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A PYTHAGOREAN APPROACH TO PROBLEMS OF PERIODICITY IN CHEMICAL AND NUCLEAR PHYSICS

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Abstract. A Pythagorean approach to numerical sequences in both chemical and nuclear physics has allowed us to show geometrical analogies based on figurate numbers (three-dimensional forms of Mendelev's periodic table and packing models for nuclei). Pascal's triangle is used to deduce analytical equations, from which magic numbers through 12,360 for atoms and through 21,400 for nuclei are calculated.

1. Introduction

Structural similarities exist between atomic nuclei and other fermionic systems such as metal clusters. In particular both of these systems exhibit specific magic numbers [e.g., 1]. In this section we briefly recall some relevant mathematical concepts.

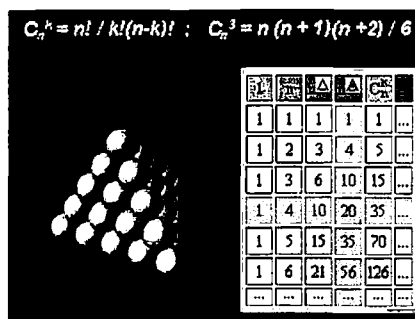
1.1. Figurate numbers

Figurate or polygonal numbers appeared in 15th-century arithmetic books, and were probably known to ancient Chinese; but they were of particular interest to ancient Greek mathematicians [2]. To the Pythagoreans (c. 500 BC), numbers were of paramount significance: they believed everything could be explained by numbers, and numbers were invested with specific characteristics and "personalities". Among the properties of numbers the Pythagoreans endowed them with "shapes" [2, 3].

1.2. Pascal's triangle

As it is well known Pascal's triangle is an arrangement of numbers such that each number is the sum of the two immediately above it in the previous row [4].

$$n\Delta = C_n^3 = (n^3 + 3n^2 + 2n)/6. \quad (4)$$



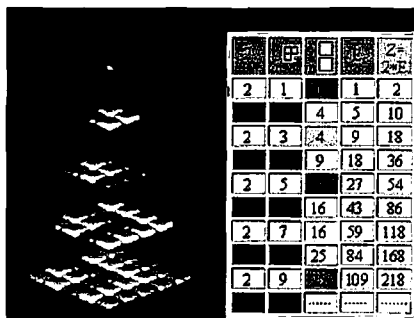


Fig. 6 - Square numbers composed from L-shaped gnomons.

After modification:

- blank lines are inserted between the lines of the table;
- a duplicate square is placed above each entry in the third column;
- the fourth column E is filled according to Pascal's triangle rule ($1, 4+1=5, 4+5=9, 9+9=18, 9+18=27$, and so on);
- doubles of the fourth column entries are entered in column 2E.

It is significant that in the fourth column are written the numbers of electron pairs in inert-gas atoms. Accordingly, in the fifth column are the charges of these atoms, or the magic numbers for atoms.

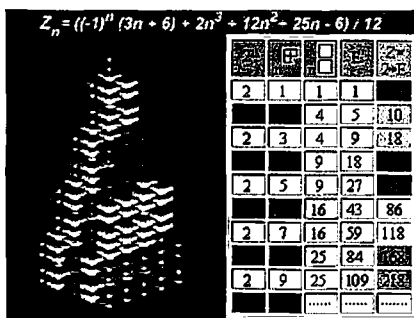


Fig. 7 - Magic numbers for atoms in fifth column of Pascal's triangle modification.

2.1.2. 3D periodic system

The manipulations performed so far result in a 3D model for Mendeleev's periodic system, to which one may give the following interpretation.

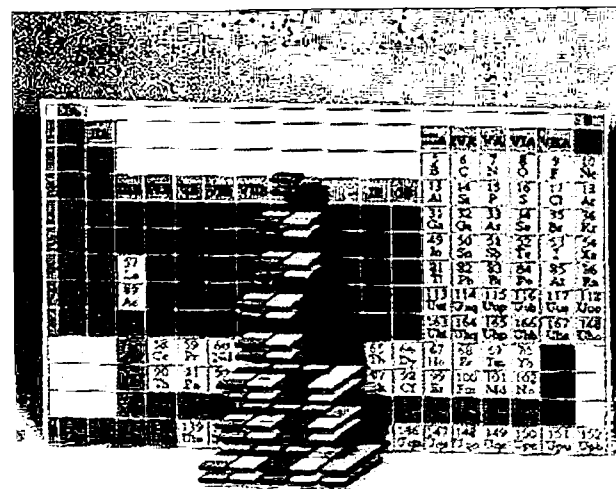


Fig. 8 - 3D Mendeleev's periodic system. Two monochromatic building blocks posed above one another in each layer correspond to an *electron orbital*.

