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CALEON, J.
2007

Sci & Educ
DOI 10.1007/s11191-007-9090-x

From Music to Physics: The Undervalued Legacy of Pythagoras

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Abstract This paper presents the early investigations about the nature of sound of the Pythagoreans, and how they started a tradition that remains valid up to present times—the use of numbers in representing natural reality. It will touch on the Pythagorean notion of musical harmony, which was extended to the notion of universal harmony. How the Pythagorean ideas have inspired many great works in physics, such as those of Galileo, Kepler and Newton, will also be presented. In exploring the legacy of Pythagoras to physics and the study of the universe, some valuable insights on the nature of science that can inspire budding physicists are extracted.

Keywords Nature of sound · Pythagoras · Music

1 From Music to Numbers

Pythagoras (about 580–500 BC), known as the ‘Master Philosopher’ of the Greeks, started a tradition that echoes to the present time—the use of mathematics (numbers) in representing the natural world. Pythagoras and his followers are thought to be the first to conduct scientific investigations on the nature of sound (Dampier 1961, p. 18). Although no writing of Pythagoras has been found, the work of some Greek philosophers contain detailed descriptions of his research. According to the writings of Boethius (about 480–525 AD), Pythagoras noticed the harmony in the sound of hammers in a blacksmith’s forge, and then found that the weight of the hammers that tend to produce pleasant sounds were, surprisingly, in ratios of simple whole numbers (Gozza 2000, p. 2). As Pythagoras tried to

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From Pythagoras to Sauveur: tracing the history of ideas about the nature of sound

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Abstract

This paper aims to supplement the scant literature on the history of ideas about the nature of sound. It presents how notions about the production and propagation of sound developed from antiquity up to the 17th century, i.e. from the time of Pythagoras to the time of Sauveur. It will highlight and examine the principles of sound that were formulated by Galileo and Newton, which are among the less well known work of these two giants in physics. The contributions of some familiar scientists, for example Hooke and Boyle, who are usually associated with scientific discoveries unrelated to sound, will also be covered. Some insights for the understanding of the nature of science and for the teaching and learning of physics will also be presented.

Introduction

The historical development of ideas in physics can provide interesting vignettes of information and insights, which can be used to enrich the process of teaching a topic. A recent article in this journal analysed the historical development of ideas on motion and assessed its implications for teaching [1].

The main goal of this paper is to supplement the scarce literature dealing with the development of ideas about the nature of sound. It intends to bring to light the ideas, be they vague, mystical, erroneous or brilliant, about sound pushed by prominent figures in the history of science, in the hope that valuable lessons can be learned for a better understanding of the concept of sound, of waves, of physics and of science. It is also hoped that the utilization of the historical perspective

that will be presented in this paper will put a humanistic touch on the presentation of normally abstract physical concepts in the classroom.

Early investigation of sound

Sound is the object of study of both music and physics. Early explorations of sound began as part of music. Music was then more of a mathematical discipline rather than a branch of art. The earliest scientific investigations of the nature of sound are attributed to Pythagoras (about 580–500 BC), a mathematician and philosopher [2]. Based on the writings of Boethius (480–525 AD), Pythagoras' interest in exploring sound was supposed to have been triggered when he was passing by a blacksmith's forge and heard that the sound of hammers hitting anvils can be at times pleasant (or consonant), and sometimes unpleasant (or dissonant) [3]. Pythagoras found that the weights of the hammers producing harmonious

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sounds were in ratios of small whole numbers. He then conducted experiments with other materials, such as glasses, vases, bells and strings, in order to determine if similar ratios could be obtained in creating consonant sounds. He was able to invent a monochord, which comprised a sounding board with movable bridge and a string stretched over it. Using the monochord, he found that two stretched strings with length ratio of 1:2 produced the same note separated by an octave, with the longer string producing the lower note [4]. He concluded that consonant sounds can be produced when the string length ratios involve the whole numbers 1, 2, 3 and 4. It is believed that Pythagoras also extended this conclusion to volumes of air in pipes and volumes of water in vases [5].

