

, D.
DEPARTMENT OF MUSIC STUDIES IONIAN UNIVERSITY

Honours Project

of Dikaiou Paraskeui/M2011077

With subject

<<Aesthetic and philosophical theories of the relationship
between music and mathematics>>

Supervisor:
Siopsi Anastasia



CORFU
ACADEMIC YEAR 2018-2019

CONTENTS

Introduction.....	p.3
First Chapter: “Music and mathematics”	p.6
Second Chapter: “The Pythagorean School”	
2.1 Historical Background.....	p.17
2.2 Theories.....	p.19
2.3 The harmony of the spheres.....	p.22
Third Chapter: “Iannis Xenakis”	
3.1 Historical Background.....	p.29
3.2 Theories.....	p.31
3.3 The stagnant music of Iannis Xenakis.....	p.36
Conclusion.....	p.43
Bibliography.....	p.44

INTRODUCTION

Music is the pleasure the human mind experiences from counting without being aware that is counting!

Gottfried Leibniz

Have you ever wondered what mathematics really is? How we do the calculations or where they might exist?

For most of people, it is a tedious process of measuring, calculating, rules we're taught in school, with rigorous reasoning and coolness. It is the field, which from an early age we hear, that in our lives we would be unable to do anything without him, since everything around us depends on them. And now I want to ask yourself, what is music?

Music contains feelings, tranquility, emotion. From ancients it is the main ingredient for education, morality, entertainment. Pretty much we all have been singing softly at some elusive moment of our lives, we have been pounding the pencil on the desk at some pointless class hour and we all have been coming across a piano. Music, like mathematics, is the last piece of memory that leaves man. Music is, therefore, a great and thorough way of expressing, treating, or practice of our remembrance.

Now, I would like you to link mathematics to music by combining to your mind all of the contradictions we have mentioned. It seems impossible but it is so obvious. It took me many hours of study to realize that in the end music is just maths with different symbols. From the first-grade fractions of Primary School, which were perfectly applied to rhythmic motifs of the conservatory, to entire statistical theories applied to contemporary music works.

In the next chapters, I will try to present you not only these theories but also the philosophy of this peculiar relationship.

In the first chapter, we will learn about the relationship between music and mathematics from ancient Greek years up to 20th century.

Some of the most well-known theories existed today, are based on this relationship and are formulated much earlier than we believe. Musical interval is something that concerns the field of musicology way before the appearance and finalization of Western music. The first clue, for example, about musical space is made by Aristoxenus "An interval [diastēma] is that which is bound by two notes which do not have the same pitch"¹. Over the centuries, reflections have escaped the musical spectrum and occupied other areas such as physics, astronomy even computer science.

Pythagoras of Samos is the first who attempt to connect music and astronomy. He was born in 572 BC² in Samos, from which therefore get his name, and was the son of an aristocratic family. A factor that helped him later travel to many countries of the known world and embrace many of the ideas and philosophies he knew. He studied close to philosopher Thales in Miletus and he might has enriched young Pythagoras with the passion of mathematics and cerebration. In 530 B.C he

¹ Spyridis, H. Charalambos, "The Musical Space through Aristoxenus" (PDF), from the National and Kapodistrian University of Athens website (<http://users.uoa.gr/~hspyridis/diastimaaristoxenos.pdf>), accessed 10-1-2019

² There are no sources for the exact date of the philosopher's birth. Most of them are between 570BC and 580 BC, e.g Maor Eli, The Pythagorean Theorem – A Story of 4000 years, Athens, Katoptro (2008) translated by Apostolopoulos Nikos, p.40

abandoned Samos because of Polycrates' tyrannical rule. So he emigrate to the Greek-populated area of Crotone³, in southern Italy, where he set up the famous Pythagorean School which initially had a political – religious purpose and subsequently became involved in science and research. And just like any vigorous and charismatic personality who feeds the world with a thought, that perhaps he should not, has acquired opponents and enemies. Years later and after riots that broke out in Crotone's area, both himself and his students, who at least survived, were scattered in various places. Pythagoras moved to Metapondio where he died. However, even after his death, the Pythagoreans continued their activity until Plato's time.

Nowadays, he is one of the most important mathematical figures in the world who urged ancient Greek mathematics, astronomy, music and philosophy.

Pythagoras was one of the first to believe and prove through time, that music is everywhere, as is mathematics, and that even on the planets there is a characteristic melody that simply does not reach our hearing. He created the great quadruple of lessons, the famous quatrivium, which includes music. First one was arithmetic followed by geometry, music and finally astronomy. This quadruple, raised by Pythagoras and later by Plato, has been a key guide to education in the Western world especially since the Middle Ages. Furthermore, was also a reason for this correlation to be made between numbers and notes, which of course did not stop in ancient Greece, not even in the Middle Ages. From Renaissance onward, we are talking about golden ratios, that is, mathematical motifs, in perfectly proportioned works of art. And from Renaissance we reach Romanticism and the 20th century, where information and technological means finally contribute to better analysis, study and understanding of all disciplines. In addition, in the last two centuries knowledge has been made freely available to all, resulting in more being aware of this peculiar relationship and also for scholars having more access to information.

Technologically, though, in the 20th century things are still steady and calm. Yes, there is an incredible breakthrough with technology, however, we are at a time that technology is still serving science, not the other way around. And Iannis Xenakis, with which this work will be completed, is exploiting this honestly and perfectly.

Iannis Xenakis was born in 1922 in Braila, Romania and was the eldest son of Klearchos Xenakis and Fotini Pavlou⁴. He spent his childhood and adolescence in Spetses, without much company until he moved to Athens in 1938 to prepare himself for the exams at the University of Athens, where he eventually succeeded at the National Technical University of Athens. With the outbreak of war and after an accident during a battle, he leaves Greece and manages to move to Paris in 1947, where he settles, while back in Greece, he's declared to be a deserter and sentenced to death. By the time he left Paris in 1974, he was developing into a genius music scientist. Connects almost all sciences with music. Laws of physics, macroeconomics, architectural drawings, but also eternal mathematical puzzles, from part of their compositions to such an extent that provokes reactions to his modernism and revolutionary thinking.

³ The area of Crotone belongs to the wider region of Calabria, a region that has been a state of Greater Greece. It was founded around 709 B.C in a safe and accessible part of the Ionian Sea, an asset that facilitated maritime trade. Our historical references show us that Crotone was created by the Achaeans, under the command of Mischelo. However, its founding is also linked to the birth of Hercules, though the date of the hero precedes the founding of the city. In 530 BC the city received and hosted Pythagoras and also his school, where they played an important role in shaping the culture and life of the city "History of Crotone (Kroton) in Italy" from: Italy this way (<https://www.italythisway.com/places/articles/crotone-history.php>), accessed 5-5-2018

⁴ She was the daughter of an industrialist from Brail who came from Lemnos. She was younger by seventeen years than Klearchos Xenakis and before she married she studied piano and foreign languages in a monastery, possibly in the area of Braila. She died of meningitis when Iannis was five years old (Makis Solomos, Iannis Xenakis: The Universe of a Peculiar Creator, translated by Plyta Tina, Athens: Alexandria, 2008 p.18)

For some people today, his work considered at least incomprehensible. Unlike Coltrane, Xenakis offers you uncertainty about the center of tone. It is also the “fashion” of the era. But also gives you the feeling that what you hear is not there in itself, only with notes. You know something is behind you even if you can’t auditory perceive it.

The purpose of this project is, therefore, to offer to the ordinary reader, as well as the researcher, the opportunity to get to know aspects of this beautiful and complex relationship. Starting with simple mathematics, those of fractions and standard mathematical operations, will come up with modern mathematical theories and proofs applied to musical works and musical theory. It is not so much the aim of the complete understanding of the mathematical theories as they are to present, as to how these two disciplines have succeeded in penetrating each other, with absolute precision, so as to reach the highest form of art.

Clearly there were difficulties during its writing and research due to information which were not valid or sometimes I needed special permission to access them or they could not cross. However, there were tools and writings that provided me with unlimited help and formed the basis for further study and research, and based on them I supplemented the structure of this project. The main guide was the online encyclopedia Britannica. Com, texts from Barker. A. The science of Harmonics in classical Greece, (Cambridge: Cambridge University Press, 2007), Jamie, J., The music of the spheres, (London: Abacus, 1995) Academia websites Edu and Mendeley. Com, with academic articles from around the world, but also articles in periodicals such as Crocker. R., “Pythagorean mathematics and music”, Journal of Aesthetics and Art criticism (1964). an important contribution to Xenakis’ chapter was the writing of Maki Solomos, Iannis xenakis: The Universe of a Peculiar Creator (Athens: Alexandria, 2008), by Matossian Mouritza, Iannis Xenakis, (Pari: fayard, 1981) and the edition of 1986, but also material from his personal interviews with Varga, Balint Andras, Conversations with Iannis Xenakis, translated by Symeonidi Aleka, Athens: Potamos, 2004). but beyond the help of writings, I would like to thank professor Anastasia Siopshy for her precious help and guidance as well as for the valuable time she provided me with and of course my family for the material, psychological and financial support all this time.

Hopefully I have been able to convey as much as I wanted, all the wealth of knowledge that I have learned and taught and to offer at readers a complete musical journey into the world of numbers and brilliant theories.

I wish you all a pleasant reading!

FIRST CHAPTER

“Music and mathematics”

*«Music is math with sound,
mathematics are silent music. »*

*Edouard Herriot
French Prime Minister*

No one can imagine the relationship between music and mathematics. Clearly, everyone knows that they are connected, but most simply stay in the relationship of rhythm or measure or even harmonics. But in the end, after so many centuries and theories, it seems that these two have come together and interact with each other deeper and deeper than what we see. Mathematics is the most abstract form of science and music is the most abstract form of art. And while the former lies in meaning, parameters and logical truth, and the latter in the sense of sound and rhythm, we have a connection from ancient times, and of course I mean Ancient Greece.

