



Principles of Palladio's Architecture: II

Author(s): Rudolf Wittkower

Source: *Journal of the Warburg and Courtauld Institutes*, Vol. 8, (1945), pp. 68-106

Published by: The Warburg Institute

Stable URL: <http://www.jstor.org/stable/750167>

Accessed: 16/07/2008 11:27

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=warburg>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We work with the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

4528  
WITTKOWER  
1945



Principles of Palladio's Architecture: II

Author(s): Rudolf Wittkower

Source: *Journal of the Warburg and Courtauld Institutes*, Vol. 8, (1945), pp. 68-106

Published by: The Warburg Institute

Stable URL: <http://www.jstor.org/stable/750167>

Accessed: 16/07/2008 11:27

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=warburg>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We work with the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## PRINCIPLES OF PALLADIO'S ARCHITECTURE—II

By Rudolf Wittkower

### 5. PALLADIO AND THE PROBLEM OF HARMONIC PROPORTIONS

The conviction that architecture is a science, and that each part of a building, inside as well as outside, has to be integrated into one and the same system of mathematical ratios, was deeply rooted in Renaissance æsthetics. This may, in fact, be called the basic axiom of Renaissance architects. But the architect is by no means free to apply to a building a system of ratios of his own choosing. The ratios have to comply with conceptions of a higher order. According to Renaissance architects and theorists a building should mirror the proportions of the human body; a demand which became universally accepted on Vitruvius' authority.<sup>1</sup> And as man is the image of God and the proportions of his body are produced by divine will, so the proportions in architecture have to embrace and express the cosmic order.<sup>2</sup> But what are the laws of this cosmic order; what are the mathematical ratios that determine the harmony in macrocosm and microcosm? They had been revealed by Pythagoras and Plato, whose ideas in this field were entirely absorbed during the sixteenth century and made applicable to architecture.

#### *Francesco Giorgio's Platonic Programme for S. Francesco della Vigna*

Hardly any more comprehensive evidence in proof of this has survived

I acknowledge gratefully the advice given to me on musical and mathematical terminology by Mr. D. P. Walker and Mr. A. Prag.

<sup>1</sup> Bk. III, i, 1: "Namque non potest aedis ulla sine symmetria atque proportione rationem habere compositionis, nisi uti ad hominis bene figurati membrorum habuerit exactam rationem."

<sup>2</sup> These ideas were so common and are so well known that it may suffice to refer in Palladio's circle to Daniele Barbaro's words (comm. to Vitruvius III, i, ed. 1556, p. 63): "La natura maestra ce insegna come havemo à reggersi nelle misure e nelle proportioni delle fabbriche à i Dei consecrate, imperoche non da altro ella vuole che impariamo le ragioni delle Simmetrie, che ne i Tempi usar dovemo, che dal Sacro Tempio fatto ad imagine, et simiglianza di Dio, che è l'huomo, nella cui compositione tutte le altre meraviglie di natura contenute sono, e però con bello avvedimento tolsero gli antichi ogni ragione del misurare dalle parti del corpo humano . . ."

Cf. also Lomazzo's *Trattato dell'arte della pittura*, etc., 1584, I, chap. 30: "Come ancora le misure delle navi, tempj, ed edifizj sono tratte dal corpo umano." A similar chapter in the same author's *Idea del tempio della*

*pittura*, 1590, chap. 34, begins thus: "il corpo umano, il quale è un opera perfetta, e bellissima fatta dal grande Iddio a simiglianza della sua Immagine, con grandissima ragione è stato chiamato mondo minore. Perchè contiene in se con più perfetta composizione, e con più sicura armonia, tutti i numeri, le misure, i pesi, i moti, ed elementi. Onde da lui principalmente, e non da altra fabbrica che uscisse dalla mano d' Iddio e dalle sue membra fu tolta la norma, ed il modello di formar i Tempj, i Teatri, e tutti gli edificj con tutte le sue parti come colonne, capitelli, canali, e simili; naviglij, machine, ed ogni sorte d'artificio."

Palladio himself (bk. II, chap. 2) compares in a brief sentence the structure of the human body with that of a building.

For the unity of the cosmological and æsthetic aspect of proportion during the Renaissance as well as for related questions cf. the fundamental article by Panofsky, "Die Entwicklung der Proportionslehre als Abbild der Stilentwicklung," *Monatshefte für Kunstwissenschaft*, 1921, p. 208 ff. See also G. Nicco Fasola's excellent introduction to Piero della Francesca, *De prospectiva pingendi*, Florence, 1942, particularly p. 15 ff.

than a document relating to S. Francesco della Vigna at Venice (Pl. 18a). On the 15th of August, 1534, the Doge Andrea Gritti laid the foundation stone of the new church, and the structure was begun in accordance with Jacopo Sansovino's design. But differences of opinion soon arose about the proportions of his plan, and the Doge commissioned Francesco Giorgio, a Franciscan monk from the monastery attached to that church, to write a memorandum about Sansovino's model.

Andrea Gritti's choice of his expert is interesting. This Francesco Giorgio had made his name by a study of the problem of proportion, in all its aspects. In 1525 he had published a large folio on the harmony of the universe<sup>1</sup> in which Christian doctrines and Neoplatonic thought were blended, and the old belief in the mysterious efficacy of certain numbers and ratios was given new impetus. The memorandum on the proportions of S. Francesco is a practical exposition based on the theories of that book.<sup>2</sup>

Giorgio suggests making the width of the nave 9 paces, which is the square of 3: "Numero primo e divino." In the Pythagorean conception of numbers, three is the first real number because it has beginning, middle and end.<sup>3</sup> It is divine as the symbol of the Trinity. The length of the nave he wants to be 27 paces, i.e. 3 times 9. The square and cube of 3, Giorgio goes on, contain the consonances of the universe as Plato has shown in the *Timaeus*; and neither Plato nor Aristotle, who knew the forces effective in nature, went beyond the number 27 in their analysis of the world. However, not these same numbers but their ratios are of importance and, that the cosmic ratios are to be regarded as binding for the microcosm also, is evident from God's command to Moses to build the Tabernacle after the pattern of the world and Solomon's resolve to give the proportions of the Tabernacle to the Temple. Giorgio also expresses the suggested proportion of width to length of the nave (9:27) in musical terms; it forms, as he says, a diapason and a diapente. A diapason is an octave and a diapente a fifth. 9:27 constitutes an octave and a fifth, if seen in the progression 9:18:27; for 9:18=1:2=an octave, and 18:27=2:3=a fifth.

To understand Giorgio's reasoning it should be recalled that it was Pythagoras who discovered that tones can be measured in space. What he had found was that musical consonances were determined by the ratios of small whole numbers. If two strings are made to vibrate under the same condi-

<sup>1</sup> *De harmonia mundi totius*, Venice, 1525. This work has hardly been noticed by modern scholars although its influence during the sixteenth century seems to have been not inconsiderable. A new edition appeared in Paris, 1545, a French translation in Paris, 1579. Panofsky, in *Monatshefte f. Kunstw.*, 1921, p. 209, was the first to draw attention to Giorgio's work; cf. also *id.*, *The Codex Huygens*, 1940, p. 113. There are short notes on Giorgio in Thorndike, *Hist. of Magic*, 1941, VI, p. 450 ff.

<sup>2</sup> Though often mentioned, the memorandum was printed only by Gianantonio Moschini, *Guida per la Città di Venezia*, 1815,

I, i, pp. 55-61. Considering its extraordinary importance, never sufficiently realized, I have given the full text in English as Appendix I (p. 103 ff.), which should be consulted for the following paragraphs.

<sup>3</sup> For the Pythagorean symbolism of "three" see Aristotle, *de coelo* I, 1 (268a) and Plutarch, *Sympos.* IX, quaest. 3. Marsilio Ficino, in his commentary to Plato's *Timaeus*, followed this definition of "three" (*Opera*, 1576, II, p. 1459): "Trinitas numerorum prima, principium et medium, finemque rerum continere videtur, atque sola inter numeros ratione quadam individua continere."

tions, one being half the length of the other, the pitch of the shorter string will be one octave (diapason) above that of the longer one. If the lengths of the strings are in the relation of two to three, the difference in pitch will be a fifth (diapente), and if they are in the relation of three to four, the difference in pitch will be a fourth (diatessaron). Thus the consonances, on which the Greek musical system was based—octave, fifth and fourth—can be expressed by the progression 1 : 2 : 3 : 4. And this progression contains not only the simple consonances octave, fifth and fourth, but also the two composite consonances which the Greeks recognized, namely octave plus fifth (1 : 2 : 3) and two octaves (1 : 2 : 4). One can understand that this staggering discovery made people believe that they had seized upon the mysterious harmony which pervades the universe. And on this was built the number symbolism and mysticism, which had an immeasurable impact on human thought during the next 2,000 years. In the wake of Pythagoras, Plato in his *Timaeus* explained that cosmic order and harmony are contained in certain numbers. Plato found this harmony in the squares and cubes of the double and triple proportion starting from unity, which led him to the two series 1, 2, 4, 8 and 1, 3, 9, 27.

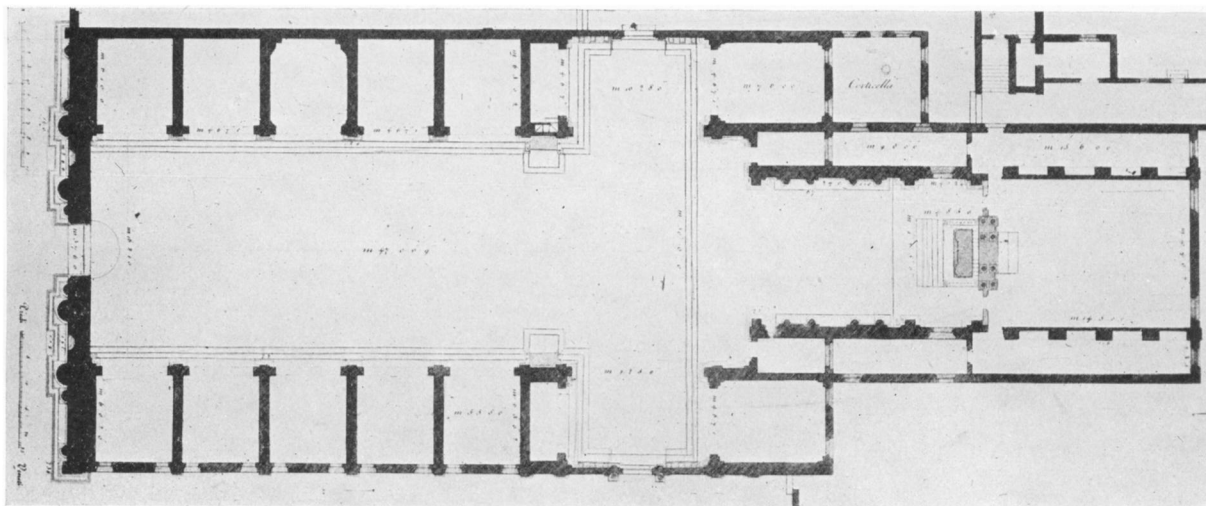
Traditionally represented in the shape of a triangle  $\begin{matrix} & 1 & & \\ & 4 & 9 & \\ 8 & & & 27 \end{matrix}$  the harmony of the world is expressed in the seven numbers 1, 2, 3, 4, 8, 9, 27 which embrace the secret rhythm in macrocosm and microcosm alike. For the ratios between these numbers contain not only all the musical consonances, but also the inaudible music of the heavens and the structure of the human soul.<sup>1</sup>

In his *Harmonia mundi* Giorgio, closely following Ficino, gave proof of his familiarity with these ideas. His fifth book deals with the Pythagorean-Platonic theory of numbers. It begins with the words: "It is absolutely evident to all Pythagoreans and Academicians that the world and the soul were defined first by Timaeus of Locri and then by Plato by certain laws and musical proportions, just as a heptachord made of seven strings (*limitibus*), beginning with unity, duplicating up to the cube of two (i.e.  $2^3=8$ ) and trebling up to the cube of three (i.e.  $3^3=27$ ). According to the writings of Pythagoras it was believed that in these numbers and proportions the fabric of the soul and the whole world was arranged and perfected. And from the odd as from the male, and from the even as from the female—from these powers together everything is generated. But in the cube of the one and the other, they said, the work was terminated. For one cannot proceed beyond the third dimension in length, width and depth. And also all power of activity and passivity is contained in these numbers and proportions, and all consonances are accumulated in them."<sup>2</sup>

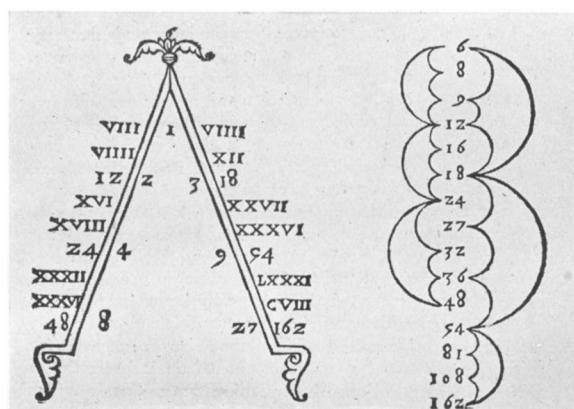
It is now clear why Giorgio does not want to go further than the number 27, and why ratios measured in space and as tones are for him synonymous. What Giorgio suggests for S. Francesco is the progression of the side of the Platonic triangle beginning with the perfect three (3, 9, 27). His further recommendations fall in line with these principal ratios. The "cappella

<sup>1</sup> Cf. *Timaeus*, 35B-36B (F. M. Cornford, *Plato's Cosmology*, London, 1937, p. 66 ff.). For the "harmony of the spheres" cf. Aristotle's survey of Pythagoreanism in *Met. A. 5* and Plato's myth of Er, *Rep. X* (616 ff.).

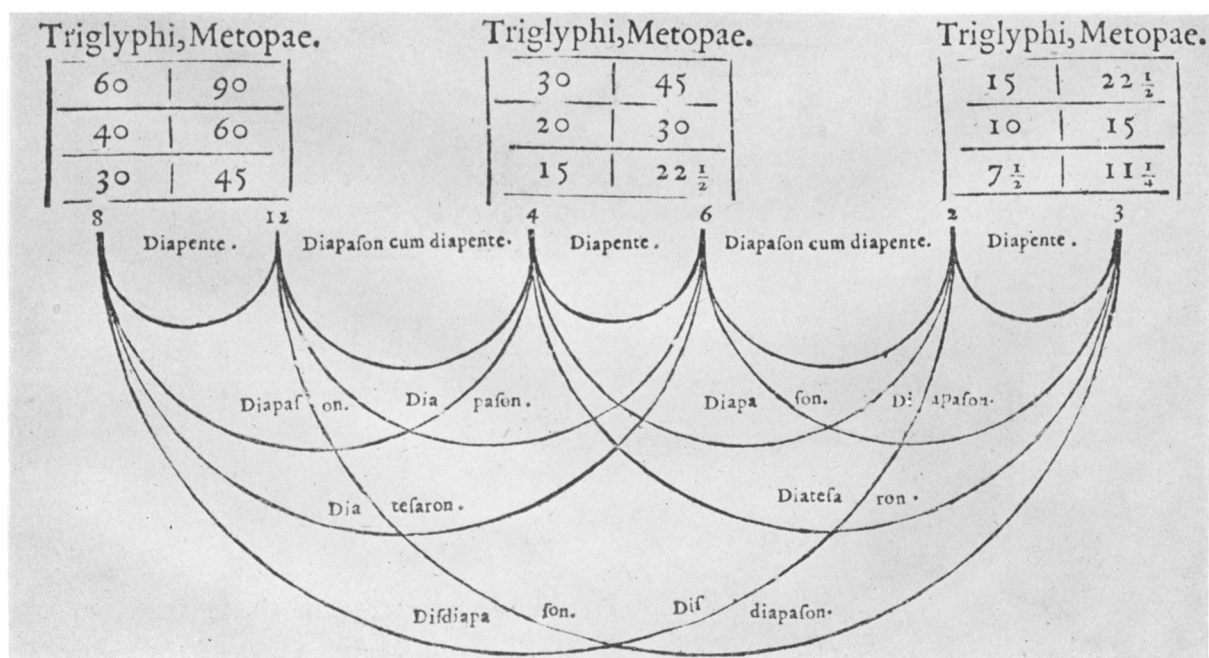
<sup>2</sup> *Op. cit.*, fol. 81 verso.



a—Plan of S. Francesco della Vigna, Venice (p. 69ff)



b—Diagram from F. Giorgio, *De Harmonia Mundi*, 1525 (p. 77)



c—Diagram. From H. Prado and G. B. Villalpando, *In Ezechielem Explanationes*, 1596-1604 (p. 83)

grande" at the far end of the nave, like the head of the body, shall be 9 paces long and 6 wide, so that its length repeats the width of the nave and its width is related to that of the nave in the ratio of 2:3 or in musical terms a diapente (fifth). At the same time this ratio of 2:3 is also valid for width to length in the cappella itself. The choir behind the "cappella grande" should repeat the measures of the "cappella grande"—6:9. The whole length of the church is therefore 9 times 5. He calls it a fivefold proportion or, in musical terms, a bisdiapason (scil. disdiapason) and diapente.<sup>1</sup> The transept should be 6 paces wide, thus corresponding to the width of the "cappella grande." He suggests making the chapels at each side of the nave 3 paces wide, which is what he calls a triple proportion to the width of the nave itself (3:9), or musically (3:6:9=) a diapason (3:6) and a diapente (6:9=2:3). The relation of the width of the small chapels to that of the "cappella grande" is 3:6, i.e. a diapason. And the relation of the width of the chapels of the transept to that of those of the nave should be 4:3, or a diatessaron "proportione celebrata."

So far Giorgio's recommendations were carried out. Most of the actual proportions do not correspond exactly to his ratios, but the divergences are small and due to the kind of irregularities which occur in practice. The three steps leading up to the chapels and the "cappella grande," which he suggested and which, indeed, Sansovino had already planned, were also executed. He was not followed, however, in the height of the ceiling which he wanted flat and coffered, and related to the width of the nave in the ratio of 4:3.