The influence of Pythagoras' ideas correlating sound to numbers on the subsequent generations of scientific thinkers should not be underestimated, even if questions can be raised regarding the authenticity of his so-called experiments owing to the scarcity and unavailability during his time of some materials that he is said to have used in such experiments. Pythagoras and his followers, who are known as Pythagoreans, should be credited for laying down some fundamental principles of the nature of sound: the generation of sound by vibrating sources, the notion of pitch and the linking of sound to numbers. Their early investigations became very good springboards for the next generations of scientists in their study of the nature of sound.

Wave versus particle views of sound

Further explorations of the nature of sound branched into two directions: towards either the wave or the particulate notion of the nature of sound. These explorations were accompanied by the determination of the mechanism by which sound propagates from source to receiver.

The notion of sound being a wave seems to have originated from observations of water waves. Early explorers of nature viewed a wave as a form of disturbance produced by a vibrating source, and as something which travels through a distance without a net transfer of water particles [6]. Aristotle's (384–322 BC) ideas on the wave nature of sound can be found in *On Things Heard* [7]. He noted that when sound is produced by a source, the air at the source is driven forcibly into the surrounding air for a finite distance—just like wind

blowing. On the generation of sound, Aristotle [7] believed that it is produced when air meets with a body: for example, when a stringed musical instrument is played, it produces sound when the air is set in motion.

Aristotle's writings indicate that he recognized the mechanical nature of sound waves being propagated in a medium such as air. He had envisaged sound waves like ripples of water when he said that sounds 'fill the space around them'. He seemed also to have given an early account of the longitudinal nature of sound waves when he wrote '[the air] is set in motion . . . by contraction or expansion or compression' [7]; this is somewhat indicative of the definition of a longitudinal wave, where the wave motion is parallel to the direction of propagation of the vibration.

A long time gap existed before further progress on the nature of sound ensued². In the 16th century, Galileo Galilei (1564–1642), famous as an astronomer and physicist, wrote an account of his wave view of sound in the 'First Day' of his book, *Dialogue Concerning Two New Sciences* [8], which was first published in 1638. He envisioned sound waves through movements of water waves.

That the undulations of the medium are widely dispersed about the sounding body is evinced by the fact that a glass of water may be made to emit a tone merely by the friction of the finger-tip upon the rim of the glass; for in this water is produced a series of regular waves.

In 1636, Marin Mersenne (1588–1648), a French mathematician, wrote in *Harmonicorum Libri* that sound is 'a disturbance in a medium' [9]. He also equated sound with movement: 'All movements that occur in the air, in water, or elsewhere, can be called sounds, inasmuch as they lack only a sufficient delicate and subtle ear to hear them . . .' [9]. He empirically determined the speed

² After the time of Aristotle, there was a big gap in the development of ideas about sound. The Roman conquest of the Greek empire began. The works of the ancient Greeks were hidden for protection. During the 17th century, the period of the Renaissance, science was at a crucial turning point, moving from theoretical to practical, emphasizing experimentation over the use of pure reason in investigating natural phenomena. It also helped that the works of the ancient Greeks were translated into other languages and were made available to other scientists.

of propagation of sound to be 316 m s^{-1} by finding the time for an echo to return after travelling a known distance [10].

Isaac Newton (1642–1727) provided an elaborate theoretical explanation of the mechanics of the propagation of sound as a wave. In his *Principia* [11], first published in 1687, he explained in Proposition XLVII that pulses propagated in a fluid medium make the particles vibrate back and forth. Using Galileo's ideas, he noted that the particles in the fluid medium 'are always accelerated or retarded according to the law of the oscillating pendulum', which is equivalent to simple harmonic motion in modern usage. He showed that the propagation of sound through any fluid was shown to depend only on measurable physical properties of the fluid, such as elasticity and density. He even calculated the speed of sound in air using theoretical considerations, although the value he got ($979 \text{ ft s}^{-1} = 298 \text{ m s}^{-1}$) significantly differed from empirical results ($1142 \text{ ft s}^{-1} = 348 \text{ m s}^{-1}$) owing to an error in assuming that the temperature of the air during its vibrations as the sound propagates remains constant [12]. In 1816, this was corrected by Laplace, who noted that the heating of the air due to its compression and expansion as sound propagates needs to be considered in the calculation; he introduced a factor γ in Newton's formula, which stands for the ratio of specific heats for air [12].