- A period corresponds to a horizontal layer. Thickness of each period is two building-blocks.
- The number of building-blocks in such a twofold layer corresponds to the number of elements in the period and then to the *principal quantum number* n .
- The top stratum with the thickness of 1 in each twofold layer corresponds generally to elements with uncoupled electrons. The bottom stratum corresponds to elements for which coupled electrons exist.
- Two monochromatic building-blocks posed one above another in every layer correspond to one *electron orbital*. The elements, the properties of which are determined by outside electron subshells: s, p, d, f, g, are grouped together in modules of individual colour.

2.1.3. Analytical representation of atomic magic numbers

The figure-numeric approach yields an algebraic expression for the atomic numbers of inert gases:

$$Z = [(-1)^n (3n + 6) + 2n^3 + 12n^2 + 25n - 6] / 12, \quad (7)$$

where $n = 1, 2, 3, \dots$ is the period number (compare with [10]).

The recurrence relation for the inert gases is:

$$Z_{n+6} = 2Z_{n+5} + Z_{n+4} - 4Z_{n+3} + Z_{n+2} + 2Z_{n+1} - Z_n, \quad (8)$$

2.1.4. Predicted atomic magic numbers

Formula 7 generates the following atomic magic numbers:

2, 10, 18, 36, 54, 86, 118 / 168, 218, 290, 362, 460, 558, 686, 814, 976, 1338, 1538, 1780, 2022, 2310, 2598, 2936, 3274, 3666, 4058, 4508, 4958, 5470, 5982, 6560, 7138, 7786, 8434, 9156, 9878, 10678, 11478, 12360,

3. Periodicity of nuclear properties

Since Pascal's triangle has proven successful in determining atomic magic numbers, the question arises as to whether it can also help to identify nuclear magic numbers – can it provide a geometrical image and an analytical formula just as for atoms?

3.1. Nuclear magic numbers

The magic numbers for nuclei are: 2, 8, 20, 28, 50, 82, 126, corresponding to the total numbers of protons and neutrons in filled nuclear shells. The magic numbers here are nuclear equivalents to the atomic numbers of the inert gases: 2, 10, 18, 36, 54, 86. The nuclear "valley of stability" is the major feature in nuclear stability. However, if this valley is subtracted out, then nuclei with magic numbers of protons and neutrons are seen to be unusually stable [8-11]. They have especially large gaps between their ground and excited states.

3.1.1. Pascal's triangle modification "B"

Table 3 - Doubled binomial coefficients in a modification of Pascal's triangle.

$2C_0^0$	$2C_1^1$	$2C_2^2$	$2C_3^3$	$2C_4^4$	$2C_5^5$	$2C_6^6$	...
$2C_1^0$	$2C_2^1$	$2C_3^2$	$2C_4^3$	$2C_5^4$	$2C_6^5$	$2C_7^6$	...
$2C_2^0$	$2C_3^1$	$2C_4^2$	$2C_5^3$	$2C_6^4$	$2C_7^5$	$2C_8^6$	...
$2C_3^0$	$2C_4^1$	$2C_5^2$	$2C_6^3$	$2C_7^4$	$2C_8^5$	$2C_9^6$	...
$2C_4^0$	$2C_5^1$	$2C_6^2$	$2C_7^3$	$2C_8^4$	$2C_9^5$	$2C_{10}^6$	...
$2C_5^0$	$2C_6^1$	$2C_7^2$	$2C_8^3$	$2C_9^4$	$2C_{10}^5$	$2C_{11}^6$	...
$2C_6^0$	$2C_7^1$	$2C_8^2$	$2C_9^3$	$2C_{10}^4$	$2C_{11}^5$	$2C_{12}^6$	...
...	...	...	...	...	...	...	...