Ancient Greek music is divided into six periods. The prehistoric period (up to 1100 BC), the geometric period (1100 -750 BC), the archaic period (750-479 BC), the classical period (479 - 306 BC), the Hellenistic period (306 - 30 BC) and finally the Roman period (30-337 AD). During the archaic period we have the greatest mobility in terms of letters and arts. Music is inextricably linked to poetry, from which it derives its rhythm. At the same time it was an essential component of young people's education along with mathematics, philosophy, rhetoric and gymnastics.

ANCIENT GREECE

At that time, a philosopher from Samos first associated it with mathematics and astronomy, setting it as the highest means of purifying the soul and body. It also played an important role in people's daily lives. All existing references to philosophical, historical, or poetic texts confirm that there is no an event or religious ceremony without music. At the same time it had a divine nature as it

healed the soul and body, relieved and calmed the sick.⁵ And while we have so many and important sources in the past, to look at, we miss the main point. The music itself. As you already know, the musical texts that we have are very few to zero, and that is because music was then passed down verbally from generation to generation. So how do we know for sure about the form of music in Ancient Greece?

Pythagoras of Samos introduces music as a philosophical tool, but also as a unit of measurement of the distances of the planets, a theory that will be confirmed centuries later by astronomers Bode and Titius⁶. So with Pythagoras, the first connection between these two disciplines begins, where others were later encouraged to develop it and to offer new theories and proofs, such as Archytas of Tarentum.

Archytas was born in Tarentum in 428 BC and belonged to the second generation of Pythagorean philosophers. He has been a teacher of many later philosophers, and in addition to his scientific and philosophical leanings, he has shown tremendous ability as a general.

He dealt with philosophy, music, mathematics and astronomy, contributing many important discoveries and theories, and this is confirmed by Aristotle himself writing a whole treatise exclusively for him, entitled *The Philosophy of Archytas*⁷. He himself was in close contact with Plato⁸ and is rumored to be the one who introduced Plato to Pythagorean thought and philosophy.

As a graduate of the Pythagorean School, he argued that mathematics is the secret to understanding the world. Hence the important theories he developed in the field of geometry⁹. He also mentions for the first time the three proportions, harmonic, arithmetic and geometric.

Philosophically, he was oriented in two directions, one arguing that one cannot always explain one's explanation of natural phenomena, and the other being organic from the inorganic world does not ultimately have such great differences. Philosophies that others, subsequently, followed.

Musically, he belonged to the flutists and was even famous for his craftsmanship. From surviving passages, he seems to have written a treatise on flute entitled "On Flutes"¹⁰ and raised concerns about musical tones, and therefore appears to distinguish genus music according to its

⁵ Asclepius' father of medicine created his own music and harmonies to heal mental and emotional disorders. See Lapatas, Kosmas A., "Introduction to Music Therapy," from Academia website Edu. (https://www.academia.edu/11462850/%CE%95%CE%99%CE%A3%CE%91%CE%93%CE%A9%CE%93%CE%97_%CE%A3%CE%A4%CE%97_%CE%9C%CE%9F%CE%A5%CE%A3%CE%99%CE%9A%CE%9F%CE%98%CE%95%CE%A1%CE%91%CE%A0%CE%95%CE%99%CE%91), accessed on 27-11-2018

⁶ See Chapter II "Music of spheres"

⁷ Unfortunately, this fact has not been able to be rescued, and we know this from very few references to his own texts or to other philosophers.
See "Archytas of Tarentum", from the Panhellenic School Network website (<http://users.sch.gr/thafounar/Genika/problemGeometry/doublingCubeArchytas/archytas.html>), accessed 14-1-2019

⁸ In the Seventh Letter Plato reveals that Archytas rescued him from the tyrant of Syracuse, where he wanted to kill him.
See trans. H. E. Corbeti, *Plato's Letter Z'. Introduction, translation, notes* (Athens, Moment, 1997).

⁹ Archyta's most well-known problem is the "Delian problem". The delian problem was the doubling of the cube, where it was finally achieved thanks to Archytas by using the semi cylinder. See Proklou Eucl. Pol, II 66, 64.

¹⁰ See: Athens. IV 184 e

characteristics, diatonic, harmonic and color¹¹. At the same time thanks to his mathematical knowledge he made the observation that depending on the length of a flute the sound produced is sometimes acute and sometimes heavy. With his mathematical knowledge, Archytas improved the field of engineering as well as action-reaction theory that helped create rockets centuries later.

He died in 347 BC in Tarentum, Italy, leaving a rich work of writing from which has unfortunately survived a few excerpts.

Aristoxenus¹² graduates with Archytas from Pythagorean School and embraces his point of view. Born in 354 BC, a student of Aristotle and raised in Tarentum, he formulated the theory of calculation of the scales, which can be achieved through mathematical relations. He held that the notes of the scale are to be judged not by mathematical ration but by the ear. He is not interested in mathematics relationships that may exist within the scale, however, he defines the half and whole tone and makes a scale, based on the twelfth of the tone. Then divide the eighth interval into twelve equal parts, which are the vocalizations, which are one from the other, a semitone. With this theory, modern music researchers, claim to be the creator of the first dimensional scale¹³.

Moreover, from his texts we conclude that he already stood out many of the rhythmic, harmonious and melodic elements that now are taken for granted, such as harmonic tonality and chromatic lifting and positioning into a rhythm, trimitonium, semi tone, potential keys that a melody can be structured, musical genres and more.

He wrote many theoretical books on harmony and music, but today only two as well as some excerpts from his other works survive. He and Pythagoras were the foundations of the composition in both the Byzantine and modern Western times.

Plato is a big part of ancient Greek philosophy. He was born in Athens in 427 BC. He traveled to Lower Sicily and the Tarentum of Italy, where he came into contact with the Pythagorean philosophers and embraced a great deal of their ideas and philosophies. That is why in 387 BC he returns to Athens, establishes a philosophy school, known as the Plato's Academy, and teaches not only philosophy, but mathematics, music and political sciences¹⁴.

His theories about music were mainly about ethics and education and not so much about mathematics. However, his belief as he illustrates it, was that in mathematical harmonies one can find the answer to the rational organization of the universe. In addition, his Pythagorean influences appear in his work "Republic" (Politeia), in which he argues that in order for man to achieve moral and mental superiority, he must deal with four specific subjects, geometry, numeracy¹⁵, astronomy

¹¹ Ptol. Arm. I, 13, p.13

¹² «Aristoxenus of Tarentum», Donald James, The chamber's encyclopaedia, (London: George Newnes, 1961), Vol. 1 pp. 593

¹³ Leousi, Linda, History of Greek Music, (Athens: Ankara, 2003), p.68

¹⁴ Diogenes Laertius, Lives of Eminent Philosophers, Book C'.

¹⁵ The theory of numbers in ancient times was called arithmetic and the practice of arithmetic, accounting. These two areas, according to Plato, belonged to world of ideas. See: Papaiwannou Andreas, Balaskas Alexandros, Thouas Konstantinos, Sotiropoulos Loukas, Korfiatis Petros, "Plato's Contribution to Mathematics", from 9th Patras General High School website (<http://9lykpatras.ach.sch.gr/autosch/joomla15/phocadownload/platon.pdf>), accessed: 9-12-2018

and music. Music for him was something ideal. That is why in Timaeus 's¹⁶ work he refers to Pythagorean theories of music, claiming that God used the proportions of the Pythagorean musical scale when creating the human soul. Plato certainly did not focus as much on the mathematical nature of music as on his educational and ethical side, which was followed by Aristotle as his student. In the course of his career, however, it has turned out that teacher and student documented argued on many issues.

Descendant of all this philosophy on music and mathematics was Euclid (325 - 265 BC). The great geometrist formulated a theory on geometric distances of the music spaces. How he manage that?

Applied musical intervals over straight lines, except that geometrically straight lines produced as numbers are defined by two letters (one at the beginning and one at the end), here defined by a letter. For example, the one known in everyone interval is generated by hemiolia¹⁷ and the semicolon. To find out the result we need to multiply the arithmetic relations of the spaces that compose it, that is, $3/2 * 4/3 = 2$. At the same time Euclid in this way created the four strings with the two whole tones, different from Pythagorean, to facilitate, because the difference in acoustics was not perceptible. With this four strings Euclid also formulates the Euclidean rule, which proved that when a chord vibrates whole, it produces the heaviest sound. If we now place a sub-directory in certain positions of the rule, then we see that the rest of the vocals are produced in turn.

But beyond all this music-mathematical philosophy he offered, Euclid discovered and offered to the later world fundamental relationships that surround and include sounds, and as a result he is considered to be the founder of what we today call acoustics and is now an independent discipline.

HELLENIC PERIOD

_Going back to history we reach the Alexandrian times or more properly Hellenistic times (323 BC - 31 BC), with Alexandria of Egypt, as a center of culture, letters and arts. At that time we have historically two persons who stand out in the scientific field, Hypatias and Didymus Alexander.

Didymus from Alexandria has been a philologist, orator and musician. His contribution to music has been decisive. Until then, Pythagoras's music scale was dominant, which, while highly mathematical, was accurate, but the divisions were detailed and difficult to analyze in its entirety. So he creates a soft music scale, with the first harmonic rings. That is, 2: 1 octave, 3: 2 5th pure, 4: 3 4th pure and 5: 4 3rd great. At a ratio of 9: 8 corresponding to the major tone, the first five strings formed, repeating it over the fifth. The result is the following scale:

DO	RE	MI	FA	SOL	LA	TI	DO
1	9:8	5:4	4:3	3:2	27:16	15:8	2

¹⁶ It is philosophical book on nature of Plato, dating back to the work of “Republic” and before the work of “Critias”. It is interactive and Socrates, Timaeus, Critias and Hermocrates participate in the dialogue. The point of the dialogue is the creation of the world, with fundamentals research tools logical probability and plausibility. The project is a reference point for researchers of the Lost Atlantis, since it is officially the oldest source in it. See: Kalfas B., Plato – Timaeus (PDF), Athens: Estia, 2014.

