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Author(s): Margaret Vardell Sandresky

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# The Continuing Concept of the Platonic-Pythagorean System and its Application to the Analysis of Fifteenth-Century Music

by Margaret Vardell Sandresky

In view of the corpus of research now available concerning the cultural and intellectual life of the fifteenth century, it has become possible to investigate in some detail the theory that analytical procedures which are rooted in fifteenth-century thought can greatly clarify the compositional practices and aesthetic views of quattrocento composers. The specific relationships which suggest such an approach can best be understood by discussing briefly some examples derived from primary sources.

In the well-known woodcut from *De Harmonia musicorum instrumentorum*<sup>1</sup> by the early Renaissance music theorist and humanist Franchinus Gafurius, Gafurius is shown lecturing to his pupils on the “harmonious discord” resulting from the two unequal consonances drawn from two dissimilar proportions, 3 : 4 and 4 : 6, or perfect fourth and perfect fifth, which to-

gether form the octave. The sonnet below the picture informs us that in order to comprehend the celestial harmonies, one must know Mercury, Apollo, Orpheus, Aristotle, Plato, and Pythagoras. By the celestial harmonies, the poet refers to Plato’s doctrine of the creation of the world as described in his dialogue *Timaeus*. In brief, Plato wrote that a Master Workman created a harmonious universe by placing the planets in their orbits around the earth according to the Pythagorean number ratios of the musical intervals.

The woodcut further demonstrates the Renaissance idea of space by showing different uses of the ratios 3 : 4 : 6. Their relation to sound is symbolized by three organ pipes on the left, to space by three parallel lines on the right, to the universe by the astronomer’s dividers below, and to time by the hour glass at the right of the lectern. Not only is the place of music theory in the quadrivium of the liberal arts here defined, but also the idea is implied that musical pitch, rhythm, and duration all find a common bond within the same ratios, a concept basic to Renaissance musical thought. So that the interval of 4 : 6 as a perfect

<sup>1</sup>See Claudio Sartori, “Franchinus Gaffurius,” *Die Musik in Geschichte und Gegenwart*, IV, col. 1238. This woodcut also appears in an earlier Gafurius treatise, *Angelicum opus Musicae*, 1508.

fifth is equated with a sesquialter rhythm as well as with a whole section of music in tempus imperfectum in relation to another section in tempus perfectum.<sup>2</sup>

Because fifteenth-century Platonic-Pythagoreanism was the reinterpretation of a concept which had a long and continuing influence from the time of classical Greece, the channels through which this concept developed need to be sketched.

A comparison of the early fourteenth-century concept of the universe which Dante describes in his *Divine Comedy* and the fifteenth-century idea of the universe as illustrated by a woodcut from the *Practica musicae*<sup>3</sup> by Gafurius illustrates the relationship between these two periods. Gafurius retains the same plan of the spheres of heaven each with its particular planet and muse, and with the same concept of a measurement of their distances that Dante describes in the *Paradiso*. On the other hand, Dante places Christian saints and the Virgin Mary in the circles nearest to God in the highest realm, while Gafurius in typical humanist fashion places Apollo and the three Graces in the highest heaven of the universe, a concept entirely foreign to Dante. However, the relationship between God and Apollo which is implied in the Gafurius woodcut is in fact a reference to the analogy between early Christian and Greek symbols. This analogy is illustrated by a mosaic from a fourth-century pre-Constantinian necropolis under St. Peter's Basilica in Rome where Christ is in fact depicted as Apollo, the sun-god in his chariot.<sup>4</sup>

An interesting medieval French illumination shows God in his role as Plato's Master Workman measuring the universe with his astronomer's dividers, and illustrates the theological impli-

cations which made possible the Church's acceptance of the liberal arts as a part of Christian thought.<sup>5</sup> The twelfth-century West Portal of Chartres Cathedral, showing the cycle of the liberal arts in which Boethius symbolizes music, also demonstrates the Church's recognition of the liberal arts as a means to the knowledge of God. Such iconographic programs in the visual arts of this period are important in illustrating the continuing influence of the Platonic-Pythagorean concept of a musical universe.<sup>6</sup>

A basic function of the new quattrocento humanistic spirit consisted of a transformation of Platonic-Pythagoreanism from a purely speculative concept into an active practical system. The harmonious ratios of the musical intervals became the basis for a new idea of beauty. They symbolized for Renaissance man the artistic truth of nature as represented by natural law, and they provided artists a means for solving the problem of proportion and perspective.

With regard to music, such an empirical process is described by Gafurius in his *Practica musicae* when he writes: "The function of numbers . . . deals with sounds according to their progression in time. . . . First, in the disposition of sounds by consonant interval, which belongs to the domain of the theorist; and second, in the temporal quality of these sounds according to the numerical value of the notes, a matter ascribed to the active or practical discipline."<sup>7</sup> In other words, the music theorist measures the intervals, but the creative artist transforms these sounds into active experience by arranging them within rhythmic and formal space according to the laws of beauty and proportion.

The same point of view is also central to the concept of beauty

<sup>2</sup>This idea is discussed by M. van Crevel in his edition of Jacob Obrecht's *Missa Maria Zart*, Volume 7 of Obrecht's complete works (Amsterdam, 1964) particularly on pages LXXI and LXXVIII.

<sup>3</sup>There are two translations of the *Practica musicae*, one by Clement A. Miller (Dallas: American Institute of Musicology, 1968) and the other by Irwin Young (Madison: University of Wisconsin Press, 1969).