By basing in this somewhat extraordinary way all the proportions of the building on the Pythagorean-Platonic philosophy of harmonic numbers, Francesco Giorgio may have created a precedent. But the three men who were consulted about the memorandum seem to have shown no undue surprise; for they approved of it. They were a painter, an architect and a humanist. This fact shows that proportion in architecture was not regarded as the province of architects alone; the unity of all arts and sciences made every initiate a trustworthy judge in these matters. And the eminence of the three people chosen as consultants is a clear sign of the importance attached to Giorgio's ideas. The painter was no less a man than Titian; the architect was Serlio, who was at that time in Venice preparing his work on architecture, the first part of which (Book IV) appeared in 1537, and who, already in 1534, seems to have been regarded as particularly well versed in the theory of architecture. The humanist was Fortunio Spira, who is now almost forgotten, but had in his own day a high reputation for his many humanist accomplishments. Francesco Sansovino<sup>2</sup> called him "Filosofo celeberrimo, di profonda scientia" and Aretino praised his "maestà nella presentia, gentilezza ne' costumi, maniera nell' attioni, gratia nei gesti, bontà nella natura, felicitade nell'ingegno, fama nelle opere, e gloria nel nome."<sup>3</sup>

<sup>1</sup> But here Giorgio seems to have committed an error which we are unable to explain without having seen the original text. 9:18:36 is two octaves, or a disdiapason, and 36:45=4:5 and not 2:3 (the diapente). Giorgio may have had in mind 36:54 which is a diapente. On the other hand, it would be

most surprising to find such an error in a man for whom the system of harmonic ratios was of such overriding importance.

<sup>2</sup> *Venetia . . . descritta*, 1581; in the ed. of 1663, p. 154.

<sup>3</sup> Aretino, *Del primo libro de le lettere*, Paris, 1609, p. 187. Spira died about 1560. A

Giorgio demands at the end of his memorandum that the ratios of the interior should be repeated in the façade, “che per esso si puosi comprendere la forma della fabbrica, et le suoe proportioni.” It appears certain that Palladio, when executing the façade a generation later, knew Giorgio’s memorandum, and derived from it the mysterious 27 moduli which constituted the width of the central portion of his façade corresponding to the nave.<sup>1</sup> Titian’s and Serlio’s approval of the memorandum suggests not only a familiarity with these ideas among artists, but also a readiness to apply them in practice, and it can be assumed that Palladio appreciated Giorgio’s Platonic speculations. His knowledge of Platonism must have been considerable. In Trissino’s circle he absorbed the spirit of the Platonic Academy, his humanist friends were steeped in the study of Plato and Aristotle, and his close association with Daniele Barbaro, particularly during the preparation of the Vitruvius edition, must have intensified his familiarity with ancient philosophical thought.<sup>2</sup>

### *Palladio’s Conception of Beauty*

Although Palladio left no written theory of proportion, there are enough scattered notes in his *Quattro libri* to test his ideas on this central theme of Renaissance architecture. In the first chapter of the first book he gives the well-known mathematical definition of beauty: “beauty will result from the beautiful form and from the correspondence of the whole to the parts, of the parts between each other, and of these again to the whole; so that the structures may appear an entire and complete body, wherein each member agrees with the other and all members are necessary for the accomplishment of the building.” This is the old demand for corresponding ratios throughout a building to which all the artists of the Renaissance subscribed, and which Palladio has formulated here in words which closely follow Vitruvius’ definition of “symmetria.”<sup>3</sup> This conception was, of course, very near to Palladio’s heart; he came back to it more than once as, for instance, in the introduction to his discussion of domestic buildings “whose parts should correspond to the whole and to each other.”<sup>4</sup> Vitruvius had shown that this correspondence can only be arrived at through a unity of measure, a fixed module, and Renaissance thought abode firmly by this idea. “Although in all fabrics” said Palladio in his discussion of temples “it is requisite, that their parts should correspond together, and have such proportions, *that there may be none whereby the whole cannot be measured*, and likewise all the other parts; this however ought to be observed in temples with the utmost care, because they are consecrated to

rich collection of material about him is to be found in Cicogna, *Iscr. Venez.*, 1830, III, p. 307 ff.

<sup>1</sup> Cf. the first part of this article in Vol. VII, p. 122. The height from the ground to the main entablature repeats the same measure.

<sup>2</sup> Cf. part I in Vol. VII, pp. 103 f., 106 ff.

<sup>3</sup> Vitruvius I, ii, 4. This definition of beauty is fundamentally Pythagorean and is

implied in Philolaos’ definition of harmony as “the unification of the composed manifold and the accordance of the discordant” (ἁρμονία δὲ πάντως ἐξ ἐναντίων γίνεται. ἔστι γὰρ ἁρμονία πολυμιγέων ἑνωσις καὶ δίχα φρονούντων συμφρόνησις) cf. H. Diels, *Die Fragmente der Vorsokratiker*, Berlin, 1934, I, p. 410, fragm. 10.

<sup>4</sup> Palladio, bk. II, chap 1.

divinity.”<sup>1</sup> For Palladio, as for earlier Renaissance architects, this conception of beauty was best expressed in circular buildings, which, in view of the perfection of this form, should be reserved for temples; a form “in which is to be found neither beginning nor end, and the one is indistinguishable from the other; its parts correspond to each other and all of them participate in the shape of the whole; and moreover every part being equally distant from the centre such a building demonstrates extremely well the unity, the infinite essence, the uniformity and the justice of God.”<sup>2</sup> Whether or not Palladio was here dependent on Plato's famous description in the *Timaeus* (33 B ff.) of the world as a sphere “equidistant every way from centre to extremity, a figure the most perfect and uniform of all” so that the world which the Demiurge brought into being “was a blessed god,” or whether these ideas reached him with the broad current of Renaissance Neoplatonism through Ficino and Francesco Giorgio, may be left undecided.<sup>3</sup> But it is beyond doubt that he imparts a Platonic quality to the Vitruvian definitions.<sup>4</sup>

The preface to the fourth book unveils something of Palladio's innermost thought. In introducing the subject of temples he insists that they should reflect the perfection of the divine being itself. And here Palladio gave rein to one of his rare, almost lyrical expositions about the order in the universe and its human-made echo. “E veramente,” he writes, “considerando noi questa bella machina del Mondo di quanti meravigliosi ornamenti ella si ripiena, & come i Cieli co'l continuo lor girare vadino in lei le stagioni secondo il natural bisogno cangiando, & con la soavissima armonia del temperato lor movimento se stessi conservino non possiamo dubitare, che dovendo esser simili i piccioli Tempj, che noi facciamo.” Therefore temples (i.e. churches) ought to be built “in such a manner and with such proportions, that all the parts together may convey a sweet harmony (he repeats: *una soave armonia*) to the eyes of the beholders.” Palladio professed here his belief in the old Platonic macrocosm-microcosm conception which was effective throughout the Middle Ages and reasserted itself with an æsthetic bias in the Platonic circles of the Renaissance. Daniele Barbaro is still more explicit on this point. He comments on the divine power of proportion, from which all beauty results, in the words: “Divina è la forza de' numeri tra se con ragione comparati,” and continues: “One can say that neither in the structure of this world nor in the microcosm is there anything more extensive and full of dignity than propriety of weight, number and measure from which time, space, movement, virtue, speech, art, nature, knowledge, in short everything divine and human is composed, has grown, and has been perfected.”<sup>5</sup>

When Palladio used expressions like “sweet harmony,” he did not think of a vague, indefinable appeal to the eye, but of the spatial consonances pro-

<sup>1</sup> *Ibid.*, IV, chap. 5. Here is Palladio's own theoretical justification for our analysis of S. Francesco della Vigna; cf. above, Vol. VII, p. 122.

<sup>2</sup> *Ibid.*, IV, chap. 2. Cf. also Scamozzi, *Idea dell'architettura universale*, 1615, II, p. 36 with reference to Aristotle.

<sup>3</sup> For the passages referring to the mysticism of the sphere in Ficino and Giorgio, cf. F.

Mahnke, *Unendliche Sphäre und Allmittelpunkt*, 1937, pp. 59 ff., 106 f.

<sup>4</sup> Cf. Vitruvius' sober description of circular temples (bk. IV, chap. 8).

<sup>5</sup> Vitruvius, ed. Barbaro, 1556, III, preface, p. 57. Our translation follows not the text of this first edition, but Barbaro's revised wording in later editions.

duced by the interrelation of universally valid ratios. The memorandum, which he was asked to write about the new design for the Cathedral of Brescia in 1567, is mainly concerned with proportions, and one sentence shows that his conception of harmony was near to that of Francesco Giorgio: "The proportions of the voices," he said, "are harmonies for the ears; those of the measurements are harmonies for the eyes. Such harmonies usually please very much, without anyone knowing why, excepting the student of the causality of things."<sup>1</sup> We may safely surmise that Palladio regarded himself as one who knew why. The parallelism of musical and spatial proportions is a commonplace, and yet, against the background of a mass of other contemporary material, it appears to be more than a simile.

Apart from these few general hints there is hardly a word on the principles of proportion in Palladio's *Quattro libri*. The work abounds in definite statements about proportions without any explanation why a particular ratio, and not another, has been chosen. Proportion is, it need hardly be said, the whole issue of any system of the orders, and Palladio opens the account of his celebrated system of the orders with the laconic remark that they must be related "con bella proportione" to the whole building.<sup>2</sup> Behind Palladio's matter-of-fact rules there is often more thought and accumulated wisdom than might be apparent to the modern reader. In one case of central importance this may be demonstrated.

### *The Mean Proportionals and Architecture*

Palladio supplies general rules of proportion for the height of rooms to their width and length, that is for the relationship of those three dimensions which constitute the shape of a room. Before discussing this important subject he gives what he considers the most beautiful ratios of width to length of rooms, that is he talks in terms of two dimensions. For various reasons it seems opportune to follow Palladio's plan although we shall concentrate on his ratios of three quantities. Palladio recommends seven shapes of rooms in the following sequence: (1) circular, (2) square, (3) the diagonal of the square for the length of the room, (4) a square and a third, i.e. 3:4, (5) a square and a half, i.e. 2:3, (6) a square and two-thirds, i.e. 3:5, (7) two squares, i.e. 1:2. With the exception of the third case, all these ratios are commensurable and as simple as possible.<sup>3</sup> However, the diagonal of a square in relation to its side is  $\sqrt{2}:1$ . The shapes of the rooms recommended by Palladio show him following in the footsteps of his predecessors. Similar lists of approved shapes for rooms were given by Alberti<sup>4</sup> and Serlio<sup>5</sup> and both mention the incommensurability of the diagonal of the square while Palladio, with his usual restraint, does not make the point. As far as we can see this is

<sup>1</sup> Cf. Magrini, *Memorie intorno Andrea Palladio*, 1845, Appendix, p. 12: "... secondo che le proportioni delle voci sono armonia delle orecchie, così quelle delle misure sono armonia degli occhi nostri, la quale secondo il suo costume sommamente diletta, senza sapersi il perchè fuori che da quelli che studiano di sapere le ragioni delle cose."

<sup>2</sup> Bk. I, chap. 11.

<sup>3</sup> Bk. I, chap. 21. For the ratios 1:1, 1:2, 2:3, 3:4 the reader can be referred back to earlier parts of this article, the ratio 3:5 will be discussed later.

<sup>4</sup> Cf. below, p. 78.

<sup>5</sup> *Libro primo d'architettura*, in the Venetian ed. of 1560-62, p. 15.

the only irrational number of importance involved in the Renaissance theory of architectural proportion.<sup>1</sup> It came straight out of Vitruvius,<sup>2</sup> where its occurrence—amidst a module system, which otherwise presupposes commensurable ratios—has been thought with good reason to be a residue of the Greek architectural theory of proportion, all but forgotten in Roman times.<sup>3</sup> Palladio—and probably all the other Renaissance architects—never used in practice the irrational proportion of his theory, and this is an argument *per negationem* in favour of the case we are going to state.

When we turn to the relations of three magnitudes, the theoretical position is surprisingly simple. Palladio declares three different sets of ratios for height to width and length to be good proportions for rooms.<sup>4</sup> For each of the three cases he gives a method of calculating the height from the length and width by a geometrical as well as by an arithmetical process. We need not follow his procedure; it will suffice to record the result. His first example: suppose a room measures  $6 \times 12$  feet; its height will be 9 feet. Second example: a room is  $4 \times 9$  feet;<sup>5</sup> its height will be 6 feet. Third example: a room is again  $6 \times 12$  feet; its height will be 8 feet. In his exposition Palladio sticks strictly to the practical side of the *métier* without mentioning what these proportions really signify. The figure for the height of the room in his three examples represents the arithmetic, geometric and 'harmonic' mean between each of the two extremes. These three types of proportion are traditionally attributed to Pythagoras<sup>6</sup> and without them no rational theory of proportion can be imagined.

In the arithmetic proportion the second term exceeds the first by the same amount as the third exceeds the second ( $b-a=c-b$ , as in  $2:3:4$ , i.e. Palladio's first example); in the geometric proportion the first term is to the second as the second to the third<sup>7</sup> ( $a:b=b:c$ , as in  $4:6:9$ , i.e. Palladio's second example). The formula for the 'harmonic' proportion, Palladio's third case, is more complicated. What we now call three terms in 'harmonic' proportion are defined in the *Timaeus* (36) as "the mean exceeding one extreme and being exceeded by the other by the same fraction of the extremes." In other words, three terms are in harmonic proportion when the distance of the two extremes from the mean is the same fraction of their own quantity (i.e.  $\frac{b-a}{a} = \frac{c-b}{c}$ ). In Palladio's example  $6:8:12$  the mean 8 exceeds 6 by  $\frac{1}{3}$  of 6 and is exceeded by 12 by  $\frac{1}{3}$  of 12 (i.e.  $\frac{8-6}{6} = \frac{12-8}{12}$ ).

Ficino in his Commentary to the *Timaeus* had discussed the three means

<sup>1</sup> For the diagonal of the square in Francesco di Giorgio's theory, cf. his *Trattato di architettura*, ed. Promis, 1841, p. 57 f.

<sup>2</sup> VI, iii, 3.

<sup>3</sup> Cf. Jay Hambidge, *The Parthenon and other Greek Temples*, 1924, p. 2 f. Also *id.*, *Dynamic Symmetry*, 1920, p. 145.

<sup>4</sup> Bk. I, chap. 23. Compare Alberti's much more complicated answer to the same problem (Bk. IX, chap. 3). Scamozzi, on the other hand, simplified Palladio still further; in the five types of perfectly shaped rooms recommended by him the height is always the arithmetic mean between width and

length (*Idea dell' arch. univ.* I, p. 308 f.). This is typical of Scamozzi's academic transformation of Palladio's precepts.

<sup>5</sup> Concerning this ratio which does not occur amongst Palladio's seven approved shapes of rooms, cf. below, p. 78.

<sup>6</sup> Cf. Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, 1907, I, p. 166; Sir Thomas Heath, *A History of Greek Mathematics*, 1921, p. 85.

<sup>7</sup> The definitions follow Porphyry's *Commentary on Ptolemy's Harmonics*, cf. Ivor Thomas, *Selections illustrating the History of Greek Mathematics* (Loeb Class. Libr.), 1939, I, p. 113.

very clearly at considerable length,<sup>1</sup> and possibly through him they became of overwhelming importance in Renaissance æsthetics. In the Venetian circle of Palladio's time they were examined by Giorgio<sup>2</sup> as well as by Daniele Barbaro,<sup>3</sup> but it seems probable that Palladio's source was Alberti who had treated of them in terms more easily accessible to an architect.<sup>4</sup>

Before explaining the three types of means Alberti discussed the correspondence of musical intervals and architectural proportions. With reference to Pythagoras he stated that "the numbers by means of which the agreement of sounds affects our ears with delight, are the very same which please our eyes and our minds,"<sup>5</sup> and this doctrine is fundamental to the whole Renaissance conception of proportion. Alberti continues: "We shall therefore borrow all our rules for harmonic relations ("finitio") from the musicians, to whom this sort of numbers is extremely well known, and from those particular things wherein Nature shows herself most excellent and complete."<sup>6</sup> Thus, for Alberti harmonic ratios inherent in nature, are revealed in music. The architect who relies on those harmonies does not translate musical ratios into architecture, but makes use of a universal harmony apparent in music.

Alberti as well as later artists were, no doubt, conscious of the fact that the musical consonances are determined by the mean proportionals;<sup>7</sup> for that the three means constitute all the intervals of the musical scale had been shown in the *Timaeus*.<sup>8</sup> Classical writers on musical theory discussed this point at great length. An exhaustive exposition is to be found in Boethius' *De musica*,<sup>9</sup> first printed in Venice in 1491-2, and of very great importance for the doctrine of numbers during the Renaissance. Francesco Giorgio, reinterpreting the *Timaeus*, summarized the position, based on the relevant chapters of Ficino's commentary.<sup>10</sup> In order to be able to find the 'harmonic' and arithmetic means

<sup>1</sup> *Opera*, Basle, 1576, II, p. 1454 f.: "Item comparationem eiusmodi esse triplicem, scilicet arithmetica, geometricam, harmonicam. Arithmeticam in numeri paritate consistere. Sic inter tria & septem medius est quinarium, numero eodem, scilicet binario alterum terminum superans, ab altero superatus, per proportionem utrinque bipartientem. Geometricam vero in rationis aequalitate sitam esse, in qua sunt multiplex atque superparticularis: quando videlicet ita comparamus, sicut se habent tria ad novem, ita novem ad septem atque viginti, nam utrobique tripla. Item quod est novenarium iuxta senarium, idem est senarium iuxta quaternarium. Nam et hic et ibi est proportio sesquialtera. Denique proportionem harmonicam in quadam similitudine collocant, per quam tribus terminis in ordinem positus, sicut maximus terminus aspicit minimum, similiter differentia inter terminos maior minorem respicit differentiam. Sic enim ponas tria, quatuor, sex, differentia inter sex and quatuor est binarius: differentia inter quatuor & tria, unitas, sicut autem inter sex

& tria dupla ratio est, ita inter duo & unum est ratio dupla. Viget hic altera quoque similitudo, scilicet portionum: Simili namque extremorum portione medius terminus excedit atque exceditur."

<sup>2</sup> *Harmonia mundi*, I, v, fol. 82.

<sup>3</sup> Comm. to Vitruvius, bk. III, preface.

<sup>4</sup> *De re aed.*, bk. IX, chap. 6.

<sup>5</sup> *Ibid.*, chap. 5.

<sup>6</sup> Ed. of 1485, fol. yii verso: "Ex musicis igitur quibus ii tales numeri exploratissimi sunt: atque ex his praeterea quibus natura aliquid de se conspicuum dignumque praestat tota finitionis ratio producetur."

<sup>7</sup> However, not every proportion with a mean results in a musical consonance. Alberti was well aware of this; cf. his introductory passages to the mean proportionals in IX, chap. 6.

<sup>8</sup> 35 C, 36; cf. Cornford, *op. cit.*, p. 70 f.

<sup>9</sup> Ed. Oscar Paul, Leipzig, 1872, bk. II, chaps. 12-17.