¹⁷ Hemiola or hemiolia is an ancient Greek term for the first time in the writings of Ippasou and Philolaos, and mainly refers to the rhythmic part of a melody. It corresponds to the mathematical ratio of 3: 2 and describes in today's European music the period of the pure fifth
See: «Hemiola, hemiolia», Don Michael Randel, The Harvard Dictionary of Music, 4th edition, (Cambridge: The Belknap Press of Harvard University Press, 2003), pp. 376

These are the frequencies that occur, in terms of the intervals we now have as follows :

DO RE MI FA SOL LA TI DO

(κε → ζω → νη → πα → βου → γα → δε → κε' →

Γ 9216 - 8192 - 7776 - 6912 - 6144 - 5832 - 5184 - 4608 “κόμμα TOY Διδύμου” or a coordinating party and is the ratio 81:80.

By talking about the East, our research could not miss the Byzantine. In the 2nd century AD, Byzantium enlightened both the East and the West with its culture. Arts, letters and music were particularly dominated by the 4th century AD. The Byzantine music that has managed to reach our days is composed of Greek texts as a melody. All historians agree that Byzantine music is a product of the evolution of ancient Greek music¹⁸.

BYZANTINE AND POST- BYZANTINE PERIOD

Musician and theoretician Cleonides, who lived in Byzantium in the 2nd century AD divided the tetrachord into 30 equal parts, 12 of which are in tune and 6 in semitone. The tetrachord of Cleonides, as it was called, consists of two tones and a half tone ($12 + 12 + 6 = 30$), but neither of them observes the physical space. That is why the tetrachord of Cleonides fitted the rules of Byzantine music better, dividing it into 28 parts and the major way being 12 parts, the minor way 9 and the semitone of 7 ($12 + 9 + 7 = 28$). So in Byzantine music this is considered the natural tetrachord.

And yet it took several centuries, from the Byzantine times, to re-write some theory about music and mathematics. The first attempt to determine the intervals in the post-Byzantine period is made by Chrysanthos of Madytos¹⁹. Chrysanthos was an avid scholar of ancient Greek culture and life. Inspired by the Pythagoreans and Aristides Coudilanos²⁰, he connects the ancient Greek musical spirit with the contemporary of his time. Thus on the scale of Ke - Ke Chrysanthos presents the following mathematical equations

¹⁸ Baynes, Norman H., Byzantium: An Introduction to Byzantine Culture, translation Sakas Dimitris, (Athens : Papadimas 1983)

¹⁹ Together with Gregory Protopsaltes and Chourmouziou Chartofylax are the three great masters of new Byzantine music, introducing the New Method, where it concerns the musical writing system and the majority of it applies to this day.
See: Wellesz Egon, A History of Byzantine Music and Hymnography, 2nd edition, (Oxford: Clarendon Press, 1961)

²⁰ Ancient Greek musician who lived in the 3rd century AD and was mainly concerned with rhythm and melody, therefore the texts On Music are spread into three books. At the same time, their content is both philosophical and aesthetic.
See: ML. West, Ancient Greek Music, translated by Stathis Komninos (Papadimas, Athens, 1999), p. 347-348

which are identified with the Pythagorean Mathematical Rule.

**9216/8132=9/8, 8192/7776=256/243, 7776/6912=9/8, 6912/6144=9/8,
6144/5832=256/243, 5832/5184=9/8 και 5184/4608=9/8.**

Thus, today it is none other than the cruel diatonic scale of Byzantine music. He then accurately calculated the distance of the intervals such as the physical forth contains 28 molecules, that is to say, based on the table above, this interval is covered by 498 cents, that is, each molecule of 28 is estimated at $498 / 28 \approx 17,7857$ cents). In other words, Chrysanthos calculates the reasons for the scale based on the scale of Zarlinos, which we will see bellow.

The other two teachers of the New Method, Gregory Protopsaltes and Chourmouzos Chartofylax, they were more concerned with the practical part of Byzantine music. They focused mainly on chanting and creating new jobs.

He argued that humanity must return to ancient Greek thought about it and translated into Latin many ancient Greek writings.

ARABIC MUSIC

Arabic music was called the music of the people of Islam, such as Persia, Syria, and North Africa. Although it appears to have originated and spread from Islam onwards, its roots are many years behind.

The Arabs were one of the first cultures to deal with the sciences, the arts and the letters. Many of the mathematics we know today have evolved thanks to their own research. So how could the mathematical nature of music be missing from their field?

From the 9th to the 13th centuries we have many studies of the tonal system, in its most famous form, valid until today. Created by Al-Farabi²¹.

The area of the Arabic scale is calculated at two octaves and is the product of the division of the primitive Arabic octave at twenty-four equal intervals. Each note has its own name and is not repeated in either the low nor the high octaves. For this to happen, his position in the octave and his distance from his "neighbor" vocalists played a role. The heaviest octave corresponds to European music from the Sol under the pentagram to the Sol within the pentagram.

The name of the vowels is as follows:

Gheyac - Usayran - Iraq - Rast, Dugyah - Segyah - Sharjah – Neva

However, until today, the theories about the magnitudes of the fundamental scale of Arabic music

²¹ Al-Fārābī, in full Muḥammad ibn Muḥammad ibn Ṭarkhān ibn Awzalagh (or Uzluḡh) al-Fārābī, he was the registered philosopher of Arabic world. He was involved in philosophy, cosmology, metaphysics and religion and wrote many scientific works. He died in Damasko at 950 AD. See Corbin Henry, Nasr Hossein, Yahya Utman, History of Islamic Philosophy, translated by Liadain Sherrard, (London: Kegan Paul, 2001), p.158-164

are different. Most likely, they based themselves on the Pythagorean theory of divorced tetrachords to create their scale, and that is why Arabs at that time were scholars of ancient Greek texts.

Today the octave of Arabic music is divided into 25 equal spaces²², that is, two tetrachords and a whole tone, which have emerged with the help of lute and lumber, as suggested by Al Farabi in the 10th century. Later, some Arab theorists tried to define the Arabic tone based on the fifth circle and the unit of measurement known as the comma²³. The scale of Syrian music is divided into 53 equal intervals, not based on the Pythagorean comma (23.46 cm), nor on the coordinate (21.306 cm), but on the Holtrian comma (22.6415 cm)²⁴.

According to the same ideas, Turkish music was built a little later. Its tonal systems are based on calculations and further development of the Pythagorean system, by Safi-Ad-Din Ardabili.

²² Touma, Habib Hassan, *The Music of the Arabs*, by Tzotzynis Basilis, (Athens: En Chordis, 2007), p. 19-22

²³ The comma is a small space used in the mixing of non-diatonic scales, such as Byzantine music, Arabic music and Gregorian music. Often such intervals are not perceived by the human ear.
Barbour, James Murray, *Tuning and Temperament: A Historical Survey*, (Mineola: Dover Publications, Inc., 2004), p. 177

²⁴ Touma, Habib Hassan, *The Music of the Arabs*, by Tzotzynis Basilis, (Athens: En Chordis, 2007), p. 23

THE MUSIC IN THE WEST

In the West, things are different. Each century and season finds its tune and scale at different intervals. For example during the Middle Ages, the perfect 5th is considered the most important interval. In the perfect 5th but also in natural perfect notes is based all the tuning of the era. On contrary, in the Renaissance, the favorite interval is the 3rd. The tunings include natural thirds or near natural (mesotonic). At this point we have to clarify that yet we have no major and minor scales, but musical ways indefinable or with few recessions in the armament or some alteration in the adapter. Therefore, the "black" keys are tuned, depending on the utility the composer or performer wants.

In both periods, we have important theorists who have developed their own version of the relationship between music and mathematics.

In the Middle Ages, a mathematician appears, presenting an original mathematical sequence for the time, based on the Pythagorean approach to harmonic series, but also on an experiment with rabbits. Of course, it is none other than Leonardo of Pisa or better known as Leonardo Fibonacci²⁵(Leonardo of Pisa). So we are talking about the most well known mathematical sequence until today, the Fibonacci sequence, which was also associated with the ratio 1.61, the ratio of the Golden intersection.

Each number of the sequence is equal to the sum of the two preceding ones 1,1,2,3,5,8,13,21,34, 55 etc.). Up there, the two successive ratios of the sequence also extract the Golden Section ($f = 1.618033989$). The discovery of the Golden Ratio has revealed that many of the most famous works in all the arts both before and after are included and based on this number. For example, Mozart divides many of his works into two parts, where the length of each part is calculated as the golden number. The painter Leonardo da Vinci designed the Mona Lisa so that her face would fit perfectly into a gold rectangle²⁶. Later musicians based on the number, such as Claude Debussy²⁷ and Bella Bartok²⁸.

However, it must be said that, as in all arts and sciences, darkness has arrived. The Middle Age was and is one of the darkest pages in history, not only for the tragic events but also for the lost wealth of knowledge. Of course there were always times or periods when "history" should not have allowed the enlightenment of people. For example, in 400 AD Saint Augustine²⁹ stated: "The good Christian must beware of the mathematicians and all those who make empty prophecies. There is already a danger that mathematicians have contracted the devil to obscure the spirit and limit people to the bonds of hell"³⁰. In this point of view, it seems not only how new knowledge was treated at

²⁵ Grimm, RE, «The Autobiography of Leonardo Pisano», Fibonacci Quarterly (1973), Vol. 11, p.99-104

²⁶ $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$. The gold rectangular rectangle whose ratio of sides is 1 / f. That is

See Tattersall, James J., Elementary Number Theory in Nine Chapters, 2nd edition, (Cambridge: Cambridge University Press, 2005), p. 28-30

²⁷ Trezise, Simon, Debussy: La mer, (Cambridge: Cambridge University Pres, 1994), pp.51-53

²⁸ Lendvai, Erno & Bush Alan, Bela Bartok: An Analysis of His Music, (London: Kahn & Averill Publishers, 2005)

²⁹ Abatzopoulou Frangiskis, Introduction: Saint Augustine and the Confessions, translated by Amapzopoulou Frangiskis, (volume A, Athens: Patakis, 1990), pp. 23-42

³⁰ Kline, M., Mathematics: a Cultural Approach, (Reading, MA: Addison-Wesley, 1962), pp. 1

times, but also how religion was able to impede many times the scientific and artistic work of each era.

