democracy: The Number of Athenian Citizens in the Fourth Century BC* (Herning, 1985), restates the case for the larger number, 30,000; smaller numbers are often preferred in the literature.

⁴⁹ 'Those 70,000 people may be thought of as 'people who did the equivalent of a modern A-level in Classical Greek'. It will be obvious that *England alone* has many more people answering to this description than Greek civilization ever had: we begin to have a sense of how much larger, numerically, modern cultures are.

Not all those readers were also authors (though the ratio of authors to readers must have been much higher than it is now). But it begins to look probable that there were no more than about 5,000 Greek authors active at any one time. Divide them by 20, say, to derive the number of Greek authors born every year, and you get 250. Most wrote poetry, among the dozens of prose writers only a minority were 'scientists'—one of whom, I argued above, was a mathematician.

The point of this somewhat cavalier exercise in numbers is to show that the 'thousand mathematicians in antiquity' hypothesis is compatible with what we know of ancient demography. But this is not just a matter of numbers. The deep historical factor behind the hypothetical numbers above is that traditional, agrarian societies put a double constraint on the size of the population participating in high culture. First, traditional agrarian economies, even in a very large (and fertile) cultural area, can support, at most, some tens of millions of people; second, since such societies are so economically polarized (and large-scale traditional agrarian societies have to be deeply polarized), most of this population is excluded from participation in high culture.⁵⁰ I gave the number of inhabitants of the Greek cultural area as 20 million, and the number of non-excluded persons as 300,000: in other words, a participation rate of 1.5 per cent. Modernity breaks both constraints, and contemporary western culture has at least about a billion people in its cultural sphere, of which some 10 per cent can be said to be at least potential participants. It is about *a thousand times* larger than Greek culture.

But I return to my main guess: there were about a thousand mathematicians in antiquity. With some difficulty, we can indeed fit them all in a single picture. I shall return below to the wider historical implications of this guess, but I now return to the picture itself. What more can we say about Greek mathematicians, as a group?

We move, as it were, from the picture to the CV. The question is: what did Greek mathematicians *do*? Were they full-time mathematicians or did they have other areas of interest? One is tempted to answer that surely Greek mathematicians were not full-time mathematicians. We have seen that they were not professional academics, and mathematics did not correspond to any clearly defined career, to a job. We now can see why—there were simply not enough mathematicians. The mathematician was not a member of a mathematical faculty, and hence, there being no faculty barriers, we should expect mathematicians to do much else besides mathematics. And clearly Greek intellectual life in general was not organized in compartmentalized,

⁵⁰ The high level of cultural exclusion in a traditional agrarian society does not derive from economic polarization alone. It is further exacerbated by the gender exclusion consequent on its traditional patterns of life as well as by its age exclusion: simply, the age distribution of contemporary Western society entails a much higher percentage of *adults*, compared to that of pre-modern societies.

neat faculties. Philosophers, from Empedocles down to Sextus Empiricus, claimed to heal, practised medicine in one way or another; the teaching of rhetoric was always indistinguishable from philosophy, or from grammar. And there is some evidence for similar overlaps with mathematics. Democritus, for instance, was one of the earliest mathematicians; but he did of course much else besides. Eudoxus is well known not only for his mathematics, but also for his views on pleasure and on chronology, to name just two subjects.⁵¹ And other examples can be adduced.

However, this must be qualified. Against the background of the Greek tendency to transgress disciplinary borders, mathematics appears as a relatively well-defined discipline. Most typically, what mathematicians do outside mathematics is cosmology, i.e. astronomy in the non-mathematical sense: people like Oenopides, Hippocrates of Chios, Euctemon, Meton, and so on to Ptolemy and beyond. But this is a development of the interest in astronomy, which is never far from the centre of mathematical attention. And in general there are many people who seem to have been predominantly mathematicians—clearly figures such as Euclid, Archimedes, and Apollonius, and with some qualifications even Ptolemy. It is symptomatic that no good authority ever gives a mathematician as a physician. Mathematicians, on the whole, then, were just mathematicians.