Although several scientists supported the wave notion of sound, the absence of detectable motion in the air (e.g. sound being observed not to affect the motion of any light body) led other scientists to think of an alternative proposition. Among them was the early 17th-century French natural philosopher and astronomer Pierre Gassendi (1592–1655), who proposed that sound was propagated in a stream of fine, invisible particles from the original source to the ear [13]. This idea sprang from the early works of Epicurus (341–270 BC) and Democritus on atomism. He posited that sound is due to the emission of a stream of atoms from a sound source, with the velocity of sound being the velocity of atoms, and frequency being the number of atoms emitted per unit time [6].

Isaac Beeckman (1588–1637), a Dutch scientist, also envisioned a particulate nature of sound. He postulated that sound travels through

air as 'globules of sonic data' [14]. He posited that any vibrating object cuts the surrounding air into little spherical corpuscles of air that are sent away in all directions by the vibrating motion of the source, which is then perceived as sound upon reaching the ear [14]. For him, vibration is not even a necessary condition for causing sound: he argued that whenever air is divided into globules, sound is thereby generated [14]. These globules of air in sound generation are reminiscent of Einstein's concept of photons in the particle theory of light. It appears that a prelude to quantum theory came in relation to sound way before it had found a better role for light!

The two contrasting ideas on the nature of sound did not experience the same intensity of controversy as that for light. It appears that the large majority of the early thinkers accepted the wave interpretation for sound. This could be due to the fact that sound waves, owing to their mechanical nature, are relatively easier to visualize and comprehend, taking a good analogy from water waves. Light, on the other hand, has a more enigmatic nature—that of being an electromagnetic wave, the comprehensive understanding of which needed to wait until the advent of Faraday's theory unifying electricity and magnetism in 1861 and Maxwell's theory of electromagnetic waves in 1865.

The role of the medium

Other scientists doubted the role of air in the propagation of sound. Athanasius Kircher (1602–1680) was the first to do an experiment with an air pump [4]. He listened to the sound of a bell in a jar while the air inside was being removed by the air pump. Otto von Guericke (1602–1686), who made a complex two-man pump that drew air from two fitted copper hemispheres [15], popularly known as Magdeburg hemispheres, also conducted a similar experiment. Both Kircher and von Guericke observed that even if air is removed from a jar, they still could hear the ringing of a bell inside it, leading them to conclude that air is not necessary for the transmission of sound [4]. This observation is perhaps due to air leakage in the pump used.

Robert Boyle (1627–1691), a well known chemist, also attempted to determine how sound propagates in a vacuum. His idea of vacuum came from Evangelista Torricelli's experiment that

involved the inversion of a tube filled with mercury, and which left a space at the top, and from the work of von Guericke [15]. With the help of Robert Hooke (1635–1703), Boyle was able to make a simple, yet efficient, air pump connected to a glass chamber [15]. Using this air pump, Boyle carefully conducted his version of the bell-in-a-jar experiment and observed that the sound of an alarm clock (or bell) placed inside the vacuum chamber faded away as air was withdrawn from the chamber [16]. He concluded that sound cannot travel in a vacuum, thereby supporting the Aristotelian perspective that a medium, such as air, is needed in sound propagation.

A revival of Pythagoras' consonance ratios

During the 16th century, there was a revival of interest in the Pythagorean consonance ratios. Vincenzo Galilei (1525–1591), the father of Galileo, conducted experiments believed to have been done by Pythagoras. The results of Vincenzo's experiments indicated that the musical ratios for generating consonant sound apply only in relation to string and pipe lengths, and not to sounds generated using different volumes of water and weights hanging on strings, with all other factors remaining the same [5]. Vincenzo's conclusions partly refuted the long-held belief on the Pythagorean consonance ratios, suggesting that these ratios depended on the properties of the vibrating source, and, thus, were not absolute.