Nevertheless, even in the dark years, there were spiritual people who fought for knowledge, enlightenment and spirit.

Around 400 AD to 500 AD, thus, around the time when the Roman Empire was overthrown and the barbarian period was inaugurated (Goths, Ostrogoths, etc.), three philosophers appeared, Boethius, Cassiodorus and Isidor.

Boethius is called "the last of the ancient Romans". He was born in Rome and was an important adviser to Emperor Theoderic, but in the end due to slander and contempt by the courtiers, he was tortured and killed in the palaces. He wrote the work "Arithmetic Lessons" where he distinguishes knowledge objects into "sets" and "sizes". Arithmetic examines sets as identical. Music examines sets in relation to other sciences, while geometry and astronomy consider sizes, one in tranquility and the other in motion. Boethius intended to write a work for each of these sciences, but unfortunately he wrote only for the first two.

Cassiodorus (Magnus Aurelius Cassiodorus) was born around 475 AD and died in 570 AD. He was from Southern Italy. After his twenties, he retired to a monastery to study, where the fruit of the study was an encyclopedia consisting of seven volumes entitled *De artibus ac disciplinis liberalium literarum*, which deals with the whole trivium and quatrivium, where due to the lack of new fields of science and tools, it is limited to existing knowledge.

Among these great persons, who had the misfortune of presenting their ideas in dark years, was born in Seville or Carthage (we cannot be sure), Isidore, about 560 AD, and because he was the bishop of the city for some years, they named him Isidore of Seville. He has compiled an encyclopedia-style book, named "Origines", which examines all the disciplines of the time and among them mathematics and music. Although there was no personal breakthrough in the field of music and maths, his love for mathematics is worth admiring. He died in 660 AD in Seville.

On the contrary, in the Renaissance, things are illuminated. The season is exactly as the word describes it. The letters and the arts bloom. We have great projects that move us until today. But we also have galloping development in science and theories. Among them is the theory of the young musicologist Giossefo Zarlino (1517-1590).

Zarlino is considered one of the most important music theorists in history. He created a variant of the Didymos' scale, which in its turn was called the Zarlino scale, and alternated the sixth and seventh chords of the scale until it was accompanied by triple chords. Didymo's rules continue to apply while this scale was called Just intonation³¹.

Followed by Jean Batist Joseph Fourier³². He was born in 1768 in Auxerre, France and died in 1830. He dealt with physics and mathematics, with pioneering observations of the time, which we study to this day, such as the mathematical lines he developed, the famous Newton's Cooling Law, etc. His interest in our field fell into theory. of the pulsating string of Pythagoras. Clearly other mathematicians had tried to solve it and explain it, as Daniel Bernouli did, but Fourier succeeded completely. In 1822 he publishes a scientific work entitled *Theorie analytique de la chaleur*, where he fully presents the general solution to the problem of pulsating string. On the basis of some mathematical uncertainties, which Pythagoras could not calculate, due to a lack of technical means, Fourier proved that

³¹ See Duffin, Ross W., "Just Intonation in Renaissance Theory and Practice," from website A journal for the society for Music theory (<http://www.mtosmt.org/issues/mto.06.12.3/mto.06.12.3.duffin.html>) accessed 26-12-2018

³² "Joseph Fourier", Francois Arago, Biographies of Distinguished Scientific Men [PDF], (Boston: HO HOUGHTON AND COMPANY, 2005), (http://www.gutenberg.org/files/16775/16775-h/16775-h.htm#JOSEPH_FOURIER)

that in case that two notes sound together and one has twice the frequency of the other, then we have the feeling that the same note sounds. You may have noticed it in the interval of the octave. In short, it is the first to prove that any periodic function f is possible to be analyzed on an infinite sum.

At the end of the 19th century, a new genre of music, jazz³³, appeared mainly in America. And while most believe that it is mainly based on improvisation, she hides an endless mathematical interpretation. The first to literally sit down with pencil and paper to discover what is hidden is saxophonist John Coltrane (1926-1967). Born in North Carolina, America, he was involved in music from a young age and by 1947 was already playing professionally. In the 1950's he focused entirely on liberated jazz. His discography is huge, but what lifted him up was the work of GiantSteps³⁴. Chord changes are unique and require special craftsmanship. The name itself gives the idea of what is about. It is worth noting that later this technique was used both in jazz improvisation and for study in the performers, and these chords were called Coltrane's changes³⁵. What is different about this work? Its first and foremost feature is the Circle of fifths³⁶. The characteristic of the fifths is that they are very easily identified with the human ear because they are included in everything we hear about us. So these fifths are the backbone of jazz music, so this



Figure 1: The first steps from the "Giant" project

³³ Steps' Hennessey, Thomas, From jazz to swing: African-American jazz musicians and their music, 1890-1935, (Detroit: Wayne State University Press, 1994), pp. 470-474

³⁴ Cook, Richard, Morton, Brian, «John Coltrane». The Penguin Guide to Jazz Recordings, (8th ed., New York: Penguin, 2006), pp. 269

³⁵ Martin, Henry, «Expanding Jazz Tonality: The Compositions of John Coltrane», Theory and Practice (2012-2013), pp. 185-219

³⁶ In the circle of fifths we find all 12 notes of the western scale. That is, one note from the other must be five notes apart. If for example we start from C major the next note is G major.
See Whittall, A., «The Circle of the Fifths», Latham, E., The Oxford Companion to Music, (Oxford: Oxford University Press, 2002), pp. 259-260

The problem, however, lies not in the constant use of the fifths, but in the fact that their changes are performed in three different keys. Now to understand the continuity, we need to imagine each tonality as an autonomous language. As is well known, many languages are similar to each other, eg Spanish to Italian, but there are also those that have no commonality, such as Greek to Serbian. So we have tones that are almost common, such as Do major with Sol major, and tones that have no common point of contact, such as Do major with Si major recession. From the 20th century onwards, it has been observed that many songs mainly on jazz and pop culture select tones from the circle and are often converted into remote tonal centers. As a result, patterns are created within the circle of the fifths, patterns discovered by Coltrane himself in the 50's - 60's, in his attempt to push jazz to its extremes.

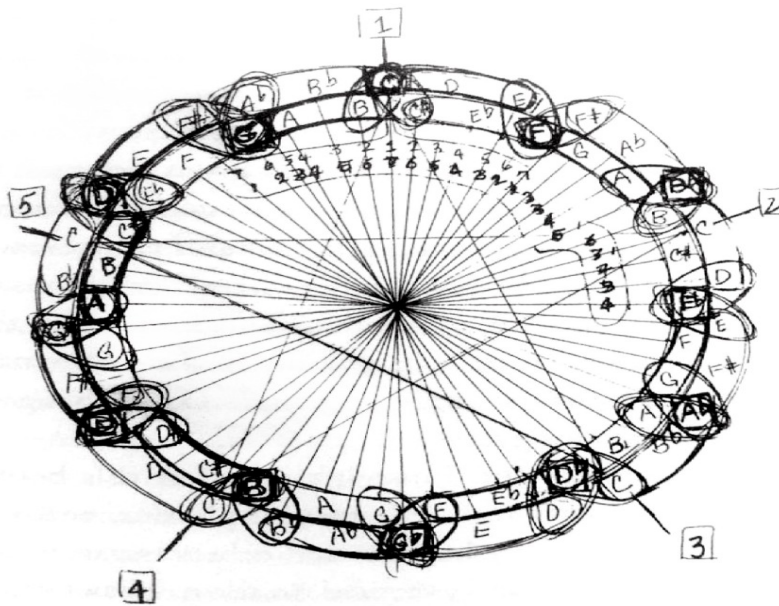


Figure 2: Coltrane's circle of fifths, according to his own blueprint

This is Coltrane's own study. Now if we look at the first steps of the Giant Steps, and apply the tones in the order that he's set them, in the above circle, we will see that gradually, a perfect star is formed. At the same time we will see the first three tones, are apart from each other by an interval of third³⁷, while the Si major and the Sol major beginning the piece belong to the scale of the Mi major. If we get to grips with the language, what is in the project, the result is certainly impressive. Essentially, the composer writes in unrelated languages, requiring both himself and the pianist to play two words per song, two words from each language, that is, two strings from each passing tone, at just 300 bpm. That is, he makes an entire musical phrase, from three or more "foreign languages". And all this through a single V-I cord progression.

So he creates a perfect musical-mathematical progression, through perfect musical triangles, where the edges are theoretically far apart, but the listener has the feeling, at least auditory, that he is always in a tonal center. But jazz is a kind of music that no one needs as much the pentagram and the pencil as the hearing, to understand it.

³⁷ Demsey, David, "Chromatic Third Relations in John Coltrane's Music", Annual Review of Jazz Studies (1991), Vol. 5 pp. 145-180

The same could be said a few decades later, in Europe with a Greek architect and musician excluded from his country, with almost revolutionary ideas on how to compose and listen to music, which until now has been the subject of study and imitation. Of course, there is none other than Iannis Xenakis (1922-2001).

He combined architecture and physics so well with mathematics that we can say that he created a new music industry. His mathematical and physical models are found in his works, many of which I heard for the first time. At the same time, he put music at the highest level of philosophical thought, with ideas such as the unity of philosophy, art and science. An idea that, if you look deeper, can succeed to some degree.

