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## A Cardboard Pythagorean Teaching Aid

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Pythagoreanism, broadly construed, seeks the mathematical structure of things; it believes that, in the words of Aristotle, things imitate numbers.<sup>1</sup> About twenty-five hundred years ago, the Pythagoreans<sup>2</sup> discovered that a string or flute twice as long as a given one will play the same note an octave lower, and that when a four-inch string and a three-inch string are strummed together, they make a harmonic chord.<sup>3</sup> In our own times both academic philosophers of science and more popular accounts note that physicists prefer theories with beautiful math. On the softer side of things it's been said the Numbers of Pythagoras begat the Forms of Plato, the Forms begat the Logos of Philo, and the Logos of Philo begat the Word of John.<sup>4</sup> That the world has a mathematical structure is a main stitch in the fabric of Western thought.<sup>5</sup>

A namesake of this vision, the Pythagorean Theorem, is far more famous than Pythagoras and the Pythagoreans. Whether or not, as a point of history, this theorem of geometry fully deserves its name,<sup>6</sup> it nevertheless abundantly exemplifies the Pythagorean spirit. Trigonometry, which unfolds the theorem, is key in building skyscrapers and space ships, as was the theorem's ancient use in marking off square corners in the wheat fields of Egypt after the Nile flooded. The theorem is prominent in the proof of relativity and it was prominent in the Hellenistic computation of the size of the earth. Astronomers, carpenters, and surveyors use it daily. High school students learn the theorem, but seldom put it in global perspective. The purpose of this lesson is to help the student see the Theorem as a pillar of civilization.

### *A Cardboard Teaching Tool*

The cardboard manipulative teaching aid here explained helps introduce these things in an effective and literally hands-on manner.

Without any mention or hint of these considerations I pass to each student one white square and four small colored triangles. *It's important*

*that I make no hint as to what I'm up to. This is the opening of the show.* I simply explain that we're going to do a little demonstration. I'm going to ask you to move the pieces in certain ways. It's not any kind of trick. It's not hard and it's not a speed test. It's a friendly little demonstration and if your hands are sticky from recent jelly doughnuts, now might be a good time to wash them because we want to keep the cardboard nice.

The square is seven inches on the side and made of white poster board. The triangles are three by four by five inches. I show the class two ways (or configurations) to put the triangles on the white square (Figures 1 & 2).<sup>7</sup>

Then I ask half the class to put their triangles on the square one way, and the other half the other way. (Perhaps the ladies the one way and the gentlemen the other. Or I divide the room in half.) When everyone has done so, I pause a little while for everyone to look at both configurations. Then I ask them to switch. And when they've done that I pause again, and then I ask that they switch back. I may introduce