Continuing what his father started, Galileo [8] conducted further experiments on sound. He was able to establish the fact that sound produced by a stringed instrument is determined more precisely by the ratios of the frequencies of the sound and not by the ratios defined by the length, size and tension of the strings, as the Pythagoreans had claimed. He noted that the frequency, which is the 'number of pulses of air waves' generated by a vibrating source [8], is the physical cause of pitch that is perceived by the ear—he found this through an experiment involving the scraping of a metal with a chisel. Although this experiment was very realistically described by Galileo in his book *Dialogue Concerning Two New Sciences* [8], slight errors in his account suggest that the experiment was unlikely to have been performed [14]. Real or otherwise, Galileo's experiments led him to an important breakthrough: linking music with the physical reality of motion and associating the

consonant ratios with a specific aspect of sound—the frequency. Despite the criticisms of the process he used in deducing these breakthrough principles, Galileo should be credited for 'reconciling nature and mechanics, mathematical demonstrations and sensate experiences, while turning the sounding number into sound' [3].

Simultaneously with Galileo's work on sound, Mersenne also conducted an independent study of the vibration of stretched strings. Mersenne discovered that a string's frequency varies inversely with its length [9] and, like Galileo, he associated frequency with pitch. Furthermore, he actually calculated the value of the frequency of vibration of a long, heavy wire that moved very slowly, and determined the frequency of a note linked to a particular pitch [10].

Robert Hooke also indicated his own way of associating frequency of vibration with the pitch of sound. Through his fine mechanical skills, he was able to devise an instrument illustrating his proposition. This instrument was composed of a toothed wheel striking a piece of metal at various speeds, thereby producing musical notes of various pitches [15].

In the late 17th century, the French physicist Joseph Sauveur (1653–1716), who first coined the term 'acoustics' to refer to the study of sound, carried out detailed investigations of the relationship between frequency and pitch of sound waves. In his book, *Collected Writings on Musical Acoustics* [17], he noted that an organ pipe of about 5 Parisian feet (1.624 m) gives out sound of frequency equal to 100 cycles per seconds (or Hz)³. He made the first frequency table of musical pitches, giving the frequency of middle C to be 256 Hz, which is rather close to the current value of 261 Hz.

Implications for understanding the nature of science and for the teaching of physics

Providing a historical account that details the development of scientific ideas, including the scientists behind these ideas and how and in what context these ideas were generated, can help in humanizing science [18] and in enhancing the significance of the achievements

³ These data were used by Isaac Newton [11] to determine the wavelength of a pulse of sound produced by the open pipe, using the length of the pipe divided by the sound frequency (Book 2, *Scholium* in Proposition L, *Principia*).

and the nature of science [19]. A historical perspective on the development of ideas about particular science concepts can help teachers understand the difficulties that their students face in giving up their pre-instructional conceptions or misconceptions [1]. The ensuing discussion describes how the history of sound provides a window through which the nature of scientific ideas and the process of knowing science can be viewed, and how it can be utilized to motivate students to restructure their alternative conceptions of sound.

On the durability and tentativeness of scientific ideas

Science is a dynamic enterprise, featuring ideas that are often subject to verification and modification. However, there are also ideas that have withstood rigorous tests and have become a relatively durable set of knowledge. In the case of sound, the ideas of Pythagoras about musical harmony and the nature of musical sound have undergone several modifications, but the original formulation, which is about the existence of simple ratios that produce consonant sounds, remains valid up to the present time. In this sense, the history of the nature of sound can be utilized to present both tentative and stable facets of scientific ideas. This helps in addressing the concerns of De Berg [20], who stressed that overemphasizing tentativeness in the nature of science may lead students to perceive that science knowledge need not be taken seriously, and of Wang and Marsh [18], who noted that giving less emphasis to the tentative nature of science will lead students to perceive scientific ideas as the final product of science. When the students' minds are conditioned such that what they study is the final form of scientific research, and thus no longer open to change, their drive to thinking of alternative ideas and their propensity for creativity can be diminished.