He characterized as a new-Pythagorean thanks to the use of mathematical models. Xenakis, however, did not use physics and mathematics to explain the function of music, but to create something new. He took advantage of these two disciplines as well as up-to-date scientific tools (UPIC system), and created new music³⁸.

There were difficulties. The groundbreaking and completely new element was not everywhere accepted. Even composers with innovative musical material such as Pierre Boulez³⁹ removed him. But his main enemies were not them, but himself. Because it's not easy at all exceeding each time the expectations and goals you set and each time to be unique and original, after all, he himself observes: "no one can create a new world. It is impossible to really create something different, there are no such examples in the history of art. It's sad: we are captives of ourselves"⁴⁰.

For this dual of his character, but also for the revolutionary formalism he has applied to his compositions, in Xenakis we find warm supporters and critics to this day. He was never interested in the criticism that was brought to him, perhaps because he awaited it from his own works and consequently from himself. Xenakis remained as passionate and idealistic with his work until the end of his life (4-2-2001) and to this day occupies the music scientific community as something inaccessible and unique.

³⁸ See Chapter III The stagnant music of Iannis Xenakis

³⁹ G. William HOPKINS, et Paul GRIFFITHS, "Boulez, Pierre", in *The New Grove Dictionary of Music and Musicians*, 2d ed., Stanley Sadie et John Tyrell (ed.), London, Macmillan Publishers, vol. IV, 2001, p. 98-108.

⁴⁰ Varga, Balint, *Conversations with Iannis Xenakis*, (London: Faber and Faber, 1996), by Symeonidou A., (Athens: Potamos, 2004), pp. 71

Second Chapter

“The Pythagorean School”

" All things are numbers "
Pythagoras

"Everything is done by numbers," this tells us the quote above, and is probably one of the most important theories of both the 6th century BC and Pythagoras and of the Pythagorean school which himself will set up as we shall see later.

Starting from the first philosophers appearing in the early period of philosophy, the so-called pre-Socratics⁴¹, it is remarkable that it is extremely difficult to have an objective evaluation and interpretation, since we do not have fully rescued philosophical works from this early period. The investigation becomes even more difficult in the case of Pythagoras. Pythagoras belongs to one of the five pre-Socratics philosophers, with a great philosophical and scientific work that, however, he never wrote it down. He represents an original philosophical thought about the facts of that time, which dates back to the middle of the 6th century BC, in Croton in lower Italy. His philosophy was based on trying to understand and interpret the world, as well as on the search for the meaning of life and death. For the first time in the history of ancient Greek thought, a philosopher combines both moral and religious beliefs and the materialistic nature of the natural world into a single and synthetic composition. For his personality we have a wealth of ancient witnesses. In fact, since the 3rd century alone, we have three extensive biographies, fully preserved, of Diogenes Laertius: *Lives of Eminent Philosophers*; of Porphyrius: *Life of Pythagoras* and of Iamblichus: *The life of Pythagoras*. On the other hand, however, the years that have elapsed since the creation of these biographies, as well as the magnificent information they provide, are key reasons for their credibility to be questioned by many. Because of the many unreliable information about his life, many such as the skeptics have even come to question his existence. Fortunately for us today, there are many later references by both philosophers and historians to Pythagoras' life that confirm the validity of certain biographies elements⁴².

2.1 Historical background

The founding of the Pythagorean school is probably about 530 BC⁴³ shortly after Pythagoras arrived in Croton. It is the first formal school to be established although descriptions and surviving elements bring us more into a rigorous religious brotherhood.

⁴¹ "Pre- Socrates", we call all those philosophers who lived and developed philosophical beliefs before Socrates (470 - 399 BC).

See Kirk, G. S., Raven, J. E., Schofield M., *The Presocratic Philosophers : A Critical History with a Selection of Texts*, Cambridge: Cambridge University Press, 1983, pp. 7-8

⁴² "Pythagoras, the son of Mnesarchus, was most indebted to research; and having chosen from these opinions, he made his own wisdom: multifaceted and fraudulent art" , Heraclitus, text 129

"They say that when he once saw a dog being tortured, he sympathized with it and said, 'Stop, don't hit it, because it's the soul of a friend; I recognized it when I heard his voice,' " Xenophanes, text 7

⁴³ See Hermann Arnold, *Think Like a God: Pythagoras and Parmenides*, Athens, Enalios, (2008), translated by Triada Pinelopi, p. 65

The Pythagorean Academy was originally created for the study of philosophy and mathematics. Over the years, this study has expanded to areas such as astronomy, physics and music. The function of the school was based on mysticism, collective thinking and work. We could liken his mentality and way of thinking to that of the early Christians.

The school accepted students by strict criteria and was not confined to the student sheet. The tests that went through the process of entering the school were called Pythagorean Mysteries⁴⁴. They were simple at first and then difficult. During the trials, Pythagoras examined their secular life, their occupations, their personal relationships with their parents and friends, and their social behavior. In addition, he relied heavily on the posture of the candidates as well as on the speech.

In the next stage they were tested in silence and loneliness if they were able to sidestep their selfishness and their secular values. The isolation lasted about three years with the aim of moderation. He then imposed a five-year silence on the students and during that time the candidates simply listened to the teachings. After this five-year ordeal, the candidates were called "indoor" and formally entered the school as his students, and they were initially attending Pythagoras, which is why they were called "acoustic" at this stage.

As soon as the candidates passed the tests, they were accepted by their peers, honored, and moved to the dorms of the school. Upon successful introduction to the school, they donated all their belongings to the school's cashiers, as Pythagoras himself believed that the commonalities should be shared. On the other hand, if a candidate failed his admission, all his existing duplicates would be returned from the treasury, and students would create a memorial like those of the dead where they were placed in school.

The student stint could last from two to five years and was divided into stages which in turn consisted of tests.

The first tier included candidates for admission who went through specific tests to be accepted by members. At the next level were the "acoustics" who had no right of visual eye contact with the teacher, but only of passive listening. At the last level were the privileged few. They could discuss with the great teacher and with the apprentices or mathematicians⁴⁵. At this stage all the research was done in all branches of the school. Students' behavior was determined by a set of prohibition and incitement rules, which were memorized by members. These rules were called "Hearings" or "Symbols".

Their diet was purely vegetarian, including dairy, nuts and pulses⁴⁶, because they believed that an animal as a living being could accommodate the soul and spirit of an ancestor. However, in addition to animals, beans were also forbidden. For the Pythagoreans, the beans were sacred, as they believed that the sacred root of the whole flora was hidden inside them. However, the consumption of broad beans was explicitly forbidden, as they were considered it a "dirty" plant⁴⁷.

Of course, just like every school, there was a specific course of study and it was based on

⁴⁴ They were at the center of the Pythagorean system and bond, combining elements from the Orphan and Kavian mysteries and flourishing in Croton during the 5th century BC. See Siettos V.G., *The Pythagorean Mysteries*, Athens: Nuclear World, 1993

⁴⁵ WKC Guthrie, *A History of Greek Philosophy, Volume I: The Earlier Presocratics and the Pythagoreans*, Cambridge, Cambridge University Press, (1962), Vol. I p. 30-31

⁴⁶ "The Pythagoreans used only bread and honey for their meal." Iamblichus, *On Pythagorean Life*, par. 97

⁴⁷ Empedocles Akragantinos strongly emphasizes: "Cowards, get your hands from the beans" See Purges, Diels, Text 141

the four basic arts of Philosophy, Mathematics, Astronomy and Harmony⁴⁸. According to the Pythagoreans these were the basic knowledge that every person had to have in order to move on with their lives. Each level contained its own teaching and lessons. The Acoustics taught the secret of the duality of the human being, psychology, exercises for the moral development of personality, self-control⁴⁹.

Those who reached the age of 28 were taught the secrets of mathematics, the mysterious power of the Golden Number. From then on, they were allowed to re-enter social life, helping their fellow citizens.

At the upper level were the Respectful or Mathematicians, and there were few who could reach it. Their main concern was the courses and supervision of the School, while at the same time, they, beside Pythagoras, were initiated into the mysteries of nature, astronomy and astrology. At this stage all the research was done in all branches of the school.

The end of the school came from an aristocratic man of the Cylon era, when he asked the philosopher to become a member of his school, and Pythagoras expelled him, knowing his deeds and character, not paying attention to his proposal. On the occasion of this offensive event, Cylon, with the help of orator Ninona, began to slander the Pythagoreans for an impending tyranny. In a very short time, the slander on the Pythagoreans erupted with the onslaught of the supporters of Cylon at Pythagoreans in the house of Milonas. Many philosophers were murdered and others expelled. As to whether the philosopher himself was there, we unfortunately cannot know for sure. We know, however, that he spent the rest of his life in nearby Metapontum⁵⁰ where he died.

2.2 *Theories*

For many, Pythagoras was a religious leader in addition to a great mathematician and philosopher. At first his school was more like some kind of religious order, with the goal of worshiping its founder as well as Apollo and the Muses, where they were considered the patrons of the spirit. In

⁴⁸ During Renaissance, the so-called quadrivium included the arts of philosophy, mathematics, astronomy and harmony. In contrast, trivium includes grammar, rhetoric, and logic. See Warren E. Preece / Robert L. Collison, Encyclopaedia, from Encyclopaedia Britannica website (<https://www.britannica.com/topic/encyclopaedia#ref193966>) accessed 20-4-2018

⁴⁹ That is why Pythagoras used the "fair mug" or "mug of Pythagoras". In addition to clever hydraulic wine limiting technology, it was intended to teach learners control and moderation. As for its operation, a line is drawn on the inner wall of the mug which defines the amount of wine. Along the way anyone can enjoy their wine. As soon as the wine crosses the line, the mug empties immediately, expelling the wine from the bottom of the glass. In the center of the cup there is a column located above a tube leading to the bottom. As the mug fills, at the same time the wine level rises inside the central column. As soon as the liquid crosses the line, its molecules drift into one another, resulting in the emptying of the cup.
See Hrwn Alexandreus, Spiritual 1.16

⁵⁰ Metapontus was an ancient Greek colony of Lower Italy and was founded by the Achaeans in 773 BC. Its name is compound (after + pontus), meaning "after the sea". Since its inception, opinions have diverged, as we have a wealth of ancient historians and philosophers of different perspectives. The city flourished around the 6th - 5th century BC. At the end of the 6th century it is reported that Pythagoras arrived there.
See "Metapontus", Structure Encyclopedia, Athens: Tegopoulos-Maniateas, Volume 10, p. 285

general, the religious beliefs of the school were very much in line with those of Orphism⁵¹.