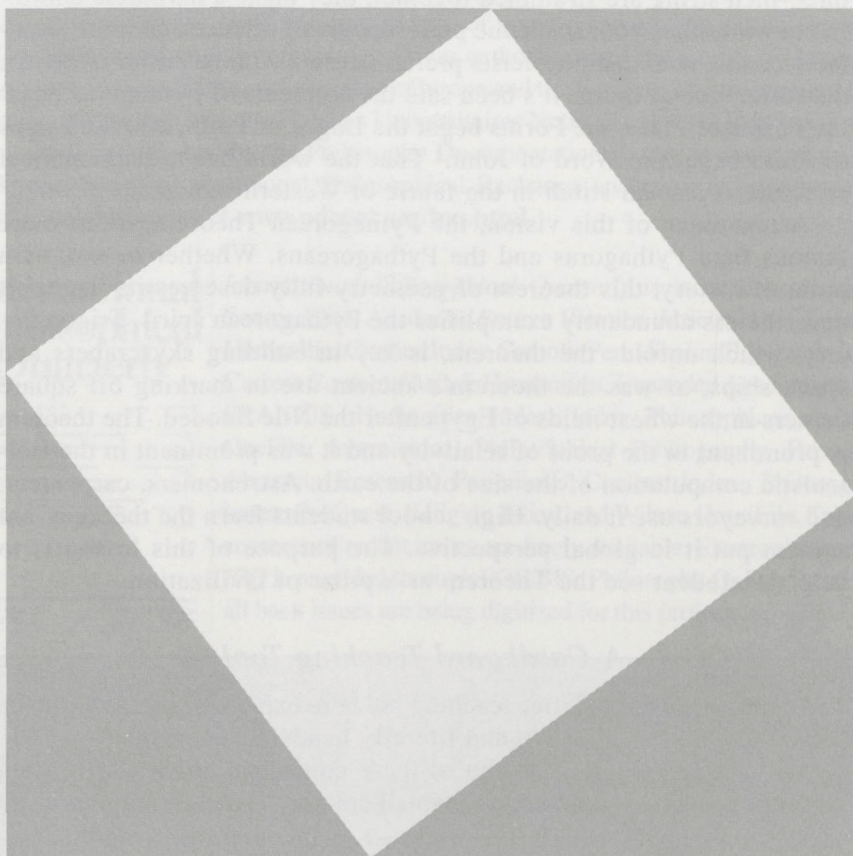


Figure 1

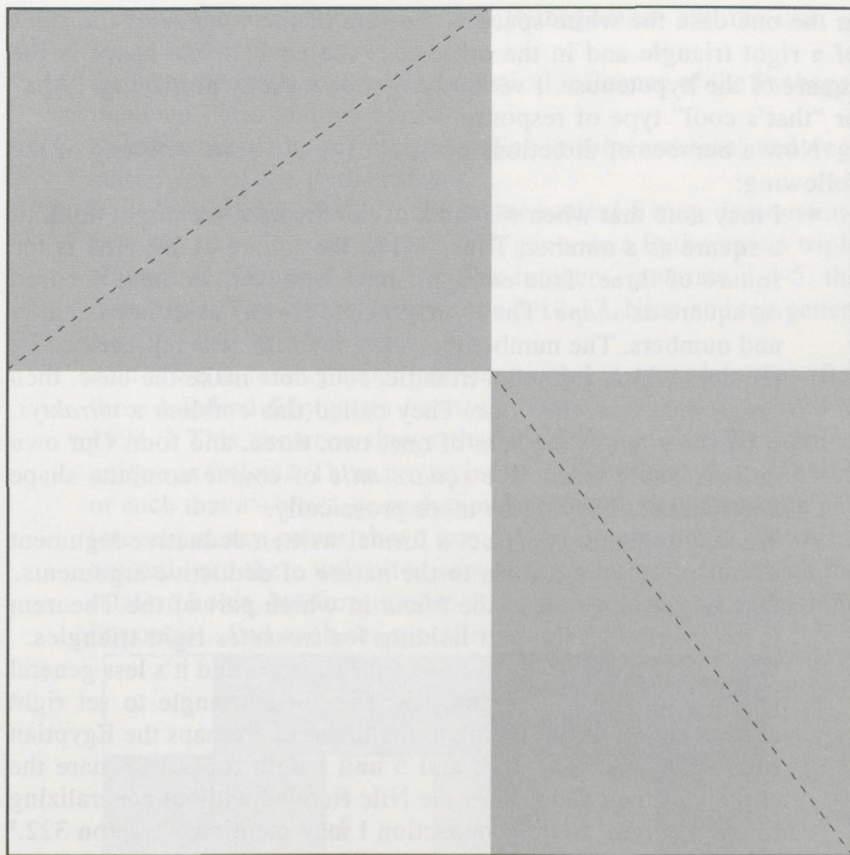


Figure 2

a third configuration. In this one any random mishmash of the four triangles is OK except that none may overlap another and none hang off the edge of the square. I then invite everyone to agree that in each configuration there is the same amount of white space left over. This might take some clarification or discussion but with a single exception has always been agreed to. (Once one student didn't agree and nobody could convince him otherwise. I admired his integrity and we moved on.) I ask if anything has been demonstrated and no one says much. I ask what they know of triangles. Not much response. (This is good. Were I to get too early a response I might ask that student to hold that thought for a moment and then give her or him the limelight when the time is right, at punch-line time as it were.) Then I ask about right triangles in particular. When finally I ask what they have remembered from high school about the sides of right triangles, some state the theorem, still without seeing what we've done. *Pay dirt.* I then remind them that the white space is the same in the two cases and show how

in the one case the white space is the sum of the squares of the sides of a right triangle and in the other case the equal white space is the square of the hypotenuse. I've always gotten a pretty gratifying, "Aha" or "that's cool" type of response.

