

# The Return of Odysseus and the Elements of Euclid

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## Introduction

It is the overall aim of this paper to obtain a new perspective on the often-treated question of the origins of Greek mathematics. For a number of reasons, an inquiry into these origins must take its start from a consideration of the Elements of Euclid. To begin with, the Elements, in actuality a late achievement of Greek mathematics, is the earliest text of its kind to survive to the present day. Secondly, Euclid himself has come to be viewed less as an original mathematician, and more as the author of a systematic compilation of a tradition of mathematical development which extends back to Plato's Academy and beyond. Thus the Elements may be considered a representative of this strong tradition, if not of Greek mathematics as a whole. Most important, however, the Elements, in its theoretical structure which is ultimately grounded upon a notion of proof, exhibits what distinguishes Greek mathematics from its ancient counterparts. In light of this last observation especially, one is justified in closely associating the origins of Greek mathematics as a whole with the origins of the Elements of Euclid.

The majority of attempts to search out these origins have been determined by two presuppositions: The first of these is that "proof" in this context means deduction from axioms, and that the origins of the mathematics of the Elements are at one and the same time the origins of this conception of proof; the second is that the field of texts which might testify to these origins includes only those texts which exemplify what we recognize today as "Greek science" (including philosophy, mathematics, logic, medicine, geography, natural history, and so on). This paper proceeds by modifying the first of these presuppositions, and by rejecting the second.

With respect to the first, there can be no doubt that proof shows itself most immediately in the Elements as a more or less rigorous procedure of deduction from axioms; a quick glance at the first few pages of the Elements is enough to establish this. I want to add that there is another mode of proof active within and essential to the unfolding of the mathematics in the Elements, proof from construction, or from diagram. Thus one of the starting points for this paper is that "proof" for Greek mathematics is two-fold in nature, on the one hand essentially mental as deduction from axioms, and on the other hand essentially visual as proof from construction, and furthermore that the origins of a distinctly Greek mathematics ought to be sought in the origins of the association of these two notions of proof. Their connection in Greek thought is preserved in two important lexical groups in the Greek language which I will indicate now and have cause to return to later. The first of these is the group of words which organize themselves around the Greek noun *noos*, which eventually comes to mean something abstract like "mind" or "consciousness", but whose primitive meaning seems have determined a special kind of seeing. The second of these is the group of words which organize themselves around the Greek verb *deiknusthai*, which can indicate precisely the act of proving by deduction from axioms, as it does in various manifestations of its prefixed form *apodeiknusthai* in the Posterior Analytics of Aristotle, but can also determine a more general kind of demonstration which we would want to call "showing" and which associated itself with the act of proving from construction or diagram.

The modern mathematician, due to a formalization in mathematics which was ultimately accomplished in the beginning of the nineteenth century as a result of the emergence of non-euclidean geometries and the subsequent re-working of the Euclidean axioms, confers evidentiary force only upon the axiomatic mode of proof, and demotes diagrammatic "showing" to the lower precinct of heuristics. Although the selection of the deductive, axiomatic method as the "true way" for mathematics had already begun by the time of Euclid and his Elements, the independent evidentiary force which the diagram held in the eyes of the Greek mathematicians is implied by the particularly constructive quality of the axioms which Euclid has chosen to motivate his mathematics, which is always at bottom geometric. The first three axioms in book I of the Elements describe the establishment of a line from points, the continuous extension of a given segment of a line along a straight line, and the construction of an arbitrary circle. These sparse axioms justify the construction of a wide variety of geometrical figures which can all be represented

axioms justify the construction of a wide variety of geometrical figures which can all be represented in a diagram, and upon which the majority of the proofs of propositions in the Elements depends. Diagrams have for the most part been subsumed under an axiomatic framework, but on the same token axiomatic proof is rarely accomplished independent of the construction of diagrams.

The lack of success of Euclid's fifth postulate, the infamous parallel postulate, provides even stronger testimony to the central role played by the diagram in Greek mathematics. If proof, for the Greeks, were only the deduction of conclusions from any given set of axioms, then the parallel postulate would be as worthy a starting point for geometry as any. But there is another criterion: The axioms by which those conclusions are deduced must be self-evident, and in Greek mathematics, this self-evidence derives from the diagram. The parallel postulate, which establishes the intersection of straight lines extended to infinity in the direction determined by the side of a transversal where the interior angles add up to less than 180 degrees, can not be represented in a diagram. It is for this reason that Proclus, the early fifth century A.D. commentator on the first book of the Elements, criticizes that this postulate "...lacks the special character of a postulate..." and, "...ought to be struck from the axioms altogether." Rather, he suggests, the postulate should be proved as a theorem because its truth is not necessary, but only probable. As early as the second century A.D. geometers had begun to set themselves to the task of proving the parallel postulate from the other four Euclidean axioms, a project which would eventually be proven impossible by Gauss and Bolyai among others in the nineteenth century. But before the problem was resolved, geometers, having failed at proving the postulate, began to look for statements logically equivalent to it, and invariably chose formulations that the Greeks themselves would have considered diagrammatically evident. Among these is the axiom invented by the eighteenth century French mathematician Alexis Claude Clairot, which establishes that rectangles exist. The choice of this axiom over Euclid's is motivated by a conception of proof which recognizes the evidentiary force of the diagram, and in so doing preserves a Greek mathematical sentiment. Indeed Euclid himself recognized the unconventional flavor of his fifth postulate, and refrained from using it until he had no other recourse.

The reverse situation arises in connection with the proof of the side-angle-side criterion for triangle equality in book one of the Elements. Here it is not the case, as with the parallel postulate, that an axiom is involved whose evidence is not intuitive in the sense that it can be shown in a diagram, but that a proof is accomplished on the grounds of this intuitive evidence alone and without axiomatic justification. Specifically, Euclid proves the proposition by hypothesizing two triangles with two corresponding sides and the included angle respectively equal, and then essentially by picking up one triangle and placing on the other so that the equal sides and angle coincide, and finally by showing that the other side and angles coincide and are equal as well. The procedure of "picking up" and "placing on" is called superposition, and as it is used in this proposition implies that a figure can be moved across the plane without suffering any distortion. But movement without distortion in turn implies, in the case of the triangle, a preconceived notion of the equality of angles and segments of lines, and this notion upon which the accomplishment of the proposition ultimately rests finds no grounding in the Euclidean axioms but derives its truth value empirically from diagrams of angles and segments. Heath writes: "The method of superposition, depending on motion without deformation, is only of use as a practical test; it has nothing to do with the theory of geometry." Its theoretical inadequacy is a direct consequence of its dependance on the visual experience of a diagram, but it is for this very reason that Proclus, who is so critical of the non-diagrammatic parallel postulate, proclaims the validity of this proposition. After declaring that, "The proof of this theorem... grows naturally out of the clarity of the hypotheses," Proclus observes:

Visible equality... in things of the same form is manifestly the ground of the entire proof. For there are two axioms here that comprise the whole procedure of this theorem. One is that things which coincide are equal to one another. This is true without qualification... The other is that things that are equal coincide with one another. This is not true in all cases, but only of things that are similar in form.

Of the two "axioms" to which Proclus here refers, the first is included among the "common notions" in the Teubner text, but Tannery and Heath argue it is an interpolation. The second is definitely not one of the Euclidean axioms, and depends on the prior notion of the similarity of form. The "visible equality" that Proclus indicates has no solid axiomatic foundation, but nevertheless he considers it sufficient grounds for the proof. Euclid, on the other hand was more wary about the validity of his use of superposition in the proof of propositions, and resorts to this procedure only here and at one other place in all of the thirteen books of the Elements. The traditional evidentiary force of the diagram was sufficiently strong to permit its use, but that use was at the same time restricted by the concurrent

demand for axiomatic rigor.

It is this combined notion of proof as the product of an essentially logical axiomatic demonstration coupled with a constructive demonstration relying upon the visual evidence of the diagram that lies at the center of the theoretical structure of the Elements. Accordingly, it is in the origins of this notion of proof that the origins of Greek mathematics shall be sought. The next question to be addressed, then, is where these origins are to be sought. As I remarked above, the traditional approach to this problem has been to presume that the roots of Greek mathematics extend no deeper than the earliest examples of recognizably scientific texts. What ought to be reconsidered here is the description "scientific". Lloyd remarks: "The explicit categories we commonly use in our highly value-laden descriptions... had a history and in most cases derive directly or indirectly from concepts invented by the ancient Greeks." Indeed, the categories which underlie the judgement that one ancient text is "scientific" and another not preserve a distinction that is Greek in origin, and one which is captured in the semantic opposition between the Greek nouns *logos*, which comes to indicate the true account, and *muthos*, the common tale infused with "error and deceit". As Lloyd suggests, this distinction is historical; the move toward the opposing semantic values of *logos* and *muthos* occurs only in the fifth century, initiated in the History of Herodotus and in the Victory Songs of Pindar and eventually instituted by Thucydides, among others. But even the first impulses in which this move can be detected occur a century later than the activity of the earliest exemplars of Greek mathematical thinking, whom the most conservative accounts place at the end of the seventh and beginning of the sixth centuries. At this time in Greece no distinction is recognized between the semantics of *logos* and *muthos* so as to justify the definition of a homogenous system of truth-oriented texts where the origins of Greek mathematics might be located, but rather truth itself is dispersed over a field of widely varying contexts. This being the case, an attempt to account for the origins of Greek mathematics must permit itself to look outside the narrow field of so-called Greek scientific texts and into textual precincts which exist for the Greeks as other and more traditional repositories of knowledge.