⁵¹ The Orphic cult is attributed to Orpheus. One of the most famous musicians and poets of ancient times and founder of the Greek Mysteries. He was the high priest of Apollo and Dionysus and his music attributed magical attributes, with that well-known myth of Euridice. Orphism dates back to about the 5th century BC. Because of the close relations between Athens and Italy, has not been detailed from where this journey began. Dionysus was the object of worship and was of great importance to the sacrifices, just like the Pythagorean school. Also another theory of Orphism was the transformation from a dead being to a living being.
See Dimitriadou Daphne, Anagnostou Evangelia, "Orphic Worship", from the Encyclopedia of Great Hellenism (<http://asiaminor.ehw.gr/forms/fLemmaBodyExtended.aspx?lemmaID=8958>) *acceesed 18-8-2018*

His two greatest beliefs are that everything is numbers and the doctrine of reincarnation⁵² in Pythagorean philosophy and school, it seems that these two concepts coexisted, yet they were different disciplines. They were divided into "Mathematicians" and "Acoustics". The first ones they were studying everything about mathematics, Pythagorean geometry, astronomy and music, however, the life for the Pythagoreans is extremely serious, and in order to live it properly it needs systematic education⁵³. The others focused on the religious and mystical nature of Pythagoreanism.

It is also true from ancient surviving texts that Pythagoras was the first to introduce the concept of reincarnation in Greece. Of course he did not invent it himself but he was influenced by the Eastern cultures where he studied⁵⁴. In Greece we had always been influenced by the Egyptians and Pythagoras had studied with them for many years, but the doctrine of reincarnation was not something that belonged to their religious beliefs. The versions of this doctrine are varied and it seems reasonable that at that time Greece came into contact with various cultures due to commerce, conquest, etc.

The doctrine of reincarnation, in addition to continuing life after death, also entails the after death penalty or reward. However, this was nothing new to the Greek context, as the testimonies of the Eleusinian Mysteries reveal that they were preparing the faithful for a better life in Hades.

The difference between the faithful of these religious ceremonies of the time and the Pythagoreans is that the first ones do this purely out of obligation to religious duty. On the contrary, the Pythagoreans do this with the aim of favoring their post-mortem life. Through the exhaustion of the body they aim at the complete purification of their soul. Which means that for the first time the soul and the body are two separate entities, self-contained and independent, and it is here that Pythagoras revolts in the religious part. The soul is autonomous, and its cultivation becomes the primary task of the Pythagorean communities. With silence, fasting, strict ritual, group life, with religious practices, the students of Pythagoras were expecting the salvation of their souls.

This holistic approach to reincarnation is inextricably linked to his mathematical nature but also to number theory. Specifically, they claimed that humans reincarnate every 216 years and the number is not accidental. The number 216⁵⁵ symbolized repetition and recovery.

For the Pythagoreans, mathematics was the way to elevate their souls and their union with the Divine. God is the unity, the universe of multiplicity, which is made up of opposing elements, and Harmony creates and sustains the opposing elements and forms them into one single thing, the World⁵⁶. Harmony is the power of God and consists of arithmetic reasons.

⁵² The soul is immortal and that it can pass on after the death of the body to other animal species. See Gogaki Constantine, Soul and Body in the Pythagorean Doctrine of Reincarnation, Speech in Athens, HELLEN Hall. AIS, 27-3-2017 (<https://www.youtube.com/watch?v=jFeMQ1W0H4o>)

⁵³ Iamblichus, On Pythagorean Life, Zitros Publications, Athens 1975, pp. 96-100

⁵⁴ See Herodotus, History 2.123

⁵⁵ 216 is also known as the "psychogenic cube" because it was formed from the number 6 in the cube, ie $6 \times 6 \times 6$ Adamakos P. "The Psychogenic Number of the Pythagoreans", from the Panagiotis Theodorou Adamakos website (<https://www.padamakos.gr/mydocs/o%20psychogonikos%20arithmos.pdf>) accessed: 30-6-2018

⁵⁶ Is the first who express the word "world", which means order and harmony. He claimed that the universe created by chaos and then by moderation and harmony it acquired the form we know.

He often resembled the universe with man. Just as man is made up of soul and body, so is the universe of measure and infinity. So, Pythagoras formulates the Theory of Numbers, that is, everything is numbers or they look like numbers.

Pythagoras's conception of numbers came from his attempt to find and understand the principle of everything. So for Pythagoras, the numbers are ultimately the beginning of all, and only among them he can find the essence of the world and the universe. So, numbers are something ideal for him. It seems that he had symbolized many elements in everyday life with numbers, such as the human with the number 250, the number 2 symbolized the woman, and the number three the man, and by multiplying them we have the number 6, which symbolizing marriage. For the Pythagoreans, number 6 was particularly important as it was the first perfect number and in addition to marriage symbolizes health and balance. They divided the numbers into odds and evens numbers and were fundamental to their philosophy as well, since the world had to be understood by the opposite pair and odds and evens numbers were the perfect natural pair of opposites. That is why they also called the odds numbers as males and even numbers as females. All numbers had their place and importance, but the highest was the "sacred quadruple" containing the first four natural numbers ($1 + 2 + 3 + 4 = 10$) which was also symbolized by a perfect triangle, which was the official emblem of the school. His later discovery was based on the sacred quadruple, that the planets produce specific musical intervals during their movement, also known as the Harmony of the Spheres.

Putting so much importance on numbers, Pythagoras and his school made important discoveries that are fundamental today in the field of mathematics and geometry. For example, his most famous discovery is none other than the Pythagorean Theorem⁵⁷. This theory studies the relation between the sides of a right triangle: "The square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides"^{58,59}. It can be applied to any field of science and today counts at least 400 evidence which are constantly increasing. Although its origin is attributed to Pythagoras, historically there are earlier references to it by the Babylonians and Egyptians, but also by Indians and Chinese of his era, but no one until Pythagoras has been able to present any logical proof, at least registered⁶⁰. Euclid⁶¹, 61 years later in his *Geometry Elements* books, will present the Pythagorean theorem with his own proof. It should be said that Pythagoras formulated this theorem as a geometrical theory of area, and later in the 16th century it was formulated in its algebraic form.

Another discovery of the school is the theory of the Golden Ratio. The first to introduce the theory of perfect proportions, the Golden Ratio, or the number "phi" is Pythagoras.

⁵⁷ According to ancient tradition, after his discovery, he offered sacrifices to the gods in Hecatomb, hence the name.

⁵⁸ Euclid, *Geometry Elements*, Book 1, 47, and 48 Theorem.

⁵⁹ $a^2 = b^2 + c^2$. - (where a = the length of the hypotenuse and b and c = the lengths of the other two sides)

⁶⁰ Christianidis G., *Topics in the History of Mathematics. Egyptian, Babylonian and Greek Mathematics*, Heraklion: University of Crete Publications, 2003

⁶¹ Stamatis E., *Euclid Geometry, Figures, Books 1,2,3,4*, Volume 1. Athens: Textbook Publishing Organization, 1975

However, Euclid brings the proof in one of his thirteen books. The Golden Ratio precisely links the harmony of mathematics with nature and art and therefore the influence of this theorem was enormous especially to the artists of later times.

In their attempt to divide a straight section they cut a straight line into two, watching the ratio of the small piece to the large to be equal to that of the large piece to the total length. The "Golden Ratio" shows the number 1,618033988749 The 15th century monk Luca Pacioli, influenced by the perception of the era that new knowledge of science had to be incorporated into ecclesiastical dogma, called it "The divine proportion". The name "gold number"⁶² is given to Leonardo da Vinci, and in the early 20th century, American mathematician Mark Barr identified it with the Greek letter phi, in honor of the sculptor Pheidias, who based his work on this number, and more specifically the Parthenon.

Today the Golden Ratio is one of the most revolutionary rules in all areas and can be found everywhere around it, from paintings, to buildings, to beautiful sculptures and even to music works⁶³.

It is truly impressive to think that 4,500 years ago, without any technology, many times without the necessary tools, the sciences have reached such a high level that they are still considered pioneers. And what we have said above is little compared to what the great philosopher⁶⁴ did. His multi-dimensional thinking and his awe-inspiring intelligence were what made him reach the universe and discover that there is still music there, and even if we cannot hear it. And no matter how absurd it sounds, even though it was verified by his own means, thousands of years later, it will be auditory confirmed, by one of the largest space stations on the planet, NASA

2.3 *The Harmony of the Spheres*

Music was one of the most important lessons in the Pythagorean school curriculum. For Pythagoras, music played an important role in the moral formation of human character, a concept that was later adopted by Plato⁶⁵. After all, music in antiquity was an essential lesson in the education of young people. But apart from its pedagogical and ethical teaching character, music also concealed its own philosophy. This is why the philosophical quests of the ancient Greek world came to this and as the relationship between philosophy and music is quite clear, so the

⁶² In mathematical terms, a golden number is the one that if we add the 1 it will give us the same result as if we raise it to the square. See Livio Mario, *The Golden Reason: The Story of Phi, the Most Amazing Number*, translated by Stavropoulou Marianna, Athens: Enalios, 2005

⁶³ Mozart divided a large number of his sonatas into two parts, equal to the number Phi. See Mantzoukidis Chr. Costas, *The Golden Ratio in Music*, Athens: Ziti, 2018

⁶⁴ Pythagoras first created the "Pythagorean table" or "Abacus", a multiplication table or otherwise a table of precedents, where he helps find the product of the first ten integers numbers. See Eliadi Amalia, "On the Life and Work of the Glorious Samos," from Olympia.gr (<https://olympia.gr/2013/12/03/%CF%80%CF%85%CE%B8%CE%B1%CE%B3%CE%BF%CF%81%CE%B1%CF%83-%CE%BF-%CE%BA%CE%BF%CF%81%CF%85%CF%86%CE%B1%CE%AF%CE%BF%CF%82-%CF%84%CF%89%CE%BD-%CE%B5%CE%BB%CE%BB%CE%B7%CE%BD%CF%89%CE%BD-%CE%BF-%CE%B5%CE%BB>) accessed 17/8/2018

⁶⁵ Strabono's Testimony, Appendix A.4, Strabnoo's Geographically, Book 10.3, C468.10

Pythagoreans and later Plato called philosophy "music."