<sup>10</sup> Ficino, *op. cit.*, II, particularly p. 1461 f., chap. 36.

as whole numbers between the terms of Plato's original series (1, 2, 4, 8 and 1, 3, 9, 27), he suggests using 6 as the lowest term. By multiplication with 6 we get the series 6, 12, 24, 48 and 6, 18, 54, 162. Into these series the means can be inserted without fractions. "For the means lying between 6 and 12 are 8 and 9, where 9 is exceeded and exceeds by the same quantity. But 8 exceeds and is exceeded by the same fraction of the extremes themselves. Between 12 and 24 the means are 16 and 18, between 24 and 48 they are 32 and 36. One set of means is 'harmonic', the others are arithmetic and geometric (namely 6, 12, 24, 48)."

So far Giorgio was only concerned with the mathematical definition of the means. Now follows their application to musical theory: "However, they all belong to harmony. For the ratio of the greater extreme to the smaller is a double proportion and makes the diapason  $/6:12/$ . From the minor extreme to the major mean is a sesquialtera proportion and makes the diapente  $/6:9/$ . But from the same extreme to the minor mean is a sesquitercia proportion and makes the diatessaron  $/6:8/$ . And the same results from the ratio of the major mean to the major extreme  $/9:12/$ . A diapente results from the minor mean to the major extreme  $/8:12/$ . From one mean to the other results a sesquioctave, which makes a tone  $/8:9/$ .<sup>1</sup> . . . In the same way is related the other side (*scil.* of the triangle of numbers) which is multiplied by 3, where the first extremes are 6 and 18 between which the means are 9 (harmonic) and 12 (arithmetic). The other extremes are 18 and 54 between which lie the means 27 and 36. On the other hand, the means between 54 and 162 are 81 and 108. . . . From the mass which has been written this much may be added about what the geometric, arithmetic and harmonic means are: The arithmetic mean is the proportion of excess, the geometric mean is the proportion *par excellence* ('proportio proportionum').<sup>2</sup> From the two, results the harmonic mean.<sup>3</sup> . . . With these rules all the intervals can be filled, short of the semitones and quarter-tones. . . ."<sup>4</sup>

Thus the geometric progression constitutes the octaves, and the 'harmonic' and arithmetic means determine the intervals of the fourth and fifth and the tone. The interlocking of these ratios is well shown in Giorgio's diagram (see Pl. 18b). Whenever one meets ratios of the series 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, etc., it is safe to presume that this is not casual but the result of reflections which depend directly or indirectly on the Pythagorean-Platonic division of the musical scale. And when Palladio recommends one of the three means—and only these—for the height of his rooms, he was, no doubt, aware that "they belong to harmony."

<sup>1</sup> 8:9 is the interval between fourth and fifth for:  $\overbrace{6 : 8 : 9 : 12}^{\text{fifth}}$ , and is  $\overbrace{6 : 8}^{\text{fourth}} : \overbrace{9 : 12}^{\text{fourth}}$  and is therefore the ratio of the major tone.

<sup>2</sup> Regarded as such in antiquity, for a geometric progression from unity to squares and cubes represents line, surface and solid.

See T. L. Heath, *The thirteen Books of Euclid's Elements*, 1908, II, p. 292 f.

<sup>3</sup> This will be understood by comparing the three formulas (1)  $a-b=b-c$  ("proportion of excess"), (2)  $\frac{a}{b} = \frac{b}{c}$  ("proportio proportionum"), (3)  $\frac{a-b}{a} = \frac{b-c}{b}$  ('harmonic' proportion).

<sup>4</sup> Giorgio, *op. cit.*, V, iii, p. 82 f.

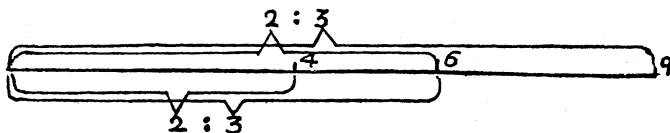
*Alberti's 'Generation' of Ratios*

The Renaissance identification of musical and spatial ratios was only possible on the basis of a peculiar interpretation of space which, as far as we can see, has not been properly understood in modern times. When Francesco Giorgio called the relation of width to length of the nave of S. Francesco della Vigna a diapason and diapente he expressed the simple ratio 1:3 (9:27) in terms of the compound ratio 1:2, 2:3 (9:18, 18:27). Is this only a theoretical application of musical ratios to space or does it imply a particular mode of space perception? If we suppose the latter it would mean that for Giorgio the length of the nave is not simply a triplication of its width, but that the length itself is charged with definite relations; for one unit (9) is seen in relation to its duplication, and the two units together (18) are visualized in relation to the whole length of three units (27). A graph of the two different approaches makes the position abundantly clear:



Giorgio perceives the length like a monochord, on which by stopping at  $\frac{1}{2}$  and  $\frac{2}{3}$  of its length respectively the octave and the fifth are produced. That for Giorgio these intervals are not theoretical breaks is proved by the fact that they coincide with important *cæsuras* in the building: the first unit of 9 with the centre of the central chapel and the second unit, 18, with the end of the nave proper. With this kind of generation of the ratio 9:27 Giorgio expressed himself in a language which was generally understood in his day. He expounded a method for which Alberti had laid the theoretical foundation.

Alberti differentiates three types of plans, small, medium and large ones.<sup>1</sup> Each type can be given three different shapes. To the small plans belong the square (2:2) and shapes of one to one and a half (2:3) and one to one and a third (3:4). These ratios comply with the simple musical consonances and need no further explanation. Medium-sized plans 'duplicate' the ratios of small plans, i.e. one to two, one to twice one and a half and one to twice one and a third. With these more complicated ratios the matter becomes very interesting. To draw a plan of one to twice one and a half, the architect puts down a unit which we may call 4, extends it until the ratio one to one and a half is reached, i.e. 4:6, and adds to this another ratio of one to one and a half, i.e. 6:9; the result is a ratio of 4:9.<sup>2</sup> In other words, Alberti anticipates exactly Francesco Giorgio's procedure, for he breaks up the ratio 4:9 into two 'basic' ratios of 2:3 in the following manner:



<sup>1</sup> Bk. IX, beginning of chap. 6.

<sup>2</sup> Cf. ed. of 1485, fol. viii.

We may now say that the ratio of 4:9 is generated from the two ratios 4:6 and 6:9. In the same way the ratio of one to twice one and a third, 9:16, is generated from 9:12:16, for  $9:12=1:1\frac{1}{3}$  and  $12:16=1:1\frac{1}{3}$ . The three classes of large plans are formed first, by adding to the double square, 2:4, one half so that the proportion 1:3 is generated from 2:4:6, secondly, by adding to the double square, 3:6, one third so that the proportion 3:8 is generated from 3:6:8 and, thirdly, by doubling the double square so that the quadruple proportion 2:8 is generated from 2:4:8. Now, the double proportion 1:2 (musically an octave) is a composite of the two ratios 2:3 and 3:4 (for  $\frac{1}{2}=\frac{2}{3}\times\frac{3}{4}$ ) so that it is generated from 2:3:4 or 3:4:6 (musically from fifth and fourth or fourth and fifth). We can now say that, for instance, the proportion of 1:4 is generated from 2:3:4:8, or 2:3:4:6:8 (i.e. from fifth and fourth, and fifth and fourth), or 3:6:9:12, or 3:4:6:9:12 (i.e. from fourth and fifth, and fifth and fourth), etc. For Alberti the splitting up of compound proportions into the smallest harmonic ratios is not an academic matter, but a spatial experience as his explanation of the architect's procedure shows when planning the proportion 4:9. Harmonic ratios like the double, the triple and the quadruple are compounds of simple consonant ratios. Alberti is very explicit that sub-ratios of a compound ratio cannot be used indiscriminately by architects; they must be exactly those ratios which belong to the compound ratio. If one wants, for instance, to build the wall of a room, the length of which is double its width, one would not use for the length the sub-ratios of the triple proportion, but those of which the double is compounded. The same is true for a room in the proportion of one to three; no other than the numerical relations of which the triple is composed, should be used.<sup>1</sup>

The splitting up of ratios for the sake of making the proportions of a room harmonically intelligible appears to us very strange. And yet, this is the way the whole Renaissance conceived of proportions. A wall is seen as a unit which contains certain harmonic potentialities. The lowest sub-units, into which the whole unit can be broken up, are the consonant intervals of the musical scale the cosmic validity of which were not doubted. In some cases only one way of generation is possible, but in others two or even three different generations of the same ratio can be carried through; as we have seen, the ratio 1:2, the octave, can be seen as fourth and fifth (3:4:6) or as fifth and fourth (2:3:4). But the ratios of the musical intervals are only the raw material for the combination of spatial ratios. Alberti's harmonic progressions 4:6:9 and 9:12:16 are a sequence of two fifths and two fourths respectively,

<sup>1</sup> Ed. of 1485, fol. yiii: "His numeris quales recensimus utuntur architecti non confuse et promiscue: sed correspondentibus utrinque ad armoniam. Utique parietes velit attollere in area fortassis: cuius longitudo sit ad sui latitudinem dupla: is istic utatur respondentibus non quibus tripla: sed iis tantum: quibus eadem ipsa componatur dupla. Acque itidem sequetur in area tripla: nam suis quoque utetur respondentibus: utetur inquam non aliis quam suis. Itaque diffiniat

diametros ternatim numeris quos recensimus: uti accommodatiores eos venire suum ad opus intelligat."

Alberti's conception of the generation of ratios, made abundantly clear in his text, has been misunderstood by the students of his theory; cf. I. Behn, *L. B. Alberti als Kunstphilosoph*, 1911, particularly p. 104, and Paul-Henri Michel, *La pensée de L. B. Alberti*, Paris, 1930, p. 454 f.

i.e. musically they represent dissonances. The ratios of musical intervals are regarded as binding and not the build-up of consonant intervals to musical harmonies. Nothing shows better than this that the Renaissance did not endeavour to translate music into architecture, but regarded the consonant intervals of the musical scale as the audible proofs for the beauty of the ratios of the small whole numbers 1:2:3:4.

In analysing the proportions of a Renaissance building one has to take the principle of generation into account. It can even be said that, without it, it is impossible fully to understand the intentions of a Renaissance architect. We are touching here on fundamentals of the style as a whole; for simple shapes, plain walls and clear divisions are necessary presuppositions for that 'polyphony of proportions' which the Renaissance mind understood and a Renaissance eye was able to see.

### *Musical Consonances and the Visual Arts*

From all that has been said so far it will be realized that the Renaissance analogy of audible and visual proportions was more than a theoretical speculation; it testifies to the solemn belief in the harmonic mathematical structure of all creation. But beyond this, music had a peculiar place in the scheme of the liberal arts. It was indeed the only respectable "liberal art" ranking amongst the *quadrivium* of the mathematical arts and with an unbroken tradition coming down from antiquity. Painting, sculpture and architecture, on the other hand, had to be raised by Renaissance theorists and through the agency of mathematics from the level of mechanical to that of liberal arts. No wonder that in the closely interrelated encyclopædia of the arts the mathematical foundation of music was regarded as exemplary for the other arts and a familiarity with musical theory became a *sine qua non* of artistic education.<sup>1</sup> The few scattered notes of this section are meant to prepare the reader for the problems which will be found when analysing the proportions of Palladio's buildings.

It comes as a confirmation of one's expectations to find that Brunelleschi, according to his biographer Manetti, studied the musical proportions of the ancients.<sup>2</sup> Manetti, writing after 1471 and under the influence of Alberti, may have read these ideas into the past; his remark, in any case, shows how acutely aware his generation was of the importance of the problem, and this is also illustrated by Alberti's famous warning to Matteo de' Pasti during the erection of S. Francesco at Rimini that by altering the proportions of the pilasters "si discorda tutta quella musica."<sup>3</sup> Nobody expressed his belief in the efficacy of harmonic ratios behind all visual phenomena with more conviction than Leonardo. We may recall in particular his well-known saying that music is the sister of painting. This was not meant as a simile but in a literal sense; for both, music and painting, convey harmonies; music does it by its chords and painting by its proportions. Musical intervals and linear

<sup>1</sup> For Palladio's musical education in Trisino's circle, cf. part I of this article in Vol. VII, p. 103 f.

<sup>2</sup> Manetti, *Filippo Brunellesco*, ed. Holtzinger, 1887, p. 16.

<sup>3</sup> Corrado Ricci, *Il Tempio Malatestiano*, 1924, p. 587.

perspective are subject to the same numerical ratios, for objects of equal size placed so as to recede at regular intervals diminish in 'harmonic' progression.<sup>1</sup>

The "exactissima harmonia" of the human body was the subject of Pomponius Gauricus' *De sculptura*, 1503. Gauricus asks "what geometrician, what musician must he have been who has formed man like that?"—thus apostrophizing the fundamental unity of geometry and music. The *Timaeus*, quoted more than once by Gauricus, seems also for him the book of wisdom in which the mystical harmony in the universe was revealed. It is well known that these ideas remained alive throughout the sixteenth century. Lomazzo, in his scholastic *Trattato dell'arte della pittura* (1584), talked of human proportions in musical terms. He carried on a habit of thought which is first to be found in Alberti's writings. Alberti interprets, for instance, the ratio 4:9 which he analysed as the product of two ratios of  $1:1\frac{1}{2}$ ,<sup>2</sup> also as a double proportion (4:8) plus one *tone* (8:9)<sup>3</sup> and the ratio 9:16, generated from two ratios of  $1:1\frac{1}{3}$ , also as a double proportion (9:18) minus a *tone* (18:16).<sup>4</sup> Failing an algebraic symbolism, musical terminology was ready at hand for an adequate description of proportions. In the same vein Lomazzo regarded the applicability of musical terms to the proportions of the body as so self-evident that he not only omitted a discussion of the common laws of musical and spatial proportions, but refers constantly to spatial ratios as if they were an acoustic experience. For instance, the distance from the top of the head to the nose "*risuona con lo spazio che è da quivi al mento in proporzione tripla, onde riesce la diapason, e diapente*; ed a detto spazio che è fra il naso e'l mento, quello che è dal mento alla fontanella, viene a *risuonare* in proporzione doppia che fa la *diapason* . . ."<sup>5</sup>

In his later *Idea del Tempio della Pittura* (1590) Lomazzo formulated theoretically what was implied in the quotation we have just given. Here he declared that masters like Leonardo, Michelangelo and Gaudenzio Ferrari "have come to the knowledge of harmonic proportion by way of music";<sup>6</sup> the human body itself is built according to musical harmonies. This microcosm "fatta dal grande Iddio a simiglianza della sua Immagine" contains "all numbers, measures, weights, motions and elements." Therefore all the buildings in the world together with all their parts follow its norm.<sup>7</sup> A story told by Lomazzo in another context shows that these ideas were common knowledge and generally approved.<sup>8</sup> The architect Giacomo Soldati added to the three Greek and two Roman orders a sixth "which he calls harmonic and which by sound he makes intelligible to the ear, but it can hardly be noticed by the eye; with this order he wanted to imitate the ancients who

<sup>1</sup> Cf. J. P. Richter, *The literary Works of Leonardo da Vinci*, 1939, I, pp. 72 f., 76 ff.

<sup>2</sup> Cf. above, p. 78.

<sup>3</sup> *De re aed.*, 1485, fol. yiii: "Excedat igitur longitudo maxima istic brevissima ex dupla atque amplius ex duplae tono."

<sup>4</sup> *Ibid.*: "Ergo hic maior linea exceditur a dupla minoris uno minus tono."

<sup>5</sup> *Trattato*, ed. 1844, I, p. 63 f.

<sup>6</sup> 2nd ed., p. 113.

<sup>7</sup> *Ibid.*, p. 117. The whole passage is quoted above in Italian, p. 68, note 2.

<sup>8</sup> It may be recalled that Lucio Faberio in his commemoration speech at Agostino Carracci's funeral tells us that the latter was a student of philosophy, arithmetic and geometry, astrology and, above all, of music; arithmetic, the foundation of music, taught him the origin of the musical consonances. Cf. Malvasia, *Felsina Pittrice*, 1841, I, p. 308.

not less by sound than by design and building made known to the world the harmony of their five orders."<sup>1</sup>

Not much is known about this Giacomo Soldati.<sup>2</sup> However, he must have been a man of great reputation in his own days. In 1561 he was, together with Pellegrino Pellegrini, "architetto della Regia Camera dello Stato" at Milan and in 1570 he belonged to the court of arbitration which had to decide about the attacks by Bassi against Pellegrini. Six years later he was appointed court architect to Emanuele Filiberto of Savoy and he seems to have spent the last fifteen years of his life in that position at Turin. He made his name particularly with engineering and hydraulic works; his whole career shows that the man was no visionary but rather a sober scientist.

Palladio himself had contacts with Soldati's circle. He was one of those architects who had sent a report in answer to Bassi's inquiry in the above-mentioned quarrel with Pellegrini;<sup>3</sup> before that date he had lived and worked in Turin<sup>4</sup> and as an expression of his gratitude for the excellent treatment at the court he dedicated the 3rd and 4th books of the *Quattro libri* to Emanuele Filiberto "who alone in our era through his prudence and valour is like the ancient Roman heroes." Under Emanuele Filiberto Turin became perhaps the most vigorous intellectual centre in Italy and it must have been on account of no common qualities that Soldati was appointed ducal architect.

Soldati's harmonic order is unknown, but the impulse for his undertaking is not too difficult to guess. A sixth order which should embrace all the qualities of the other orders and express more clearly than they did the basic harmonies of the universe became a preoccupation of architects. This order was believed to have been originally inspired directly by God when He ordered Solomon to build the Temple, and architects attempted to re-create the perfection of this lost archetype from which all the other orders were thought to be derived. Shortly after Soldati this idea was developed with an almost unbelievable amount of scholarship in Giovan Battista Villalpando's reconstruction of the Temple, a work which had a continuous and international influence on architects.<sup>5</sup> The Temple of Jerusalem was an obvious focusing point for the cosmological-æsthetic theories of proportion.<sup>6</sup> Here was a test-case for the philosophical endeavour of the Renaissance of reconciling Plato and the Bible; for was it not God Himself who had enlightened Solomon

<sup>1</sup> *Ibid.*, p. 30: "... il sesto (ordine) ... che egli chiama armonico, e col suono facilmente lo fa sentir a l'orecchie, ma agli occhi stenta rappresentarlo, volendo in questo imitar gl'antichi che non meno sonando, che disegnando, e fabbricando fecero conoscere al mondo l'armonia dei suoi cinque ordini."

<sup>2</sup> Cf. Thieme-Becker, *Künstler-Lex.*, for further literature.

<sup>3</sup> Dated July 3, 1570 and often printed; first in Martino Bassi's *Dispareri in materia d'architettura, et prospettiva*, Brescia, 1572, pp. 42-45. For an analysis of this letter cf. Panofsky, "Die Perspektive als 'symbolische Form'," *Vorträge der Bibl. Warburg*, 1924-25,

p. 325 f.

<sup>4</sup> Cf. Temanza, *Vita di Andrea Palladio*, 1762, p. 45. Magrini, *op. cit.*, pp. 112, 249, xlviii.