The values of all numbers in Table 3 correspond to doubling the numbers in the conventional Pascal's triangle. From this modified table it is possible to obtain:

a. *oblong plane numbers*, given by the formula:

$$n\Box = 2n\Delta = n^2 + n; \quad (9)$$

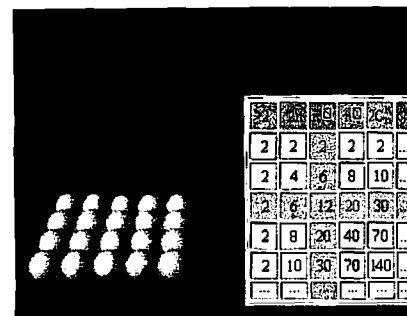
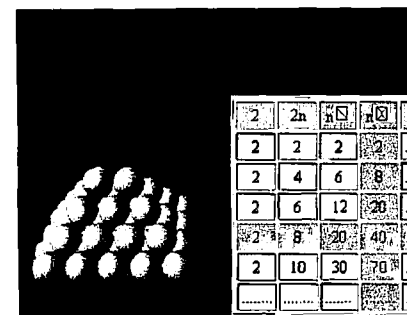


Fig. 9 - Oblong plane numbers.

b. *oblong pyramidal numbers*, given by the formula:

$$n\Box = 2n\Delta = (n^3 + 3n^2 + 2n)/3. \quad (10)$$



[11]. Similar diagrams and formulas have been found on a few pages of Linus Pauling's notebooks [12] and in some other works [13, 14].

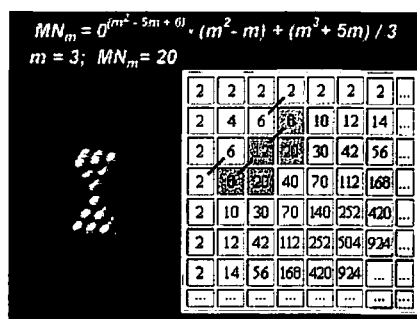


Fig. 11 - 3D representation of nuclear magic numbers 2, 8, and 20.

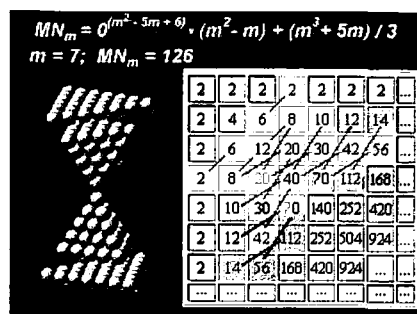


Fig. 12 - 3D representation of nuclear magic numbers 28, 50, 82, and 126. The number of spheres changing to each other is equal to the magic numbers 2, 8, 20, 28, 50, 82, and 126. The segments of dark blue lines bridge the numbers whose sum is equal to the next magic number.

3.1.3. Analytical representation of nuclear magic numbers

The lowest values (2, 8, 20) correspond to the independent nucleon motion in a single particle harmonic oscillator potential [15]:

$$MN_m = (m^3 + 3m^2 + 2m)/3, \quad (11)$$

where $m = 1, 2, 3$. The numbers 28, 50, 82, 126 are given by the formula:

$$MN_m = (m^3 + 5m)/3, \quad (12)$$

where $m = 4, 5, 6, 7, \dots$

Analytical representations of nuclear magic numbers can be found in the literature [13, 14]. A formula valid for all nuclear magic numbers is:

$$MN_m = k * (m^2 - m) + (m^3 + 5m)/3, \quad (13)$$

where $m = 1, 2, 3, \dots$ and $k = 1$ if $m = 2$ or 3 , $k = 0$ otherwise.

Here is a recurrence relation for the (11) and (12) series:

$$MN_{m+4} = 4MN_{m+3} - 6MN_{m+2} + 4MN_{m+1} - M_m, \quad (14)$$

3.1.4. Predicted nuclear magic numbers

Formula 13 generates the nuclear magic numbers:

2, 8, 20, 28, 50, 82, 126, 184, 258, 350, 462, 596, 754, 938, 1150, 1392, 1666, 1974, 2318, 2700, 3122, 3586, 4094, 4648, 5250, 5902, 6606, 7364, 8178, 9050, 9982, 10976, 12034, 13158, 14350, 15612, 16946, 18354, 19838, 21400,

4. Periodicity of cluster properties

It is possible to show that our Pythagorean approach can also be used to rationalize the magic numbers discovered in the analysis of the stability properties of clusters formed by either inert-gas [16] or alkali-metal [17] atoms.

Let us now consider the magic number sequences: (a) 7, 29, 66, 118, 185; (b) 7, 19, 37, 61 [18]; (c) 2, 8, 20, 40, 58, 92, 138, 196, 260, 344, 440, 558 [19].

4.1. Sequence 7, 29, 66, 118, 185

4.1.1. Method of finite differences

The method of finite differences, which is involved in the building of Pascal's triangle, can sometimes be used to conjecture a formula $f(n)$. Table 4 shows the first and second differences obtained from sequence (a).