But no one so far has brought to light the mathematical nature of music, until Pythagoras.

For Pythagoras, music was also a number, that is, it could be expressed fully in numerical relations. Through single chord and precise mathematical calculations he determined the reasons for musical intervals. In short terms, Pythagoras proved what had existed and perpetuated in the theory of musical experience through mathematics.

But before we proceed to the basic music-mathematical theories of the philosopher, we must identify basic objects and terms used by the philosopher in his research.

Initially the single string was an instrument created by Pythagoras for his research to music intervals. It consisted of an elongated loudspeaker with a string that was stretched above a graded rule and a removable rider. He noticed that if he struck the whole string he would produce a sound A. But if he struck three quarters of the string the fourth of the sound would be produced. Thus he discovered the intervals, a perfectly constant relationship between the length of the instrument strings and the key chords ($1/2$ for the eighth, $3/2$ for the fifth, and $4/3$ for the fourth). These harmonic relations were based on the fact that they included the first four natural numbers (1, 2, 3, 4), whose sum equals 10, the sacred Quadruple.

But what defines interval in Pythagorean music theory?

The relation between two numbers, that is to say what we call today in algebra and geometry ratio⁶⁶, in Pythagoras's musical theory, is called "interval" or "harmonic ratio"⁶⁷.

However, the word "interval" was ambiguous. Interval was the numerical relation with which the ratio of the musical interval was expressed, but it was also consistent with its everyday meaning and the "straight line", that is, the distance between two points. In this way Pythagoras discovered the relationship between the length of the strings and the tonal height they give. The "sacred quadruple" also played an important role in this, as it contained all the musical reasons in the top ten numbers:

2:1	Octave
3:2	Interval of fifth clean
4:3	Interval of fourth clean
3:1	Octave and fifth clean
4:1	Two Octaves

So, by simple mathematical actions Pythagoras was able to lay the foundation for almost all musical couples, pentatonic, colored western and Arabic. Examining this system, that is, the musical scale, he found frequencies and mathematical relationships among them, and on them he formulated the theory of Harmony of the Spheres. This system will

⁶⁶ The mid-ratio is a quantity that has a median value between two other values and is calculated by a special formula or a set of conditions. The formula of harmonic ratio is as follows: $H = 2\alpha\gamma / (\alpha + \gamma) \cdot (\alpha - \beta) / \alpha = (\beta - \gamma) / \gamma$
See Nexus Network Journal, Architecture and Mathematics Online, The Geometer's Angle No5: Geometric and Harmonic Means and Progressions

⁶⁷ Harmonic ratio, is the ratio of the square of the geometric ratio to the arithmetic ratio $H = G^2 / A$. Later the "Harmonic ratio" was renamed by Arctytas and Hippasus and into "Harmonic circle", as they observed that it produces both harmonic and tonal ratios.
See Henle Jim, "Classical Mathematics", The American Mathematical Monthly (1996), 103 (1), p. 19

later named by Newton spectroscopy however Pythagoras would have invented it by accident without knowing it.

Harmony of the Spheres argues that both nature and celestial bodies obey the same rules that govern the harmony of sounds in music.

From his studies on the peoples of the East, he knew that for each known planet and star certain numbers were assigned, that is, a particular numerical value representing the interplanetary distances. But he, in addition to claiming that the Earth is spherical and part of a set of other concentric spheres, also argued that planetary bodies can be accurately measured in their circular motion⁶⁸. And since everything was a number and music belonged to them, therefore the universe had to be musical. All he had to do was to prove the relationship between music and the universe⁶⁹.

So he matched the known planets with a musical scale. Nicomachus Gerasinos in Handbook 3 argues that the names of the seven musical vocalists came from the seven planets and their position relative to the Earth. The moon, which was closer, was the lower note and Saturn, which was farthest, was the highest note. The names of the known planets back then were:

Moon	Nete (νήτη)
Mercury	Paranete (παρνήτη)
Venus	Paramese (παρμέση)
Sun	Mese (μέση)
Mars	Lichanos (λιχανός)
Jupiter	Parypate (παρυπάτη)
Saturn	Hypate (υπάτη)

The basic belief of the Pythagoreans was that there is a central axis⁷⁰, around from which the Earth, the sun, the planets and the various stars move in circular motions. However, the sequence that the planets had was corresponding to a particular set of notes. So in this way every starry night, the harmony of the spheres emerged, and each time varying according to the positions of the planets. Below is the table with the correspondence of the notes, as Pythagoras defined it.

SYMBOLISM	PLANETS	CHORDS	NOTES
A	Moon	Nete (νήτη)	Re
E	Mercury	Paranete (παρνήτη)	Do
H	Venus	Third	Si major
I	Sun	Mese (μέση)	La

⁶⁸ See
Iamblichus, On Pythagorean Life, 938-958

⁶⁹ Homeric poets had somehow formulated this theory. In the Homeric hymn in honor of Mars, the planets invoke it as a kind of chorus. Also, according to testimonies in the 7th century BC, the poet Terpanndros added the seventh string to the lyre to produce planetary music.
Spyridis, Charalambos C., "The Music of spheres of the Pythagoreans", from the National and Kapodistrian University of Athens website (<http://users.uoa.gr/~hspyridis/kallipateira.pdf>) accessed 20-08-2018

⁷⁰ They claimed that the Earth is stable and all other bodies move around it, the so-called geocentric system.
See Gavroglou K., Diatetis ., Christianidis C, "Perceptions of the Earth's Movement in Ancient Greek Astronomy", Proceedings of the 4th Panhellenic Congress of Aristarchus of Samos 17-19 dec. 2005

SYMBOLISM	PLANETS	CHORDS	NOTES
O	Mars	Lichanos(λιχανός)	Sol
Y	Jupiter	Parypate(παρυπάτη)	Fa
Ω	Saturn	Hypate(υπάτη)	Mi

Assume, then, that in order to exist "planetary" notes somehow they had to be produced.

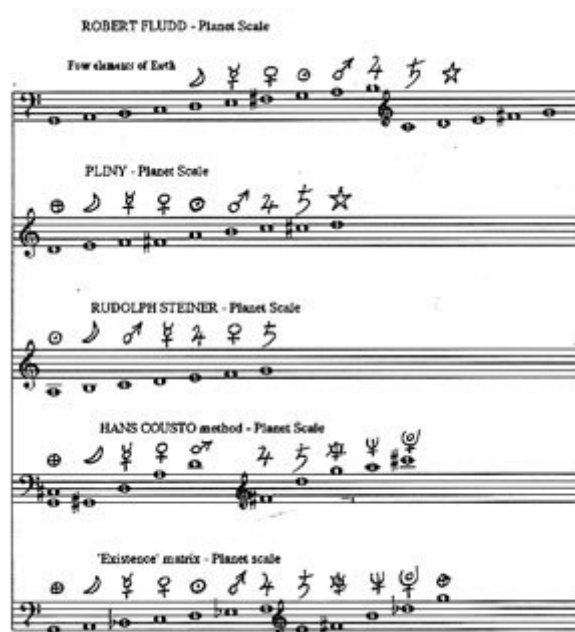


Image 3: The scales of the planets

Both he and his students came to the conclusion that the rotation of each planet on its orbit in relation to the ether's friction produces a specific sound, where its tonal height is determined by the radius of each orbit and the speed of rotation of each planet⁷¹. That is, they observed that for the sound to occur there had to be some friction or collision between the planets and since this could not be done because of the different motion and frequency of each, it was achieved during their encounter, either by slowing them down or by passing them. This means that for each celestial body, the music that produced is different.

The celestial bodies are arranged in such a way that they relate to each other on the basis of musical proportions and produce a melody that vibrates through their movement. Pythagoras claimed that he could hear these melodies when he concentrated on his hearing⁷². However, there was a problem in his theory that he could not know it, but for us it is now proven.

There is a gap in space and the sound does not propagate in the void. Therefore in the case where

⁷¹ To propagate a sound from one point to another requires the interference of any material between the two points, eg air.

See Papadogiannis Nektarios, Wave Physics: For Audio and Acoustic Engineers, Athens: Hellenic Academic Libraries Association, 2015

⁷² Porphyrios, Live of Pythagoras, 30.3-7

the planets did produce music, it is almost unlikely to reach us. In space, sound inhere like electromagnetic vibration. So we assume that by the word "listened," he meant that only he could find this musical relationship between the planets.