One very important such textual precinct is the "mythical" content of epic poetry. Epic poetry, preserved in the Homeric poems, is a primary location for the encoding of the Greek concept of truth, and in the following discussion it is to the Homeric poems that the problem of the origins of Greek mathematics shall be referred. Specifically in the Odyssey thematic content shall be discussed which develops a concept of truth as certainty deriving from demonstration by construction and simultaneously concerns itself with the ground of the construction, the integrity of the surface upon which the diagram is inscribed. This thematic content expresses the essential structure of the notion of proof which is adopted into the practice of Greek mathematics as represented by the Elements of Euclid. Along the way it will be possible to show that the move of this concept of truth out of its original epic environment and into the environment it eventually inhabits in the Elements is at the same time its dedication to an innovative positive project, and its departure from its primary epic context -- a context I will want to call dialectical. This last observation will indicate the sort of discussions which a new perspective on the origins of Greek mathematics can provoke.

## **Mathematics Before Euclid**

At the outset it ought to be recognized that by the third century B.C. when the Elements were written the epic themes which I have indicated were only latently preserved in the structure of Greek mathematics. Euclid himself would certainly not have recognized the relation between his concept of proof and the content of the Homeric poems. Rather, according to the categories of *logos* and *muthos* which by his day had been well established, he more likely saw epic poetry as archaic story-telling in contrast to which he could legitimate his truth-oriented mathematics. There were, however, a number of philosophers of the sixth century in Greece whose work can be shown to constitute the beginnings of the geometrical tradition to which the Elements belong, and who either ally themselves directly with the Homeric poems or at least are likely to have been strongly influenced by epic material. It is by locating these philosophers that a continuous stream of thought flowing out of eighth century Homeric epic and into third century Euclidean mathematics is established.

The first of these philosophers to be considered belong to the old Ionian school, the most renowned member of which is Thales of Miletus. Proclus, in the prologue to his commentary on the first book of the Elements, includes a catalogue of geometers whose work precedes that of Euclid, in which Thales is the first person to be named. After claiming that Thales, "...was the first to introduce [geometry] into Greece," Proclus writes, "He made many discoveries himself and taught the

principles for many others to his successors, attacking some principles in a general way and others more empirically." Proclus, who wrote in the middle of the fifth century A.D., uses for his source in this instance, and for much of the information in the following catalogue, a lost book on the history of Greek mathematics written in the third century B.C. by Eudemus, who was a pupil of Aristotle. As early as 423 and the performance of Aristophanes' *Clouds* Thales is already recognized as having achieved some prominence in geometry. In one scene in that play, Socrates is shown preparing to perform a geometrical demonstration and in the process stealing the cloak of a student at the wrestling school. Upon hearing a report of this, the dull-witted Strepsiades remarks: "Why do we continue to marvel at old Thales?" The remark calls attention to the fourth century reputation of Thales the geometer, but perhaps also questions the genuineness of that reputation by likening Thales to Socrates, who is pictured here as a second rate thief only pretending to geometrical ability.

There are no writings by Thales upon which a more reliable estimation of his contributions to Greek geometry could be based, but there are reports from late antiquity which attribute to Thales the proof of two geometrical theorems. The first of these comes from Proclus, and probably has Eudemus for its source. Proclus writes of Thales that "He is said to have been the first to prove (apodeiksai) that a circle is bisected by its diameter." Here, as Heath notes, Proclus means that Thales proved this proposition in the Euclidean sense, for the other two propositions which Proclus attributes to Thales were not, according to Proclus, proven, but only discovered or merely stated by Thales without scientific demonstration (epistemonikes epideiksios). Modern commentators, including Heath, have doubted that Thales could have proven this statement with any rigor since it shows up in the *Elements* not as a theorem, but a definition, and they therefore conclude that Thales only "showed" its truth with a diagram. The method of proof which Proclus assumes that Thales used is superposition, not a rigorous deductive method, but still a marginally acceptable procedure in the mathematics of the *Elements* and entirely diagrammatic in essence. The second theorem which Thales is said to have proven is more complicated, and the attribution, if correct, implies that Thales was in command of some sort of deductive method as well as the diagrammatic procedure of superposition. Diogenes Laertius cites Pamphile, who claims that Thales, "was the first to inscribe a right-angled triangle in a circle, whereupon he sacrificed an ox." The implication seems to be that Thales understood the general proposition that the angle inscribed on the semi-circle is always right, which implies, in turn, that he knew among other things that the angles of a triangle are together equal to two right angles, from which fact he could have deduced the theorem suggested by Pamphile's remark. The accuracy of the remark has been called into question on two grounds: first that Pamphile may have this story confused with a similar one which Apollodorus tells about Pythagoras; and second that it is anachronistic in that the proposition about the angle sum of triangles has traditionally been attributed to Pythagoras or the more or less nebulous "Pythagoreans" and in either case would not have been available to Thales. However the case may be, it is clear that by the middle of the sixth century in Ionia the Greeks had begun to formulate general geometrical propositions, and had developed a notion of proof which was diagrammatic if not yet deductive.

Thales died in the vicinity of 547 B.C. and left behind him in Ionia an intellectual heritage which would soon be assumed by Hecataeus of Miletus who was active at the end of the fifth century. Hecataeus is best known as the author of the map of the world (*Periodos Ges*) to which Herodotus alludes in book four of his *Histories*:

I am amused to see those many who have drawn maps of the world and not a one of them making a reasonable appearance of it. They draw Ocean flowing around an earth that is as circular as though traced by compasses, and they make Asia of the same size as Europe.

Before publishing the map shortly before 500 B.C., Hecataeus had made extensive travels into Asia and Europe. Rather than organize his map according to particular information collected empirically in the course of these travels, he chose to construct it around pure geometrical objects like the circle and equal areas, and in so doing betrayed a conviction that such objects provided a surer foundation for geography than the evidence of the senses. The evidentiary force of these objects derives, without a doubt, from the early and successful practices of geometry in Ionia, and the concern which Hecataeus displays over structural elements and foundations shows a developing theoretical orientation which most likely held sway over the practice of early Ionian geometry as well, and eventually manifests itself as the demand for axiomatic rigor in the mathematics of the *Elements*.

The demonstrative potential of the map so constructed is made clear by the role it plays in the course of events which surround the beginning of the Persian war. As Herodotus recounts, the city of

Miletus on Samos is threatened by advances by the Persians, and Aristagoras, Prince of Miletus, sails off to Sparta to implore the tyrant Cleomenes to abandon his Peloponnesian campaign against the Messenians and to join in with the Milesians in fighting back the Persians on Asia minor. Aristagoras argues that the Spartans have much to gain from this course of action, for to the west and south of Samos lie concentrations of Persian wealth in close proximity to one another for Cleomenes to plunder. To prove this last point, Aristagoras points to a bronze tablet on which a map of the world is inscribed. The map was that of Hecataeus, who serves as advisor to Aristagoras during the Persian war. By pointing (deiknus) to the map, Aristagoras means to secure the truth of his claim and thus betrays his conviction that Hecataeus' radically geometrical geography proffers evidence of the most irrefutable authority.

Hecataeus' map preserves an Ionic tradition of cartography which seems to have been originated by Anaximander of Miletus, a contemporary and compatriot of Thales. Agathernus, citing Eratosthenes, writes, "Anaximander of Miletus, the student of Thales, was the first to attempt to record the inhabited world on a tablet; after him Hecataeus of Miletus, a widely travelled man, scrutinized [the tablet], and was amazed at the work." Anaximander was best known in antiquity for his innovations in speculative astronomy and cosmology, which, like the work of Hecataeus, was radically geometrical in structure. Specifically, Anaximander conceived the universe as having the Earth at its center in the form of either a sphere or a cylinder and explained that it holds its place there because the center of the universe is "spatially indifferent", equidistant from every extremity of a spherical universe. Anaximander is said to have constructed a spherical model of this universe which must have preserved the geometrical quality of his cosmology. Most likely Anaximander's map, which Diogenes mentions in connection with this model of the heavens, was similar in its theoretical structure, and it is this map which instructed the geography of Hecataeus.

The theoretical quality of Anaximander's cosmology is evident not only from its geometrical structure, but also in that it is grounded upon a strict notion of "first principles". The first principle of Anaximander's cosmology is the apeiron (the unbounded), which is described as both an arche (ultimate beginning) and stoicheion (primary substance). The apeiron as arche, as that which casts the universe into time, has a transcendental quality which resembles the self-evidence of the Euclidean axioms. As stoicheion, the apeiron becomes not only that out of which the universe springs, but also that of which the universe is constituted. In a like manner, theorems in the Elements are built of prior theorems, which are ultimately built of the axioms themselves; accordingly the plural stoicheia, which is translated into English as "Elements", becomes the title of Euclid's book on geometry. The meaning of apeiron reveals a final similarity between Anaximander's first principle and Euclid's axioms. It is generally agreed that apeiron, for Anaximander, did not signify a spatial or quantitative infinity, but rather a qualitative indeterminateness, an abstract principle which manifests itself in the world in the circular reciprocity of generation and destruction. The image to which it corresponds is not the infinitely extended three-dimensional space which prevails in modern cosmologies, but rather the continuous curvature of the surface of a sphere or the perimeter of a circle, both of which are fundamental structural elements in the universe of Anaximander. The apeiron thus conceived constitutes a cosmos which, unlike the Newtonian universe, can be contained in a finite model. By inventing a cosmological first principle which is at once transcendental, elemental and constructible, Anaximander exhibits the theoretical disposition which comes to determine the choice of axioms upon which Greek geometry will be eventually grounded.