<sup>4</sup>See Jean Lassus, *The Early Christian and Byzantine World* (London: Paul Hamlyn, 1969), p. 3.

<sup>5</sup>See Arthur C. Clarke, "Space and the Spirit of Man," *Horizon*, January, 1959, p. 26.

<sup>6</sup>For a detail of the West Portal of Chartres, see *Twelfth Century Europe and the Foundations of Modern Society*, ed. Clagett, Post, and Reynolds (Madison: University of Wisconsin Press, 1966), p. 11.

<sup>7</sup>Franchinus Gafurius, *Practica musicae*, trans. and ed. Irwin Young, p. 166.

in the visual arts. An excellent primary source for this viewpoint is provided in the work of Leon Battista Alberti, the fifteenth-century humanist and architect. Alberti, the first to write about architecture since the Roman Vitruvius, brought out his work *De re aedificatoria*<sup>8</sup> in 1452. One of Alberti's most perceptive interpreters in modern times has been the scholar Rudolf Wittkower; and from his book *Architectural Principles in the Age of Humanism*, it is possible to reconstruct this relationship between architectural and musical space which was central to the fifteenth-century idea of beauty.<sup>9</sup>

Wittkower says that for Alberti "music is geometry translated into sound, and that in music the very same harmonies are audible which inform the geometry of the building."<sup>10</sup> He quotes Alberti as follows: "The numbers by means of which the agreement of sounds affects our ears with delight, are the same which please our eyes and our minds. . . . We shall therefore borrow all our rules for harmonic relations from the musicians to whom this kind of numbers is extremely well known."<sup>11</sup>

According to Wittkower's analysis, Alberti's design for the facade of Santa Maria Novella in Florence "can be exactly circumscribed by a square. A square of half the size of the large square defines the relationship between the two storeys. The main storey can be divided into two such squares, while one encloses the upper storey. In other words, the whole building is related to its main parts in the proportions of one to two, which is in musical terms an octave."<sup>12</sup>

Piero della Francesca's painting *The Flagellation of Christ* dates from approximately the same period. Kenneth Clark has

shown that the basic unit of measure is the black bar above the bearded figure in the foreground and that mathematical ratios relate this unit to the ceiling panels, pavement squares, columns, figures, and to the circle in which Christ is bound.<sup>13</sup> Marilyn Aronberg Lavin in her monograph on this painting came to the conclusion that the abstract design of the picture, quite apart from the visual representation of figures, "must, in its own time, have formed part of its meaning."<sup>14</sup>

Such a point of view when translated into musical terms leads to the hypothesis that the design of a musical composition, quite apart from its surface patterns of pitch and rhythm, possibly formed an integral part of its meaning. Indeed, such an assumption seems more closely related to musical form than visual design, since it is based on the concept of measuring musical space.

Example 1 illustrates the development of this Platonic-Pythagorean geometric-musical space from the ratios of an octave, fifth, and fourth to those based on the square and cube of two and three. From these the three mean proportionals—harmonic, arithmetic, and geometric—are derived. If the two series are multiplied by six, then the mean proportionals can be calculated in terms of whole numbers rather than fractions.<sup>15</sup> From the mean proportionals, the other musical intervals are measured as shown in Example 2. This is a chart of Pythagorean measurements which are not stated in terms of vibrations per second but are in terms of number relationships. The multiples of the series based on the square are all octave pitches and are placed vertically below the letter names of the notes, while the

<sup>8</sup>See *The Architecture of Leon Battista Alberti in Ten Books*, trans. James Leoni (London, 1726) reprint (London: J. Rykwert, 1955).

<sup>9</sup>The first edition was published in 1949. My quotes are taken from the fourth edition (New York: W. W. Norton, 1971).

<sup>10</sup>*Ibid.*, p. 9.

<sup>11</sup>*Ibid.*, p. 110.