Table 4 - Finite differences (a).

Magic numbers	First differences	Second differences
7	22	15
29	37	15
66	52	15
118	67	
185		

Note that the second differences are constant (i.e., 15). If and when we reach a row that contains a constant value, we can write an explicit expression for $f(n)$. In Pythagorean terms one would say that if the method of finite differences is applicable, each difference can be considered as a *gnomon* for a previous series. We can consider numbers in Table 4 as Pascal's triangle modification "C".

4.1.2. Analytical representation of series (a)

Explicit form:

$$C_n = (15n^2 - n)/2, \quad (15)$$

Recurrence relation:

$$C_{n+3} = 3C_{n+2} - 3C_{n+1} + C_n, \quad (16)$$

4.2. Sequence 7, 19, 37, 61

4.2.1. Method of finite differences

The method of finite differences yields Table 5.

Table 5 - Finite differences (b).

Magic numbers	First differences	Second differences
7	12	6
19	18	6
37	24	
61		

The resulting modified Pascal's triangle allows prediction of heavier clusters of this kind, as shown below.

Table 6 - Pascal's triangle modification "D".

6	6	6	7
6	12	18	19
6	18	36	37
6	24	60	61
6	30	90	91
6	36	126	127
6	42	168	169
6	48	216	217
...	...	...	...

4.2.2. Analytical representation of series (b)

Explicit form:

$$C_n = 6n\Delta + 1 = 3n^2 + 3n + 1, \quad (17)$$

Recurrence relation:

$$C_{n+3} = 3C_{n+2} - 3C_{n+1} + C_n, \quad (18)$$

4.3. Sequence 2, 8, 20, 40, 58, 92, 138, 196, 260, 344, 440, 558

4.3.1. Method of finite differences

The method of finite differences yields Table 7.

Table 7 - Finite differences (c).

Magic numbers	First differences	Second differences
2	6	6
8	12	8
20	20	-2
40	18	16
58	34	12
92	46	12
138	58	6
196	64	20
260	84	12
344	96	22
440	118	
558		

Here second differences are unequal. One may think that some clusters are combinations of others: e.g., $58+138=196$. On the other hand, numbers 2, 8, 20 are doubled pyramids (magic numbers for nuclei).

One may assume that, first, the series could involve approximate values and, second, the series could be non-uniform. Using a modification of Pascal's triangle that we will call "E" let us then construct an "ideal" series of magic numbers based on the second difference 12. It can be seen on Table 8 that the resulting **ideal numbers** are usually identical or very close to the *observed numbers* (usually within less than 2 %). Again this procedure can be used to predict magic numbers for heavier clusters of this kind.

4.3.2. Analytical representation of series (c)

The explicit form for the "ideal" numbers given in Table 8 is:

$$C_n = 2(3n^2 + 2n + 13). \quad (19)$$

This equation is a second-degree polynomial corresponding to 2D forms, while the numbers 2, 8, 20, as well as 40 (badly described by the above formula), are represented by a third-degree polynomial corresponding to 3D forms:

$$MNn = (n^3 + 3n^2 + 2n) / 3. \quad (20)$$

Table 8 - Pascal's triangle modification "E".

			+26	
	10	10	36	40
12	22	32	58	58
12	34	66	92	92
12	46	112	138	138
12	58	170	196	196
12	70	240	266	260
12	82	322	348	344
12	94	416	442	440
12	106	522	548	558
12	118	640	666	...
...	...	...	...	...

The recurrence relation corresponding to Eq. (19) is:

$$C_{n+3} = 3C_{n+2} - 3C_{n+1} + C_n. \quad (21)$$

It then appears that clusters at the beginning and in the continuation of the series correspond to different gnomons.

Conclusion

In this paper we have attempted to couple modern atomic theories with an ancient guiding principle. In particular, a parallel between atomic, nuclear and cluster shells is carried out using a fundamental concept of the Pythagorean school known as the *gnomon*.

Dedication

This work is dedicated to an astonishing person, Abolina Olesia. I would like to see her name given to element 118 if a valid discovery of such an ele-

ment is ever made. It could be the ideal gas "Olesium"! I do not know whether the scientific community would accept this idea.