This Geometry, which is ultimately exhibited in the Elements and which is anticipated by the sixth century work of Thales, Anaximander and Hecataeus, seems to have been already firmly in place by the middle of the fifth century in Ionia and had begun to spread about Greece through the diaspora of Ionians which occurred as a result of the Persian war. Hippocrates of Chios, who sometime around 450 B.C. moved from his birthplace in Ionia to Athens, published the first book known to be called Stoicheia which is believed on the authority of Eudemus to have contained a complicated procedure for the quadrature of the lune. The title of the book and the advanced nature of its content implies that the Greeks had finally arrived at an axiomatic system for geometry and a self-conscious method of proceeding deductively toward less-than-obvious conclusions. Although the axiomatic system and geometrical orientation of Hippocrates' book certainly originate out of the "old Ionian school" represented by Thales, Anaximander and Hecataeus, the thinking of those early Ionians presents no firm analogue for the formal deductive method which motivates the procedure from axioms to conclusions in the geometry of Hippocrates.

To uncover the origins of the application of deductive logic to geometry the Greeks themselves

recognized that one had to look elsewhere, and it was traditional, at least in the commentaries of late antiquity, to attribute these origins to Pythagoras and his immediate disciples. Proclus writes:

After these (Thales and Ameristus or Mamercus) Pythagoras transformed the study of geometry into a liberal education, examining the principles of the science from the beginning and probing the theorems in an immaterial and intellectual manner...

In this instance, Proclus' source is not Eudemus but Iamblichus, an enthusiastic neo-pythagorean of the fourth century A.D. Burkert points out that Eudemus, like his teacher Aristotle, never mentions Pythagoras in connection with developments in geometry, but only so-called "Pythagoreans". And Heidel remarks that, "...some of these Pythagoreans were earlier than Plato; how much older, we do not know, and one is strongly inclined to infer from the polemical tone of many of the references, that the questions at issue were, at least for the most part, the subjects of debate in the schools of Plato and of Aristotle and therefore not dating back a century or more." Accordingly, Heidel makes the middle of the fifth century B.C. the earliest possible date for any significant Pythagorean contribution to Greek geometry, at which time advanced geometry like that of Hippocrates was already being produced. Burkert attributes the legend of Pythagoras the geometer, which Iamblicus and Proclus invoke, to internal political developments in the Academy after the death of Plato. In the Middle Academy under the direction of the skeptic Archytas all that was dogmatic and doctrinal of the Old Academy was discounted as deriving from Pythagoras and the "Pythagoreans" so that what remained was an Academy that was predominantly "Socratic." Thus Pythagoras, by no act of his own, was credited with developments in science and mathematics which actually came out of the later writings of Plato. Then, in late antiquity, when neoplatonists such as Proclus sought to revive the discredited doctrines of the Old Academy, they effected a parallel elevation of the status of the augmented character of Pythagoras. In turn a self-styled Pythagoreanism, exhibited by Iamblicus, became the rage, a Pythagoreanism which Burkert characterizes as "Platonism with the Socratic and dialectic element amputated."

To say that there was no significant Pythagorean geometry before the time of Hippocrates of Chios is not, however, to deny the existence of any early Pythagorean mathematics. On the contrary, early Pythagoreans of the beginning of the fifth century, and perhaps even Pythagoras himself, entertained a conception of number representable by configurations of dots or pebbles. These pebble, or psephoi numbers and their interrelations were interpreted symbolically to derive a number of relatively quaint cosmological maxims, but at the same time encouraged the pronouncement of some non-trivial arithmetical conjectures. The conjectures were more algebraic than geometric in character, and were arrived at by a less than rigorous inductive procedure which ultimately fell short of proof. As Burkert and van der Waerden point out, the algebraic, inductive quality of early Pythagorean arithmetic more resembled Babylonian mathematics than it did the mathematics of the Elements, in which numbers are not represented as collections of discreet psephoi, but as continuous line segments of varying lengths. And although the psephoi numbers and the arithmetical mathematics they engender are preserved into the third century and beyond, they are preserved as a minor tradition and never attain the prominence of the tradition of geometrical mathematics which the Elements embody. One reason for the exclusion of Pythagorean arithmetic from the mainstream of Greek mathematics is that psephoi constructions were not diagrammatically strong enough to exhibit the truth of a given proposition in general, but were only able to provide specific examples which could no more than merely suggest that a statement holds in all of an infinite number of particular cases. But as early as Anaximander and almost a century before the activity of the first "Pythagoreans", Greek mathematics had already begun to form itself around the demand for the constructibility of global truths. This demand is fulfilled in connection with the Euclidean conception of number and at the expense of the Pythagorean psephoi arithmetic.

The absence of an early fifth century Pythagorean geometry and the markedly unscientific quality of the early Pythagorean arithmetic make it practically impossible that Pythagoras or his immediate disciples were responsible for the logical turn which Greek mathematics underwent between Hecataeus in the end of the sixth century and Hippocrates in the middle of the fifth. Burkert observes that Eleatic ontology of Parmenides and his successors, on the other hand, presents a much more likely source for the formal deductive logic which is essential to the success of the "elemental" geometries of Hippocrates and Euclid. Although Parmenides was a citizen by birth of the Italian town of Elea, his cultural heritage was predominantly Ionian. Elea was founded in the vicinity of 540 B.C., no more than fifteen years before Parmenides was born, by a group of Phocaeen immigrants who had fled their Ionian city after it had come under siege by an army of Medians under the command of the

general Harpagus. Nothing is known of Parmenides' parents, but it is most probable that they were among the Ionian refugees, and that Parmenides, therefore, was steeped from a young age in Ionian tradition. At some point in his education, according to Aristotle, Parmenides came under the instruction of Xenophanes, an exile of the Ionian town of Colophon whose travels included a stay at Athens and whose eclectic interests extended to poetry, philosophy and theology. As a result of his association with Xenophanes, if from no other place, Parmenides must have been exposed to the philosophies of early Ionians such as Anaximander and Thales, and consequently his philosophy reflects the Ionian emphasis on first principles. Parmenides' innovation, which turns out to be monumental, is to embrace an ontological first principle whose evidence is a direct consequence of its logical necessity. This is that being (to eon) is:

It is necessary to say and to conceive (noein) that being is, for there is being, but Nothing is not.

From this premise Parmenides deduces a number of necessary conclusions:

...that being is ungenerated and imperishable, entire, unique, unmoved, and perfect; it never was nor will be since now it is all together, one, indivisible.

The truth of these conclusions does not derive from empirical intuition, for indeed they contradict everyday experiences of change in the world, but instead rests upon the impossibility of their negation: That there be generation, destruction, divisibility, multiplicity, motion, or imperfection ultimately implies in one way or another that not-being is, which contradicts the necessary first principle, and hence leads to a logical impossibility. This is an explicit presentation of formal deduction in utter abstraction, and a new development in Greek philosophy.

Parmenides describes "great bonds" of "strong necessity" which hold being to itself and prevent the encroachment of negation in any of its various manifestations, and he develops this notion of bondage in close connection with the concept of limit (peiras). Being, as that which is at one and the same time perfect and limited, gives itself over to a diagrammatic representation:

Since now its limit is ultimate, [being] is in a state of perfection from every viewpoint, like the volume of a spherical ball, and equally poised in every direction from its center.

This construction of being is remarkably similar in structure to Anaximander's model of the cosmos, which has a spherical (or perhaps cylindrical) earth at its center equidistant from every extremity of a spherical universe. Parmenides' ontology, which is essentially an exercise of pure intuition, nevertheless manifests itself in a visible model; the formal logical procedure which is an innovation of Parmenides emerges in close connection with the traditional Greek conception of truth as construction -- a conception fundamental to early Ionian philosophy.

By the middle of the fifth century, Parmenidean ontology had spread as far as the Ionian island of Samos, and made a notable impact on philosophers in Athens after Parmenides visited that city somewhere in the vicinity of 450 B.C.. Parmenides' student Zeno, who traveled along on the trip to Athens, devoted himself to the task of refuting objections which were being voiced against the new Eleatic ontology. His refutations, as one would expect, were deductive after the fashion of his teacher. Specifically, he formalized a method of *reductio ad absurdum* which he employed on more than forty different occasions in defence of Parmenidean doctrine. Hippocrates of Chios, who happened to have been in Athens around the time of Parmenides' visit, relied heavily on the Eleatic procedures of proof by cases in his construction of the quadrature of the lune and presumably also in his *Stoicheia*, and Zeno's method of *reductio ad absurdum* is essential to all Greek proofs about irrationality. In the first half of the fifth century Greek mathematics incorporates the formal logical techniques of Eleatic ontology into its theoretical structure, and this structure, which was now both axiomatic and deductive, is essentially unchanged as it appears in the *Elements* of Euclid.

To summarize the preceding survey of pre-Euclidean mathematics, in the period of time between the middle of the sixth century and the beginning of the fifth, two main roots of the theoretical structure around which the *Elements* of Euclid is organized were located. The first of these roots is to be found in the cosmologies and geographies of early Ionians represented by Anaximander and Thales in the middle of the sixth century, and Hecataeus of Miletus in the early years of the fifth. Anaximander and Hecataeus constitute their presentations out of geometrical first principles which derive their truth

value and evidentiary force from being constructible in a model or representable in a diagram. These first principles anticipate in close detail the axiomatic framework of the mathematics in the *Elements*. The second root was located in the Eleatic ontology of Parmenides and his student Zeno. Eleatic ontology combines the Ionian concept of first principles with formal deductive techniques which provide geometers of the middle to late fifth century with methods for deducing the truth of propositions from a set of axioms. The combination of these two roots made possible in particular the production of the geometry in the *Stoicheia* of Hippocrates which was similar if not identical in structure to Euclid's book of the same name.