Now a number of directions open up. I might start with any of the following:

- I may note that when we think of the theorem we might think of a square as a number. Thus, 361 is the square of 19, nine is the square of three. True enough. Here, however, we have focused on square *as shape*. The Pythagoreans closely associated shapes and numbers. The number ten, a big favorite, was represented by ten dots which formed a triangle: four dots make the base, then three, then two, then one. They called this emblem a *tetraktys*. It pictures ten as the sum of one, two, three, and four. Our own indispensable terms like *square mile* of course combine shape and number. They do so more prosaically.
- We may together construct a formal written deductive argument of our proof as a lead-in to the nature of deductive arguments.
- One might move on to the *Meno* in which part of the Theorem is demonstrated, the part holding for *isosceles* right triangles.
- We discuss the probable origin of the theorem and its less general predecessors such as using, say, the 3-4-5 triangle to set right angles without generalizing to the theorem. Perhaps the Egyptian rope stretchers used 3, 4, and 5 unit length ropes to square the corners of their fields after the Nile flooded without generalizing to the theorem. In this connection I may mention Plimpton 322.<sup>8</sup> This is the name of a five-thousand-year-old Babylonian clay cuneiform tablet containing a table of fifteen sets of three numbers. One set is 25, 16, and 9. Another is 169, 144, and 25. Each set is a square followed by two squares the sum of which equals the first number. Such a table shows a Babylonian farmer that two fields, one of three units square and one of four, together produce the crops equal to one field five units square. Although the table says nothing about triangles, if it is applied to a right triangle one can easily derive the Theorem (without proof) from it.
- Discuss the Pythagorean character of the movie "Evolution" and its use of the periodic table. The gimmick in this comedy is that arsenic is poison to carbon-based life forms like us. From carbon, arsenic is a (chess) knight's move right/down on the periodic table. These alien enemies are nitrogen-based, and a knight's move right/down on the table from nitrogen is selenium. Ergo selenium (Head and Shoulders shampoo) might be poison to these nitrogen life forms.

- Explain the musical views of the Pythagoreans and the idea of the music of the spheres.
- Discuss the mathematical and mystical influence of the Pythagoreans on Plato and his forms.
- Speak a bit of the mysticism, belief in reincarnation, and vegetarianism of the Pythagoreans.
- If we're feeling a little more mathematical I may demonstrate how any two *different* integers will generate a Pythagorean triple (Figure 3). One and two, the first integers, generate 3-4-5, the first triple. Two and three generate 5-12-13. Nine and ten generate 19-180-181. (See Figure 3.)<sup>9</sup>
- And if we're feeling super mathematical, I may explain that after three-hundred-fifty years *Fermat's Last Theorem* was proved in 1994.<sup>10</sup> This theorem shows that the Pythagorean Theorem is unique as follows: There are no integers  $a$ ,  $b$ ,  $c$  such that  $a^3+b^3=c^3$ , or such that  $a^4+b^4=c^4$ , or such that the relation holds between any integers at any power above two. At the power of two, however, not only are there unlimited sets of three integers for which the relation holds, but furthermore that relation expresses something important. And each set specifies its own triangle.

|   | 1 | 2           | 3             | 4             | 5              |
|---|---|-------------|---------------|---------------|----------------|
| 1 |   | 3<br>4<br>5 |               | 8<br>15<br>17 |                |
| 2 |   |             | 5<br>12<br>13 |               | 20<br>21<br>29 |
| 3 |   |             |               | 7<br>24<br>25 |                |
| 4 |   |             |               |               | 9<br>40<br>41  |
| 5 |   |             |               |               |                |