Conversely, very few people who were not active mathematicians ever bothered with mathematics—a fact to which I have already alluded in the demographic discussion above. More than half of the occurrences of *γεωμετρία* and cognate words in the Greek corpus occur in Aristotle and his commentators, another fifth in Plato and his commentators. Otherwise mentions of geometry tend to occur as random examples, mere flukes. It is remarkable how little educated people in antiquity were aware of what may have been the most enduring intellectual development in their midst. Take, for example, Thucydides. He is unsurpassed in sheer intelligence, he is the Archimedes of history. His description of the plague shows him indebted to Hippocratic medicine. The Melian dialogue is an important text for understanding what is known as the sophistic movement. Yet when he comes to estimate the size of Sicily, he does so by giving an estimate related to its *circumference* (6. 1. 1)—not a silly thing to do, perhaps, but it shows that the author (and the audience) do not approach questions such as 'the size of Sicily' as questions of geometry;⁵² and in general, there is no

⁵¹ Pleasure: Arist. *Eth. Nic.* 1. 12. 5, 1101^b27–32; 10. 2. 1–2, 1172^b9–26. Chronology: Pliny, *HN* 30. 3.

⁵² Fowler (n. 39), 281, notes a similar phenomenon in Hdt. 1. 170; 5. 106; 6. 2, where Sardinia is evidently treated as the largest island in the world because it has a larger *perimeter* than Sicily (cf. R. J. Rowlands, 'The Biggest Island in the World', *CW* 68 (1975), 438–9). The point at issue (that equal perimeters can enclose unequal areas) was liable to be neglected even in local land measurement: cf. Netz (n. 34), 300, and Dem. 42. 5, where the speaker assesses an estate by assigning it a 40-stade circumference, perhaps hoping to deceive the jury into

real hint in Thucydides' work that anything like mathematics was known to him.

Polybius' case is much more interesting, and calls for a detailed study. Look at how *he* approached the geography of Sicily:

τὸ δὲ σχῆμα τῆς Σικελίας ἐστὶ μὲν τρίγωνον, αἱ δὲ κορυφαὶ τῶν γωνιῶν ἐκάστης ἀκρωτηρίων λαμβάνουσαι τάξεις. (1. 42. 3)

Sicily is triangular in shape, the vertices of all its angles being formed by capes.

This language is unmistakably geometrical, and Polybius may be used to reveal what an educated Greek of his time, who *was* aware of mathematics, would be aware of: not only because Polybius' writings show an awareness of several sciences, but mainly because he is deeply concerned with methodological questions: *what makes a good general*, and *what makes a good historian*? The two are interconnected, and Polybius asserts that generals should write history and vice versa, idealizing 'historian-generals' on the explicit model of Plato's 'philosopher-kings' (12. 28. 2–4). Now this is a useful comparison from our point of view: Plato's philosopher-kings were given an education with two components, of practical experience and of theoretical studies, the theoretical studies predominating; and, within those theoretical studies, mathematics was dominant. Both relations are reversed by Polybius. The main theme of the methodological discussion in book 12 is the crucial role of personal experience, not just for becoming a general (more on this later) but even for writing a history: a necessary condition for a good writer on battles, say, is to have experienced battles (Timaeus, an earlier historian, is being criticized). As for the role of mathematics within the theoretical studies, we shall soon see that.

For clearly some theoretical study *is* necessary: book 34, for instance, was a self-contained geographical survey, and to some extent Polybius' approach to geography is theoretical (though he keeps mentioning the fact that he has *personally travelled there*). Most notably, there is a geometrical argument, where Polybius criticizes a measurement given by Dicaearchus. (It is typical that these methodological discussions are driven by polemical concerns.) Dicaearchus gave the distance from the straits of Messina to the Pillars of Heracles (i.e. the length of the western Mediterranean) as 7,000 stades, which Polybius considers a gross underestimate. His method is sound:⁵³

thinking it bigger than it really was (G. E. M. de Ste Croix, 'The Estate of Phaeonippus', in E. Badian (ed.), *Ancient Society and Institutions* (Oxford, 1966), 109–14). But it seems to have become a paradigmatic mathematical paradox: see Quint. 1. 10. 39–45 (something an orator should be aware of); Papp. *Coll.* 5. 1, pp. 304. 1–308. 8 Hultsch (on honeycombs, from a book aimed at quite a general audience: Cuomo (n. 2), 58); Procl. *In Eucl.* 403. 4–14 Friedlein (in a context of public utility and ethical propriety: Cuomo (n. 2), 54).