Primary to both the Ionic and Eleatic roots of Euclidean geometry is the conception of proof as visible construction, and the simultaneous demand that construction precede from worthy beginnings. It is my claim that this conception of proof is explicitly developed in the thematic content of epic poetry, and especially the *Odyssey*, and that the Homeric poems belong to a tradition of Greek thought which includes the *Elements* of Euclid. Presuming that these thematic elements of epic poetry can be identified, the continuity of this tradition is established by showing the influence the Homeric poems had on early Ionic and Eleatic thought. In the case of Eleatic ontology and its founder Parmenides, the importance of Homeric material is undeniable. Parmenides' poem, fragments of which are all that survive of his writings, is composed in the Homeric hexameter and draws its vocabulary almost entirely out of the Homeric lexicon. At certain points in Parmenides' poem, entire lines of hexameter are reproduced out of the Homeric corpus. And as Coxon observes, Parmenides' subtle use of epic imagery require that the reader of his poem be thoroughly familiar with the content of the *Iliad* and the *Odyssey*. In the case of Early Ionic thought the evidence is not so direct. The main reason for this is the scarcity of text which survives from sixth century and early sixth century Ionia: Thales is thought to have written nothing at all, nothing survives of the writings of Hecataeus, and the writings of Anaximander survive in only five short fragments. Nevertheless, the strong likelihood that the Homeric poems were influential to the thought of the early Ionians can be suggested. In the first place, the oral tradition out of which the Homeric poems grew seems to have been centered in Ionia. This is indicated by the preponderance of Ionic forms in the Homeric dialect and by the associations made by historians and biographers as early as the fifth century B.C. of Homer with various places in Ionia. The most traditional accounts describe Homer as having been born in Smyrna and then moving to Chios where he would have composed epic poetry in the middle of the eighth century. Although these accounts are based almost entirely on conjecture and often include such fantastic details as the story that Homer was born of the river Meles and the nymph Krytheis, there is some supporting evidence for associating Homer with Chios; this is that as early as the late sixth century there lived on Chios a group of bardic poets who called themselves *Homeridae* and claimed direct descentance from Homer. From archaeological evidence dating from the middle of the eighth century it is clear that the epic material out of which the Homeric poems were composed was at that time already a fundamental element of Ionian culture, and fragmentary literary references of the middle of the seventh century indicate that the epic material was frequently encountered in a distinctively Homeric context. Although there is no evidence of a direct correspondence between the *Iliad* or the *Odyssey* and the philosophies of Anaximander, Thales, or Hecataeus, it is almost certain that an intellectual of the sixth century in Ionia would have been familiar with the Homeric poems. Furthermore, the sixth century was a time when the categorical distinction between *logos* and *muthos* did not yet prevail, so that there was no active conceptual bias to prevent an early natural philosopher to look to epic for instruction. I turn now to a discussion of thematic material in the *Odyssey* which instruct the notion of proof which I have discussed in connection with early Ionian and Eleatic thought on the one hand, and the *Elements* of Euclid on the other.

### **Nostos: The Nautical Stage**

The *Odyssey* is the tale of Odysseus' *nostos* (return) from Troy to his neglected household on Ithaca. The *nostos* occurs in two stages. The first is the sea voyage from Troy to Ithaca in the course of which Odysseus and his companions become embroiled in a number of perilous encounters with powerful forces of nature and with uncivilized monsters. By acts of cunning trickery Odysseus removes himself and the more fortunate of his companions from danger and resumes the homeward journey. The second stage of the *nostos* consists of Odysseus' return to Ithaca. At Ithaca he finds his home overrun by a band of marauding suitors who seek to win the hand of Odysseus' wife Penelope. Odysseus' task is two-fold: First he must depose the suitors; second he must establish himself as the rightful master of his house, family, and property by demonstrating that he is the same Odysseus who had left Ithaca for Troy twenty years earlier. To depose the suitors Odysseus performs more cunning acts of trickery. As Odysseus demonstrates his identity, on the other hand, his cunning undergoes a

transformation. Trickery, or false persuasion becomes true persuasion, or demonstration. The scenes of demonstration occur in the last five books of the *Odyssey*, and in them the concept of proof which I have discussed in connection with Greek mathematics is developed. The demonstration scenes constitute the culmination of themes of cunning and return which run through the entire *Odyssey*. In the opening line of the tale Odysseus is described as *polutropos*, which means both "wandering" and "cunning". Twelve lines later in the first book this "wandering" is explained not as an aimless drifting, but a directed effort, Odysseus' longing for return and reunion with his wife. I find it convenient to take up these themes as they present themselves in book nine, where Odysseus tells the Phaeacian court of his trials on the sea. This is a central scene in the story, for after the tale is told the Phaeacians are to transport Odysseus to Ithaca, and the nautical stage of the *nostos* then concludes. In a very real sense Odysseus wins his way home with a tale, and he is the subject of his tale. The self-referential quality of Odysseus' song, which is at bottom an act of cunning designed to please the audience and win favor for the performer, is reflected in the events described in the song. The events are the conflicts at sea resolved by cunning trickery, and just as Odysseus recognizes himself in the song, he recognizes himself in each of these acts of cunning.

Early in the song, Odysseus comes to describe his encounter with the man-eating cyclops Polyphemus. Odysseus and a number of his companions are held captive in the cave of the cyclops. Odysseus effects his escape from the cave and country of the cyclops for himself and those of his companions whom Polyphemus has not already devoured by way of a cunning act of dissimulation: Odysseus tells Polyphemus that his name is Outis (Nobody) so that Polyphemus, when Odysseus blinds him with a sharpened olive branch, suffers the scorn of the neighboring cyclopes when he calls out for their help against the attacks of "Nobody." As he sails away from the island of the cyclops, Odysseus calls out to the blinded Polyphemus:

Cyclops, if any mortal man should ask about the shameful blinding of your eye, then tell them that the man who gauged you was Odysseus, ravager of cities: one who lives in Ithaca--Laertes son.

In this instance, the cunning act refers to Odysseus in that it consists of a pun on his name. The pun is of a special kind: it casts Odysseus over into negation (Outis). Recognition soon follows as Odysseus proclaims his given name to Polyphemus and to himself as he sails to safety along the continuing path of return. The turns to negation and over again into recognition are dialectical moves which accompany each act of cunning in Odysseus including those acts which belong to the recognition scenes at the end of the *Odyssey*.

Odysseus describes himself as cunning in connection with the episode with the cyclops. While Polyphemus is running about his fields bemoaning his recent injuries, Odysseus and his companions make their escape from the cave bound beneath the bellies of the sheep of their debilitated captor. Last to lumber away from the cave is a great ram burdened by the weight of Odysseus. Odysseus tells the tale at the court of Alkinous:

Last of all a ram made its way through the door encumbered by its wool and me with my cunning plots.

What is important here is the adjective *pukinos*, the basic meaning of which is "thick-packed", or "dense". In the Homeric poems it is commonly applied to dense things like stones or logs, and then to artificial structures which are sturdy as a result of being tightly fitted. Drawn by metaphor toward the intellect, *pukinos* comes to describe a cunning quality of a mind that looks to deceive. It is this sense that *pukinos* conveys in the excerpt above, and that occurs again in book II of the *Iliad* as Agamemnon enters the Assemble of homesick Achaians.

Assembling them he advanced his thick plan.

The plan is to implore the host of Achaians to quit the campaign against Troy, only that they be shamed by their captains into entering into battle with renewed vigor. The irony of the plan is that it only barely comes off, and that the battle which Agamemnon designs leads to his own undoing. Agamemnon considers his plot cunning, when it is actually only dense.

Agamemnon's misfortune, it turns out, is the design of Zeus, who advises the battle to Agamemnon by way of a dream. The dream which delivers the deceptive council of Zeus is *oulos* which often

means "close in texture", and perhaps in this instance carries the same sense as pukinos. In any case, cunning mindfulness frequently attends the father of gods, as can be seen in book XV of the Iliad as Zeus guards the life of Hektor when Hektor rages against the palisade before the ships of the Achaeans. Here the Achaian archer Teukros strings his deadly arrow and takes aim at Hektor.

One more arrow Teukros drew for Hektor helmed in bronze, and would have stopped the battle for the ships if that shot had dispatched him in his triumph. But he did not escape the close-packed mind of Zeus who guarded Hektor...

Zeus cunningly exercises his "close-packed mind" to postpone the fall of Hektor and advance his larger plot which is to bring disgrace to a defeated Agamemnon and honor to Akhilleus. It is a cruel deception for Hektor, for what appears to him to be imminent glory contains and magnifies his coming defeat.

The crucial association in the above passage is that of pukinos and noos (intelligence), which again determines a frame of mind disposed to cunning deceit. In the Iliad it already plays an important role in that it is by virtue of the cunning mind of Zeus that the war at Troy arrives at its proper end. But the internal cause of the events of the Trojan war is the hyperbolic rage of Akhilleus, far more passionate than intellectual. In the development of the Odyssey, on the other hand, the pukinos noos is no Deus ex Machina, but inheres in the hero Odysseus as a force which drives his adventures from beginning to end.

The events which constitute the nautical stage of Odysseus' nostos and which provide a stage upon which Odysseus can perform his cunning are frequently occasioned by another peculiar quality which prevails in the mind of Odysseus: his powerful curiosity. In Laestrygonia, Odysseus' entire fleet save his own ship and crew is destroyed at the hands of man-eating giants, all because of because of Odysseus' ill-concieved desire "to know what men -- bread-eaters -- there were in that land." And earlier in the tale, soon after Odysseus and his escort enter the cave of Polyphemos who is out tending his flocks, the companions urge their commander to make off quickly with a few cheeses and some livestock. But Odysseus, to the eventual distress of the four men whom Polyphemos devours, pays them no heed:

But I was not persuaded, although that would have been of more profit, for it was my intention to see the man and know if he would welcome me as a guest.