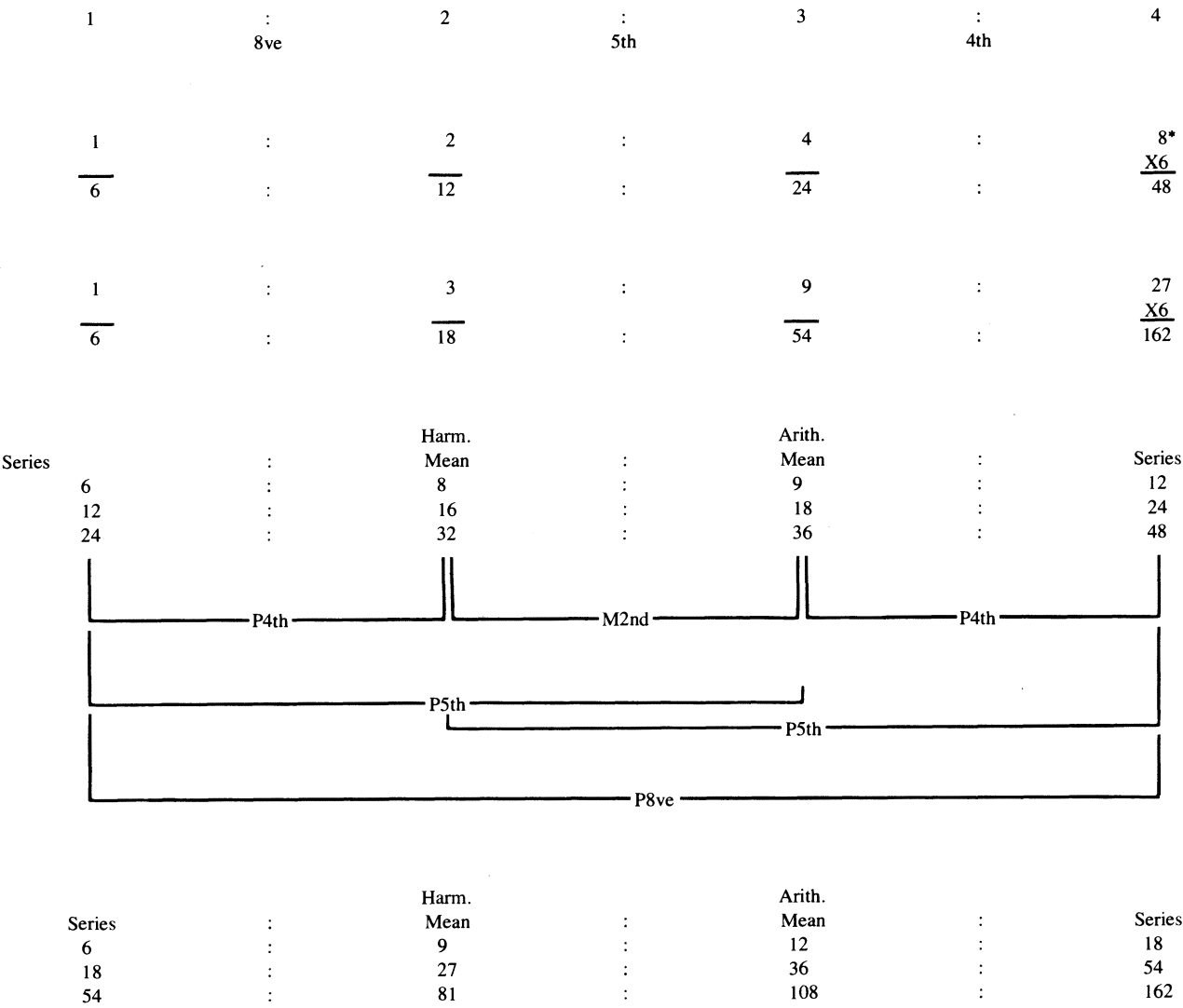
<sup>12</sup>*Ibid.*, p. 46.

<sup>13</sup>Kenneth Clark, *Piero della Francesca*, 2nd edition, London and New York: Phaidon, 1969), p. 35.

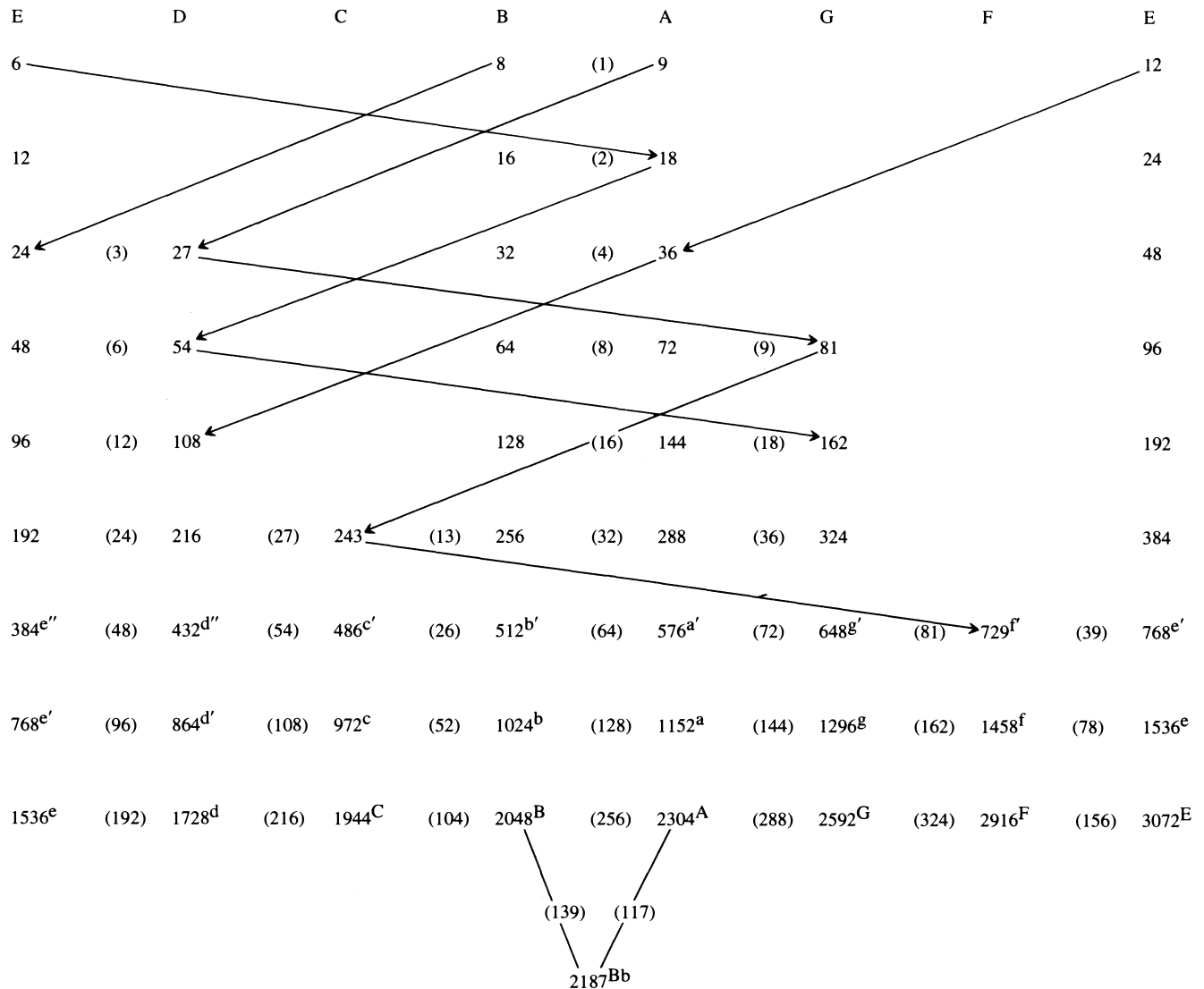
<sup>14</sup>Marilyn Aronberg Lavin, *Piero della Francesca: The Flagellation* (New York: Viking Press, 1972), p. 13.