On the personal, psychological and social context of scientific investigation

Scientific investigations are conducted by people neither in a vacuum nor exclusively in a laboratory. They exist in a particular social context. Often, scientists need to overcome various obstacles and be ready to stand up for their ideas that may not conform to the accepted ones. One

scientist worth mentioning for his courage and determination in challenging the prevailing belief is Vincenzo Galilei. His boldness in verifying and challenging a long-held belief on musical ratios has largely contributed to the identification of the key to the underlying mystery behind the 'sounding numbers' of Pythagoras. Good filial connections, such as between the father-and-son tandem of Galileo and Vincenzo, who had a common interest in music, also help to facilitate the growth of scientific ideas.

On knowing about the process of knowing in science

The path of events leading to the modern view of harmony and consonance provides a framework by which students can experience the process of science and engage in restructuring long-held views. The historical development of sound would help students realize that there must be a good interplay between reason, sense experience, and explanation, in order to arrive at the truth about nature, with theory and experiment complementing each other in leading to scientific truths. This was illustrated in the section where the error in Newton's theoretical calculations of the speed of sound was identified after comparison with the empirical results of Mersenne and through reflections of how the personal experiences of Pythagoras, Hooke and Galileo led them to correct generalizations about the nature of sound.

In this article it is also shown that novel ideas can be generated in various ways. One way is through thought experiments. How Galileo imagined his way through the key principle that links the frequency of sound and harmony in music, despite the subtle mental errors he committed, is just amazing. The congruence between Galileo's main finding and that of Mersenne adds credibility to the common idea that they discovered independently. However, the eventual acceptability of Galileo's propositions does not deny the danger that goes with thought experiments. This is a good instance for emphasizing to students the value of careful verification of scientific ideas before they are accepted, citing reproducibility as the hallmark of validity. Another good thing to point out to students is the fact that in the process of verifying ideas, such as during replication of previous experiments, interesting discoveries can

also be made. When Vincenzo Galilei repeated the so-called experiments of Pythagoras, he not only detected the errors in these experiments but he was also able to deduce that the properties of the source of sound are crucial in musical harmony.

On dealing with alternative conceptions

It has been reported that students' preconceptions about scientific phenomena resemble those of pre-modern science thinkers, and the presentation of relevant excerpts from the history of science that mirror the students' difficulties may provide motivation for the students to realize the inadequacy of their ideas, appreciate modern concepts and eventually restructure their own ideas [19]. Researchers have reported that students have difficulty grasping the wave nature of sound, tending to use an object-like mode for understanding sound phenomena [21, 22]: this seems to agree with Gassendi's [13] and Beeckman's [14] view of sound. It was also found that students, and even teachers, have difficulty understanding the connection between pitch and frequency of vibration [23]. Noting that it took around 1600 years before the link between pitch and frequency had been fully clarified, this difficulty amongst students is justified. If students are made aware of the aspects of the history of sound that are linked to their preconceptions, they would have a sense of consolation that they are not alone in their struggle. Knowing how Galileo and others derived and reasoned out the correct conceptions may help students in altering their alternative conceptions of sound.

Conclusion

The history of sound, just like the history of any other idea, is a history of dreams, creative imagination, obstinacy, error and enlightenment. It also shows one important facet of science, which is not commonly found in other human activities: the systematic criticism of errors often leading to a better version of nature's truth. The history of sound can be used as a window through which the nature of science can be seen and analysed, and in the process may provide flesh to the usually abstract concepts involved in physics.

Acknowledgment

We are grateful to the Nanyang Technological University for the award of a Research Scholarship

to the first author and a research grant (RI 9/06 RS) to the second author.

Received 26 September 2006, in final form 17 November 2006
doi:10.1088/0031-9120/42/2/007

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