Odysseus' heedless curiosity stands as antithesis to his premeditative cunning in the dialectical unfolding of his noos. As I just now have suggested, Odysseus' curiosity induces his nostos by occasioning its constituent episodes. It is Odysseus' cunning, on the other hand, that produces his nostos by resolving the conflicts which arise out of these episodes, and by restoring Odysseus and the more fortunate of his companions to the homeward path. The cunning mind (pukinos noos) displays itself to itself by negating itself in the positive pursuit of blind curiosity which it must in turn overcome by a strong act of cunning. In the affair with Polyphemos this dialectical progression develops at once into death and into the preservation of life, into Outis and Odysseus.

The encounter with the Sirens and their spellbinding song in book XII provides Odysseus with another opportunity for the realization of noos. The goddess Circe warns Odysseus that the price which one must pay for the pleasure of hearing the song of the Sirens is death and the forfeiture of nostos. But she also informs him that he alone among men may hear the song without penalty. To this end she gives him further instruction which he follows as his ship begins to pass by the island of the Sirens. He commands his companions to bind him securely to the mast of the ship and plug their ears with softened wax so that when the Sirens begin to sing he may listen, but they, in their oblivion to the song, transport him to safety. The cunning mind of Odysseus willfully forces its negation by exposing itself to the charm of the song of the Sirens. Once again the negation of cunning effects in Odysseus an overpowering curiosity, and it is to this curiosity that the Sirens address their song, which ends with the following promise:

And when he leaves, the listener has received delight and knowledge of so many things. We know the Argives' and Trojans' griefs: their tribulations on the plain of Troy, because the gods have willed it so. We know all things that come to pass on fruitful earth.

For the curious Odysseus, the prospect of attaining this knowledge is overwhelming. Casting aside all

care for life and return, he implores his companions to free the ties which bind him to the mast of his ship. But his companions, who have been forewarned against the pleas of their commander, only strengthen his bonds. Odysseus' cunning preserves itself in the face of its negation by transforming itself into ties which bind him to nostos and the internal production of noos. Up and against these ties stands the song of the Sirens, which aims to distance Odysseus' noos from itself by attaching to it knowledge of the manifold of "things that come to pass on fruitful earth." Odysseus' cunning, his pukinos noos, realizes itself negatively by repressing this knowledge of particulars. What Horkheimer and Adorno identify as a "linguistic appropriation to death" performed by Odysseus upon his name in the episode with Polyphemus is extended in this instance for Odysseus into an epistemic appropriation of the death of the particular in the world around him. When the mind, as it progresses dialectically toward its own realization, turns away from the things in the world, it turns toward itself in the form of recognition.

The reflective mind which develops as a result of the dialectic of cunning in Odysseus is a necessary psychological precondition for the type of abstract thinking that Parmenides performs in his ontological philosophy. Indeed, Parmenides, most likely intentionally, incorporates imagery and vocabulary of the nautical stage of Odysseus' nostos into his philosophical poem. Parmenides' poem recounts a philosophical journey away from the realm of darkness and the plethora of things which are said to occur in time, and into the realm of light, contemplation and inquiry into the notion of being. The traveler in the poem, following a path "that leads the knowing man through all towns," is readily compared to the wandering Odysseus, who "has seen the towns of many men and learned their mind (noon)." The verb Parmenides uses for "contemplation" is noein, a verbal form of the noun noos, which he almost always uses intransitively and in its nominal infinitive form to indicate the pure exercise of the mind. As the traveler proceeds along the road to being, his thinking becomes increasingly more reflective, since, for Parmenides, thinking and being are identical:

...for being and thinking (noein) are the same.

Both Odysseus and the traveler in Parmenides' poem follow a path that leads to the internal realization of noos.

Parmenides' traveler is set upon that path by a goddess who dwells in the realm of light and is served by maidens of the Sun. She addresses him as he passes through the gates which guard the passage from night to day:

Come now, I will tell what paths (hodoi) there are for thinking, and after you hear it, guard the story well.

Circe, daughter of Helios, delivers a similar speech to Odysseus and his weary crew who have sailed out of the impenetrable mists of Hades and arrived at the island of Aeaea, the home and county of new-born Dawn:

But come now, eat food and drink wine and restore yourselves to day. But at the glint of dawn you will set sail. Meanwhile I shall show your path (hodon), and tell its every sign.

The journey that Circe proceeds to describe to Odysseus is his nostos (return home) past the Sirens and the horrific Scylla and Charybdis to the Island of Thrinacia where the Cows of the Sun pasture, and finally to the shores of Ithaca. When the goddess greets Parmenides' traveler, she too refers to the path he travels as a return:

Welcome, O youth, arriving at our dwelling as consort of immortal charioteers and mares which carry you; no ill fate sent you forth to return (neesthai) on this path (hodon), which is far removed indeed from the step of men...

The verb for return, neesthai, is related to the noun nostos by the hypothesized indo-european root \*nes, "return to light and life." The return path, which marks out a solitary journey "far removed indeed from the step of men," resembles the nostos of Odysseus, which he alone achieves after the last of his companions are deprived of return after foolishly slaughtering and feasting upon the cows of the Sun.

The journey of return to noos, for Parmenides' traveler, is at the same time an epistemic movement

away from "however many things mortals suppose to be real and to come to pass (gignesthai) and fade away." The arrival at a conception of such things is the final stage of the false path of inquiry which takes its start by consigning being to that which is not. The Goddess admonishes Parmenides' traveler to shun this path of inquiry:

But you bar your thought from this path (hodou) of inquiry, and let not habit force you to exercise an unseeing eye and a noisy ear and tongue on down this much tried path (hodon).

The images employed in this imperative recall Odysseus before the island of the Sirens. By having himself bound to the mast, Odysseus bars himself from knowledge of "the things that come to pass (gignetai) on fruitful earth." His ears are filled with the beguiling song, and, desperately longing to hear, he repurposes his eyes away from the act of vision so that they may signal his desire to his deaf companions:

I wanted to listen, and urged my companions to free my ties; I signaled this with my eyes.

Confused hearing and especially the absence of the faculty of vision signal in Odysseus, as they do in Parmenides' traveler, the departure of noos. In the event of this departure, no-mind prevails in Odysseus as a positive curiosity that propels him onto death. For Odysseus, as I have said, this move is crucial in the dialectal process by which his noos achieves self-recognition. This is always the affirmation of noos in its positive and negative transformations -- a situation which recurs in the double entendre of Odysseus' name: the name which names Odysseus always at the same time names his negation, Outis.

For Parmenides, on the other hand, the statement of identity "being is" is set out at the outset as a postulate. As such is not subject to dialectical scrutiny. Its negation, that non-being is, is strictly forbidden as precisely that which is not to be named:

Now it has been decided, as was necessary, to leave the one way (that not-being is) unconceived and nameless, since it was not a real way, and for the other (that being is) to be a way and authentic.

The positive logic of Parmenides' ontology is based on the mind presumed to be in firm possession of itself. But just beneath the surface of this presumption lies the dialectical process by which the mind becomes conscious of itself in the first place. This is the process described in the nautical stage of Odysseus' nostos. When Parmenidean logic came to be applied to the Ionic conception of proof as a grounded geometrical construction, the theoretical structure of Greek mathematics was complete. In the Odyssey, this conception of proof is developed in connection with the events which surround Odysseus' return to his home on Ithaca.

### **Odysseus at Ithaca: the Final Stage of Nostos**

Odysseus' arrival at Ithaca preserves the dialectical quality of nostos developed in the nautical stage of the return from Troy. In the opening lines of book XIII the conditions are described which attend the hero after the Phaeacians deliver him asleep to the shore of the island of his home:

But bright Odysseus awoke from his sleep on the land of his father, and no one recognized the one who had long been away. For the goddess Athena had poured a mist all round, that she might make him unrecognized.

Shortly thereafter the Athena lifts the veil of mist and Ithaca is laid out in all its splendor before the eyes of Odysseus. Sleep and mist are images which often attend death in epic, and Odysseus' homecoming is, therefore, described as a symbolic return to life. For Odysseus, nostos always proceeds out of his dialectical alternation between stages of life and death, and as the excerpt indicates, this dialectic is bound up with the act of recognition.

Here the alternation from death to life is closely related to the move from concealment to revelation. This is seen earlier in book X in Odysseus' voyage out of the mists of Hades and into the light of Aeaea, and earlier still in book IX in his passage out from the cave of Polyphemus. This last incident is paralleled by his alternation from Outis to Odysseus, which is the basic formula for recognition. In turn, what necessarily precedes the recognition scenes which come in the last book of the Odyssey is

Odysseus' return to a state of concealment. At the end of the thirteenth book, Athena, the wily Olympian double of Odysseus, casts a spell over her hero:

With this Athena touched him with her wand. She shriveled the smooth skin on his lithe limbs; she took away the light hair from his head; she sheathed him round with an old man's flesh; she dimmed his eyes, the brightness of his glance.

What Athena imparts to Odysseus is no mere disguise, but a dialectical transformation: Odysseus moves from hero to anti-hero, from commander to buffoon. Before regaining control of his home and family, Odysseus so transformed will suffer the ridicule of servants and will be forced to wrestle with beggars for the entertainment of the suitors who will reward him by hurling scraps of meat at his head. As in the episode with Polyphemos, Odysseus takes on his own negation to win his return. The other road, which moves directly to life and return without first symbolically appropriating death, is the one taken by the unfortunate Agamemnon, who dies an irrevocable death after learning too late of the murderous intent of his wife Klytemnestra and cousin Aigisthos. Similarly there is the example of the ill-fated companions of Odysseus, who are penalized with death after foolishly seeking life by slaughtering and feasting upon the cows of the sun; Odysseus, on the other hand, abstains from food and wins return.