<sup>5</sup> H. Prado and G. B. Villalpando, *In Ezechielem Explanationes*, 1596-1604. The second volume by Villalpando contains the reconstruction of the Temple. On the influence of this work cf. Wittkower in this *Journal*, VI, 1943, p. 221. Villalpando's new order was incorporated into a number of seventeenth and eighteenth century treatises on architecture.

<sup>6</sup> Cf. above, p. 69.

to incorporate the numerical ratios of the celestial harmony into his building? We therefore find in Villalpando what we would expect: his system of harmonic architecture is absolutely watertight. After discussing the three mean proportionals and after insisting on harmonic proportions throughout the building, he winds up with the familiar reference to music.<sup>1</sup> He follows explicitly Barbaro's Commentary on Vitruvius in accepting only the Pythagorean three simple and two composite consonances<sup>2</sup>—diatessaron, diapason, diapente, and diapason cum diapente, disdiapason—and rejecting Vitruvius' sixth consonance, diapason cum diatessaron.<sup>3</sup> An impressive example of the orthodox use of these five consonances is the relation of the parts of the entablature and of the triglyphs to the metopes "in domo domini," "in atriis" and "in domo regia" of the Temple. A glance at the diagram (Pl. 18c)<sup>4</sup> showing the ratios between triglyphs and metopes reveals the harmonic interrelation not only inside one order, but also between the orders of the three parts of the Temple. Simple consonances were chosen for the ratios of the same order (2:3) and those between triglyphs of one order and the next, between metopes of one order and the next (1:2), and between the triglyph of a larger and the metope of a smaller order (3:4). The ratio between a metope of a larger and a triglyph of a smaller order is based on composite consonances (1:3), and so is the ratio between the largest and the smallest order (1:4). Thus, the ratios between the triglyphs and metopes of the three orders express the five musical consonances, and no other ratios than these are possible.

It may be argued that the speculations of a counter-reformatory theologian,<sup>5</sup> in which the spirit of the Middle Ages was peculiarly revived, have nothing in common with the work of architectural practitioners and that an unbridgeable gulf separates them from a book like Palladio's *Quattro libri*. Admittedly, there lies a whole world between the architect of the Venetian nobility of the mid-sixteenth century and the Spanish Jesuit of the next generation; and yet, the difference is one of emphasis rather than of fundamentals. It is therefore not surprising that the bulkiest architectural treatise written in Italy by Palladio's pupil Scamozzi in the same Venetian *ambiente*, but at Villalpando's time, is heavy, dogmatic and scholastic and reads, compared with Palladio's *Quattro libri*, like a mediaeval exposition of the subject. In the system of the Liberal Arts with moral and natural philosophy as "nutrice di tutte le scientie" music is given its old place in the *quadrivium* "delle Matematiche." It is divided into "musica theoricale" which is concerned with the harmonies of the spheres, and "musica naturale" which is concerned with the sound of voices and instruments. A knowledge of music makes the architect acquainted with the reasons for the consonances and dissonances of sounds. The whole circle of related ideas is revived by Scamozzi;

<sup>1</sup> *Op. cit.*, II, p. 458: "Illud vero huius aedificij proprium est, ac maxime observandum, idque ad harmonicam proportionem spectat; quod non alia ratione visus huius aedificij partibus contemplandis delectari videatur, quam auditus vocum, aut instrumentorum suavi modulatione perfruatur."

<sup>2</sup> Barbaro, ad Bk. V, iv, 7. Cf. also Alberti,

*de re aed.*, IX, chap. 5.

<sup>3</sup> Based on Ptolemy; cf. I. Düring, *Ptolemaios und Porphyrios über die Musik*, Göteborg, 1934, p. 29 (5, ii).

<sup>4</sup> *Op. cit.*, p. 449.

<sup>5</sup> About Villalpando cf. A. de Backer, *Bibl. des écrivains de la Compagnie de Jésus*, 1876, III, p. 1407.

he dwells at length on the importance of the Platonic numbers, on the anthropomorphic character of architecture, and, with reference to Aristotle, on the "regola homogenea," the modulus, which must be effective throughout the building, inside as well as outside.<sup>1</sup>

Not only Palladio himself but also the other architects of his generation were less eloquent than the late Mannerist Scamozzi. Vignola's *Regole delle cinque ordini* (1562) has no text at all and only a short introduction. But here too we find the analogy of musical and architectural proportions. His effort at systematizing the orders was centred in the problem of finding even for the smallest members "certa corrispondenza et proportione de' numeri insieme." The satisfaction attained by such a system has been proved by the theory of music.<sup>2</sup> In spite of a hundred years of theoretical studies by architects, Vignola was still convinced that music as a science was better founded than architecture. The leading musical theorist of the period, Gioseffo Zarlino, maintained in the dedication of his *Dimostrazioni harmoniche* (1571) that "per la certezza della Dimostrazione" music without any doubt was superior to architecture. It was Vignola's aim to give architecture a "certezza" of ratios equal to those in music.

The certainty inherent in mathematical deduction had always been the basis of musical theory. Franchino Gafurio, the famous Renaissance musical theorist, made this the subject of the frontispiece to his *De harmonia musicorum instrumentorum* of 1518 (Pl. 19a). He is shown lecturing to his pupils; on the left are three organ pipes of different length, marked 3, 4, 6, illustrating the ratios of the octave divided by the harmonic mean 4 into fourth and fifth. On the right there are three lines, repeating the ratios 3, 4, 6, and a pair of dividers, thus indicating that musical harmony is geometry translated into sound. At the same time the picture propounds the old thesis that harmony results not from the consonance of two tones, but from two unequal consonances which are drawn from dissimilar proportions<sup>3</sup> (i.e. 3:4 and 2:3, fourth and fifth which together form the octave). That is the reason why Gafurio preaches to his pupils: "Harmonia est discordia concors" which appears written on a scroll near his mouth. Gafurio accepted Philolaos' Pythagorean definition of harmony,<sup>4</sup> which had such a far-reaching influence on Renaissance thought, and he regarded in a truly Platonic spirit this principle of harmony as the basis of macrocosm and microcosm, body and soul, painting, architecture and medicine.

Gafurio's earlier *Theorica Musicae* of 1492 had a striking frontispiece with a fuller illustration of musical consonances (Pl. 19b). The top left picture shows Tubalcain, the biblical founder of music, presiding over a forge where six smiths are busy hammering iron on the anvil. In the next picture Pythagoras beats bells and glasses filled with liquid to different heights. In

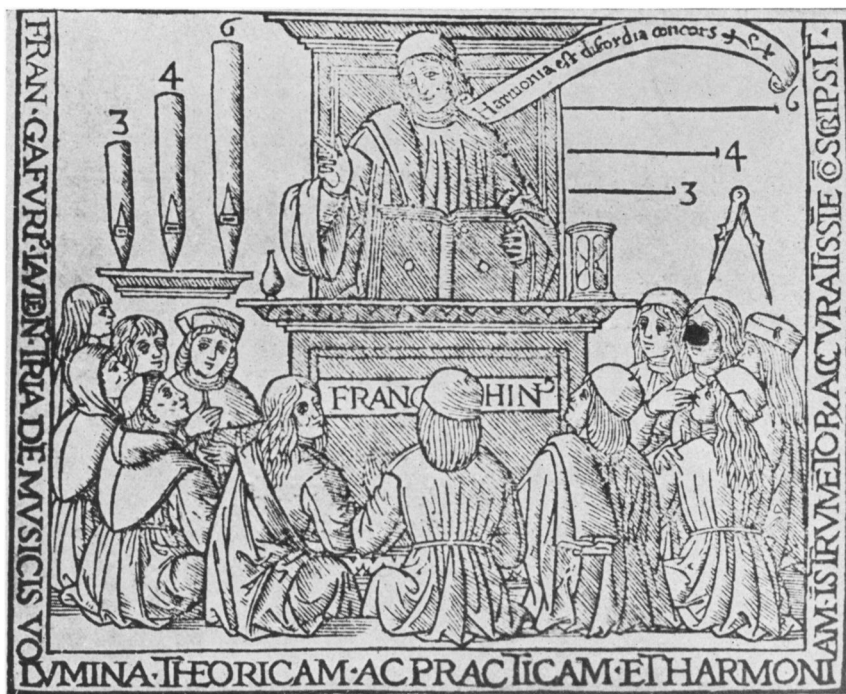
<sup>1</sup> Cf. Scamozzi, *Idea dell'architettura universale*, 1615, I, pp. 3, 23, 307 f., II, pp. 4, 31 f. and *passim*.

<sup>2</sup> From the preface to the *Regole*: Those orders are most beautiful which have "certa corrispondenza et proportione de' numeri insieme . . . Laonde considerando più adentro quanto ogni nostro senso si compiacchia in

questa proportione, et le cose spiacevoli essere fuori di quella come ben provano li Musici nella lor scienza . . ."

<sup>3</sup> Bk. III, chap. 11: Harmony "nempe duabus consonantiis inaequalibus constat, quae ex dissimilibus proportionibus . . . conducuntur."

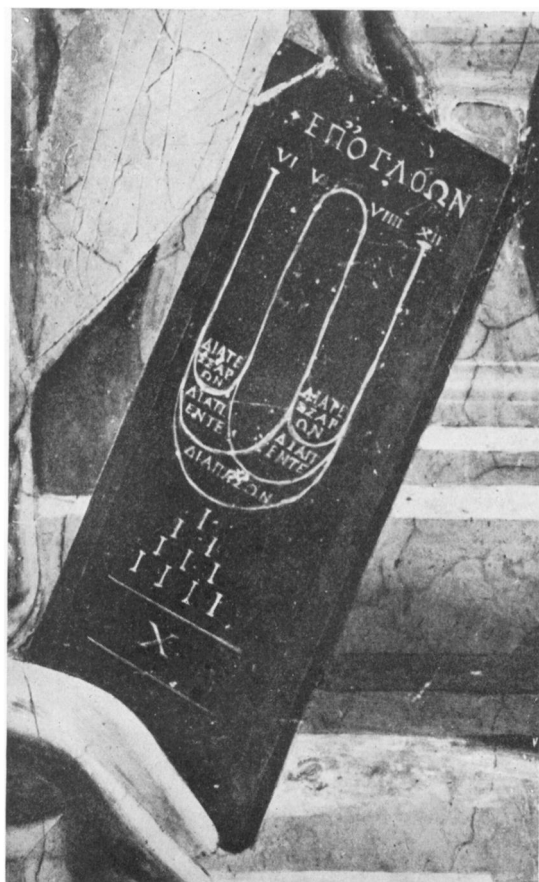
<sup>4</sup> Cf. above, p. 72, note 3.



a—Gafurio Lecturing. From F. Gafurio, *De harmonia musicorum instrumentorum*, 1518 (p. 84)



b—Tubalcain, Pythagoras, Philolaos. From F. Gafurio, *Theorica Musicae*, 1492 (p. 84f)



c—Pythagorean Musical Scale, Detail from Raphael's *School of Athens* (p. 85)

the lower row Pythagoras is shown beating strings to which weights of different size are fixed, and in the last picture Pythagoras and Philolaos appear with flutes. In all these cases the objects through which sound is produced bear the figures 4, 6, 8, 9, 12, 16, and the heads of the hammers, the bells, the liquid, the weights, the length of the flutes illustrate these ratios by the gradation of size. The figures comprise two octaves, the "Greater Perfect System" of the Greeks,<sup>1</sup> with their fourth and fifth and the major tone (8:9). Pythagoras is shown in the picture testing the consonance of the octave 8:16; in the last picture he does it in concert with Philolaos, one blowing a flute half as long as that of the other (8 and 16), while Philolaos holds two flutes expressing a fifth (4 and 6) and Pythagoras two others expressing a fourth (9 and 12). The whole page is an illustration of the discovery of the musical consonances by Pythagoras, and the designer followed almost verbally the story as reported in Boethius' *De musica*.<sup>2</sup>

What is shown with delightful naïveté in this somewhat barbaric woodcut was represented by Raphael on the tablet facing the figure of Pythagoras in the School of Athens. He gave here in an ingenious diagrammatic design of the four strings of the ancient lyra the whole system of the Pythagorean harmonic scale (see Pl. 19c).<sup>3</sup> This representation is interwoven with and expressive of Raphael's complex programme; however, it must suffice here to say that above the teacher Pythagoras appears the heroic figure of the great pupil carrying the *Timaeus* in one hand and pointing upward with the other. This is Raphael's interpretation of the harmony of the universe which Plato had unriddled in the *Timaeus* based on Pythagoras' discovery of the ratios of musical consonances.

We are back in the intellectual atmosphere which prompted Francesco Giorgio to apply the Pythagorean-Platonic system of harmonic ratios directly to architecture. And when Raphael mentions in a letter of 1514, that the Pope had appointed the aged Fra Giocondo as his architectural adviser so that he may learn, "whether he has some *bello secreto* in architecture," it seems not far-fetched to believe, that these secrets were more than mere technicalities.<sup>4</sup>

### *Palladio's 'fugal' System of Proportion*

In the eyes of the men of the Renaissance musical consonances were the

<sup>1</sup> Cf. above, p. 70.

<sup>2</sup> Bk. I, chaps. 10, 11.—It is not surprising to find that Gafurio seems to have been regarded by his contemporaries as a critic in architectural matters. In 1490 he was sent to Mantua to consult Luca Fancelli about the structure of the *tiburio* of the cathedral.

<sup>3</sup> At the bottom inside the "lyra" and in the arches connecting the first string with the second and the third, the fourth with the third and with the second and the first string with the last, are inscribed the words ΔΙΑΤΕΣΣΑΡΩΝ ΔΙΑΠΕΝΤΕ, and ΔΙΑΠΑΣΩΝ; at the top are the numbers VI, VIII, VIIII, XII, which show the ratios of the intervals. There

is an arch between 8 and 9 and above it stands the word ΕΠΙΟΓΛΩΩΝ, i.e. the tone. No more convincing diagrammatic system of the Pythagorean scale could be devised. It is worth recording that sheer logic led Zarlino in his *Istitutioni harmoniche*, 1558, p. 59, to represent the basic consonances in exactly the same manner.

Under Raphael's musical diagram is the representation of the perfect Pythagorean number 10. As can be seen in Raphael's tablet, 10 is the sum of the first four figures which constitute all musical harmonies.

<sup>4</sup> Cf. V. Golzio, *Raffaello*, 1936, p. 32.

audible tests of a universal harmony which had a binding force for all the arts. Although Palladio restricted himself, as we saw, to occasional hints in that direction, he left in the illustrations of his *Quattro libri* a practical system of proportions which, if properly interpreted, is no less important a key to the whole question than Alberti's writings. The second book contains Palladio's own buildings in elevations, plans and sections. The many discrepancies between the plates and the actual buildings were generally attributed to careless publication.<sup>1</sup> Yet it is obvious from the plan of the whole work<sup>2</sup> that Palladio did not publish his buildings as an autobiographical contribution. He made a statement to this effect in the preface to the *Quattro libri* with these words: "In the second (book) I shall treat of the quality of the fabricks that are suitable to the different ranks of men: first of those of a city; and then of the most convenient situation for villas. . . . And as we have but very few examples from the antients, of which we can make use, I shall insert the plans and elevations of many fabricks I have erected . . ." In this light many differences between buildings and plates can be explained.<sup>3</sup>

The illustrations were to him a means of expounding his conceptions not only of planning but also of proportion, and hence his theoretical measurements could deviate from the executed ones. If this is a right deduction, the hypothesis seems justified that Palladio wanted his inscribed measurements to convey ratios of a general character and of universal importance beyond the scope of the individual buildings. In most of his plans ratios of width to length of the rooms are prominently placed and easily readable, while—with the exception of a few large-scale details—it is generally more difficult to read the elevations. For heights of rooms, which appear only in the few sections, he often refers in his text to the method employed. These arrangements seem to reveal a definite scheme which we propose to follow by confining ourselves to an examination of some of Palladio's plans.

What kind of proportion did Palladio exemplify with his inscribed measurements? The early Villa Godi at Lonedo contains the gist of the story in a simple form. Each of the eight small rooms—four at each side of the hall—measures  $16 \times 24$  feet, i.e. the length is equal to  $1\frac{1}{2}$  width which is one of the seven shapes of rooms recommended by Palladio.<sup>4</sup> The ratio of width to length is 2:3. The portico has the same size of  $16 \times 24$ , while the hall be-

<sup>1</sup> Cf. for instance O. Bertotti Scamozzi, *Les Bâtimens et les desseins de André Palladio*, 1776-83, I, p. 8, and T. Temanza, *Vita di Andrea Palladio*, 1762, pp. 15, 44.

<sup>2</sup> Cf. above, part I, Vol. VII, p. 105.

<sup>3</sup> Mistakes obviously occurred, as for instance when the width of the hall of Villa Saraceno at Finale is given as 18 feet instead of 28. But apart from such slips Palladio had often very good reason for changes between building and plate. One reason was that he did not want to hand down to posterity the designs of structures which he had built a long time ago and which no longer satisfied him. The most conspicuous case is that of the Villa Godi Porto at Lonedo, begun in 1540

(i.e. thirty years before the publication of the book), the front of which was retrogressive in style. This was 'overhauled' in the plate and the principles of the late style were grafted on to the early building. In other cases Palladio adjusted irregularities which were forced upon him by circumstances. The Palazzo Valmarana had to be planned for an irregular site. In the plate he shows a regular plan such as he would have built on an ideal site. In his text he does not even mention that he gave the plan as he would have liked to build it, and not as it was built.

<sup>4</sup> Cf. above, p. 74. For the plan of the Villa Godi cf. part I, vol. VII, pl. 25c.

hind it measures  $24 \times 36$ ; its ratio —  $1:1\frac{1}{2}$  or  $2:3$  — is therefore equal to that of the small rooms and the portico. The use of the same ratio throughout the building is apparent. But beyond this, the equation  $\frac{16}{24} = \frac{24}{36}$  shows that rooms and hall are, one might say, proportionately firmly interlocked. The progression 16, 24, 36 underlies the plan as a whole, which we know from Alberti's analysis of the ratio 4:9 as 4:6:9 and which can be expressed in musical terms as a sequence of two diapente. Thus, for those who understood the language of proportions, Palladio's meaning was made abundantly clear by the conspicuous inscriptions of measurements on the plan; without them the reader would be left with no key to the architect's ideas. And an inscription of the measurements which were actually built would not convey a picture of the intended harmonic proportions. Instead of 16 feet, the depth of the portico is 14.9 feet and the widths of the two adjoining rooms are 15.5 and 17.3 feet.