Figure 3

Any two different numbers generate a Pythagorean triple. Twice their product, the difference of their squares, and the sum of their squares each determine one member. Students can satisfy themselves that shaded boxes either designate a single number instead of a pair, or they designate an already designated pair, or they designate a higher iteration. In the last case, take the example 2 and 4. This generates 12/16/20, which reduces to 3/4/5. 3 and 5 produce 16/30/34 (8/15/17).

|    | 1 | 2           | 3             | 4             | 5              | 6              | 7              | 8                | 9                | 10               |
|----|---|-------------|---------------|---------------|----------------|----------------|----------------|------------------|------------------|------------------|
| 1  |   | 3<br>4<br>5 |               | 8<br>15<br>17 |                | 12<br>35<br>37 |                | 16<br>63<br>65   |                  | 20<br>99<br>101  |
| 2  |   |             | 5<br>12<br>13 |               | 20<br>21<br>29 |                | 28<br>45<br>53 |                  | 36<br>77<br>85   |                  |
| 3  |   |             |               | 7<br>24<br>25 |                |                |                | 48<br>55<br>73   |                  | 60<br>91<br>109  |
| 4  |   |             |               |               | 9<br>40<br>41  |                | 33<br>56<br>65 |                  | 72<br>75<br>97   |                  |
| 5  |   |             |               |               |                | 11<br>60<br>61 |                | 39<br>80<br>89   |                  |                  |
| 6  |   |             |               |               |                |                | 13<br>84<br>85 |                  |                  |                  |
| 7  |   |             |               |               |                |                |                | 15<br>112<br>113 |                  | 51<br>140<br>149 |
| 8  |   |             |               |               |                |                |                |                  | 17<br>144<br>145 |                  |
| 9  |   |             |               |               |                |                |                |                  |                  | 19<br>180<br>181 |
| 10 |   |             |               |               |                |                |                |                  |                  |                  |

Figure 3a (expanded fig 3)

In short, the manipulative provides an opening to bring out a number of historical and philosophical issues.

### *Making Triangles*

Each three-by-five index card makes two 3-4-5 triangles with two easy cuts. The all-important square corners are factory perfect.

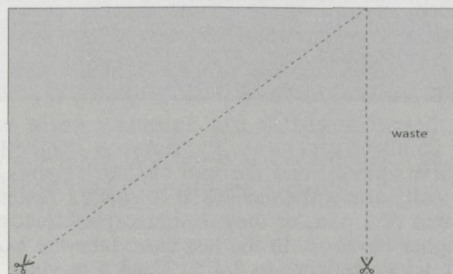


Figure 4

### Notes

I thank Erica Hughes for making the drawings and Joseph S. Socher for several suggestions.

1. "For the Pythagoreans say that things exist by imitation (*mimesis*) of numbers" (Metaphysics 987b 11).

2. I'll use the term loosely to cover the original followers of Pythagoras and/or any who used that name or are called by that name for the one- to two-hundred years after his death. See Charles Kahn, *Pythagoras and the Pythagoreans* (Indianapolis: Hackett, 2001).

3. Although we have since learned that pitch is a matter of the rate of vibration, the ratios between rates of vibration are the same as between the lengths of the vibrating strings. Thus the older discovery is extended and further confirmed and not at all superseded.

4. Cf. Harry Austryn Wolfson, "Extradeical and Intradeical Interpretations of Platonic Ideas," in *Religious Philosophy, A Group of Essays* (New York: Atheneum, 1965), pp. 67–68.

5. Other examples making a prima facie case that things imitate numbers are found in Amir D. Aczel, *Fermat's Last Theorem* (New York: Four Walls Eight Windows, 1996), and include the Golden Section and Fibonacci numbers.

6. Kahn explores this controversy.

7. Both configurations are derived from a single diagram from Alec Fisher, *The Logic of Real Arguments* (Cambridge: Cambridge University Press, 1988), p. 3.

8. Aczel, *Fermat's Last Theorem*.

9. I thank Owen Leibman for this information.

10. By Andrew Wiles. See Aczel, *Fermat's Last Theorem*, pp. 14, 15.

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