The recognition scenes, which occur within the walls of his house on Ithaca when Odysseus emerges for the last time from concealment, mark the fulfillment of his nostos. As in the nautical stage of Odysseus' return, nostos at Ithaca develops as a function of Odysseus' cunning. In the nautical stage cunning shows up as trickery, as the wily ability to present false demonstration, to dissimulate and evade. At Ithaca cunning in Odysseus preserves this original meaning, but at the same time undergoes a transformation: it moves over into its double and becomes the ability to induce true persuasion, to produce incontrovertible demonstration. Cunning as trickery in Odysseus is always dedicated to the end of concealing the identity of Odysseus. Conversely, cunning as a demonstrative faculty always demonstrates that the hero is Odysseus, the one who left for Troy twenty years before. Odysseus, by proving that he is himself, gains the recognition of his servants and family and returns to his proper place as the master of his home on Ithaca. As I shall discuss, the demonstrations which induce this recognition are of a particular kind, and their structure anticipates that of the notion of proof I described earlier in connection with Greek mathematics.

The transformation of cunning in Odysseus from dissimulation to demonstration occurs in book XIX. Odysseus, disguised as a beggar, has entered his home and now shares words with troubled wife, Penelope. She, unaware that she is speaking with her husband, tells him of the troubles she has endured in the absence of Odysseus, and asks him to tell of his land and the family. Odysseus tells a false tale: he gives his name as Aethon and explains that he came from a noble family on Crete, but has since endured many misfortunes. He proceeds to tell that eleven years past he met the hero Odysseus who had been blown off course on his way to Troy. Penelope wishes to test the truth of the story and asks her guest to describe the clothes that Odysseus wore. The disguised Odysseus gives the following account:

Bright Odysseus wore a shimmering thick wool cloak--he wore it doubled over. The brooch on it had been constructed of gold and with double clasps; its surface was cleverly wrought. A hound held pinned in its forepaws a struggling speckled stag. All were amazed at this, that though in gold, the hound choked the stag and stared, and the stag struggled vehemently to flee with his feet.

Just before giving this description, Odysseus is described as "telling many lies that are like to truths." Odysseus himself signifies the duplicitous character of his speech when earlier he describes the Cretan land from which he comes as being a place where "the tongues of many are mixed." This is reinforced by the image of the cloak which is "worn doubled over" (*diplen*) and by the image of the brooch with double (*didumos*) clasps. The account of the cloak cloaks the truth; and like the brooch, the cleverly construed tale makes artifice appear like reality. Odysseus encodes himself in the image of the dog and the stag. As the stag struggles for release, Odysseus yearns to emerge out from his disguise and announce his return. At the same time he is the hound holding a steady eye and a firm check on hasty flight. This dual representation recalls the image of Odysseus bound voluntarily to the mast of his ship before the island of the sirens, and its two elements correspond to the antithetical stages of curiosity and cunning in the mind of Odysseus.

The description is put forth by Odysseus as a demonstration, as proof that Odysseus was sighted in Crete on his way to Troy. The demonstration is false, but like the scene portrayed on the brooch which, though only an image, is on the very verge of bursting into truth, the false demonstration is an image of the true demonstrations of his identity which Odysseus is soon to present. The false tale mimics true demonstration in structure. Odysseus will eventually prove his identity by indicating signs. These signs, like the brooch, will have been constructed (*tetukto*), and the constructed signs will have a special evidentiary force. In the case of the cloak and the brooch, these signs have evidentiary force, they consist of proof for Penelope that Odysseus had been seen some time ago on Crete, because she herself had given the cloak and brooch to Odysseus before he left for Troy. Odysseus chooses these signs for his description of the fictitious Odysseus at Troy for this very reason, that in them Penelope may recognize her husband. The episode of recognition follows immediately upon the description:

Thus he spoke, and yet more did desire for wailing rush upon her, when she recognized the sure signs that Odysseus had spoken.

The passage reinforces the basic structure and vocabulary of proof and recognition in the *Odyssey*. Proof consists of a telling of signs (*semata*) which have some evidentiary force. The evidentiary character of the signs is implied, in this context, by the adjective *empeda* which means "stable" or "secure". In the later demonstration scenes this adjective will be replaced by stronger one, but for now it is enough to say that the special quality of the demonstrative sign is emphasized in Homeric language. Finally, the completion of the act of demonstration is marked by a recognition, expressed in some form of the verb *gignoskein*, and most often, as in this case, by its prefixed form *anagnoskein*. This verb, which is functionally related to the verb *noein* and which is conceptually bound up with the overall theme of *nostos*, appears repeatedly in connection with the demonstrations in the last books of the *Odyssey*. In these contexts it signifies the powerful recognition which is induced by a formal demonstration involving the showing of signs. That the recognition is particularly profound is emphasized by the pathos which always accompanies the experience of *anagnoskein*. In the passage above this pathos manifests in Penelope an increased desire for wailing.

From Odysseus' description of the cloak and brooch, then, it is already possible to detect the basic outlines of the structure of the demonstrations that soon follow. But at the same time the description is essentially untrue; it is a lie dressed up in the garb of proof, a telling of false signs. I turn now to a number of places in the text where Odysseus produces true demonstration, where he reveals his identity by way of proofs involving the showing of true signs. In these examples it will be possible to determine precisely what constitutes true, demonstrative signs in contrast the signs of the cloak and the brooch, and then to show the relation between the demonstrations built around such signs in the *Odyssey*, and the notion of proof at work in Greek mathematics.

The first true sign which Odysseus produces as proof of his identity is a scar on his leg, and he cleverly alludes to this sign in the description of the cloak. He describes the cloak as being *oulen*, which means thick or dense. The adjective may contain some echo of the psychological meaning of *pukinos* (thick and dense, and then cunning), but more importantly, in the feminine accusative form it takes in describing the cloak, it is a pun on the noun *oule*, which names Odysseus' scar. Odysseus' use of pun to conceal truth in deceptive speech is paralleled earlier by his alternation between the names *Outis* and *Odysseus*. The one who can make lies like truths can also encode the truth in a lie.

In book XXI the conflict between Odysseus and the suitors draws near. Odysseus seeks the aid of his cowherd and his shepherd, and to gain their confidence he reveals his identity to them with a demonstration built around the scar:

Come now! I will show (*deikso*) this unmistakable (*ariphrades*) sign so that you will recognize (*gnoton*) me well and be persuaded in your hearts; it is a scar (*oulen*), from that time when the boar struck me with his white tusk when I was in the region of Parnassus along with the sons of Autolycus.

The sign that Odysseus produces in this true demonstration is different in character than the signs of the cloak and the brooch. First of all, its special demonstrative quality is indicated by a new adjective: the sign of the scar is not *empedos* (sure) like the cloak and the brooch, but *ariphrades*, or "easily discerned." The adjective is formulaic as a modifier of the noun *sema*, and is used in every instance

where Odysseus produces a sign as proof of his identity. The special quality of the ariphrades sema is emphasized by the role it plays in other epic cycles. In book XXIII of the *Iliad*, Nestor tells an ariphrades sema to his son Antilochus who prepares to compete in a horserace run in honor of the recently fallen Patroclus. The sign he tells is the turning point in the racecourse which Antilochus must negotiate carefully to return safely to the and victorious to the mark which doubles as starting point and finishing line. The showing of the ariphrades sema, the running of the race, and the funeral rites of Patroclus immediately precede Agamemnon's return to the battle at Troy, and thus mark the completion of a major thematic cycle around which the plot of the *Iliad* is built. In the cycle suggested by the prophesy of the oar which Teiresias tells to Odysseus in Hades in book XI of the *Odyssey* the ariphrades sema appears again. Odysseus is told that, after returning to Ithaca, he must travel inland by foot carrying an oar of a ship on his back, and spread the fame of Poseidon among men who do not know the sea and use no salt with their food. The completion of his inland journey will be signaled by an ariphrades sema: Odysseus will encounter a man on the road who will mistake the oar for a winnowing fan. As in the *Iliad*, in the unwritten epic cycle of the oar, the showing of an ariphrades sema marks completion and return. Likewise in the *Odyssey* the showing of the ariphrades sema marks the end of a cycle and return. Soon after showing the scar to his henchmen Odysseus slaughters the suitors and resumes dominion over his home. His wanderings are over and what remains is to restore himself to his wife and father, which he does by producing other ariphradea semata.

In the *Odyssey*, the force of the ariphrades sema is evidentiary. By producing it in the scene quoted above, Odysseus designs to induce in the cowherd and the shepherd recognition and persuasion. The evidentiary force of the ariphrades sema derives first from its having been constructed, from its being the product of a unique sequence of recollectible events. The story of the scar is given in book XIX of the *Odyssey*: When Odysseus was an infant he was named by his maternal grandfather Autolycus, who at that time promised that he would honor Odysseus with gifts when he reached the prime of his youth. Odysseus, grown, travels to the home of Autolycus to collect these gifts. There he partakes in a hunting expedition in the region of Mt. Parnassus. In the course of the hunt Odysseus receives an injury on his thigh from a charging boar which he kills with a display of courage and might. Considering the dialectical character of Odysseus' identity, the story is important in so far as the story behind the attribution of the name Odysseus is tied up with the account of the young hero's suffering a nearly mortal wound. Symbolically, the scar from the wound is equivalent to the name, and its construction is essentially the construction of Odysseus' identity. By showing the sign, then, Odysseus recalls the unique sequence of events the recognition of which is at the same time the recognition of the identity of Odysseus. This is readily contrasted with the false signs of the cloak and the brooch, which when told are of ambiguous origin. Odysseus, disguised, concludes the telling of these signs with the following disclaimer:

I know not whether he wore these things about his skin at home, nor whether one of his companions gave them him as he travelled on his swift ship, nor still if it were some stranger of another place, since Odysseus had many friends; for few among the Achaeans were his equal.