The ratios of Palladio's later structures are somewhat more complicated as can be illustrated in the Villa Malcontenta (Pl. 20a). The smallest room on either side of the cross-shaped hall measures  $12 \times 16$  feet, the next one  $16 \times 16$  and the largest  $16 \times 24$ , while the width of the hall is 32 feet. Thus, the harmonic series 12, 16, 24, 32 is the key-note to the building. As if in an overture the first and last members of this series appear in the ratio 12:32 of the portico, which is a diapason and diatessaron (for 12:24:32). The intercolumniation of the centre (6 ft.) is related to the depth of the portico (12) as 1:2. The smaller intercolumniations are  $4\frac{1}{2}$  feet; they are related to the central one as 3:4 which, incidentally, is the ratio of the smallest rooms. Finally, the diameter of the columns, 2 ft., represents the smallest unit, the module, and by a process of multiplication beginning with two all the ratios of the building can be derived.<sup>1</sup>

In such an organism built up with the "regola homogenea," the module, there is no room for incommensurable quantities; however, the application of the module does not necessarily mean that the ratios throughout a whole building must be harmonic.<sup>2</sup> But the systematic linking of one room to the other by harmonic proportions was the fundamental novelty of Palladio's architecture, and we believe that his wish to demonstrate this innovation had a bearing on the choice and character of the plates and the inscription of measurements. Those proportional relationships which other architects had harnessed for the two dimensions of a façade or the three dimensions of a single room were employed by him to integrate a whole structure.

The demand that "the parts should correspond to the whole and to each other" was generally adhered to in churches, for the relation of nave, aisles and chapels, and here the Renaissance could build on mediaeval traditions. But for domestic buildings the decisive step was taken by Palladio.<sup>3</sup> He formu-

<sup>1</sup> The only measurement in this building which is not easily intelligible is the length of the hall measuring  $46\frac{1}{2}$  feet, where one would have expected 48 feet. The measurement can be analysed in more than one way, for instance as  $6 \times 7$  plus  $4\frac{1}{2}$  (6 and  $4\frac{1}{2}$  being the widths of the intercolumniations), but I cannot offer a fully satisfactory explanation.

<sup>2</sup> Cf. below, p. 91, note 1.

<sup>3</sup> It would not be difficult to give earlier examples which show similar tendencies. But as far as we can see, none of Palladio's predecessors developed this problem systematically. Francesco di Giorgio seems to be the only one who attacked it theoretically in his treatise on architecture; cf. Promis, *Trattato di archi-*

lated his views on this point in one very important sentence which will add weight to the analysis of the two buildings which we have given: "But the large rooms ought to be so related (*compartite*) to the middle ones, and these to the small, that, as I have said elsewhere, one part of the building may correspond with the other, so that the whole body of the edifice may have in itself a certain harmony (*convenienza*) of members which may make it entirely beautiful and graceful."<sup>1</sup>

A thorough acquaintance with Renaissance ideas on proportion is often necessary to understand the legitimacy of the ratios given by Palladio. In the Villa Emo (Pl. 20c) rooms of  $16 \times 16$ ,  $12 \times 16$ ,  $16 \times 27$  frame the portico (also  $16 \times 27$ ) and the hall ( $27 \times 27$ ). The ratio  $16:27$  can only be understood by splitting it up in the way Alberti has taught us; it has to be read as  $16:24:27$ , i.e. as a fifth and a major tone ( $=2:3$  and  $8:9$ ) and similarly the compound ratio  $12:27$  can be generated from  $12:24:27$ , i.e. an octave and a major tone ( $=1:2$  and  $8:9$ ). Thus the figures 27, 12, 16 which, written one under the other, strike the reader's eye, are perfectly intelligible through the medium of generation. Ratios of the same order are to be found in the wings; 12 is again the middle term, this time with 24 and 48 as the extremes. The harmonic character of this series is obvious ( $2:1:4$ ,  $1:4$  being two octaves  $=1:2:4$ ). The whole building appears now like a spatial orchestration of the consonant terms 12, 16, 24, 27, 48.<sup>2</sup>

The same theme was developed in other structures with different measurements. The Villa Thiene at Cícogna (Pl. 20d) has 4 as module (diameter of columns) and the rooms are based on the harmonic series 12, 18, 36. In the four corners are square rooms measuring  $18 \times 18$  feet; they flank a double square room,  $18 \times 36$ , and this ratio is repeated in the two porticos which flank the hall, being the square of the small corner rooms,  $36 \times 36$ . The progression  $18:18$ ,  $18:36$ ,  $36:36$  is broken between the small squares and the porticos by rooms measuring 12 feet in width, so that the sequence 18, 12, 18 ( $3:2:3$ ) is repeated four times.<sup>3</sup>

Progressions similar to  $1:1$ ,  $1:2$ ,  $2:2$  of the Villa Thiene occur in other buildings. Rooms of  $20 \times 20$ ,  $20 \times 30$ ,  $30 \times 30$  form the core of the Palazzo Porto-Colleoni, and ratios based on the series 12, 16, 18, 24, 27, 32, 36 are frequent.<sup>4</sup> All these spatial proportions have their equivalent in the consonances of the Greek musical scale. But we are far from suggesting that Palladio, while planning these buildings, was consciously translating musical

*tettura civile e militare di Francesco di Giorgio Martini*, 1841, pls. 1, 2. See also E. Langenskiöld, *Michele Sanmicheli*, Uppsala, 1938, p. 191 and figs. 92, 93.

<sup>1</sup> Bk. II, chap. 2: "Ma le stanze grandi con le mediocri, e queste con le picciole deono essere in maniera compartite, che (come ho detto altrove) una parte della fabrica corrisponda all'altra, e così tutto il corpo dell' edificio habbia in se una certa convenienza di membri, che lo renda tutto bello, e gratioso."

<sup>2</sup> The depth of the rooms in the wings is 20; the ratios of the depth to the widths of

the rooms (12, 24, 48) will be explained in the next section.

<sup>3</sup> This description follows the inscribed numbers. One can, of course, also read 18, 12, 36 (length of the portico).

<sup>4</sup> For instance Villa Pojana (18, 36), Villa Trissino at Meledo (12, 18, 36), Villa Sarego at Santa Sofia (12, 18, 24, 36), Villa Cornaro at Piombino (16, 24, 27, 32). In all these buildings one finds with this basic series other figures which would need further explanation.

into visual proportions. Francesco Giorgio, in his memorandum, did not set out to prove the applicability of musical consonances to architecture, but worked with them for the design of S. Francesco della Vigna as a matter-of-course procedure. "The rules of arithmetic," said Daniele Barbaro, elaborating Vitruvius,<sup>1</sup> "are those which unite Music and Astrology; for proportion is general and universal in all things given to measure, weight and number."<sup>2</sup> We have Palladio's own word for it, that for him the proportions of sounds and in space were closely related, and he must have been convinced of the universal validity of one and the same harmonic system. These were convictions which belonged to the general intellectual make-up of the Renaissance, and it needed no particular sophistication to turn them into practice.

*Palladio's Ratios and the Development of Sixteenth Century Musical Theory*

It should now be said that ratios based on the small integral numbers of the Greek musical scale (1:2:3:4) are by no means the only ones to be found in Palladio's plans. Palladio showed a predilection for rooms measuring  $18 \times 30$  or  $12 \times 20$ , i.e. for a ratio of 3:5. There are buildings with ratios of 4:5 and 5:6<sup>3</sup> and these and similar ratios occur not only in the proportions of one room but also in the relation of one room to another: 4:5 in the Villa Valmarana at Lisiera, 5:6 in the Villa Ghizzole, 3:5 in the design for the Palazzo Angarano, 5:9 in that for Count della Torre at Verona, and this list could be considerably prolonged.

Here are problems which cannot be understood without considering the fundamental changes in the conception of proportion during the sixteenth century. In the course of this century ratios became perceptible which were outside the grasp of fifteenth century artists. The development of musical theory during that period, particularly in Northern Italy, is a reliable guide. It was Ludovico Fogliano of Modena who, in his *Musica theorica* of 1529, first protested against the authority of the Pythagorean consonances, and according to him experience teaches that, apart from the five Pythagorean consonances, minor (5:6) and major third (4:5), minor (5:8) and major sixth (3:5), and major (2:5) and minor tenth (5:12), eleventh (3:8), and minor and major sixth above the octave (5:16 and 3:10) are all consonances.<sup>4</sup> But it was Zarlino, the great Venetian theorist of the mid-sixteenth century, who with his rigorously scientific approach<sup>5</sup> classified the entire harmonic material which had come down from antiquity. It is a phenomenon which Zarlino calls "veramente maraviglioso"<sup>6</sup> that the consonances are determined by the arithmetic as well as by the 'harmonic' mean. The arithmetic mean 3 between

<sup>1</sup> To bk. I, i, 16.

<sup>2</sup> This had also a theological connotation, cf. *Wisdom of Solomon*, XI, 20.

<sup>3</sup> Such ratios occur in some of the structures which we have discussed. In fact, they are so frequent that no detailed list need here be given.

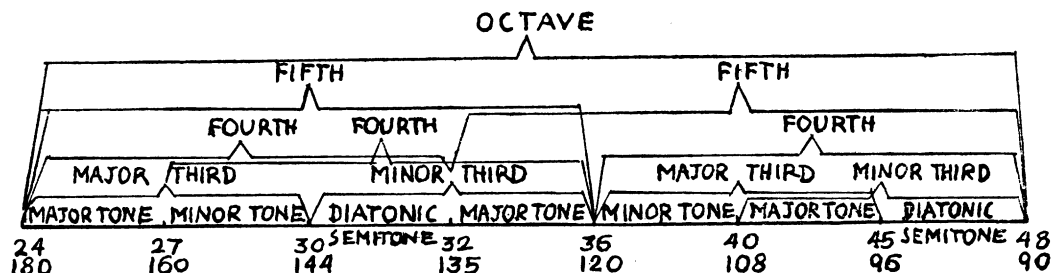
<sup>4</sup> *Musica theorica*, Venice, 1529, II, chap. 1, fol. xi verso. Cf. H. Riemann, *Geschichte der Musiktheorie im IX.-XIX. Jahrhundert*, Leipzig,

1898, p. 326: "haec autem quae addimus sunt consonantiae quae a *practicis* appellantur (follow the consonances mentioned in the text) . . . quae omnia intervalla esse *veras et valde delectabiles consonantias non potest negari*."

<sup>5</sup> *Le Istitutioni harmoniche*, Venice, 1558, p. 21: "la Musica è scienza, che considera li Numeri, e le proportioni."

<sup>6</sup> *Ibid.*, p. 161.

2 and 4 divides the octave into fifth and fourth (2:3 and 3:4); the same result, inverted, is achieved by the 'harmonic' mean 8 between the extremes 6 and 12 ( $6:8=3:4$  and  $8:12=2:3$ ). Zarlino could show that the same law applies to the division of the fifth, for 2:3 or 4:6 with the arithmetic mean 5 determines the ratios of major and minor third (4:5 and 5:6) and with the 'harmonic' mean—as in 10, 12, 15—the ratios of minor and major third. A further division of the major third is possible; the insertion of the arithmetic mean between 4 and 5 leads to the ratio 8:9:10, 8:9 being the major tone and 9:10 the minor tone, while the harmonic mean 80 between the extremes 72 and 90 divides the series into minor and major tone. Zarlino can now show in a diagram the "*divisione harmonica della Diapason nelle sue parti*":<sup>1</sup>

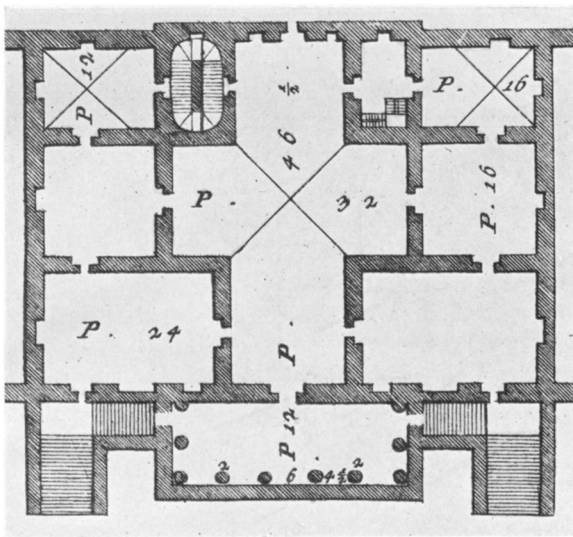


If we take this new development into consideration, most of the problematic ratios in Palladio's buildings become intelligible. A comparatively simple example for the combination of the old with the new consonances is the Villa Pisano at Bagnolo (Pl. 20b). The smallest room measures  $16 \times 16$  feet, the middle one  $16 \times 24$  and the largest one  $18 \times 30$ , while the cross-shaped hall is  $32 \times 42$  feet. We have met with the sequence  $16 \times 16$ ,  $16 \times 24$  in the Villa Malcontenta. The measurement 30 of the largest room is not inscribed, but Palladio says in his text that these rooms are "*lunghe un quadro e due terzi*." This shape, recommended by Palladio as one of the seven "*più belle e proportionate maniere di stanze*" is musically a major sixth (3:5) and it can be divided into  $18:24:30$ , i.e. 3:4:5, a fourth and a major third. The figures 16 of the square room and 18 of the largest room express the firm proportional relationship of the major tone (8:9); in addition 18 and 16 are linked to the 24 of the room between them by the ratios 3:4 (fourth) and 2:3 (fifth). All these affinities are suggested by the inscription of the numbers 16, 24, 18 in the right-hand side of the plan. Moreover, the length of the central room, 24, is related to the (uninscribed) length of the largest room, 30, as 4:5 (major third). The length of the hall, 42 feet, results from the addition of 18 and 24 (the lower part of the hall forms a square of  $18 \times 18$ ); and the figures 18, 24, 32 (width of the hall) represent two ratios of 3:4.

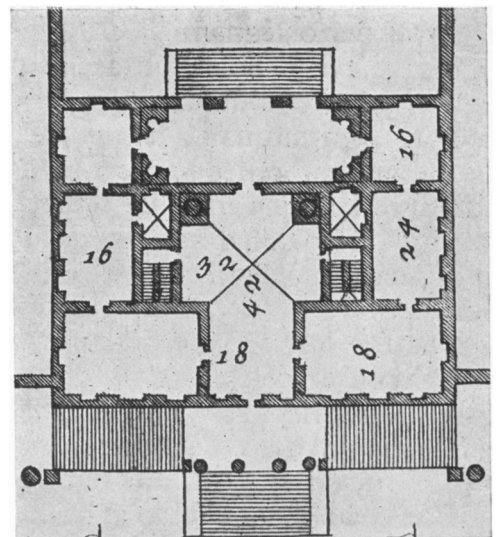
In the Villa Sarego at Miega (Pl. 21b) the sequence 12:16, 16:16, 16:27, which we have met in a different order in the Villa Emo, is to be found again. But the central part of the building, between the three framing rooms of each

<sup>1</sup> *Ibid.*, part II, chap. 39, p. 122. We have slightly altered Zarlino's diagram by translating his terms and also by quoting the arithmetic series 24, 36, 48 together with

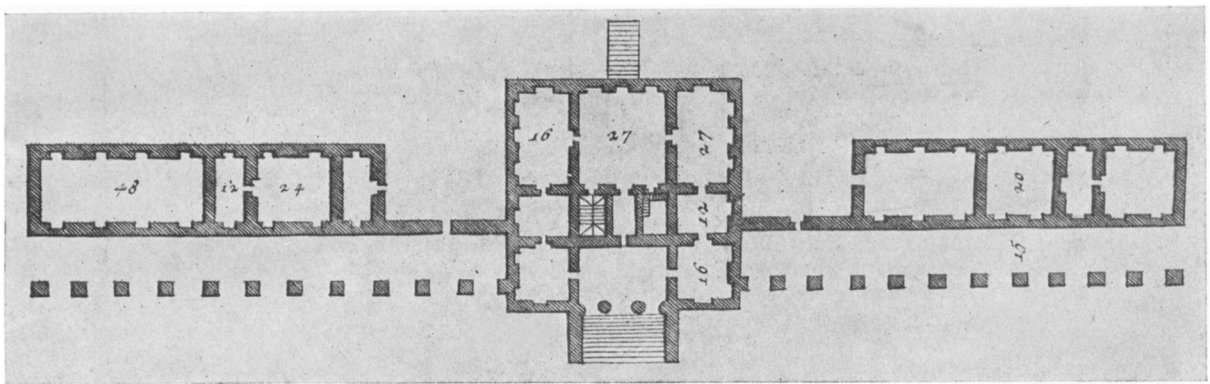
Zarlino's harmonic series 180 to 90. The diatonic semitone 15:16 is necessary, for without it one cannot proceed from the major third to the fourth.