⁵³ Polyb. 34. 6. 2–8. The passage derives from Strabo 2. 4, but it seems possible to reconstruct Polybius' original argument as well as, to some extent, his original words. In general, much of

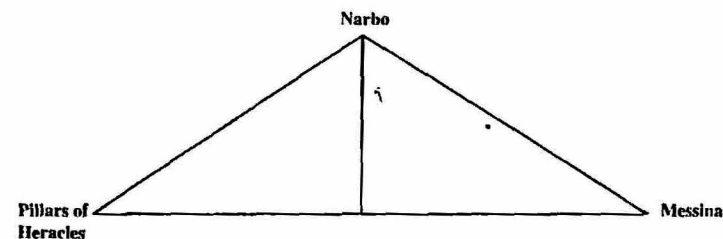


FIG. 11.1. Estimating the distance from Messina to the Pillars of Heracles

- Represent the western Mediterranean as a triangle, with the line from Messina to the Pillars of Heracles as its base and Narbo (at the mouth of the Rhone) its vertex (Figure 11.1).
- Estimate the distances Messina–Narbo and Narbo–Pillars at 11,200 and 8,000 stades respectively, these figures being in all probability based upon journey lengths.
- Drop a perpendicular from Narbo to the base of the triangle, and estimate its length as 3,000 stades (see below).
- Use Pythagoras' theorem to measure the two segments of the base—which turn out, of course, to be much greater than Dicaearchus' account suggests. Significantly, Polybius (who does not seem to have given an explicit calculation) is reported to have referred to this as 'the school-boy's measurement' (ἡ παιδικὴ μέτρησης).

So this is the 'elementary geometry' a historian requires: to be able to extrapolate geographical measurements on the basis of geometrical theorems. But notice the main mistake of Polybius' account. He estimates the length of the perpendicular by the length of travel by sea ('now the longest distance from Europe to Africa across the Tyrrhenian Sea is not more than 3,000 stades; across the Sardinian Sea it is somewhat shorter'),⁵⁴ but seems to have failed to understand that there is a theoretical distinction between measuring latitudinal and measuring longitudinal distances, viz. that with latitudinal (north–south) distances—and with them alone—it is possible to use astronomical data to calculate exact ratios between the separate distances. Thus the distance from Narbo to Africa is in principle the most secure piece of data available to Polybius in this measurement, yet he does not even mention astronomy and, in fact, this is the distance he gets most wrong. Bear in mind that in the second half of the second century BC all the foundations of Greek theoretical geography were already available. Polybius after all explicitly criticizes Eratosthenes, in the same book 34—and

Polybius' work survives as (usually long and apparently verbatim) excerpts taken out of context. Fortunately, excerptors were especially interested in Polybius' methodological observations.

⁵⁴ Polyb. 34. 6. 6. Here and below I follow the Loeb translation, with minor amendments.