Just as anyone may have given the cloak and brooch to Odysseus, anyone may have worn it. Because their construction is ambiguous the signs have no essential connection to the identity of Odysseus; the signs fail as proof, and are therefore effectively deceptive.

The evidentiary force of the scar as a sign derives, in a similar way, from the essential quality of the surface to which it is attached. In the course of his adventures Odysseus undergoes alteration in height, the loss of hair, diminution and augmentation of youth, the loss and restoration of agility, and changes of pigment and wrinkling of the skin. He suffers each of these variations as he passes in and out of disguise. What remains unaltered in the course of these transformations is the region of Odysseus' body which displays the scar of his youth, the mark by which Odysseus can be identified in any of his various incarnations. The location of the scar is precisely determined in the story of the hunting trip on Parnassus:

The boar attacked first and struck [Odysseus] above the knee (gonu); lurching to the side it flayed much flesh with its tusk, but did not reach his bone (osteon).

The knees and the femur were considered by Homeric Greeks to contain vital fluids and to be the seat of the soul or psyche. Hence the reference to the tusk which "did not quite reach the bone" means that

the wound was almost, but not quite, mortal. In the Iliad, the sanctity of the knees is indicated when Hector, impaled through the neck by the spear of Akhilleus, says: "I beg you by your soul (psyche) and your knees (gounon) and your parents (tokeon), do not let the dogs devour me by the ships of the Achaians." The plea appeals to those things with which Akhilleus would most readily identify himself. The psyche is the hero's personal allotment of life force, and after death it is the disembodied psyche with which the hero is identified in the underworld. Likewise, parentage is an essential component of a person's identity in epic: When Odysseus sails away from the island of the cyclopes, he declares himself to the blinded Polyphemus as "Odysseus, sacker of cities, son of Laertes." Also, it is formulaic in epic for a stranger to be asked to identify himself by his parents. Hektor's appeal to the knees of Akhilleus understands a close association between the knees and the psyche and parentage. The knees were not only believed to contain the psyche, but along with the thigh bone were thought to have reproductive powers. This is reflected in the Greek language by a group of cognates which includes gonu (knee), and genos, geinomai, gignomai, goneus, gonos, gone, and gonar. It is further exhibited by such stories as the birth of Dionysos out of the thigh of Zeus. Hence the story of Odysseus' birth and naming contains an account of his having received an injury above the knee but not quite to the bone. The zone of the body marked out by the knee and the femur is an essential location, intimately bound up with the hero's identity. Unlike other parts of his body, Odysseus's thigh, to which is affixed the scar of his youth, is never disguised. The scar derives much of its evidentiary force from its attachment to this essential surface.

The demonstrative sign of the scar connected to the thigh is readily compared with the false sign of the brooch fixed to the cloak. The cloak, unlike the thigh of Odysseus, is an inessential surface. A gift from an unknown donor, it comes into Odysseus' possession only by accident. The changeable nature of the cloak is further suggested by the adjective porphureos, which is most often used to describe the glancing quality of light reflected off of the disturbed fluid surfaces of rivers, oceans and clouds. The cloak as a surface is vague and nomadic, and the sign affixed to it has no true demonstrative powers; the Odysseus whom the brooch signifies is a fabrication.

In the contrast between the signs of the scar and the brooch a Homeric theory of demonstration begins to emerge. True demonstration consists of the showing of signs constructed through a sequence of unique and recollectible events, and at the same time fixed to some permanent surface. In the scene with the cowherd and the shepherd there is a lexical correspondence between this theory of demonstration and the one which shows up in later in the practice of Greek mathematics. The verb which is used for Odysseus' showing his scar is deiknumi:

Come now! I will show (verb: deiknumi) this unmistakable sign...

The showing of the scar is at the same time the formal proof of Odysseus' identity, and the verb deiknumi communicates both the showing and the proving. This is the same verb which, in its variety of prefixed forms, denotes formal proof in Greek mathematics at least as early as the late fifth century when Demokritos is said to have boasted of his skill at putting together lines with proof (epideiksios). Deiknumi is, of course a common Greek verb, and I do not suggest that the correspondence is in any way conclusive; I only mention it now to anticipate the thorough comparison of Homeric demonstration and the theory of demonstration in Greek mathematics which is soon to come. In the meantime I want to return to the scene with the cowherd and the shepherd to work out in more detail the Homeric theory of demonstration. The following excerpt described Odysseus showing his scar to his henchmen:

As he spoke, he pulled off his ragged cloth and exposed the great scar; the two inspected and pondered its every detail, and then wailed as they threw their arms about crafty Odysseus.

As was seen earlier with Penelope, the demonstration induces a pathos in those it persuades: The two henchmen wail as they recognize their returned master. The excerpt indicates another characteristic of the demonstrative sign: This is that it can at any time be produced for visual inspection. This is once again contrasted by the false signs of the brooch and the cloak. Before Odysseus, disguised, describes the brooch to Penelope, he remarks on the amount of time has past since he saw it with Odysseus:

O woman! It is ill-advised to speak after so much time has passed; for now is the twentieth year since he went to that place and took leave of my fatherland.

The signs of the brooch and the cloak can not be produced for visual inspection, but only remembered over a span of twenty years. As Odysseus remarks, the recollection of signs which can not be experienced visually is ill-fated, and the implication is that the persuasive force of such signs is deceit. The immediate visual quality of the sign of the scar, on the other hand, contributes to its evidentiary force.

The demonstrative sign is now determined by three criteria: First it must have been constructed through a sequence of recollectible events; second, it must be affixed to a permanent, essential surface; and finally, it must appeal immediately to the faculty of vision. The first such sign which Odysseus produces is the scar on his leg. Later on in the course of events which unfold within the walls of Odysseus' house on Ithaca Odysseus finally reveals himself to Penelope, and he does so by producing another sign of his identity. The sign is the bed which he built for himself and his wife when they were first married, and this sign, like the scar on his thigh, is demonstrative according to the criteria I have outlined. The sign of the bed is alluded to by Penelope as she sits at the hearth with Odysseus whom she does not recognize on account his beggar's disguise. Odysseus has recently slaughtered the suitors and has come to reclaim his wife, but she refuses to acknowledge him. Telemachus rebukes her for her insolence, and she delivers the following response:

If truly it is Odysseus and he is come home, then surely we shall recognize (verb: gignosko) one another far better. For there are signs (semata) which only we have seen (verb: \*eido)-- they've been hidden away from others.

In this speech, Penelope describes the precise form which a demonstration of identity must take. She requires that Odysseus produce special signs which the two of them together have looked on in the past. The signs she has in mind all refer to the wedding bed. By suggesting that the bed be placed outside the bedchambers where Odysseus might sleep, she tests her husband's knowledge of the signs. Then Odysseus, perhaps a little slow to catch on, reveals the signs:

Who put my bedstead in another place? It would certainly be a difficult thing, even for one with craft, when the god himself, should he happen along, would not easily displace it to his liking. For no one alive among men, not even one young and strong, could easily change its place, since a great sign (sema) has been built into the unmoving bedstead. And I made it, and no one else.

The crucial features of Odysseus' speech are first, that sign of the bed is constructed (tetuktai), and second, that the location of the bed is fixed. The secret sign which has been built into the bed is the fact that it is built around an olive tree which had sprung up within the walls of the Odysseus' home. The olive tree is still rooted to the ground, and thus the bed cannot be moved. As he did earlier with the sign of the scar, Odysseus recounts the construction of the bed. There are a number of details of this construction which are worth mentioning. First is the recurrence of the adjective pukinos. Odysseus tells of his building the walls and ceiling of the bedchamber around the olive tree, and that the walls and ceiling were made of pukinesi lithadessi (stout stones). And the doors of the bedchamber he describes as pukinos araruias (tightly fitted). The making of a tightly fitted structure is analogous to the production of an incontrovertible demonstration, and Odysseus signifies the truth value of his speech with the image of the well-built bedchamber. The metaphor is reinforced by the dual meaning of pukinos. The "dense" quality of the building translates into the "clever" construction of proof. The adjective which signifies Odysseus' deceitful cunning in the nautical stage of his return, also describes his cleverness in producing true demonstration at Ithaca. Second, there is a certain geometrical bent to the construction of the bedchamber which was not present in the description of the sign of the scar: Odysseus recounts how he skillfully prepared the olive tree, lopping off its top, stripping it of its branches, and smoothing its trunk with an adze until it was "straight to the line" (epi stathmon ithuna). The trunk of the Olive tree then becomes the structural support for the bed, and the integrity of the bed, the demonstrative sign, is a function of the regular linearity of the crafted trunk of the tree. The sign of the bed, as constructed, fixed, immediately visible, and at least in part geometrical, is demonstrative. It induces recognition in Penelope, and the following excerpt describes her reaction:

As he spoke her knees and dear heart went weak; The sure signs Odysseus had told her she recognized.

Once again there is the showing of signs, recognition, and pathos. And the bed still stands in the

middle of the bedchamber as evidence of the identity of Odysseus. A few lines down in the poem Penelope concedes to having been persuaded on account of the *ariphradea semata* which Odysseus told. And as I discussed earlier, the *ariphrades sema* is the formulaic tag of the demonstrative sign.