<sup>15</sup>Example 1 is derived from Wittkower's summary of this material in *Architectural Principles*, pp. 111, 112; but see also Crevel, p. LXXXIV for historical background on the use of the number six as a multiplier.

Ex. 1. The mathematical structure of Plato's universe in Pythagorean measurement (\* geometric mean)



Ex. 2 Numbers in parentheses are the distances between pitches



multiples of the series based on the cube are all octave plus fifth pitches and are indicated by arrows.<sup>16</sup>

The remaining examples present a summary of the Platonic-Pythagorean number patterns found in a group of fifteenth-century cantus firmus Masses by Guillaume Dufay. In order to show the development of the form, these mature Masses of Dufay are introduced by one of the earliest extant cantus firmus Masses, *Alma redemptoris mater* by Leonel Power. The patterns shown in these examples were derived from a tabulation of the number of semibreves contained in different structural elements of the works.<sup>17</sup> The various elements consist of musical materials with cantus firmus underlay compared to those with no cantus firmus, one mensuration compared to another mensuration, or a comparison of the lengths of cantus firmus phrases.

The examples imply that the compositional procedure must have been first to outline the total form and its substructure in terms of Pythagorean ratios before actual composition of pitches and rhythms began. The graphs show that the composer must have considered the various elements listed above as separate entities and that an integral part of this pre-compositional process must have consisted in arranging these entities in a pattern of interwoven designs by means of the proportional number ratios. For instance, Example 3 shows that Leonel Power's cantus-firmus-based material extends for forty-eight semibreves in all tempus perfectum sections of not only the Gloria but also of the entire Mass. Furthermore, the non-cantus firmus sections are always the duration of twelve or twenty-four semibreves, and the ratio of these elements is 48 : 12 : 48 : 24 or 4 : 1 : 2 : 1; in Pythagorean terms, the octave and the double octave. These are the proportions Alberti used in his design for the facade of Santa Maria Novella.

The structural ideas shown in Example 4, *Missa Se la face ay*

<sup>16</sup>Example 2 is an extension of the work of Jacques de Liège as presented in Crevel, pp. LXXXII and LXXXVIII.

<sup>17</sup>In tempus imperfectum diminutum, I equated the breve with a semibreve of tempus perfectum.

*pale*, are very close to *Alma redemptoris mater*. While *Missa Se la face ay pale* is a mensuration Mass, the cantus firmus is still regularly broken up by non-cantus firmus material and the entire pattern is either doubly or triply augmented by means of mensural canons. There is, however, some flexibility introduced into the form by means of an occasional difference in the lengths of non-cantus firmus material, and the cadences are sometimes extended. The large design is nevertheless quite clear.<sup>18</sup>

In *Missa Caput*,<sup>19</sup> since the cantus firmus was taken from a melisma, it was necessary for the composer to decide on the note values and lengths of phrases. The cantus firmus appears twice in each movement, once in tempus perfectum and once in tempus imperfectum, and it never varies except for two phrases which are omitted in the Agnus dei. While the cantus firmus is broken up in a regular pattern by insertions of non-cantus firmus material, the lengths of this material vary widely from 112 semibreves in the Credo to as few as three. For these reasons, the plan of the work may be more clearly introduced by Example 5, a graph of the cantus firmus alone.<sup>20</sup> Here a study of the example reveals an interwoven pattern of Pythagorean ratios (reading both vertically and horizontally) between phrases, groups of phrases, and mensurations.

Example 6 illustrates the design for the entire Mass. While the non-cantus firmus material seems to lack structure, it also has its own pattern. The total length of all non-cantus firmus or three-voice writing in all tempus perfectum sections of the Mass is 378 semibreves. These may be divided into three equal parts each containing 126 semibreves. The first part extends from the beginning of the Mass through the first twenty-four semibreves

<sup>18</sup>For further study of architectural parallels in Dufay's work see Charles W. Warren, "Brunelleschi's Dome and Dufay's Motet," *Musical Quarterly*, 59 (1973), 92-104.

<sup>19</sup>Dufay's authorship of *Missa caput* has recently been questioned.

<sup>20</sup>The cantus firmus phrase structure used here agrees with Manfred Bukofzer's analysis in his *Studies in Medieval and Renaissance Music* (New York: W. W. Norton, 1950), p. 260.