Eratosthenes' estimate of the size of the earth assumes a mastery of the relations between astronomy and geography. Nothing of that is to be seen in Polybius, whose geography might as well have been that of a flat earth. His mathematics does not go beyond the 'schoolboy's measurement', Pythagoras' theorem—and the point of the 'schoolboy' is, of course, polemical: 'even a child knows *that!*'⁵⁵

This impression is confirmed by examination of the second half of Polybius' methodological equation: what theoretical knowledge a *general* ought to have. This is discussed most explicitly in 9. 12–20. The superiority of personal experience over theoretical understanding is stressed once again (9. 14. 1–5), but experience itself requires some theoretical understanding (not too much, Polybius insists), in astronomy and geometry (9. 14. 6). Here are the things one needs to know:

Astronomy: the different length of days, in order (a) to estimate correctly the distance one may cover in a single day's march (9. 14. 6–15. 3) and (b) to know how to subdivide day and night (e.g. so that one can sound the reveille at the right time: 9. 15. 4–5). This is followed by good practical advice on how to estimate the subdivisions of the night, by observing the stars and the moon, and Polybius then gives several historical exempla of bad timing in warfare and its disastrous consequences.

Geometry: to know what length of ladder is required to scale a given wall (9. 19. 5–9). (More on this remarkable Polybian concern below.)

Geometry: to know how to enlarge camps in proportion to the number of soldiers (9. 20. 2–3). On this subject Polybius merely refers us to his work on tactics, which is unfortunately lost. But we did not lose too much: the discussion closes with Polybius insisting, once again, that he is against too much theoretical study—anything which goes beyond immediate practical application.

Let us instead concentrate on those ladders. A theoretical discussion of ladders was promised by Polybius earlier in the *History*, following a story about the Macedonian king, Philip:

[Philip], pushing on vigorously all night without stopping . . . arrived before Melitea at daybreak, and setting up his scaling-ladders, attempted to storm the town. He terrified the Meliteans so much by the suddenness and unexpectedness of the attack that he could easily have taken the town; but the attempt was foiled by the ladders being far too short for the purpose. (5. 97. 5–6)

Whereupon Polybius explodes, and the following chapter is a remarkable methodological outburst, insisting upon the importance of careful planning

⁵⁵ A more extreme claim than even e.g. Strabo 1. 3. 11 (the proposition that the surface of a liquid at rest is spherical, the sphere having the same centre as the earth, is 'known by all who have even touched mathematics').

and understanding on the part of generals. Ladders, in brief, are no trivial matter.

So here is how the length of ladders is to be calculated:

If any of our collaborators can give us the height of the wall the required length of the ladders is evident. For if the height of the wall be, let us say, ten of a given measure, the length of the ladders must be a good twelve. The distance from the wall at which the ladder is planted must be half the length of the ladder, for if they are placed further off they are apt to break when crowded and if set up nearer to the perpendicular are very insecure for the scalers. (9. 19. 6–7)

Polybius goes into some sort of 'theoretical discussion' concerning applied mechanics (anachronistically speaking) rather than mathematics. The mathematical element is the same application of Pythagoras' theorem we saw already, and *it is wrong*. Given the ratio 1:2 between the smaller side and hypotenuse, the ratio of the larger side to the hypotenuse is (in a modern notation) about 10:11.5, not Polybius' 10:'a good twelve'. Clearly what we have here is not a theoretical *derivation* of the right length of ladders, but a clumsy attempt to rationalize an established practice.

So there he is, the Greek intellectual from the second half of the second century BC, writing at the end of three great centuries of Greek mathematical expansion: well-read, universal in the scope of his historical interests—and a major source on Archimedes!⁵⁶ He claims to have the right grasp of the theoretical knowledge required by a historian; insists that this is limited; and duly exhibits this limitation in his superficial grasp of mathematics, which does not go beyond a few technical terms and a knowledge of Pythagoras' theorem. It seems to me certain that Polybius never set eyes on the works of Archimedes or anything of that sort: perhaps he glanced at Euclid, but that too is doubtful. This then is the mathematical knowledge of the educated Greek of Polybius' time, and once again we see the *isolation* of Greek mathematicians. A function of the small number of mathematicians, to some extent. But in other ways, we can see that mathematics did not appeal to non-mathematicians. We are looking for 'popularized' mathematics—i.e. attempts to bring the contents of Archimedes and his like to the general public—and they are very rare, which is in itself significant. One example of the popularization of technical mathematics appears in a fragment of Eratosthenes preserved in Eutocius' commentary on Archimedes.⁵⁷

This fragment is a letter sent by Eratosthenes to King Ptolemy III. It describes a mean-finding instrument produced by Eratosthenes, and it refers to a dedication Eratosthenes had set up with that instrument, describing the dedication and citing it in full. We thus have access to two

⁵⁶ 8. 3–7 reports his contribution to the defence of Syracuse against Marcellus' siege engines—on which see also Livy 24. 34. 1–16; Plut. *Marc.* 14–17.