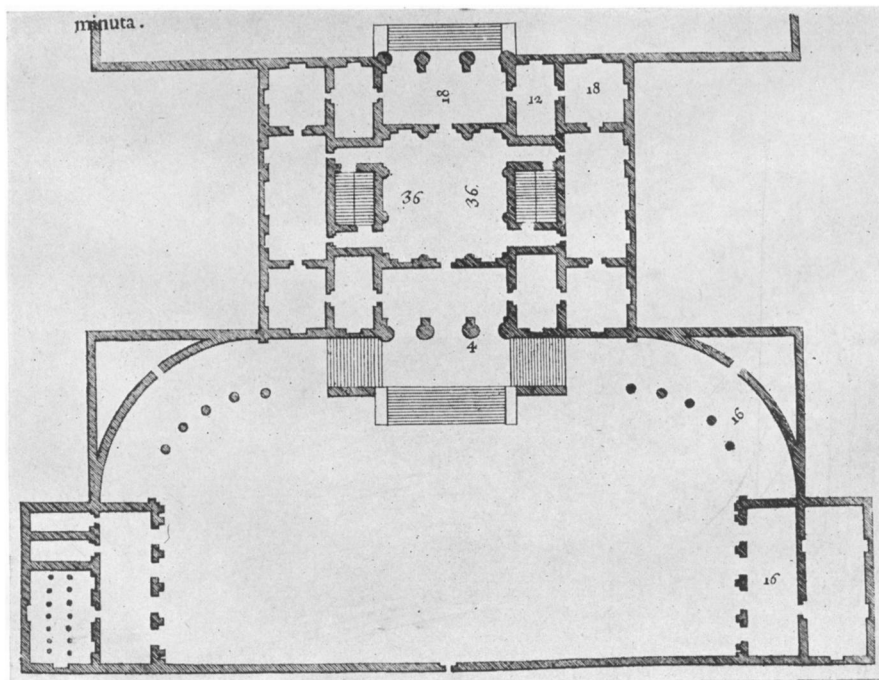
a—Villa Malcontenta. From Palladio's *Quattro libri* (p. 87)



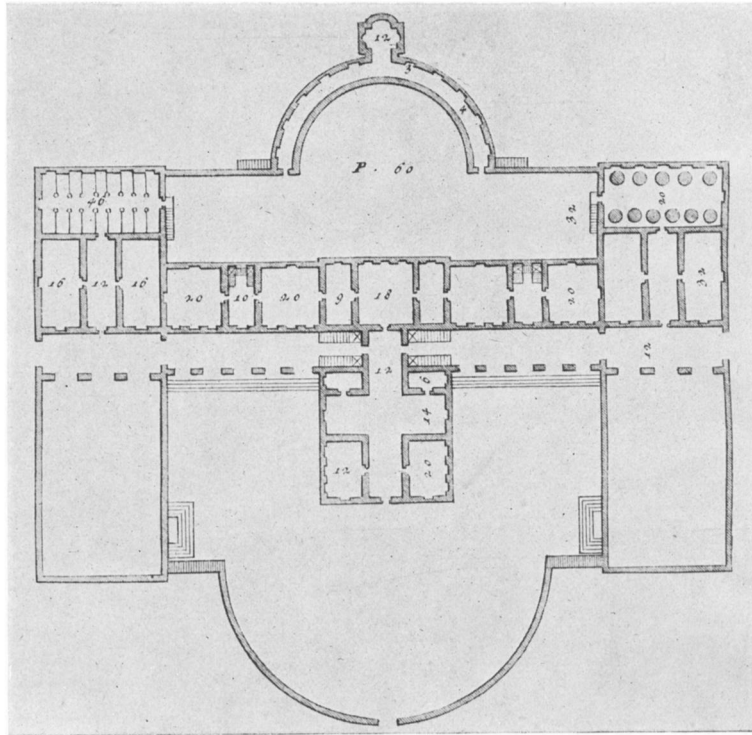
b—Villa Pisano at Bagnolo. From Palladio's *Quattro libri* (p. 90)



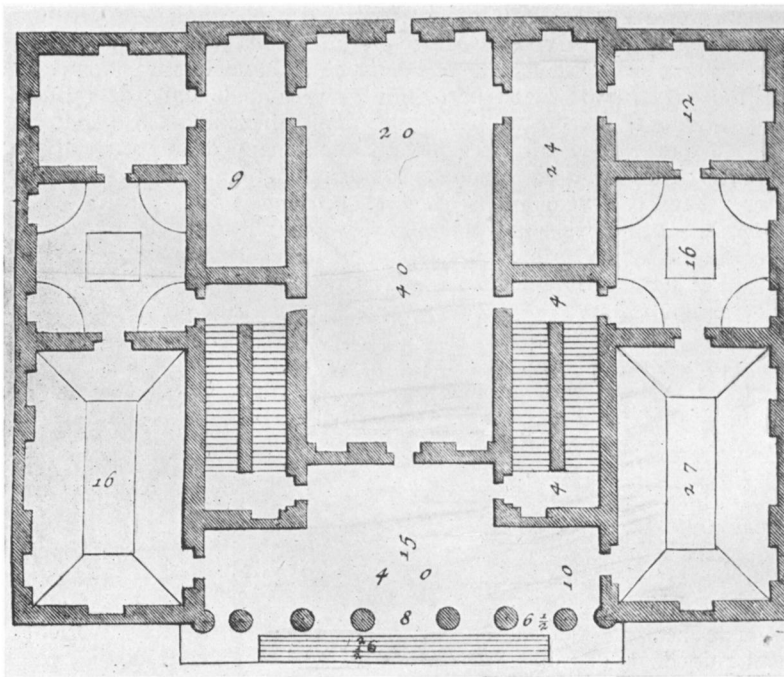
c—Villa Emo. From Palladio's *Quattro libri* (p. 88)



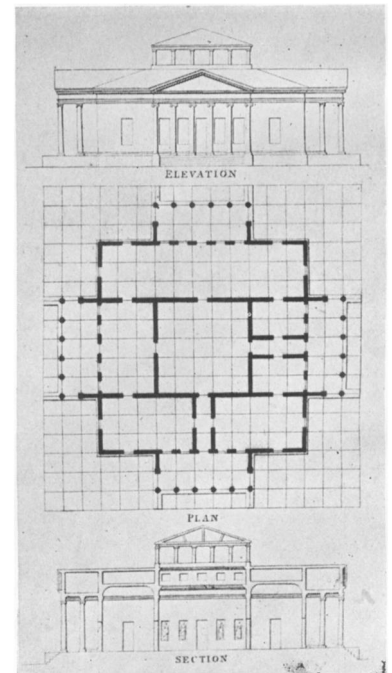
d—Villa Thiene at Cicogna. From Palladio's *Quattro libri* (p. 88)



a—Villa Maser. From Palladio's *Quattro libri* (p. 91)



b—Villa Sarego at Miega. From Palladio's *Quattro libri* (p. 90f)



c—From J. Gwilt, *Rudiments of Architecture*, 1826 (p. 102)

side, seems to follow a different system of ratios. In the portico are inscribed the numbers 10, 15 and 40, in the hall 20 and 40 and in the rooms connecting the hall with the wings 9 and 24. The numbers 10, 15, 20, 40 form a series of the order known to us (2, 3, 4, 8). 9:24 is an octave and a fifth (9:18:24), and both terms are linked in many ways not only with the adjoining rooms but they bridge the series of terms in the outlying rooms with those in the centre: 9:12 is a fourth, 9:20 an octave and a minor tone (9:18:20); 24 is related to 12, 16 and 27 as octave, fifth, and major tone, and to 20 and 40 of the hall as minor third (5:6) and major sixth (3:5; this proportion can also be expressed as fourth and major third, i.e. 24:32:40). All this, however, does not exhaust the relations which a Renaissance mind could here envisage; and with the development of sixteenth century musical theory in mind we can now grasp something of the harmonic "cross-currents" in this building. The ratios of 9 and 10, 10 and 12, 15 and 16, 12 and 20, although not occurring in adjoining rooms, must be understood as parts of the same organism.

Instead of carrying this analysis further, we may turn to another building, the Villa Maser (Pl. 21a), in which the basic harmonic unity of all the inscribed numbers may be demonstrated in detail. The long wings behind the main building contain three groups of three rooms each—two of these groups are repeated at each side of the third central one—the widths of which are inscribed as 16, 12, 16; 20, 10, 20; 9, 18, 9. It is obvious that the ratios in each set of rooms are consonant (4:3:4; 2:1:2; 1:2:1). But one can go a step further. In the front of the main building are three rooms—of which the middle one is part of the cruciform hall—all 12 feet wide (together 36); in the corresponding part of the wing the three rooms reappear with the different orchestration 9, 18, 9 (together 36). The 12 is the harmonic mean between 9 and 18 and divides the octave into fourth and fifth; the two inscribed figures 12 and 18, one above the other, are indicative of Palladio's intentions. We find the figure 12 again in the outside group of rooms of the wing, while the depth of the front rooms, 20, is repeated in the width and depth of the middle group of the wings. In other words, the three groups of rooms of the long wing repeat and develop the theme of the main building. At the same time, the figures inscribed in the three groups of the wings are interrelated, the smaller class of rooms as 9:10:12 (minor tone and minor third), the larger class of rooms as 16:18:20 (major tone and minor tone). The relation between the lengths of rooms, 20 and 32 (farthest group), is 5:8 (minor sixth, or minor third and fourth—20:24:32). The stables, the courtyard, the colonnade all form part of this symphony.<sup>1</sup> 12, the basic term of the building, returns in the width of the fountain in the main axis at the

<sup>1</sup> However, two measurements occur—14 (width of horizontal arm of hall) and 46 (length of stables)—which do not fit into the harmonic pattern. For Zarlino all harmonies are contained in the progression 1, 2, 3, 4, 5, 6, which includes even the minor sixth (5:8, generated from 5:6, minor third, and 6:8, i.e. 3:4, fourth), but the 7 forms no part of the harmonic series. And yet, both figures, 14 as well as 46, are firmly linked to the rest

of Palladio's terms. This will be evident by reading the whole series from which the proportions of the building were generated: 2, 3, 4, 6, 8, 9, 10, 12, 14, 16, 18, 20, 32, 46, 60.—14 and 46 are the arithmetic means between 12 and 16, and 32 and 60. Moreover, the distance of the mean 46 from the extremes 32 and 60 is 14.

The 'dissonant' ratio 6:7 as well as the arithmetic mean are to be found in other

farthest end of the yard and also in the width of the colonnade. This figure is further divided and appears as 6 in the smallest room of the house and as 3 and 4 in the niches and the passage of the exedra leading to the fountain. The width of the cortile, 32, corresponds to the length of the farthest group of rooms, and the width of the exedra, 60, is a fivefold proportion of 12 or a triple proportion of the equally important term 20. As the ratios of this building are evolutions of one and the same harmonic pattern, the proportional affinities could be stated in still greater detail.

However the reader may doubt whether Palladio's inscribed figures are really so full of implications. In spite of our close adherence to Palladio's own inscribed figures he may accuse us of the mistake, so often made by modern writers on proportion, of interpreting into a building relations which were never intended by its architect. Yet nobody will doubt that Palladio's numbers were meant to be suggestive of certain ratios, and it is not this fact but only the degree of interpretation which may be questioned. Now the position in architecture is exactly paralleled by that in musical theory and practice. Such a brilliant student of musical theory as Matthew Shirlaw stated the case as it was in Zarlino's time with these words:

The older art, although it was not on harmony alone that it depended for its æsthetic effect, was nevertheless capable of a very high degree of harmonic expressiveness. Composers of that time did not consider that there was any lack of harmonic material; for them a rich means of harmonic variety existed in the various consonances, and in the various ways of combining them. Not only so, but by different arrangements of these consonances it was possible to obtain a great many different tone-combinations which varied in harmonic effect and expressiveness: a delicate and subtle art which has since been to a great extent lost.<sup>1</sup>

There is a further means of solving any uneasiness about Palladio's intentions by reading the relevant chapters in the Commentary to Vitruvius by Daniele Barbaro, the very man for whom the Villa Maser was built. Vitruvius' work contains no real theory of proportion. Barbaro could not let this pass and so, after Vitruvius' preface to the third book, he inserted a circumstantial discourse on proportion.<sup>2</sup> He considered it of such importance that he introduced it with a formal address to the reader who is desirous to see "più a dentro, à ritrovare la verità delle cose."

The discourse begins with that complicated classification of numerical ratios which was in use from the times of Nicomachus' Arithmetic<sup>3</sup> to the

buildings. The Villa Zeno at Cesalto is generated from the terms 12, 14,  $21\frac{1}{2}$ , 29;  $21\frac{1}{2}$  being the arithmetic mean between 14 and 29, but no two numbers correspond here to musical intervals.

<sup>1</sup> Matthew Shirlaw, *The Theory of Harmony*, London, 1918, p. 37.—It would be suggestive and full of interesting implications if one were to find that Zarlino, Daniele Barbaro and Palladio belonged to the same circle. No

definite proof of this could be discovered, though Zarlino in his *Sopplimenti musicali* (Venice, 1588, pp. 179, 288, *passim*) quotes Barbaro's Vitruvius edition more than once and with great personal respect for the author.

<sup>2</sup> Ed. 1556, p. 57 ff.

<sup>3</sup> Cf. the table of Nicomachus' terminology in Heath, *A History of Greek Mathematics*, 1921, I, p. 101 ff.; Barbaro still uses this.

seventeenth century, when the modern fractional notation was generally introduced. After this, Barbaro turns to "proportionalità" which he declares to be the essence of every work of art. "In queste proportionalità consisteno tutti i secreti dell'arte," he exclaims. Following the traditional usage of "proportio" and "proportionalitas," "proportione" for Barbaro is the ratio of two magnitudes, while "proportionalità" consists "in the comparison not of one magnitude with another, but of one proportion with another."<sup>1</sup> Barbaro then explains in great detail methods of subtraction, addition, multiplication and division of ratios and of finding the common denominator of two and more "proportioni." He winds up with compound ratios which he considers of the utmost importance and for which he follows, as he professes himself, the system of "Alchindo."<sup>2</sup>

Barbaro's discourse contains nothing new for the theory of numbers. What is important about it is that he regarded his expositions as fully applicable to architecture; and he comes to the conclusion that "the possibilities of using now one, now another proportion are unlimited, for instance in subdividing the bulk of buildings ('i corpi delle fabbriche'), or in *atria*, *tablina*, halls, loggias, basilicas, and other things of great importance."<sup>3</sup> It would be entirely wrong to interpret this sentence, without regard to its context, as if the architect were free to handle proportion without the firm basis of science—on the contrary, the system is so complex and the definitions are so detailed that there is no room left for arbitrary proportions. Moreover, the proportions to be chosen should be consonant. "Every work of art must be like a very beautiful verse, which runs along according to the best consonances one followed by the other, until they come to the well ordered end."<sup>4</sup> The proportions of the human body are consonant and harmonious like the chords of a guitar. Of singers it is expected that their voices should be in tune, and the same refers to the parts in architecture. "Questa bella maniera si nella Musica, come nell'Architettura è detta Eurithmia, madre della gratia, e del diletto . . ."<sup>5</sup> The theme of proportions runs through Barbaro's whole commentary like a guiding thread and he returns to it with renewed emphasis. Perhaps the most illuminating passage is that following the part from which we have just quoted. In commenting further on Vitruvius' notions of "symmetria" and "eurythmia" Barbaro says:

Symmetry is the beauty of order as "eurythmia" is the beauty of disposition. It is not enough to order the measurements singly one after the

<sup>1</sup> Barbaro, p. 58: "si come la proportione è rispetto, & convenienza di due quantità comprese come due estremi sotto un'istesso genere, così la proportionalità è rispetto, & comparatione non d'una quantità all'altra, ma d'una proportione all'altra." Compare with this definition Boethius' "proportionalitas est duarum vel plurium proportionum similis habitudo" (*De musica*, ed. Friedlein, p. 137), which is probably Pythagorean; cf. Aristotle's "Proportion is equality of ratios" (*Eth. Nic.* V, 6, 1131 a 31). Barbaro's admira-

tion of "proportionalità" was firmly rooted in Renaissance thought; see Ghiberti's "Ma la proportionalità solamente fa pulcritudine" (J. von Schlosser, *Lorenzo Ghiberti's Denkwürdigkeiten*, 1912, p. 105).

<sup>2</sup> Cf. Al-Kindi (9th cent. A.D.), *De medicinarum compositarum gradibus investigandis libellus*, printed 1531, 1541 and after.

<sup>3</sup> Barbaro, *op. cit.*, p. 101.

<sup>4</sup> Barbaro, p. 24, ad Vitruvium I, ii, 3.

<sup>5</sup> *Ibidem*.

other, but it is necessary that those measurements be related to each other, that is to say that there must be some proportion between them.

This sentence seems to imply a recommendation of Palladio's "symphonic" principle of proportion. And the conclusion drawn by Barbaro in the next sentence reveals his sympathy with Palladio's practice:

Thus, where there is proportion there can be nothing superfluous. And as nature's instinct is the ruler of natural proportion, so the rule of art is master of artificial proportion. From this it results that proportion belongs to form and not to matter, and where there are no parts there cannot be proportion.

Here Barbaro touches on the manner in which proportion was perceived during the Renaissance.<sup>1</sup> Falling back on Aristotle's notions of matter and form, he regards it as the pre-requisite of "formed matter" that it should consist of parts which are proportionately related to each other.

For proportion originates from composite parts and their relationship to each other; and, as has been shown, there must be at least two terms in each relation.

He ends with a panegyric on proportion, and throws light on the ideas which guided him when writing, in the third book, his discourse on proportion.

One cannot sufficiently praise the effect of proportion, on which is based the glory of architecture, the beauty of the work and the miracle of the profession. This will become apparent when we talk about proportion and explain the secrets of this art demonstrating the innate quality of proportion, its terms, use and effect and by what power it determines the appearance of things.<sup>2</sup>

Those who work through Barbaro's chapter on proportion—not an easy task for a modern reader—will put it aside with the conviction that this man expected and saw in a building proportional relationships which are outside

<sup>1</sup> Cf. above, p. 78 ff.

<sup>2</sup> The whole passage (p. 24) runs as follows: "La Simmetria è la bellezza dell'Ordine, come la Eurithmia della Dispositione. Non è assai ordinare le misure una dopo l'altra, ma necessario è, che quelle misure habbiano convenienza tra loro, cioè sieno in qualche proportione, & però dove sarà proportione, quivi non può essere cosa superflua; & si come il maestro della natural proportione è lo instinto della natura, così il maestro dell'artificiale è l'habito dell'arte, di qui nasce che la proportione piu presto della forma, che della materia procede (later edition: è propria della forma, & non della

materia), e dove non sono parti, non può essere proportione: perchè essa nasce dalle parti composte, & dalla relatione di esse, & in ogni relatione è forza, che ci sieno almeno due termini (come s'è detto) ne si può lodare abastanza l' effetto della proportione, nella quale è posta la gloria dell'Architetto, la fermezza (later ed.: bellezza) dell'opera, & la maraviglia dell'Artificio, come si vedrà chiaramente, quando ragioneremo delle proportioni, & apriremo i secreti di questa Arte, dimostrando qual rispetto s'intende essere nella proportione, quali termini siani i suoi, qual' uso & quanti effetti, & di che forza essa faccia le cose parere."

our range of perception. With the analysis of some of Palladio's buildings we have tried to show that he was a master in the application of "proportionalità" throughout a whole structure, and we believe that he shared Barbaro's opinion that "it contained all the secrets of art." If one considers Palladio's friendship with Barbaro and their community of interests,<sup>1</sup> it is evident that the former was predestined to realize in Barbaro's own villa those subtle principles of proportion in which patron and architect equally believed.

After all that has been said, it will hardly be doubted that Palladio controlled and corrected his innate sense of proportion by a rational theory of proportion. There exists, moreover, an interesting proof for this. In his letter to Martino Bassi in which he stated his reasons for supporting Bassi's case against Pellegrino Pellegrini,<sup>2</sup> Palladio mentions that he wanted to hear the opinion of "huomini intendenti." He had therefore shown Bassi's suggestions to the painter Giuseppe Salviati, a specialist on perspective, and to Silvio Belli, "the most excellent geometrician whom we have here." This Silvio Belli, in whose judgment Palladio had such confidence, was the author of a work on proportion, entitled *Della Proportione, et Proportionalità*, which appeared in 1573 and which covered much the same ground as Barbaro's discourse. The lucidity and simplicity of Belli's presentation is congenial to Palladio's conception of architecture. Belli was not only a mathematician, he was above all a practical man. He had won laurels as an engineer; and as a co-founder of the Olympic Academy he must have been in close contact with Palladio for many years.<sup>3</sup>

A contemporary scholar coupled the names of Palladio and Belli in a remarkable passage at a time when the two men were still alive. It appears to us a strange and unexpected tribute when he says in their praise: "Certainly everybody knows how much talent and nature means even without learning; or else if he does not know it, let him turn to Andrea Palladio and Silvio Belli. For these with a minimum of erudition, but a maximum of meditation and skill call back into use the measurements, forms and works according to the rules of Archimedes, Euclid and Vitruvius and embellish our age with very beautiful buildings."<sup>4</sup> Measured by our standards Palladio's learning appears to us as the foundation of his art. To absorb Euclid and Vitruvius and other classical knowledge was for the mid-sixteenth century architect no more than what in our time would be a young architect's university training. It was the ground-work upon which talent could build.