Ex. 3. Leonel Power, *Missa Alma redemptoris mater*

| O       |     |     |     |     |     |     | ♢   |     |     |     |     |     |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|         | NCF | CF  | NCF | CF  | NCF | CF  | NCF | CF  | NCF | CF  | NCF | CF  |
| Gloria  |     | 48  | 12  | 48  | 24  | 48  |     | 40  | 16  | 76  | 16  | 28  |
| Credo   | 108 | 48  | 12  | 48  | 24  | 48  | 36  | 40  | 16  | 76  | 16  | 28  |
| Sanctus | 27  | 48  | 12  | 48  | 24  | 48  |     | 40  | 16  | 76  | 16  | 28  |
| Agnus   |     | 48  | 12  | 48  | 24  | 48  | 96  | 40  | 16  | 76  | 16  | 28  |
| <hr/>   |     |     |     |     |     |     |     |     |     |     |     |     |
| Totals  |     | 192 | 48  | 192 | 96  | 192 |     | 160 | 64  | 304 | 64  | 112 |

CF O : ♢ each mvt. 144 : 144  
 NCF O : ♢ each mvt. 36 : 32 (9.8)  
 Total CF O : ♢ 576 : 576  
 Total NCF O : ♢ 144 : 128

Ratios of irregular NCF:  
 Credo 108 : Sanctus 27 = 4:1  
 Credo 108 : Credo 36 = 3:1  
 Credo 36 : Sanctus 27 = 4:3  
 Agnus 96 = geometric progression  
 from regular CF:NCF = 12:24:48:96

of the Gloria prelude and a *clausula vera* on G. The next section of 126 semibreves begins immediately after this cadence and extends through the first eighteen semibreves of the Sanctus prelude ending at the penultimate semibreve to a *clausula vera* on D. The last section consists of all the non-cantus firmus material from this point to the end of the Agnus dei tempus perfectum.

The total non-cantus firmus material in all tempus imperfectum sections<sup>21</sup> is 448 semibreves. These are divided into two

<sup>21</sup>Signs for both tempus imperfectum as well as diminutum are found in the sources, and both work out to Pythagorean ratios. I have chosen the former, because the ratio seems more logical and more balanced: 1458 : 1296 over 162 where the divider is equal to the difference, as opposed to the diminutum which would make the ratio 1456 : 648. For a discussion of this mensuration sign, see Alejandro Planchart's edition of the work in *Missae caput* (New Haven: Yale University, 1964), pp. 159, 160.

large equal sections of 224 semibreves. The first section extends from the beginning of this mensuration at the Christe through the non-cantus firmus tempus imperfectum sections of the Gloria and through the first fifty-six (a quarter of 224) semibreves of the Credo tempus imperfectum prelude and a *clausula vera* on C. The second section begins immediately after this cadence with the text "Crucifixus etiam pro nobis" in the superius and includes all non-cantus firmus material in tempus imperfectum from this point to the end of the Mass.<sup>22</sup>

<sup>22</sup>One further result of this analysis is of interest. Because the Kyrie was copied into the choir books of Cambrai in 1463, a date which is late for the style of the work, it has been suggested by Bukofzer and others that this movement was written some years later than the other movements of the Mass. In such a case, the composer would then have been compelled to return to his earlier style and to imitate this in order for the Mass to be a unified work. I believe that the

Ex. 4. Dufay, *Missa Se la face ay pale*

|         | 1        | 2   | 3    | 4    | 5   | 6    | 7    | 8    | 9     |
|---------|----------|-----|------|------|-----|------|------|------|-------|
|         | Canon    | NCF | CF A | Cad. | NCF | Cad. | CF B | Cad. | Total |
| Kyrie   | Duplo    |     | 108  | 2+6  | 78  | 2    | 72   | 2+9  | 279   |
| Gloria  | Triplo   | 54  | 162  |      | 27  |      | 108  | 2+6  |       |
|         | Duplo    | 36  | 108  |      | 18  |      | 72   | 4    |       |
|         | Ut jacet | 18  | 54   |      | 9   |      | 36   | 2+9  | 725   |
| Credo   | Triplo   | 54  | 162  |      | 27  |      | 108  | 2+6  |       |
|         | Duplo    | 36  | 108  |      | 18  |      | 72   | 4    |       |
|         | Ut jacet | 18  | 54   |      | 9   |      | 36   | 2+9  | 725   |
| Sanctus | Duplo    | 36  | { 54 | 3+6  | 117 | 6    |      |      |       |
| Osan. I |          |     | { 54 | 2+6  | 36  | 2+8* |      |      |       |
|         |          |     |      |      | 36  | 8*   |      |      |       |
|         |          |     |      |      | 54  | 4    | 72   | 2+4  | 510   |
| Agnus   | Duplo    | 18  | 108  | 2+6  | 132 | 4    | 72   | 2+9  | 353   |
| Total   |          | 270 | 972  | 33   | 561 | 34   | 648  | 74   | 2592  |

NCF column 2 = 270

Cad. column 4 = 33

NCF column 5 = 561

Total = 864 = pitch d'

Cad. columns 6 & 8 = 108

NCF Total = 972 = pitch c

CF A = 972 = pitch c

CF B = 648 = pitch g'