⁵⁷ Eutoc. 88. 3–96. 27 Heiberg = 64. 5–69. 11 Mugler. I follow W. Knorr, *Textual Studies in Ancient and Mediaeval Geometry* (Boston, 1989), in considering this fragment to be genuine.

acts of popularization of mathematics, one addressed to the king, the other addressed to the Alexandrian public. Both have this in common: that they inscribe the mathematical work into the Greek literary tradition. The letter to the king opens like this:⁵⁸

Eratosthenes to King Ptolemy, greeting.

They say that one of the old tragic authors introduced Minos, building a tomb to Glaucus, and, upon hearing that it was a hundred feet long in every direction, saying:

*You have mentioned a small precinct of the holy tomb;
Let it be double, and, not losing this beauty,
Quickly double each side of the tomb.*

...

And this was investigated by the geometers, too: in what way could one double the given solid, the solid remaining in its own shape, and they called this problem 'duplication of a cube'.

So this is how Eratosthenes introduces the mathematical problem of 'duplicating a cube', a problem which in turn gives rise to the need to find means. As for the public dedication, this consisted mainly of the following epigram:⁵⁹

If you plan, of a small cube, its double to fashion,
Or—good sir—any solid to change to another
In nature: it's yours. You can measure, as well:
Be it byre, or corn-pit, or the space of a deep,
Hollow well. As they run to converge, in between
The two rulers—seize the means by their boundary-ends.
Do not seek the impractical works of Archytas'
Cylinders; nor the three conic-cutting Menaechmics;
And not even that shape which is curved in the lines
That Divine Eudoxus constructed.
By these tablets, indeed, you may easily fashion—
With a small base to start with—even thousands of means.
O Ptolemy, happy! Father, as youthful as son:
You have bestowed all that is dear to the Muses
And to kings. In the future—O Zeus!—may you give him,
From your hand, this, as well: a sceptre.
May it all come to pass. And may he, who looks, say:
'Eratosthenes, of Cyrene, set up this dedication.'

I quote extensively because nothing briefer can convey the certain sense of absurdity in this mode of presentation of mathematics. Reflected in the mirror of the dominant Greek literary tradition, mathematics has a strange

⁵⁸ Eutoc. 88. 3–16 Heiberg = 64. 5–18 Mugler. This and the following translation are mine—not literal translations, as I find it important to stress the 'literary' character of this text.

⁵⁹ Eutoc. 96. 10–27 Heiberg = 68. 17–69. 11 Mugler.

face which we can hardly recognize. Mathematics did not lend itself to popularization, in the Greek context—that of a culture where the literary mode was much more important than the scientific mode.

Briefly, mathematics was not popular—and the main explanation for the unpopularity of mathematics is much more simple than the arguments delineated above. Mathematics is difficult. So is philosophy, no doubt, as so many other ancient disciplines, but mathematics has a major disadvantage peculiar to it, its all-or-nothing nature: the most rugged Roman general can spend time with Greek philosophers, apparently finding some satisfaction in his dim understanding of their utterances on Truth and the Good Life. But what satisfaction is there for him in Euclid? Only the frustration of the feelings of inferiority, so well known to anyone who has passed through our educational system. We invest enormous social and economic capital in forcing children against this obstacle, and still most fail to make it. Lacking these forces, the ancients did not try. Or, to be more precise: we all know the fate of books which suddenly become best-sellers, after being turned into a movie—in the version 'according to the film'. This originated in southern Italy in the late fifth century, but it was Plato who turned this, '*Mathematics: The Movie*', into a compelling vision. This vision remained to haunt Western culture, sending people back again and again to the '*book according to the film*'—the numerology associated with Neoplatonism. But, especially in the Aristotelian tradition, a few people went to the original, until, emerging from the last Platonic revival of the Renaissance, mathematics exploded in the sixteenth century and left Platonism behind it with the rest of philosophy and the humanities. We now take this centrality of mathematics for granted; we should not project it into the past.