Palladio reserved for himself the right to break the rules. This should be emphasized at the end. It is an important ingredient of his art and theory. He concludes his survey of rules for the proportion of rooms with the words:

<sup>1</sup> Cf. above, part I, p. 106 ff.

<sup>2</sup> Cf. above, p. 82.

<sup>3</sup> On Belli cf. Angiolgabriello di Santa Maria [Calvi], *Biblioteca, e storia di . . . scrittori . . . di Vicenza*, 1772-82, Vol. IV, pp. 103-107.

<sup>4</sup> Sebastiano Montecchio, *De Inventario haeredis*, Venice, 1574, p. 163 (quoted after Magrini, *op. cit.*, p. 2): "Nemo quippe

ignorat quantum valeat ingenium et natura etiam sine disciplina; vel si ignorat, respiciat in Andream Palladium et Sylvium Bellum. Hi enim ut minimum eruditioni, ita plurimum meditationi artificioque attendentes, Archimedis, Euclidis, Vitruvii regulis, dimensionibus, figuras, opificia in usum revocant, nostramque aetatem pulcherrimis exornant substructionibus."

"There are still other heights for rooms which fall under no rule, and the architect has to use them according to his judgment and need."<sup>1</sup>

Similarly, the next chapter on "The Measurements of Doors and Windows" begins with the statement that "one cannot give a certain and absolute rule about their height and width." Such unorthodox statements punctuate Palladio's treatise, and in the stress laid on individual judgment and practical experience, reflected also in his work,<sup>2</sup> one might see the typically North Italian Aristotelian accretion<sup>3</sup> to the Platonic substance with which the foregoing pages were concerned.

#### 6. THE BREAK-AWAY FROM THE LAWS OF HARMONIC PROPORTION IN ARCHITECTURE

It seems appropriate to conclude with a few notes on the chequered history of proportion in post-Palladian architecture. Palladio's work remained canonical for those academic architects who abode by the conception of harmonic ratios. However, in the long run time was against them and the classical ideas on proportion were completely reversed.

It need hardly be recalled that the doctrine of a mathematical universe which, with all its emanations, was subject to harmonic ratios, was triumphantly reasserted by a number of great thinkers of the seventeenth and eighteenth centuries. We find this conception of the world fully expounded in Kepler's *Harmonia mundi* (1619), we find it in Galilei,<sup>4</sup> and later in Shaftesbury, for whom, truly Platonic, the laws of musical harmony are effective also in human nature: "*Virtue* has the same fix'd Standard. The same *Numbers, Harmony, and Proportion* will have place in *Morals*; and are discoverable in the *Characters and Affections* of Mankind."<sup>5</sup>

The poets echo these ideas. Dryden thinks in terms of the Greek musical scale in "A Song for St. Cecilia's Day":

From harmony, from heav'nly harmony  
This universal frame began;  
From harmony to harmony  
Thro' all the compass of the notes it ran,  
The diapason closing full in man.

Reflections of a knowledge of this universal harmony are to be found in seventeenth and eighteenth century writers on art. In England Sir Henry Wotton in his *Elements of Architecture*, 1624, pointed to the importance of harmonic proportions. As a student of Vitruvius, Alberti, Palladio and the French theorists like Philibert de l'Orme, he could write: "In truth, a sound piece of good Art, where the *Materials* being but ordinary Stone, without any garnishment of Sculpture, do yet ravish the beholder (and he knows not how)<sup>6</sup> by a secret *Harmony* in the *Proportions*." In his chapter on doors and

<sup>1</sup> Bk. I, p. 50.

<sup>2</sup> Cf. above, p. 87, n. 1 and p. 91, n. 1.

<sup>3</sup> Cf. Part I of this article, p. 107.

<sup>4</sup> Cf. E. Cassirer, *Das Erkenntnisproblem*, 1911, I, p. 383 ff.

<sup>5</sup> In "Advice to an Author," *Character-*

*istics*, ed. 1737, I, p. 353. See also the hymn on harmony in nature in "The Moralists," *ibid.*, II, p. 284 ff.

<sup>6</sup> See Palladio's wording of the same idea, above p. 74.

windows he is more explicit reminding the reader that Vitruvius himself wishes the architect "to be no superficial, and floating *Artificer*; but a *Diver* into *Causes*, and into the *Mysteries* of *Proportion*." And following Alberti's interpretation of Pythagoras he explains how to reduce "Symmetry to *Symphony*, and the *Harmony* of *Sound*, to a kind of *Harmony* in *Sight*."<sup>1</sup> A student of the classical art theory like Reynolds, 150 years later, still believed in the basic unity of all the arts, and the validity of the same proportions in music and architecture was to him a truism on which he remarked in the 13th Discourse: "To pass over the effect produced by that general symmetry and proportion by which the eye is delighted, as the ear is with music, architecture certainly possesses many principles in common with poetry and painting."

But there was an important classicist current, the representatives of which kept alive the Platonic conception of numbers in a doctrinal and didactic sense. François Blondel was perhaps the first architect who gave this academic turn to the old Italian ideas on proportion. Almost a whole book of his *Cours d'architecture*, 1675-83,<sup>2</sup> deals with musical proportions in architecture. His approach to the problem is historic and apologetic, for, in contrast to his Renaissance predecessors, he has to prove a case of which many of his contemporaries were ignorant. Alberti's theory and Palladio's buildings were, as one would expect, used as test-cases for the theory; a whole chapter is devoted to an analysis of façades by Palladio, for the proportions of which Blondel found the key in the simple consonances 9, 6, 4; 6, 4, 3; 4, 2, 1, etc.<sup>3</sup> The answer to Blondel was given by Claude Perrault in his *Ordonnance des cinq espèces de colonnes*, 1683, who broke decisively with the conception that certain ratios were *a priori* beautiful and declared that proportions which follow "the rules of architecture" were agreeable for no other reason than that we are used to them. Consequently, he advocates the relativity of our æsthetic judgment and quite logically turns against the idea that musical consonances can be translated into visual proportions.

Blondel's treatise was the result of his teachings at the "Académie royale de l'architecture," to which he was appointed first director in 1671. Eighty years later Briseux wrote his *Traité du Beau essentiel dans les arts*, 1752, in defence of Blondel's principles against Perrault. The author is well versed in Platonism, and he even harnesses Newton's theory of colour in support of the ancient truth. Much of his material was based on Blondel whom he follows entirely in the choice and interpretation of the Palladian examples. But in spite of Briseux's claim to universality for harmonic ratios, he is much concerned with the demonstration that "les mêmes proportions produisent les mêmes effets," and the attack against this somewhat pedestrian logic was soon to follow.

In his work Briseux tried to revive a tradition which was in danger of being forgotten. In fact, the chain was broken and proportion in architecture

<sup>1</sup> Sir Henry Wotton discourses here at length on the nature of the fifth and the octave, "the two principall *Consonances*, that most ravish the eare." Ed. of 1624, p. 53 f.

<sup>2</sup> Vth Part, 5th book, p. 727 ff.

<sup>3</sup> Blondel's exposition culminates in a summary of Ouvrard's *Architecture Harmonique*, ou

*l'Application de la doctrine des proportions de la Musique à l'Architecture*, a book which I was unable to trace in London. Ouvrard was a musician, and his work was undoubtedly an important link in the revival of what Blondel calls the "ancienne doctrine."

was regarded as a mystery the knowledge of which had to be rediscovered. Robert Morris, an architect associated with the Burlington group, believed, in his *Lectures on Architecture*, 1734-36, he had found the secret "which was by the Antients found out, and but by a few Moderns known and practis'd."<sup>1</sup> For this classicist Palladio was, of course, the chief restorer of ancient wisdom<sup>2</sup> and, guided by his works, he developed a system of hard and fast rules of harmonic proportions based on the "only seven distinct notes in music" the ratios of which "produce all the Harmonick Proportions of Rooms." From ready-made tables the reader or architect can pick out the shape of rooms, façades, doors and chimneys with the correct harmonic proportions.

The complete break with the great tradition of the sixteenth century and the isolation of the problem of proportion is also to be observed on Italian soil. Perhaps the most penetrating student of Palladio, the architect Octavio Bertotti Scamozzi, declared that he had found that Palladio used musical ratios; but he made this discovery only after his work *Les Bâtimens et les desseins de André Palladio* (1776-83) was well on the way. In the preface to his third volume he submits his ideas on that point to the judgment of the critics. After having carefully studied the proportions of Palladio's buildings, he declares he had come to the conviction that they depend "des principes beaucoup plus solides que ce qu'on appelle bon goût dans le sens vulgaire." These "principes solides," the musical ratios, are discreetly pointed out by him in the descriptions of Palladio's buildings. It is apparent that Bertotti Scamozzi had no idea of the general principles which directed a Renaissance mind. Though his results are often convincing, because they are obvious to those who are familiar with Palladio's methods, he developed his thesis entirely in a void. Briseux's book had come to his knowledge just before his work was finished, and he notes with satisfaction the correspondence of their conclusions. What he did not know, however, was that discussions on harmonic proportions had been going on all the time and that in the neighbouring Treviso they were even practised in his own days.

In 1762 appeared Tommaso Temanza's *Vita di Andrea Palladio*, which is still one of the most important sources for Palladio's life. Temanza states that Palladio in the ratios of length, width and height of his rooms made clever use of the arithmetic, geometric and 'harmonic' means "as is clearly manifest in his works."<sup>3</sup> On this point began a controversy with the Trevisan architect Francesco Maria Preti, which is worth recording because it throws light on the ideas about proportion in architecture and music during the late eighteenth century. Preti advocates "una legge stabile e ferma"<sup>4</sup> which is alone guaranteed by the musical progression 1, 2, 3, 4, 5, 6 (octave, fifth, fourth, major and minor third). He concludes dogmatically that there is no beauty outside these proportions, for—and here we hear the old pattern—the same consonances "che diletano l'orecchio diletano anche la visione."<sup>5</sup> Temanza in

<sup>1</sup> Preface.

<sup>2</sup> p. 52.

<sup>3</sup> p. 81.

<sup>4</sup> Bottari, *Lett. pitt.*, 1822, VIII, p. 277. Preti's letter dates from May 1, 1762 (publ. by Bottari with the wrong date 1760).

<sup>5</sup> Preti also maintains that these consonances are universal. But his wording is interesting: "Per le osservazioni da me fatte entra la musica non sola in architettura, ma nel sistema universale del mondo" (p. 280).

his long-winded answer<sup>1</sup> agreed that in the widest sense numbers regulate buildings as well as music. He still insists on commensurability throughout a structure; but apart from that, proportion in music and architecture are widely different.<sup>2</sup> His criticism of the general applicability of musical consonances in architecture boils down to two objections which reveal an entirely new standpoint. The one objection is that the eye is not capable of perceiving simultaneously the ratios of length, width and height of a room;<sup>3</sup> the other that architectural proportions must be judged from the angle of vision under which the building is viewed.<sup>4</sup> In other words, architectural proportions cannot be absolute but must be relative. The emphasis has shifted from the objective truth of the building to the subjective truth of the perceiving individual. It is for this reason that Temanza regards the use of the mean proportionals as "più misterioso che ragionevole."<sup>5</sup> It will be noticed that Temanza's theoretical position is not quite clear; for in spite of his introducing revolutionary factors into the problems of proportion, he still cannot get away from traditional notions. In a later letter addressed to Bottari in which Temanza again states his case, he insists that the use of harmonic proportions in architecture would lead to sterility.<sup>6</sup> For an eighteenth century classicist this was a very sound observation.

Francesco Maria Preti, Temanza's opponent, had grown up in Treviso in a tradition in which the 'harmonic' mean for the height of rooms was recommended with an academic stubbornness. Giovanni Rizzetto (b. 1675), mathematician and architect, had worked out the theory. His son Luigi Rizzetto, Ottavio Scotti, Andrea Zorzi, Jacopo Riccati and his sons Vincenzo, Giordano and Francesco and, last but not least, Francesco Maria Preti, all of them interested in mathematics and music, had consolidated the harmonic system of proportions, convinced that musical consonances had to be applied to architecture. Preti (d. 1774) was perhaps the most prolific theorist and practitioner of the school which left a number of buildings in Treviso and the province, erected strictly according to the rules of musical harmony.<sup>7</sup>

But the school of Treviso was no more than a late and provincial survival of the Renaissance tradition, without that cosmic vision which had given it breadth and universality.<sup>8</sup> The new ideas swept all this remorselessly away.

<sup>1</sup> Dated June 29, 1762, cf. Bottari, *op. cit.*, V, pp. 462-80.

<sup>2</sup> *Ibid.*, pp. 473, 472, 470.

<sup>3</sup> *Ibid.*, p. 465: "quel piacere che dal giudizioso accozzamento delle consonanze musicali risulta, convenebbe che l'uomo avesse l'organo della vista accomodato in modo di vedere nello stesso tempo, ed in un'occhiata sola, tutte e tre le dimensioni . . ."

<sup>4</sup> *Ibid.*, p. 474.

<sup>5</sup> *Ibid.*, p. 478.

<sup>6</sup> Letter dated March 19, 1768. *Ibid.*, VIII, pp. 293-306, particularly p. 302.

<sup>7</sup> For detailed information cf. P. Federici, *Memorie Trevigiane*, 1803, II, pp. 144 ff., 173 ff. A late advocate of the validity of the harmonic mean in architecture was Leopoldo

Cicognara in his work *Del Bello*, Milan, 1834, p. 77 ff.

<sup>8</sup> It should be mentioned that the speculations about the applicability of musical proportions to architecture never died out during the eighteenth century. In Rome, for instance, the painter Niccolò Ricciolini and the architect Antoine Derizet made "profondi studi, ricerche, esami, e scoperte sopra l'applicazione delle proporzioni musiche all'Architettura" (cf. Galiani's translation of Vitruvius, 1758, quoted by A. Prandi in *Roma*, XXI, 1943, pp. 18 ff.), but their treatise was never published. Derizet was a friend of Anton Raphael Mengs, and from that fact we may perhaps derive an idea of the trend of his thought. Mengs' friend, and

Milizia, the foremost Italian theorist of the late eighteenth century, subordinated the rules of proportion to those of perspective, as buildings are seen at different distances and in different situations. He goes a decisive step further on the path shown by Temanza twenty years before; his theory of proportion is sensuous, based on the impression which a building makes on the eye. Logically he refutes the efficacy of the three mean proportionals, and even the necessity of commensurable dimensions. Proportion for him is a matter of experiment and experience.<sup>1</sup> The modern approach of the architect to the problem of proportion is taking shape.

It was, however, in England, that the whole structure of classical æsthetic was overthrown from the bottom. Hogarth was only the mouthpiece of the new tendencies when he rejected any congruity between mathematics and beauty.<sup>2</sup> Without an idea of the universality of the classical conception of proportion, he comments on the "strange notion" that because "certain uniform and consonant divisions upon one string produce harmony to the ear," "similar distances in lines belonging to form, would, in like manner, delight the eye. The very reverse of which has been shown to be true . . . yet these sort of *notions* have so far prevail'd by time, that the words, *harmony of parts*, seem applicable to form, as to music."

The man in whom the new ideas found the most marked expression, was Hume. Just as he declared that "all probable reasoning is nothing but a species of sensation," so he turned objective æsthetic into subjective sensibility. In his essay *Of the Standard of Taste*, first published in 1757, he broke with unprecedented boldness with the basic axiom of all classical art-theory, according to which beauty is a quality inherent in the object. "Beauty," he said, "is no quality in things themselves: It exists merely in the mind which contemplates them; and each mind perceives a different beauty . . . To seek the real beauty, or real deformity, is as fruitless an enquiry, as to pretend to ascertain the real sweet or real bitter;" and later: "To check the sallies of the imagination, and to reduce every expression to geometrical truth and exactness, would be the most contrary to the laws of criticism; because it would produce a work, which, by universal experience, has been found the most insipid and disagreeable."<sup>3</sup>

In the same year 1757 appeared Burke's *Enquiry into the Origin of our Ideas of the Sublime and Beautiful*. With his sensual and emotional approach and his exaltation of sublimity he subjected the classical conception of proportion to

the editor of his writings, Giuseppe Niccola D'Azara, reports how he found the painter whistling and singing while painting the Annunciation, his last picture. When asked for the reason, Mengs explained that he was repeating a sonata by Corelli, for he wanted his picture to be in Corelli's musical style (cf. *Opere di Antonio Raffaello Mengs*, Bassano, 1783, I, pp. LXVI, LXXI). This "materialistic" eighteenth century approach to the translation of music into painting throws a strong light on the change which had come about since the days of the Renaissance, when

the conception of one universal harmony towered behind both music and architecture.

<sup>1</sup> *Memorie degli Arch. ant. e mod.*, 1785, I, p. xli ff. It is exceedingly important to find such views in a dogmatic neo-classicist like Milizia.

<sup>2</sup> *Analysis of Beauty*, 1753, p. 76 f.