Ratio A : B = 3 : 2

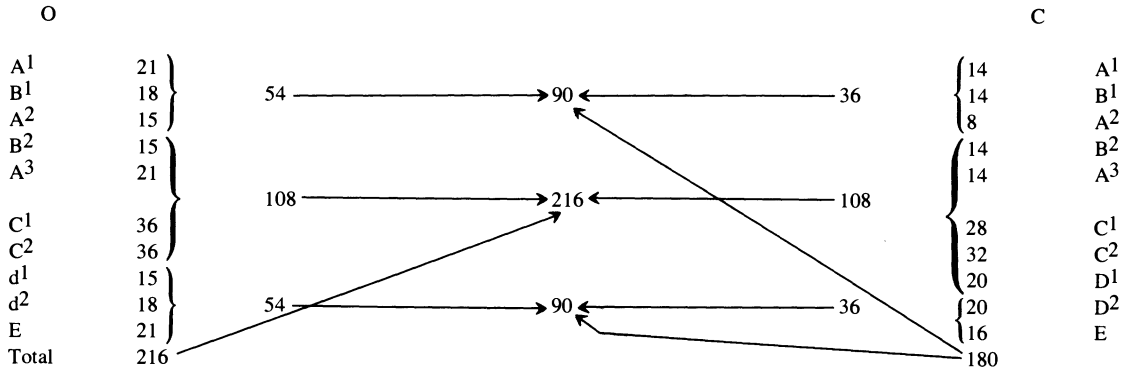
(\* fermata in O equals 9; in C, 8)

Finally, it should be noted in Example 6 that the ratio of the total number of semibreves between the two mensurations of the cantus firmus is 6 : 5 while the ratio of the total number of semibreves between the two mensurations of the entire Mass is 9 : 8. Since these are the ratios of the intervals between the first five pitches of the cantus firmus, we are led to the conclusion

analysis presented here proves that the Mass was definitely conceived and written in its entirety during one time span.

that the composer may have derived the large plan for his work from the opening cantus firmus pitches.

*Missa Ecce ancilla domini—Beata es Maria* shown in Example 7 uses two cantus firmi from different sources. *Ecce* is set in tempus perfectum and *Beata* in tempus imperfectum diminutum, except for the Kyrie, in which *Beata* is also set in tempus perfectum. Note in the graph that the lengths of cantus firmus phrases as well as those of non-cantus firmus insertions begin to be treated with more freedom and that the pattern is less

Ex. 5. Guillaume Dufay (?), *Missa Caput*. Organization of cantus firmus in each movement

formal. An examination of the music shows that the cantus firmus is not invariably stated in long note values but contains points of embellishment. Every movement ends on a complete authentic cadence in C major with an open fifth tonic. These triads are made up of the tones G, B, D and C, which are also found as Pythagorean numbers in the large sections of the mass.

In *Missa L'homme armé*, the many exceptions to a pattern necessitate a graph of great detail, and I have therefore excluded it from my summary. A study of the music shows that the cantus firmus itself is frequently embellished or extended by melismas which make the tenor assume the quality and shape of the other voices. *Missa L'homme armé* has been a very difficult work to analyze, and until only recently I believed that my procedures would not apply to this Mass. After all, would a high embellished, free, and irregular cantus firmus be channeled into a neat pattern of number ratios? I have found two designs, one summarized in Example 8a which follows the analytical procedures of the previous Masses. The other, shown in Example 8b, treats the musical materials as a continuum, but separates the three

canons and the section with three simultaneous time signs from the main body of the Mass.

In Example 8b the musical events which delineate structural divisions are as follows. In the Kyrie there are 210 semibreves. These are exclusive of the canon consisting of twenty-three semibreves shown in the right hand column. In the Gloria tempus perfectum<sup>23</sup> there are seventy-eight semibreves to the close of cantus firmus phrase A<sup>1</sup>, 144 to the close of phrase C<sup>1</sup>, and forty-six to the cadence; in the tempus imperfectum section there are 170 semibreves to the beginning of the Amen in the contratenor and seventeen to the close of the movement. In the Credo tempus perfectum section there are 216 semibreves to the close of cantus firmus phrase B<sup>3</sup> and the beginning of a duet between the superius and contratenor to the text "Deum verum de deo vero." Then there are thirty-three semibreves to the

<sup>23</sup>According to Leo Treitler's analysis of the cantus firmus. See "Dufay the Progressive," *Dufay Quincentenary Conference*, ed. Allan W. Atlas (Brooklyn College, 1976), p. 115.

Ex. 6. Guillaume Dufay (?), *Missa Caput*

|         | NCF | A <sup>1</sup> | B <sup>1</sup> | A <sup>2</sup> | B <sup>2</sup> | A <sup>3</sup> |   | NCF | C <sup>1</sup> | NCF | C <sup>2</sup> | D <sup>1</sup> | D <sup>2</sup> | E  |
|---------|-----|----------------|----------------|----------------|----------------|----------------|---|-----|----------------|-----|----------------|----------------|----------------|----|
|         |     |                |                |                |                |                | O |     |                |     |                |                |                |    |
| Kyrie   | 63  | 21             | 18             | 15             | 15             | 21             |   | 12  | 36             | 27  | 36             | 15             | 18             | 21 |
| Gloria  | 48  | 21             | 18             | 15             | 15             | 21             |   | 3   | 36             | 6   | 36             | 15             | 18             | 21 |
| Credo   | 57  | 21             | 18             | 15             | 15             | 21             |   | 6   | 36             | 9   | 36             | 15             | 18             | 21 |
| Sanctus | 51  | 21             | 18             | 15             | 15             | 21             |   | 12  | 36             | 9   | 36             | 15             | 18             | 21 |
| Agnus   | 48  | 21             | 18             | 15             | 15             | 21             |   | 15  | 36             | 9   | 36             | 15             | 18             | 21 |
|         |     |                |                |                |                |                | C |     |                |     |                |                |                |    |
| Kyrie   | 80  | 14             | 14             | 8              | 14             | 14             |   | 0   | 28             | 40  | 32             | 20             | 20             | 16 |
| Gloria  | 76  | 14             | 14             | 8              | 14             | 14             |   | 0   | 28             | 0   | 32             | 20             | 20             | 16 |
| Credo   | 112 | 14             | 14             | 8              | 14             | 14             |   | 0   | 28             | 32  | 32             | 20             | 20             | 16 |
| Sanctus | 48  | 14             | 14             | 8              | 14             | 14             |   | 0   | 28             | 12  | 32             | 20             | 20             | 16 |
| Agnus   | 40  | 14             | 14             | 8              | 14             | 14             |   | 8   | 28             | 0   | 0              | 0              | 20             | 16 |