I sum up my results. Greek mathematicians formed an inward-looking group. Relatively speaking, they were interested in their mathematics and not in much else beyond. They were few in number, a tiny group at the extreme fringe of the Gauss curve. This is why they were a motley group, why so little can be said to characterize them in general, but also why they must have been rich and influential citizens. If Greek mathematics involved few people, it means that the very access to mathematics was extremely complicated. How many ancient potential Newtons must have passed their lives unnoticed back on their Lincolnshire farm! Under normal conditions, papyrus is not an enduring material. For a work to survive, it must be copied almost every generation. If a city had only a single mathematician, his mathematics would die with him, and would have to be reimported from elsewhere to be born again. Such rebirths must have happened again and again. Continuities were the exception in Greek mathematics. We hear of a few people teaching others: Hippocrates of Chios taught a practically unknown Aeschylus, Theodorus taught Theaetetus (according to the Platonic dialogue of that name), Neoclides taught Leon, and Eudoxus taught

Menaechmus (whose brother was Dinostratus) and Helicon.⁶⁰ There are other names, sometimes just names: Oenopides, Andron and Zenodotus, Timocharis and Aristyllus.⁶¹ And of course we know of Archimedes' father (above, p. 200). The lists never get beyond a single generation; and often not even that. For instance, we know that Arcesilaus and Philonides studied mathematics, but we also know that they did not become mathematicians. It should be borne in mind that traditions and transmissions are a literary topos in antiquity. Ancient histories of philosophy, such as that of Diogenes Laertius, are structured by genealogies: 'A taught B who taught C'. There is a great element of legend in this, of course. It is now understood, for instance, that the great schools of philosophy, such as the Academy, did not have the continuity ascribed to them by the ancients.⁶² But this only serves to stress the relative absence of similar genealogies for mathematics: mathematicians did not belong to the 'schools', not even in the imagination of later commentators.

The best symbol is the list of astronomical observations mentioned in the *Almagest* (n. 11 above). It is a set composed of intermittent explosions. No site of observation was kept for more than a few consecutive decades. There is only one exception to this—the Babylonian set. Here we begin to see a meaningful pattern. Greek mathematics is not a guild, it is not like the Babylonian family-guild 'Scribes ENUMA ANU ENLIL'.⁶³ It is an enterprise pursued by *ad hoc* networks of amateurish autodidacts—networks for which the written form is essential; constantly emerging and disappearing, hardly ever obtaining any institutional foothold. So it is not just the accidental absence of ancient cameras which prevents us from having the ancient equivalents of the modern group pictures of faculties and conferences, mentioned at the beginning of this article. The Greek mathematicians did not have faculties and conferences, a fact reflected, as I have argued elsewhere, by their form of presentation and ultimately by the contents of their mathematics.⁶⁴

⁶⁰ Arist. *Mete.* 1. 6, 343^a1; Pl. *Thet.* 145 c; Procl. *In Eucl.* 66. 18–67. 12 Friedlein, Pl. *Ep.* 360 c.

⁶¹ Procl. *In Eucl.* 66. 2; 80. 15–20 Friedlein; Ptol. *Synt.* 7. 1, p. 3. 3–4 Heiberg; Plut. *Mor.* 402 F.

⁶² See especially J. Glucker, *Antiochus and the Late Academy* (Göttingen, 1978).

⁶³ See O. Neugebauer, *Astronomical Cuneiform Texts* (London, 1955), i. 13 ff.

⁶⁴ This argument is pursued in Netz (n. 34).

IN: SCIENCE AND MATHEMATICS
IN ANCIENT GREEK CULTURE