The *ariphrades sema* shows up one more time in a very similar recognition scene between Odysseus and his father Laertes. Like Penelope, Laertes is incredulous when Odysseus announces to him his return. Also like Penelope, Laertes demands proof:

If indeed you are my child Odysseus returned to this place, tell me a unmistakable  
(*ariphrades*) sign, so that I may be persuaded.

As ought to be expected by now, the *ariphrades sema* is once again a fixed construction present to be viewed upon. Odysseus tells of the orchard his father planted for him when he was a boy. Odysseus describes the planting in detail and names the trees one by one. The orchard, like the bed, is rooted to the ground. Furthermore, the design orchard has both numerical and geometrical properties: the orchard consists of thirteen pear trees, ten apple trees, forty fig trees, and fifty rows (orchous) of grape vines. The precise number of each type of tree, and the arrangement of the vines in rows make the orchard unique and recognizable as a sign of Odysseus identity. Laertes recognizes the sign, and this is his reaction:

As [Odysseus] spoke, his knees and beloved heart went weak; the sure signs Odysseus had told him he recognized.

The passage is formulaic and contains almost exactly the same language as the description of Penelope's recognition of Odysseus. Of particular interest is the phrase GREEK ([his] knees and beloved heart went weak). The scenes with Penelope and Laertes are the only ones in the entire epic corpus in which this formula is used to connote recognition. In the seven other places in the *Iliad* and the *Odyssey* where the formula appears it is used to describe the experience one has in the face of death. A clue as to how it came to be applied to the experience of recognition can be found in Book V of the *Odyssey* when Odysseus is shipwrecked and the wrath of Poseidon is upon him:

Euros and Notos and blustering Zephyr, and aether-born Boreas came on together, and a great wave rolled up. Then the knees and dear heart of Odysseus went weak. He spoke in distress to his strong breast...

Odysseus tells himself:

Now it is certain that steep death is mine.

The adverb *sos* (certain) is crucial. What Odysseus experiences in the midst of the storm is the recognition of the certainty of his oncoming death. It is this recognition which induces his pathetic condition described by the formulaic phrase *lueto gounata kai philon etor* When the poet used this formula in the final recognition scenes in the *Odyssey*, he abstracted it out of its original context of oncoming certain death expressed in the excerpt above. The formula was attractive on two counts: First, it describes the powerful experience of certain recognition, a description which is useful in connection with the theme of demonstration which was, perhaps, a late development in epic diction. Second, it casts the shadow of death upon the demonstrative project, a poetic move which is consistent with the dialectical quality of the theme of *nostos* in the *Odyssey*. When Odysseus conceals himself, he symbolically moves over into death. When he reveals himself, he brings death to others. This second death is expressed symbolically in the disarticulation of the knees and hearts of Penelope and Laertes when they experience the certain recognition of the returned Odysseus.

The similarities between the Homeric theme of demonstration and the theoretical structure of proof which underlies the practice of Greek mathematics should, by now, be quite transparent. In both cases, proof consists, on the one hand, of the exhibition of a sign so constructed as to appeal immediately to the faculty of vision. The evidentiary force of such a sign in Greek mathematics was seen first in the *Elements* of Euclid in the procedure of superposition by which the side-angle-side criterion for triangle equality is established. This procedure of superposition was then identified with the early geometrical investigations of Thales of Miletus. The importance of the constructed sign appeared again in connection with early Greek cosmology and cartography. In sixth century Ionia the desire to produce true accounts of the natural world led to the production of Hecataeus' map of the

world and Anaximander's account of the universe. Both of these works were built around simple geometrical constructions from which they derived their validity. Similarly, the first principle of Parmenidean ontology ultimately manifests itself in the visible model of the solid sphere. This fundamental notion of the evidentiary force of the visible, constructed sign -- a notion essential to Greek mathematics -- is anticipated in the recognition scenes which conclude the story of the Odyssey. Odysseus proves his identity by producing the signs of the scar on his thigh, the wedding bed, and the garden of Laertes. Each of these signs are described in connection with an account of their construction. Furthermore, each of them have the characteristic of being immediately visible, and in the case of the bed and the orchard, they are described in connection with rudimentary arithmetical and geometrical ideas. All these signs derive their demonstrative power from the precise details of their construction.

On the other hand, and again in both the Homeric description of demonstration and the structure of the Greek mathematical notion of proof, the demonstrative project is launched in close connection with the demand for worthy beginnings. In the Elements, this demand manifests itself in the production of a system of axioms in which the geometrical constructions are grounded. And in early Ionian philosophy, this demand shows up in the notion of first principles. Anaximander's apeiron is such a first principle; he posits its truth and makes it the beginning of the cosmos and the ground out of which the universe springs. Likewise, Parmenides grounds his ontology in the first principle "that being is." The logical quality of this first principle ultimately leads to the development of the formal deductive techniques employed in the mathematics of the Elements. This concern with the ground of a demonstration, the surface upon which the demonstrative sign is inscribed, is prevalent in the Odyssey. The strength of the sign of the scar is that it is fixed to a zone on Odysseus' body marked out by his knee and femur. This physiological location is as essential to Odysseus as are his name and his psyche. Odysseus thigh is a permanent surface, and the scar affixed to it is a demonstrative sign. Similarly, the signs of the bed and the orchard are grounded. Their roots extend into the earth, and it is their intractability, their permanence, that gives them evidentiary force.

The arrival at these demonstrative themes in the Odyssey began with a consideration of the Elements of Euclid. I want now to return to the Elements and briefly discuss the remarkable similarity between the proof of one of the early construction problems in the first book of Euclid's text, and the proof of his identity that Odysseus presents to Penelope in the wedding chamber. The problem in the Elements is the simple and beautiful construction of the equilateral triangle. The problem begins with an account of a construction.

Let there be given the line segment AB. Let the circle BGD have been drawn with center A and radius AB; And again let the circle AGE have been drawn with center B and radius BA. And from point (semeiou) G defined by the intersection of the circles, to points (semeia) A, B let the lines GA, GB have been extended.

The verbs all take the perfect passive imperative tense which places the time of the construction in the past. When Odysseus proves himself to Penelope, he too refers to a construction: it is the construction of the wedding bed which likewise was built long ago in the time before the war at Troy. Like the wedding bed, the Euclidean construction contains signs: These are the points which define the lines and the circles. The greek word for "point" in geometry is semeion, a special form of the noun sema, which in the Odyssey names the sign of the olive trunk at the center of Penelope's bed. In the Elements, a point is defined as that which has no part, like the olive tree shorn of its branches and bark. For Euclid, the problem is to prove that the figure before him is an equilateral triangle. As with Odysseus, the question is one of identity, and for both Odysseus and the triangle, the key to the demonstration of identity lies in the details of the construction. The triangle is equilateral because its sides are equal to the radii of equal circles; Odysseus is Odysseus because he alone could recall the detailed construction of the wedding bed. The construction of the equilateral triangle and the proof of the wedding bed have so much in common because they both stem from the same basic notion true of demonstration.

I hope to have shown that this notion is a basic element of Greek culture transmitted through at least five centuries, and represented in at two major literary genres: epic poetry and formal mathematical proof. I also hope to have shown along the way that the original epic context for this notion of proof is dialectical. It is always employed toward the end of a hero's self-recognition. Every positive move in the hero's dialectical self-realization is shadowed by negation. Epic poetry understands the consequence of the positive rational project; this is, perhaps, because formal demonstration in theory

and practice was an intellectual innovation in the eighth century, when Greek civilization was undergoing a number of other monumental disturbances, such as the introduction of the polis, the emergence of the Olympic games and pan-Hellenism, and the adoption of a northern Semitic alphabet which initiated a gradual transition from a culture based in orality to one which expressed itself in letters. However that may be, what is clear is that by the sixth century, proof essentially of the same kind as that found in the *Odyssey* was no longer understood dialectically. The negative shadow of positive rationality had been forgotten, and a new era of enlightenment had begun.

## Epilogue

In my mind, the important part of this paper is the basic association of Euclidean proof structures with the demonstrative themes in the *Odyssey*. The value of this association is that it opens up a rich new field of evidence bearing upon questions concerned with the origins of Greek mathematics and the history of rational thought in general. In discussing the dialectical context of Homeric demonstration as opposed to the positive proofs of the sixth through the third centuries in Greece I have tried to imitate with different evidence the historical analysis performed by Horkheimer and Adorno in their impressive book *The Dialectic of Enlightenment* (trans. 1989). Whether or not the reader accepts my conclusions, the more important association between epic poetry and Greek mathematics may nevertheless be preserved, improved upon, and put to other uses. Two such uses have come to my mind in the process of writing this paper, and I will list them with no further comment.

(1) Whereas a mathematical text is more or less univocal, epic is microcosmic. The function of proof in Greek society at large could be studied by analyzing the interaction between demonstrative themes in the *Odyssey* and other themes. For example, what is the relationship between proof in the *Odyssey* and the education of Telemachus? This could lead to some early formulation of the Greek association of mathematics with pedagogy. Or again: What is the relationship between proof in the *Odyssey* and the execution of law? This could lead to a new perspective on Athenian culture in particular.

(2) In concluding the paper I suggested that formal demonstration may have been a relatively new intellectual development in Greece in the eighth century. I have wondered if the adoption of the Phoenician alphabet by the Greeks at that time could have encouraged such innovations. An alphabet does provide a certain analytical approach to language which is not available to oral cultures. Plus, there is the fact that letters were sometimes called *stoicheia* by the Greeks, and that the study of the alphabet was closely connected to the study of numbers in Greek education. This sort of speculation into the origins of Greek mathematics is made possible by the Homeric evidence which I have attempted to establish.

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