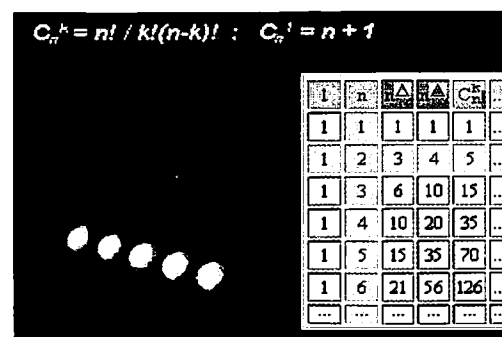
Acknowledgments

Special thanks are due to Pr Ray Hefferlin of Southern Adventist University (Collegedale, TN, USA) for useful advice about this work, and to a referee for pointing out the relevance of atomic clusters.

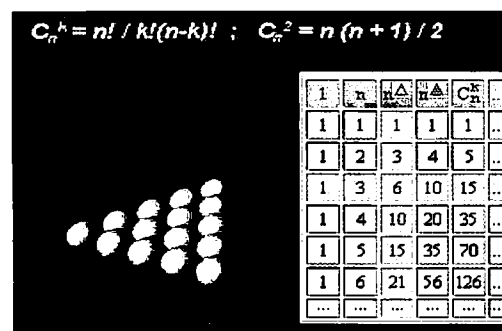
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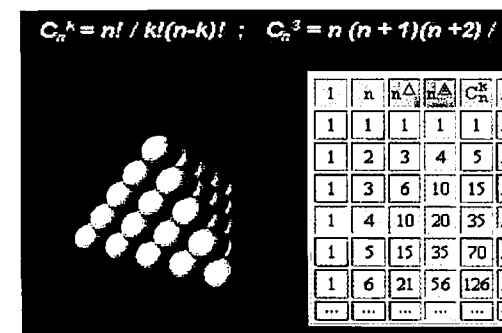
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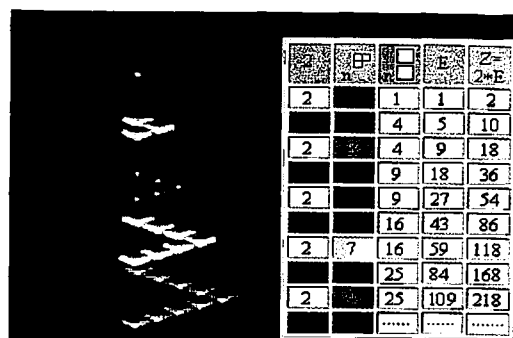
D. Weise, Fig. 1.



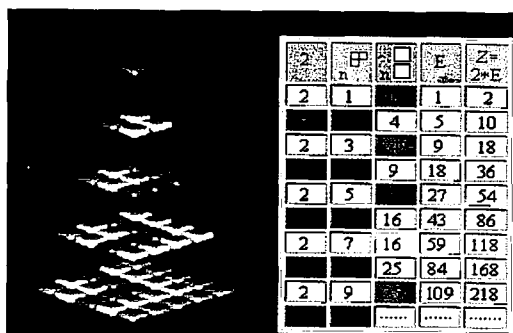
D. Weise, Fig. 2.



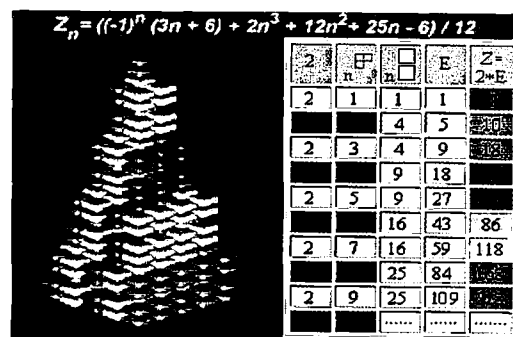
D. Weise, Fig. 3.



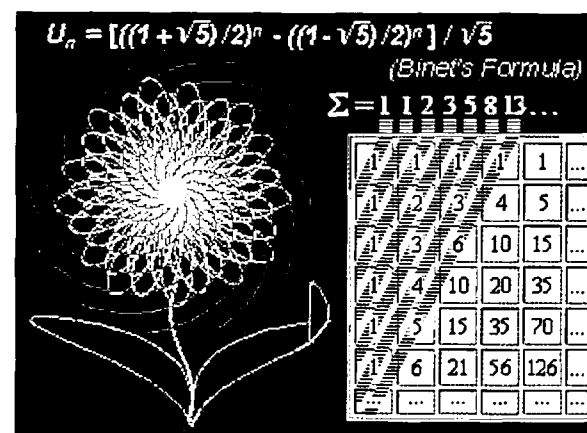
D. Weise, Fig. 5.



