

Analogia: The Pythagorean Unity of the Liberal Arts and Professions

Plato, the greatest Pythagorean of Antiquity, is remembered as the creator of higher education in the West and founder of the first university, the Academy at Athens (c.385 BC-529 AD). He also designed the first curriculum of academic studies, which originally consisted of just four 'mathematical arts' – arithmetic, geometry, astronomy and music. After Plato's death the Academy introduced three preliminary studies of 'verbal arts' – grammar, rhetoric and logic (or 'dialectic') – thus completing the classical system of the 'Seven Liberal Arts'. This syllabus of general education, which remains a powerful influence on modern universities, became standard for the Roman Empire and much of Medieval Europe. The course of studies was originally designed to introduce students to a Pythagorean world-view in which the mathematics of music is the key to understanding the Kosmos or universal system and, in post-graduate training, becomes the theoretical basis to the practice of all the liberal professions.

The Greeks viewed the cosmos as a hierarchy of similarly ordered systems, ranging from the human soul and body, to the family and the city-state: all of these were part of the 'Microcosm' or smaller, human order and this in turn was seen as a reflection in miniature of the 'Macrocosm', the celestial system of Earth, Sun, Moon, stars and planets. Linking the system at every level was the unifying principle of 'Harmony'. The term 'armonia' was originally used in joinery and other arts to refer to the process of binding, joining or fitting together; but, in the philosophy of the Pythagoreans, the harmony of the cosmos (or 'music of the spheres') became a system of mathematical ratios which they believed was found in the natural structure of the musical scale as well as the entire world-system. This universal harmony came to be formulated as the 'analogy of the Macrocosm and the Microcosm': the technical term 'analogia', a central concept of Greek mathematics, meant 'identity (or similarity) of ratios'.

General Education

Plato's *encyclopaedia* ('cycle of studies') was a tightly integrated curriculum of the four mathematical arts organized by a logic that was unquestionably Pythagorean in origin and aim (the modern sense of 'encyclopaedia' arose when Plato's successors at the Academy recognized the need for reference books to support the teaching). For the Pythagoreans arithmetic is the fundamental study, the science of numbers which they conceived as metaphysical units, the atoms of all things. Combinations and aggregations of these units form lines, planes and solids, whose laws are studied in the next mathematical art, Geometry. The Pythagoreans had originally assumed that any form or object must consist of a finite number of units and, therefore, that geometry would be a subset or application of arithmetic; but the proof of 'Pythagoras's Theorem' (which was certainly not discovered by the Master himself) demonstrated that the hypotenuse of the right-angled isosceles triangle was incommensurable with the other two sides and could not, therefore, consist of a finite number of units. The classic proof revealed that some geometrical dimensions could not be measured arithmetically and, consequently, that in some cases

numbers cannot be simple integers, units or whole numbers. To preserve the logical sequence of his curriculum, Plato had to insert an additional study of irrational numbers.

The third mathematical art is Astronomy (Cosmology or Spherics), the study of the *Kosmos* (another conception ascribed to Pythagoras): here again, Plato envisaged this third art as a subset or logical application of the preceding one: that is, geometry as applied to analyzing the motions of the 'spheres' or celestial bodies. According to the Pythagoreans, this analysis will reveal the universal harmony: that is, the series of musical ratios that define the structure of the cosmos and preserve it from dissolving into disharmony and chaos. Thus the fourth and final discipline of the mathematical curriculum is Music and its subject-matter the 'Harmony of the Spheres'. Music (or Harmonics), for the Pythagoreans, was the key to understanding the universe; but the music studied at the end of the higher education was not the practical art but the mathematical theory of 'musica speculativa'. This remained the core study of scientific astronomy until the seventeenth century AD.

The encyclopaedic curriculum features prominently in two of Plato's greatest works, the *Republic* and the *Laws*, though, in the latter work, Plato appears to place less emphasis on the final training in music. The content and rationale of the curriculum, however, is reviewed again in the *Epinomis*, an unfinished summary of Plato's general philosophy which obviously emanated from the Academy, even though Plato's authorship has been questioned. The *Epinomis* reads very much like an appendix to the *Laws* and many students have accepted the *Epinomis* as a genuine writing of Plato and his final philosophical testament.

If this attribution is correct, then the *Epinomis* (991A-B) reveals the secret of Plato's harmonic system, the very special 'analogy' or musical module of 6:8:9:12: this set of interlocking ratios specifies the proportions of the structural intervals of the musical scale - of the octave (1:2), the fifth (2:3) and the fourth (3:4). Pythagoras, on his deathbed, is said to have urged his followers to apply themselves to the study of the monochord, the instrument on which these and other musical ratios can be accurately demonstrated and measured. The Pythagorean Plato appears to have followed this precedent and injunction in finally revealing the module necessary for the tuning of the musical scale and understanding the harmony of the universe as a whole. The crucial role of this musical module in Plato's thought and practice was generally overlooked or underestimated until the publication of Ernest G. McClain's *The Pythagorean Plato* (1978), a brilliant decoding of the musical mathematics in Plato's political allegories.

From General Education to Professional Practice

The graduates of the Hellenistic system of general education emerged with a mastery of rational thought and expression in both word and number and specific expertise including a detailed knowledge of the theory of *proportio*, the Roman equivalent of *analogia*. The finished encyclopaedic or liberal arts graduates would all understand that 'a single bond naturally unites all things':¹ Theon of Smyrna is evidently referring here to the 'single bond of natural inter-connection' that Plato saw as the ultimate encyclopaedic vision of

the ideally educated graduate (*Epinomis* 992A): if so, that bond must be the harmonic analogy or module 6:8:9:12 which had just been revealed in the preceding section of the dialogue (991A-B). Thus equipped with the key to the Pythagorean world-view, the finished student would then proceed to post-graduate specialisation in the higher study of pure philosophy and science (particularly cosmology and, later, theology) or in more practical applications. In Pythagorean terms, the harmonically informed graduates would become either pure or applied harmonists, employing their common knowledge of proportion either in the mathematical study of the Macrocosm or in dealing with the practical harmonies of the earthly and human microcosm.