<sup>3</sup> Hume, *Essays Moral, Political, and Literary*, ed. T. H. Green and T. H. Grose, 1889, I, p. 268 ff. Following the passages quoted in the text Hume develops his theory of experience on which "all the general rules of art are founded."

a detailed analysis and tore it to shreds. He denied that beauty had "anything to do with calculation and geometry." Proportion is, according to him, solely "the measure of relative quantity," a matter of mathematical enquiry and "indifferent to the mind." His further analysis shows again that his generation had lost the faculty of understanding even the most general principles of the classical conception. He does not see that the beauty of the classical theory has its roots in the idea of an all-pervading harmony, which is an absolute and mathematical truth, and he is therefore unable to grasp that, for instance, ratios of parts of a body remote from each other may be compared. Nor can he understand the relation between the human body and architecture which was, as will be remembered, at the basis of Renaissance thought on proportion. What he says on this point reveals most clearly the complete break with the past which, for the perception of proportion, the age of empiricism and emotionalism had brought about. "I know that it has been said long since, and echoed backward and forward from one writer to another a thousand times, that the proportions of building have been taken from those of the human body. To make this forced analogy complete, they represent a man with his arms raised and extended at full length, and then describe a sort of square, as it is formed by passing lines along the extremities of this strange figure.<sup>1</sup> But it appears very clearly to me, that the human figure never supplied the architect with any of his ideas. For in the first place, men are very rarely seen in this strained posture . . ." Burke winds this point up with the remark: "And certainly nothing could be more unaccountably whimsical, than for an architect to model his performance by the human figure, since no two things can have less resemblance or analogy, than a man, and a house or temple."<sup>2</sup>

Lord Kames, in his *Elements of Criticism*, 1761, is perhaps 'reactionary' as compared with Burke, and yet he launched a frontal attack against the translation of musical consonances into architecture. He begins his discussion with the words: "By many writers it is taken for granted, that in buildings there are certain proportions that please the eye, as in sounds there are certain proportions that please the ear; and that in both equally the slightest deviation from the precise proportion is disagreeable." From this it is evident that he too was unaware of the deeper bond that for a Renaissance mind united ratios in music and visible objects. He argued against the doctrinal Blondel-Briseux-Morris-Preti interpretation. It is therefore only logical when he carries on: "To refute the notion of a resemblance between musical proportions and those of architecture, it might be sufficient to observe in general, that the one is addressed to the ear, the other to the eye; and that objects of different senses have no resemblance, nor indeed any relation to each other." In support of this he refers to the octave which is the most perfect musical concord; but a proportion of one to two, he asserts, is very disagreeable in any two parts of a building. Here his eighteenth century taste contrasts with that of the Renaissance when, as we have seen, a ratio of 1:2 in architecture was regarded as faultless. His main line of attack is not dissimilar to that of the

<sup>1</sup> This refers, of course, to Vitruvius' famous and often illustrated description in bk. III, chap. 1.

<sup>2</sup> For the passages here quoted cf. the 9th ed., 1782, pp. 175 ff., 181 ff.

Italian critics. Judgment of proportion rests with the percipient. As we move about in a room the proportions of length to breadth vary continuously, and if the eye were an absolute judge of proportion one "should not be happy but in one precise spot, where the proportion appears agreeable." Therefore we can congratulate ourselves that the eye is not "as delicate with respect to proportion as the ear is with respect to concord"; if it were, this "would not only be an useless quality, but be the source of continual pain and uneasiness." Thus, apart from the subjective approach to proportion, Lord Kames introduced as a new element the limitations of human sight—an idea utterly foreign to Renaissance theory.<sup>1</sup>

Alison's theory of association, anticipated by Burke, exposes perhaps most clearly the significance of the revolution which had occurred in the course of the eighteenth century. He maintains that any abstract or ideal standard destroys the function of a work of art. It is the "trains of thought that are produced by objects of taste," the spontaneous stimulus to the imagination which make a work beautiful and sublime. "The sublimity or Beauty of Forms arises altogether from the Associations we connect with them, or the Qualities of which they are expressive to us."<sup>2</sup> In Alison's footsteps Richard Payne Knight in his *Analytical Inquiry into the Principles of Taste*, 1805, declared that proportion "depends entirely upon the association of ideas, and not at all upon either abstract reason or organic sensation; otherwise, like harmony in sound or colour, it would result equally from the same comparative relations in all objects; which is so far from being the case, that the same relative dimensions, which make one animal beautiful, make another absolutely ugly . . . but the same proportionate combinations of sound, which produce harmony in a fiddle, produce it also in a flute or a harp."<sup>3</sup> Thus, a pseudo-logical proof was found to show that musical harmony and spatial proportions cannot have anything in common.

Against the background of a new conception of the world the whole structure of classical aesthetics was systematically broken up, and in this process man's vision underwent a decisive change. Proportion became a matter of individual sensibility and in this respect the architect acquired complete freedom from the bondage of mathematical ratios.<sup>4</sup> This is the attitude to which most architects as well as the public unconsciously subscribed right down to our own days. It is hardly necessary to support this truism with authoritative statements; but brief reference may be made to two authors who interpreted the general feeling on this point. Ruskin declared that possible proportions are as infinite as possible airs in music and it must be left to the inspiration of the artist to invent beautiful proportions.<sup>5</sup> Julien Guadet

<sup>1</sup> Cf. the 8th ed., 1807, II, pp. 460 f., 463 and *passim*.

<sup>2</sup> Archibald Alison, *Essays on the Nature and Principles of Taste*, Edinburgh, 1790, 5th ed., 1817, I, pp. 13 ff., 317, also II, pp. 20 ff., 33 ff.

<sup>3</sup> p. 169.

<sup>4</sup> However, mathematical ratios survived in a degenerated form as a teaching expedient for architectural students and without any

connection with their original meaning. Cf. our plate 21c which is taken from Joseph Gwilt's *Rudiments of Architecture*, London, 1826. Gwilt's plate is based on J. N. L. Durand, *Précis des leçons d'architecture données à l'Ecole Royale Polytechnique*, 1819-21, a book which contains a large number of similar designs.

<sup>5</sup> *The Seven Lamps of Architecture*, 1849, in "The Lamp of Beauty."

in the *Eléments et théorie de l'architecture*, the often re-printed handbook of the students of the "Ecole des Beaux-Arts" in Paris,<sup>1</sup> explains that in order to establish a dogma of proportions, authors of the past had invoked science. But "elle n'a rien à voir ici; on a cherché des combinaisons en quelque sorte cabalistiques, je ne sais quelles propriétés mystérieuses des nombres ou, encore, des rapports comme la musique en trouve entre les nombres de vibrations qui déterminent les accords. Pures chimères . . . Laissons là ces chimères ou ces superstitions. . . . Il m'est impossible, vous le concevez bien, de vous donner des règles à cet égard. Les proportions, c'est l'infini."

While thus the harmonic mathematical conception of architecture was philosophically overthrown in the age of "nature and feeling" and disappeared from the practical handling of proportion, scholars began investigating a subject which had become historical. With unlimited patience and resourcefulness a great number of contradictory systems, often with a claim to exclusive validity, were worked out which were meant to lead to an understanding of proportion in antiquity, the Middle Ages and the Renaissance.<sup>2</sup> The contribution offered in these pages, though limited in scope, aims at being less speculative than some previous writings, for it is strictly based on one of the few certain indications about proportion which have come down to us from the Renaissance: Palladio's own inscribed measurements of his villas and palaces.

## APPENDIX I

### *Francesco Giorgio's Memorandum for S. Francesco della Vigna*

April 1, 1535. In order to build the fabric of the church with those fitting and very harmonious proportions which one can do without altering anything that has been done, I should proceed in the following manner. I should like the width of the nave to be 9 paces (1 pace=ca. 1.8 m.) which is the square of three, the first and divine number. The length of the nave, which will be 27, will have a triple proportion which makes a diapason and a diapente. And this mysterious harmony is such that when Plato in the *Timaeus* wished to describe the wonderful consonancy of the parts and fabric of the world, he took this as the first foundation of his description, multiplying as far as necessary these same proportions and figures according to the fitting rules and consonances until he had included the whole world and each of its members and parts. We, being desirous of building the church, have thought it necessary and most appropriate to follow that order of which God, the greatest architect, is the master and author. When God wished to instruct Moses concerning the form and proportion of the tabernacle which he had to build, He gave him as model the fabric of the world and said (as is written in Exodus 25) "And look that thou make them after their pattern, which was shewed thee in the mount." By this pattern was meant, according to all the interpreters, the fabric of the world. And rightly so, because it was necessary that the particular place should resemble His universe, not in size, of which He has no need, nor in delight, but in proportion, which He wills should be not only in the material places,

<sup>1</sup> Four vols. First ed. 1901-4, 4th ed. 1915, I, p. 138 f. Cf. also F. Hoeber, *Orientierende Vorstudien zur Systematik der Architekturproportionen*, Frankfurt, 1906, who declares that proportion in architecture as the result of an unconscious process should be the object of study of an æsthetic psychology. For a

modern view of an English practitioner cf. T. W. Simpson, *Essays and Memorials*, 1923, p. 54 ff., according to whom "proportion cannot be reduced to any mathematical or geometrical formula."

<sup>2</sup> Cf. Appendix II.

in which He dwells, but particularly in us of whom Paul says, writing to the Corinthians: "Ye are the Temple of God." Pondering on this mystery, Solomon the Wise gave the same proportions as those of the Mosaic tabernacle to the famous Temple which he erected. If we, then, follow the same proportions, we shall content ourselves for the length of the nave of the Church with the number 27, which is three times that of the width, and the cube of the ternary number; beyond which [number 27] Plato, in the description of the world, would not go, nor would Aristotle in his first book of "De Caelo"—having command of the measurements and forces of nature—allow this number to be transgressed in any one body. The truth is that one can increase the measures and numbers, but they should always remain in the same ratios. And whosoever should presume to transgress this rule would create a monster, he would break and violate the natural laws. To this perfect and complete body, we shall now give the head, which is the 'capella grande.' As for the length, it should be of the same proportion, or rather symmetry which one finds in each of the three squares of the nave, that is 9 paces. I consider it advisable that it should be of the same width as the nave (which as we have said should not be longer than 27); but [I prefer] that its width be 6 paces, like a head, joined to the body proportionately and well balanced. And to the width of the nave it will be in the ratio of 2:3 (sesquatera) which constitutes the diapente, one of the celebrated harmonies. And, as the architects usually approve of the symmetry between the chancel and the transept, we want to make these 'wings' 6 paces wide, in conformity with the capella grande. And returning to the length: adding the length of the said capella to the nave, it forms a quadruple proportion in relation to the width, which forms a bisdiapason, a most consonant harmony. From this symmetry the choir will not be excluded. It will be another 9 paces in length and will form a quintuple proportion in relation to the width: which gives it the most beautiful harmony of a bisdiapason and diapente. The width of the chapels will be 3 paces in triple proportion to the nave of the church which is a diapason and diapente; and with the width of the capella grande it will be double, which results in a diapason. The chapels will not fail to be proportionate to the other chapels which will be near and adjoining (placed against: *scontri*) the capella grande; they will be 4 paces in the sesquitercial ratio, which forms the diatessaron, a celebrated proportion. Thus all the measurements of the plan, lengths as well as widths, will be in perfect consonance, and will necessarily delight those who contemplate them, unless their sight be dense and disproportionate. Now for the altars of the chapels. I recommend that they be outside the square of the chapels, separated from the latter by balusters or railings like a sanctum sanctorum, into which none but the priest with his acolyte can enter. And this will be the case in all the chapels except the two false ones, for which one can not adhere to this order. I recommend that the church should be kept above the street, and even more so the chapels, where there will be three steps, by which one will ascend to them. This has always been the opinion of everybody, and has already been begun at the capella grande and the choir. I recommend to have all the chapels and the choir vaulted, because the word or song of the minister echoes better from the vault than it would from rafters. But in the nave of the church, where there will be sermons, I recommend a ceiling, (so that the voice of the preacher may not escape nor re-echo from the vaults). I should like to have it coffered with as many squares as possible, with their appropriate measurements and proportions; which squares should be treated in a workman-like manner with grey paint, a colour which we deem agreeable, and more impressive and durable than others. And these coffers, I recommend, amongst other reasons, because they will be very convenient for preaching: this the experts know and experience will prove it. Turning now to the height, I commend the same as that which M. Giacomo Sansovino has given to his model, namely 60 feet or 12 paces, in the sesquitercial proportion to the width, which results in a diatessaron, a celebrated and melodious harmony. And finding in that model all the other heights of the capella grande, the medium, and the small chapels thus proportionate, I shall not enlarge on these by going into particulars. Similarly, I recommend the orders of the columns and pilasters to be designed according to the rules of the doric art, of which I approve in this building as being proper to the Saint to whom the church is dedicated and to the brethren who have to officiate in it. Lastly it remains to speak of the front, which I wish should be in no way a square, but it should correspond to the inside of the building, and from it one should be able to grasp the form of the building and all its proportions. So that inside and outside, all should be proportionate. And this is our final intention, in which there agree with us the generals (of the Order) and the undersigned

fathers, i.e. the most reverend "padre ministro" and the "Diffinitori." So that nobody will be able to dare, nor be any more at liberty, to change anything.

Given in our place S. Francesco a Vigna, Venice, April 1st: authenticated in our place S. Lodovico a Ripa, the 25th of the same month, A.D. 1535.

I, F (rate) Francesco Georgio, at the request of the most serene P(adre) have made the above description so that everybody may understand that what one undertakes in this church, is done in accordance with good principles and proportions and so I commend and pray that it may be done.

## APPENDIX II

### *Bibliographical Notes on the Theory of Proportion*

The literature on proportion in architecture has been very extensive during the last hundred years. The purpose of the following synopsis is to give the reader a few general bibliographical hints. Many of the systems which apply a mathematical formula or a geometric configuration to historical buildings are mutually exclusive. There is no bridge between Pennethorne's<sup>1</sup> findings of harmonic ratios in Greek architecture and Hambidge's "dynamic symmetry."<sup>2</sup> Zeising believed he had discovered the golden section as the central principle of proportion in macrocosm and microcosm;<sup>3</sup> Odilio Wolff found the solution of the secret in the hexagon;<sup>4</sup> Lund in the pentagon;<sup>5</sup> and Moessel in the "geometry of the circle."<sup>6</sup> Viollet-le-Duc's triangulation<sup>7</sup> was modified by Dehio<sup>8</sup> whose results stimulated a younger generation. Hay developed a comprehensive system based on a revival of the Pythagorean law of numbers<sup>9</sup> to which later authors returned, sometimes without a knowledge of Hay's work; quite recently the Pythagorean conception re-emerged in an American "Primer of Proportion" for practical use of architects.<sup>10</sup> Thiersch's finding of the analogy of geometric figures throughout a building as the essence of harmonic proportion<sup>11</sup> was approved of by men like Burckhardt,<sup>12</sup> Woelfflin<sup>13</sup> and Giovannoni;<sup>14</sup> Thiersch's discovery was certainly of the greatest importance for the history of proportion during the Renaissance, but what was in fact a result was here taken as a cause. In spite of the great variety of purpose and the

<sup>1</sup> John Pennethorne, *The Geometry and Optics of Ancient Architecture*, 1878.

<sup>2</sup> Jay Hambidge, *The Parthenon and other Greek Temples. Their Dynamic Symmetry*, New Haven, 1924.

<sup>3</sup> A. Zeising, *Neue Lehre von den Proportionen des menschlichen Körpers*, Leipzig, 1854. Zeising's "historic survey of the previous systems" is very useful as an introduction. Zeising had a large following, see F. X. Pfeifer, *Der goldene Schnitt und dessen Erscheinungsformen in Mathematik, Natur und Kunst*, 1885. Interesting bibliographical material is to be found in R. C. Archibald, "Notes on the Logarithmic Spiral, Golden Section and the Fibonacci Series" in Hambidge's *Dynamic Symmetry*, p. 152 ff.

<sup>4</sup> Odilio Wolff, *Tempelmasse*, Vienna, 1912.

<sup>5</sup> F. M. Lund, *Ad Quadratum*, London, 1921. Cf. Theodore A. Cook's criticism of the theories of Lund, Hambidge and Samuel Colman: "A new Disease in Architecture," *The Nineteenth Century*, 1922, pp. 521-32.

<sup>6</sup> Ernst Moessel, *Die Proportion in Antike und Mittelalter*, Munich, 1926; *id.*, *Urformen des Raumes als Grundlagen der Formgestaltung*, Munich, 1931.

<sup>7</sup> Viollet-le-Duc, *Dict. rais. de l'architecture*, article "symétrie" and *Entretiens sur l'architecture*, Paris, 1863, Vol. I.

For further studies written in French on proportion see E. Lagout, *Esthétique nombree*, Paris, 1863; E. Henszlmann, *Théorie des proportions*, Paris, 1860; A. Aurès, *Nouvelle théorie du module, déduite du texte de*

*Vitruve*, Nîmes, 1862; L. Cloquet, *Traité d'architecture*, 1901, Vol. V, pp. 49 ff., 165 ff., with further literature; M. Borissavliévitch, *La science de l'harmonie architecturale*, 1925.

<sup>8</sup> Georg Dehio, *Untersuchungen über das gleichseitige Dreieck als Norm gotischer Bauproportionen*, Stuttgart, 1894, and enlarged in scope: *Ein Proportionsgesetz der antiken Baukunst und sein Nachleben im Mittelalter und in der Renaissance*, Strassburg, 1895.

<sup>9</sup> D. R. Hay, *The Science of Beauty, as developed in Nature and applied in Art*, 1856, contains a summary of the author's previous detailed elaboration of his thesis in several volumes. Of these see particularly: *The Natural Principles and Analogy of the Harmony of Form*, London, 1842.

<sup>10</sup> R. W. Gardner, *A Primer of Proportion in the Arts of Form and Music*, New York, 1945. The notion "area scale" is here introduced as the visual counterpart to the musical scale.

<sup>11</sup> August Thiersch in *Handbuch der Architektur*, IV, I, 1883, p. 39 ff. Cf. also John Beverley-Robinson in *Architectural Record*, 1898, p. 297-311.

<sup>12</sup> Jacob Burckhardt, *Geschichte der Renaissance in Italien*, 6th ed., 1920, p. 98 ff.

<sup>13</sup> Heinrich Woelfflin, *Renaissance und Barock*, Munich, 1888, pp. 53-57.

<sup>14</sup> Gustavo Giovannoni, *Saggi sulla architettura del Rinascimento*, Milan, 1931, p. 12 ff.

diversity of systems the combined effort of the last hundred years has put at our disposal a rich and comprehensive material which has helped to clear the way for an understanding of what the builders of the temples, cathedrals and Renaissance palaces thought about proportion; and more recent studies by men like Ueberwasser,<sup>1</sup> Ghyka,<sup>2</sup> Hautecoeur,<sup>3</sup> and Frankl<sup>4</sup> have thus much in common, that they aim at the correct interpretation of historical material rather than at the advocacy of an exclusive system.

<sup>1</sup> Of Ueberwasser's writings cf. particularly "Nach rechtem Masz," *Jahrbuch der Preuss. Kunstsammlungen*, 1935, pp. 250-272.

<sup>2</sup> Matila C. Ghyka's books (*Esthétique des proportions dans la nature et dans les arts*, Paris, 1927 and *Le Nombre d'or. Rites et rythmes pythagoriciens* . . . Paris, 1931), though difficult to digest, contain much valuable material; see particularly his lecture delivered during the International Congress of Art History at Stockholm, 1933: "Influence de la mystique pythagoricienne des

Nombres sur le développement de l'architecture occidentale," summary in *Résumés des communications présentées au congrès*, Stockholm, 1933, p. 263 ff., which has much in common with the theme of the present article.

<sup>3</sup> L. Hautecoeur, "Les proportions mathématiques et l'architecture," *Gazette des Beaux Arts* XVIII, 1937, p. 263 ff. A brilliant summary of the problem.

<sup>4</sup> Paul Frankl, "The Secret of the mediaeval Masons," *Art Bulletin*, XXVII, 1945, pp. 46-60.