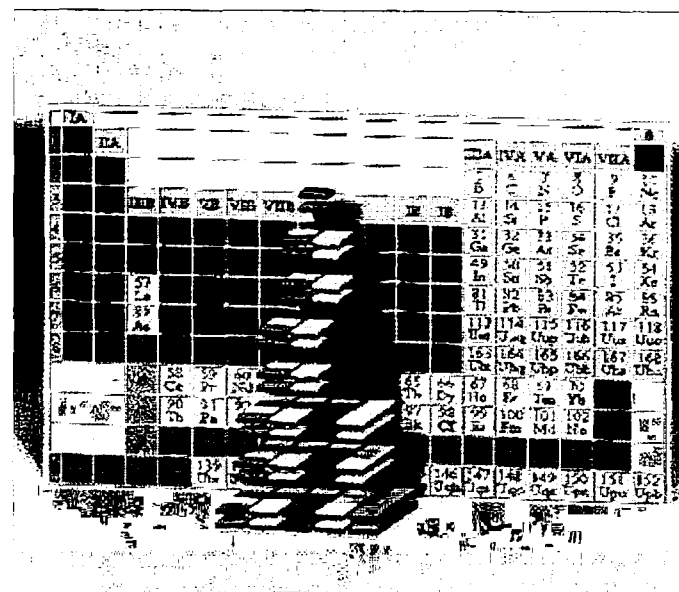
D. Weise, Fig. 6.



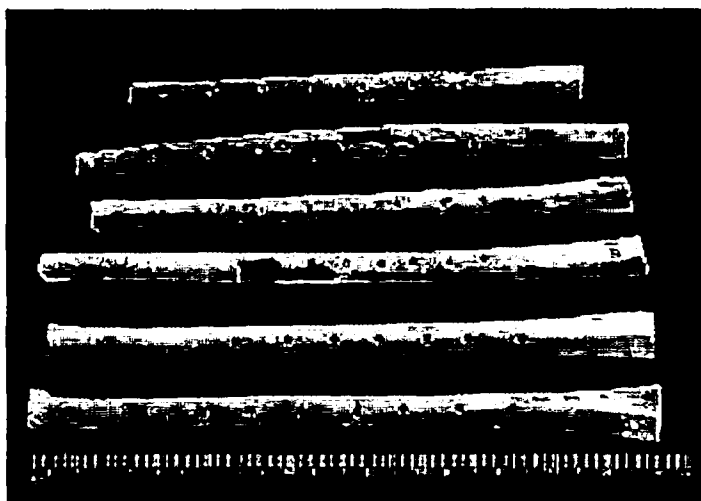
D. Weise, Fig. 7.



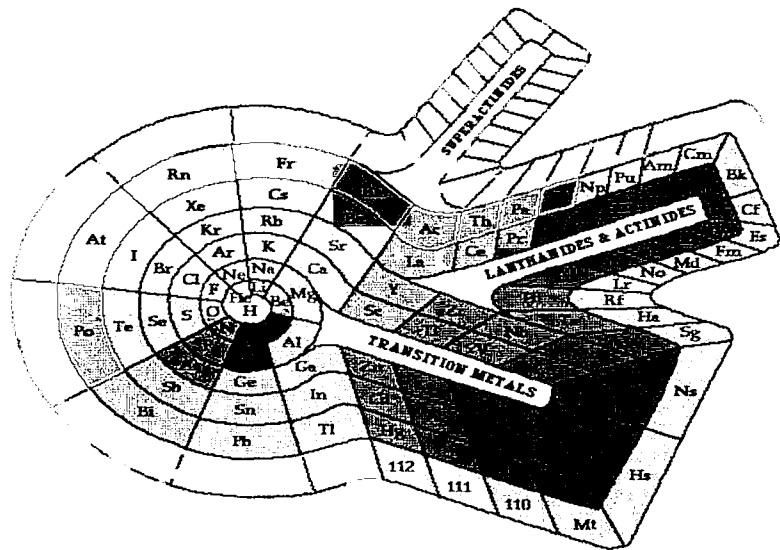
D. Weise, Fig. 4.



D. Weise, Fig. 8.



J. Maruani, R. Lefebvre, and M. Rantanen, Fig. 1 - Chinese flutes.



J. Maruani, R. Lefebvre, and M. Rantanen, Fig. 3 - Theodor Benfey's periodic table.