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CF O : C each mvt. = 216 : 180 (6:5)  
 CF plus NCF total for entire Mass equals 1458 : 1296 (9:8)  
 Opening pitches of CF: B<sub>6</sub>:5D<sub>9</sub>:8C D B

Ex. 7 Guillaume Dufay, *Missa Ecce ancilla domini-Beata es Maria*

| Ecce    |   |     |     |     |        |     |        |     |   |     | Beata |    |     |    |     |    |     |    |    |     |
|---------|---|-----|-----|-----|--------|-----|--------|-----|---|-----|-------|----|-----|----|-----|----|-----|----|----|-----|
|         |   | NCF | A   | NCF |        |     |        |     |   |     |       |    |     |    |     |    |     |    |    |     |
|         |   | NCF | A   | Cad | NCF    | B   | NCF    | Cad |   | NCF |       | A  | NCF | B  | NCF | C  | NCF | D  | E  | Cad |
| Kyrie   | O | 12  | 30  | 0   | 60     | 36  | 0      | 11  | ☐ | 66  | O     | 18 | 0   | 18 | 39  | 15 | 0   | 21 | 54 | 9   |
| Gloria  | O | 60  | 48  | 0   | 48     | 24  | 0      | 9   | ☐ | 42  |       | 16 | 8   | 10 | 0   | 10 | 0   | 16 | 6  | 3   |
|         |   |     |     |     |        |     |        |     |   |     |       |    |     |    |     |    |     |    |    |     |
| Credo   | O | 90  | 48  | 0   | 48     | 24  | 6+12c  | 0   | ☐ | 44  |       | 16 | 12  | 10 | 0   | 10 | 8   | 16 | 8  | 2   |
|         | O | 42  | 48  | 0   | 84     | 24  | 0      | 12  |   |     |       |    |     |    |     |    |     |    |    |     |
|         |   |     |     |     |        |     |        |     |   |     |       |    |     |    |     |    |     |    |    |     |
| Sanctus | O | 48  | 48  | 6   | 84+6c  | 30  | 18     | 11  | ☐ | 46  |       | 10 | 0   | 10 | 0   | 10 | 0   | 16 | 6  | 3   |
| Agnus   | O | 18  | 42  | 0   | 6      | 30  | 0      | 11  | ☐ | 64  |       | 10 | 0   | 10 | 0   | 10 | 0   | 16 | 6  | 3   |
|         |   |     |     |     |        |     |        |     |   |     |       |    |     |    |     |    |     |    |    |     |
|         |   |     |     |     |        |     |        |     |   |     | O     | 18 | 0   | 18 | 39  | 15 | 0   | 21 | 54 | 9   |
|         |   |     |     |     |        |     |        |     |   |     |       |    |     |    |     |    |     |    |    |     |
| Totals  | O | 270 | 264 | 6   | 330+6c | 168 | 24+12c | 54  | ☐ | 262 | ☐     | 52 | 20  | 40 | 0   | 40 | 8   | 64 | 26 | 11  |

CF *Ecce* A plus B equals 432 (pitch d'') plus final cadences 54 equals 486 (pitch c')

NCF *Ecce* sections O plus cadences equals 648 (pitch g')

CF plus NCF *Beata* Φ equals 512 (pitch b')

Beata O (Kyrie):

$$A + B = 36$$

$$C + D = \underline{36}$$

$$72$$

interval between

648 and 576

$$E = 54$$

interval between 486 and 432

$$\text{NCF} + \text{cadence} = 48$$

interval between 432 and 384

Ex. 8a. Guillaume Dufay, *Missa L'homme armé*

Kyrie —————→ O+♢=233

|         | O      | ♢         |
|---------|--------|-----------|
| Gloria  | 268    | 187       |
| Credo   | 307    | 18 ♢ =362 |
| Sanctus | 422    | 120       |
| Agnus   | 299    | 70        |
| Total   | 1296=g | 972=c     |

Total 2268 is not a Pythagorean number

contratenor change of meter into tempus imperfectum. This is the beginning of the passage shown in the right hand column consisting of three different mensuration signs at once. Next there are fifty-eight semibreves from here to the close. In the Credo tempus imperfectum there are 256 semibreves to the beginning of the canon "Scindite pausas," shown in the right hand column, consisting of eighty-eight semibreves to the end of the movement. In the Sanctus tempus perfectum section there are 149 semibreves to a highly embellished cadence beginning with the word "Sabaoth" in the superius and consisting of eighteen semibreves. The Pleni consists of 144 semibreves and the Benedictus of 120. In the Osanna II there are seventy-eight semibreves to the beginning of phrase C and 33 to the close. After the Agnus dei I prelude of twelve semibreves, 108 remain. In the Agnus II there are seventy semibreves. In Agnus III the cancrizan canon shown in the right hand column opens this section after which there are sixty-five semibreves to the close of the Mass.