Little is known of ancient post-graduate and professional education before the writings of Marcus Terentius Varro whose missing *Disciplinarum libri novem* (c.48 BC) added Medicine and Architecture to the Seven Liberal Arts, thus indicating their common theoretical and pedagogical foundation. But, with the gradual sharpening of the distinction between general education and professional training (which goes back to Plato and Aristotle), Medicine, Architecture and Law came to be distinguished from the liberal arts proper and recognized in their own right as *liberal professions*. Though the history of this important educational development is still obscure, the actual process and its results are strongly indicated by the surviving evidence of the theoretical and practical role of *analogia* or *proportion* in the ancient liberal professions.

The technical term *analogia* was created by Greek mathematicians - probably Pythagorean music theorists² - to refer to identity or similarity of ratios in the Macrocosm or Microcosm generally and specifically to the mathematical attunement of harmonic systems. Long before the technical term was adopted into medical theory, Pythagorean physicians had employed comparable terminology, as in Alcmaeon's concept of *krasis* - that musical harmony or balance of the cosmic elements which was also the doctor's model of health in the human microcosm. The term 'analogia' is not found in the Hippocratic corpus; but, according to the on-line *Thesaurus Linguae Graecae*, 'analogia' appears no fewer than 232 times in the writings of the last of the classical physicians, Galen (c.129-210 AD). He was educated by his father (an architect) in the liberal arts for twelve years, made a synopsis of Plato's *Timaeus* and wrote a book arguing that 'the best physician must also be a philosopher'. Since Galen was a Pythagorean, who claimed to have read the 'Golden Verses' every evening, it is not surprising that he made extensive use of 'analogia'; but, by this time, the term had acquired some of its modern, non-mathematical meanings. To determine the extent to which Galen's medical theory employed the original sense of 'proportionality' - similarity or identity of ratios - would be a considerable undertaking, beyond the limits of this preliminary survey.

Galen's liberal education resembles that described by the Roman architect Vitruvius in his *Ten Books of Architecture* (c.30-20 BC). His opening chapter explains the student's need of a general education, particularly in geometry, music, history, philosophy - and even medicine and law - before proceeding to professional practice. Vitruvius also emphasises the fundamental importance of *analogia* or proportion, especially in the human frame and its architectural analogue, the temple (de Architectura, III.I.1). While

the continuing influence of Pythagorean tradition is confirmed by Vitruvius' numerous references to *harmony*, his writing is often obscure, reflecting the Roman inferiority in philosophy and mathematics.

Before Law became a liberal profession, Aristotle proposed an important corollary to the theory of analogy: 'the just is a species of the proportionate';³ and injustice is 'what violates proportion'.⁴ As Aristotle makes clear, he is referring to analogy or proportion in the original sense of 'equality of ratios', which involves at least four terms: A:B::C:D. He divides the Just into two species, distributive and rectificatory.

In distributive justice, the judge has to resolve a dispute between two parties about dividing an honour, reward or proceeds of an investment by finding a geometrical proportion such that A (first person): B (second person):: C (first portion): D (second portion). According to this formula, the prize or profit is divided in the same proportion as is found between the relative merits or investment of the parties involved.

In rectificatory justice, the judge acts as a mediator to find a formula which restores equality to a situation where one party has been responsible for an unjust gain or damage, to the detriment or loss of another party. In an unjust situation one party has 'more of the good and less of the evil' and the judge's task is to restore equality by finding a mean between the unfair extremes of loss and gain. Injustice can be represented by a line divided into unequal parts: the judge restores equality by taking away that portion 'by which the greater segment exceeds the half, and add[ing] it to the smaller segment' (1132^a25-28). The point at which the judge cuts the line is the arithmetic mean between the extremes of the original unequal segments.

Even though he criticizes the Pythagoreans for defining justice as 'reciprocity' (an eye for an eye, etc), Aristotle's proportionate treatment of justice is very Pythagorean in spirit and method: his judicial application of the geometrical and arithmetical means strongly suggests that he, like Plato, was relying on the mathematics of the monochord. This inference seems to be confirmed by Aristotle's comment that 'the judge (*dikastes*) is one who bisects (*dichastes*)'⁵: like the modern expression 'interval', the term for the line that is divided, *diastemma*, could be understood in both a geometrical and a musical sense.

So, in equating the just (*dikaion*) with the divided (*dichaion*), Aristotle was evidently envisaging the judicial decision as a *cutting of a line* in the same sense as a monochord string is precisely divided to demonstrate the ratio of a given harmony (*sectio canonis*). If so, he was interpreting the art of the judge (*techne basilike*) as another form of applied harmony, employing the same musical mathematics that ideally informed the practice of all the liberal professions of the Roman Empire.

¹ Theon of Smyrna, *Mathematics useful for understanding Plato*, trans. R. & D. Lawlor (San Diego, 1979), p.55.

² On the early history of *analogia* and the musical module 6:8:9:12, see Árpád Szabó, *The Beginnings of Greek Mathematics*, trans. A.M. Ungar (Budapest, 1978), pp. 154-161.

³ *Ethica Nicomachea*, trans. W.D. Ross (Oxford, 1925), 1131^a 30.

⁴ *Ibid.*, 1131^b15.

⁵ *Ibid.*, 1132^b30

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