*Missa Ave regina caelorum* carries the development shown in the previous Masses further and is Dufay's last work in this form. My detailed analysis is still incomplete, but the large

structure illustrated in Example 9 is shown as the ratio of the three mensurations to each other, 1296 : 1458 : 162. Note the similarity of these proportional ratios to those of *Missa Caput* in Example 6.

In closing I should like to discuss some of the questions which needed to be answered during the analytical process. One of the first which presented itself was whether or not small rests occurring in the cantus firmus were conceived as an integral part of its design. The procedure which I adopted was to count all rests of a breve or less as part of the cantus firmus, with rests of more than a breve carrying enough weight to be considered as non-cantus firmus material.

The problem of cadences is twofold. First, there is the question of whether or not to count the cadences as a definite duration, and, second, if they are counted, what the duration should be. While Apel says the duration of the final long "was not considered an exact value,"<sup>24</sup> it is a significant fact that number patterns do emerge when cadences are counted as a

<sup>24</sup>Willi Apel, *The Notation of Polyphonic Music*, 2nd edition (Cambridge, Mass., Mediaeval Academy of America: 1944), p. 104.

Ex. 8b Guillaume Dufay, *Missa L'homme armé*

|              |      |   |     |   |     |   |     |   |     |                           |
|--------------|------|---|-----|---|-----|---|-----|---|-----|---------------------------|
| Kyrie O+♢    | 210  | } | 288 | } | 432 | } | 648 | } | 972 | canon: 23                 |
| Gloria O     | 78   | } |     | } |     | } |     | } |     |                           |
|              | 144  | } |     | } |     | } |     | } |     |                           |
|              | 46   | } |     | } |     | } |     | } |     |                           |
| Gloria ♢     | 170  | } | 216 | } |     | } |     | } |     |                           |
|              | 17   | } |     | } |     | } |     | } |     |                           |
| Credo O      | 216  | } |     | } |     | } |     | } |     |                           |
|              | 33   | } | 324 | } |     | } |     | } |     |                           |
|              | 58   | } |     | } |     | } |     | } |     | *18,5                     |
| Credo ♢      | 256  | } |     | } |     | } |     | } |     | canon: 88                 |
| Sanctus O    | 149  | } |     | } |     | } |     | } |     |                           |
|              | 18   | } |     | } |     | } |     | } |     |                           |
| Pleni O      | 144  | } |     | } |     | } |     | } |     |                           |
| Benedictus ♢ | 120  | } |     | } |     | } |     | } |     |                           |
| Osanna II O  | 78   | } | 243 | } |     | } |     | } |     |                           |
|              | 33   | } |     | } |     | } |     | } |     |                           |
| Agnus I O    | 12   | } |     | } |     | } |     | } |     |                           |
|              | 108  | } |     | } |     | } |     | } |     |                           |
| Agnus II ♢   | 70   | } | 243 | } | 288 | } | 486 | } | 648 |                           |
| Agnus III O  | 65   | } |     | } |     | } |     | } |     | canon: 105 (plus 9 rests) |
| Totals       | 2025 |   |     |   |     |   |     |   |     | 216+27=243                |
| Grand Total  | 2268 |   |     |   |     |   |     |   |     |                           |

\*See Charles Hamm, *A Chronology of the Works of Guillaume Dufay* (Princeton, N.J.: Princeton Univ. Press, 1964), pp. 144-45.

Ex. 9 Guillaume Dufay, *Missa Ave regina caelorum*

|         | O      | C      | "3"  | Total  |
|---------|--------|--------|------|--------|
| Kyrie   | 344    | 268    | 20   | 632    |
| Gloria  | 193    | 366    | 14   | 573    |
| Credo   | 333    | 358    | 40   | 731    |
| Sanctus | 301    | 192    | 40   | 533    |
| Agnus   | 125    | 274    | 48   | 447    |
| Total   | 1296=g | 1458=f | 162* | 2916=F |

\*162 is the difference between 1296 and 1458

$$O : C = \frac{1296 : 1458}{162} = 8 : 9$$

definite duration. This does not mean that when one adds up a series of perfect longs and compares it to a series of imperfect longs one is bound to come up with a 3 : 2 ratio, because in many instances Dufay extends the cadence in one or two voices. I counted these extensions as additional semibreves to the final long in order to obtain the results presented here. A good illustration of this is shown in Examples 4 and 7, where the charts show cadential lengths in a separate column.

In dealing with the duration of the fermata, I followed Charles Warren's research and his citation of De Grocheo as the authority for holding the note under a fermata in tempus perfectum for the duration of a perfect long consisting of nine semibreves.<sup>25</sup> However, no reference is made to the meaning of a fermata written over a note in tempus imperfectum, such as one finds in *Missa Ave regina caelorum*. In this case, the note was counted as an imperfect maxima consisting of eight semibreves.

Finally, in setting up a system for analysis, there is always the problem of defining a set of guidelines to which the theorist feels

he can adhere with honesty. When all the evidence is assembled for building principles of construction, there are undoubtedly problem areas and seeming inconsistencies which have to be worked out or perhaps can never be solved. Nevertheless, I feel it is important to work toward developing procedures which reveal a composer's ideas with greater precision and insight, and I hope and believe that the process outlined in this paper is one further step in that direction.

<sup>25</sup>Charles W. Warren, "Punctus Organi and Cantus Coronatus in the Music of Dufay," *Dufay Quincentenary Conference*, p. 134-36.