But in addition to the universal music he offered to humans, Pythagoras was able to calculate first intra-planetary distances, with the help of musical intervals, into delphian stadium⁷³. He calculated all the distances of the stars and planets, whose shape claimed to be spherical like Earth's. Each constellation or planet has its own motion, sound, and influence, both to it and to the surrounding constellations⁷⁴. For example, the musical distance between the Earth and the Moon was calculated by a single tone, and their metric distance is 126,000 Delphian stadium, with the current unit being 22,371,300 meters⁷⁵. The distance between the Moon and Mercury is half a ton, while the planet Venus is just one and a half tons away from the Sun. However, astronomers of the time noticed something more. Because they failed to explain the stability that governs these celestial bodies, they connected them to a constant. That is, they believed that they were all connected by a black sphere that defined the edge of the universe. In turn, the moving stars were each connected by a sphere. We have to emphasize that the known planets back then were the Earth, the Moon, Mercury, Mars, Venus, Jupiter and the most distant Saturn. Number seven was not accidental for Pythagoras. Seven was the number of the Sacred Quadruple, and since there were seven notes on his scale he also linked them with planets.

This theory has influenced for many years many of the well-known names of physics, mathematics and astronomy that we know today. For example, Kepler⁷⁶ tried to adapt the trajectory of the planets to the musical scale. Many scientists were the ones who began their scientific course by first studying music to understand physics and astronomy.

Nevertheless, Pythagoras left behind many things that were left unanswered, either because he did not think of them from another perspective or because he could not find an answer. For example he never told us why frequency is inversely proportional to length or why we tend to show interest in these intervals? He provided, though, important bases for the subsequent study of these questions.

And if I were to tell you that all this theory and laws were not only related to their scientific side but also to the inner nature of man?

⁷³ The delphic stadium was a unit of length during Antiquity. It was calculated as a stadium length of 600 feet and today stands at 195.15 meters. The length of the stadium varied from city to city, depending on the proportions of the area's sports stadium. Delphic stadium had length 177.55 m..
See Newer Encyclopedian Dictionary of Helium, vol.17 p.196

⁷⁴ Here was based the view of many later people, that human character and behavior can be affected by constellations, depending on who you belong to and when you have born, today's so-called "zodiacs". This theory, however, was called Theory of Influences . See Haritaki Maria, "Theory on the World of Pythagoras", from Theosophical Society of Greece website (<http://www.theosophicalsociety.gr/index.php/2013-08-31-19-16-39>) accessed 30-8-2018

⁷⁵ Hundreds of years later, Johann Bonde and Titus, German astronomers, will confirm and discover more distances almost completely. His rule ceases to apply to the newly discovered planets
See Nieto, MM, The Titius-Bode law of planetary distances: its history and theory, Oxford: Pergamon Press, 1972

⁷⁶ Johannes Kepler was born on 27 December 1571 and was of German descent. He was a mathematician, physicist and astronomer. In 1596 he published his first work on cosmology under the title "Mysterium Cosmographicum". He introduced and founded many theories of planetary motion and nature and function of our solar system in general. He died on November 17, 1630 See Westman Robert S., 'Johannes Kepler', from Encyclopaedia Britannica website (<https://www.britannica.com/biography/Johannes-Kepler>) accessed 20-8-2018.

How was all this a way to teach the peace of mind that the human soul needs?

For the Pythagoreans, the laws of music impact on the inner world of man through harmony. Since the harmony of music is identified with the harmony of the universe, so the universe is identified with the harmony of the soul, that is, the inner human universe, the soul. If the balance is disturbed, mental illnesses appear. Therefore health itself had to do with the proper harmonization of body and soul, with the universe, diet, music, and divine laws.

Of course, Pythagoras was not the only one who sought its deeper scientific interpretation of music. Subsequent Pythagorean philosophers presented and proved important theories. Heppasus of Metapontus in the first half of the 5th century BC precisely determined the mathematical relationships of intervals. Philolaos in the late half of the 5th century BC divided the harmony, that is, the octave. Finally Plato (428-347 BC) formulated the theory of ideas “ Harmony is something invisible and without physical- material substance and something beautiful and divine in the well-tuned lyre ”.⁷⁷

In the Harmony of Spheres today, the rule works up to the planet Saturn and even contributed to the discovery of asteroids between the planets Mars, Jupiter and Uranus in the late 18th century. Until 1977 all this was only theoretical and confirmed only visually and mathematically. On August 20, 1977, NASA sends spacecraft "Voyager" 1 & 2 to record visual and acoustic data. The first elements are sent 20,000,000,000 kilometers from our planet. Through specially manufactured instruments such as Injun, Isee 1 and Hawkeye special space probes and using a wave recording antenna, all vibrations that the human ear can hear (20-20,000 Hz) were recorded. The recorded sounds are the result of complex reactions of charged electromagnetic particles from space wind, magnetosphere and the ionosphere. Below we will hear some sounds from the recorded planets.



THE SUN



JUPITER



MERCURY



EARTH



MARS



VENUS



SATURN

⁷⁷ Plato, Phaedon, (chapter A.5, 85E.)

NEWTON



PLATO



URANUS



Someone could characterize these sounds as unrealistic or even non-existent, yet the preceding sounds are the translation and rendering of the electromagnetic waves emitted by each planet, in order to understand what the great philosopher really meant.

Getting to this point and concluding this chapter, I realize that by studying the philosophical side of music science there is a great deal of interaction with their mathematical nature, even if they are two different disciplines. Mathematical problems and their solutions to the theory of music affect the philosophical perspective on music and vice versa. Because until now, the seven planets and the seven notes were just a coincidence.

It seems that in the end the philosopher was trying to reject the autism of music without mathematics and maybe the same thing happen with mathematics and that is something that we certainly cannot discover and know for sure, this is also the beauty of philosophy. Nothing can be proven and nothing is what it really looks like.

Third Chapter :

“ IANNIS XENAKIS ”

***“ Music, probably, is the most tied to
mathematics piece of art,
with mathematical thought
by its very nature.”***

Iannis Xenakis

Music can be one of the few arts that can be combined with other fields and arts, such as painting, math and even architecture. Music is the art that organizes the sounds that surround us, with the aim of composing, producing, performing and researching them.

The 20th century, was the one that offered and favored all forms of subversion and avant-garde, not only in the arts, but in everything that concerned human life, from technology to politics. Because everything that is happening in the world is reflected in the artwork, and so music reflected all the change, the new currents and the new life that it brought to life. After all, it is no coincidence that the 20th century was named as the century of "New Music"⁷⁸.

These changes came from persons who could be said to have been intended for this purpose from the beginning, and in turn have inspired other artists to this day. Arnold Schenberg takes the first step by rejecting tonality in his works, which is impressive and shocking at the time, to end up with Anton Weber, Aaron Coupland, John Cage and many others rejecting traditional composition of music, traditional artist and musical work.

Among them stands a Greek composer, not only for his atonal work, but also for his daring to combine his music with other arts, such as architecture and mathematics. Iannis Xenakis comprised and inspired entire generations as much at this specific era as later, and systematically introduced mathematical thought and composition into his works, as we shall see later.

3.1 Historical Background

Xenakis was born in Braila of Romania, by Greek parents. After the death of their mother, the children were sent to study at Korgialeneio School⁷⁹ in Spetses, Greece. one of the best institutions of that time.

⁷⁸ "New music" was also called the period of Middle Ages (14th century) with the foreign terminology "Ars Nova", originating in France. This title was taken by the composer Philippe de Vitry in about 1320.
See "Ars Nova", from Encyclopaedia Britannica website (<https://www.britannica.com/art/Ars-Nova-music>)
accessed 13-9-2018

⁷⁹ The Anargyros and Korgialenios School was a private institution, which began operating in 1927. The curriculum was Greek in combination with a faint English education. In 1983 it ceased operation and since 1986 it has been operating as a cultural center.
See Anargyreio and Korgialeneios School of Spetses, from GRECT website ((<http://www.grect.com/el/anargyreios-kai-korgialeneios-sholi-spetson/paroyysi-kai-egkatastasei>) accessed 12/5/2018

At this school, he is given the opportunity to get back in touch with music by taking piano lessons and music theory⁸⁰. He writes in autobiographical notes that he did not have good relations with his new classmates, so he was isolated and living with the books of Jules Vern and through astronomy books.

After being sentenced to death in 1951 for being a deserter from the National Army, he moved to Paris where he was permanently stationed until 1974⁸¹ where his conviction was overturned by a legislative decree on Karamanlis government. After being sentenced to death in 1951 for being a deserter from the National Army, he moved to Paris where he was permanently stationed until 1974 where his conviction was overturned by a legislative decree on Karamanlis government. In Paris, because of his bad financial situation, he was subsisted, either on friends who belonged to the Communist Party, or from an organization that helped refugees. Of course, his difficult financial situation was responsible for the greatest and undoubtedly remarkable collaboration of his career, that of Le Corbusier, when Xenakis was sent for work as an engineer by his friends at the Technical University. They eventually collaborated for twelve full years (1947-1959), on the architectural design of major projects such as the Philips Pavilion at the Brussels International Exhibition in 1958, the Baghdad Stadium in 1957 and more.

And as with any artist, once the financial issue is solved, and not obliged to create "custom-made", he begins a permanent engagement with the composition, each time projecting a different mathematical model, trying to accept this kind of projects. In 1948 he enrolls and starts classes at the Ecole Normale de Musique⁸², but the composition teacher will disappoint him, by disagreeing with Xenakis' particular way. The same will happen to Nadia Boulanger, who can neither understand nor follow his composure. The decisive step to break free from stereotypes and create his own freedom was given by two years of continuous meetings with Olivier Messiaen.

The truth is, from at least his early works, there seems to be no specific musical taste like the other composers of his time. One can clearly see the influence of Debussy and Ravel, composers who retained the ancient Greek ideal in their works, and perhaps this was what drew him to their music. As he declare later, but without being part palpably to his works.

By looking all his life, we realize that until then he had neither the means nor the ideal environment to act on his musical spirit. He would later state that the only way to create was to be able to remove and subtract from reality. After all, who else could have perceived at that time, a demonstration and a murmur as a flawless musical composition?⁸³ Yet it was this distancing from reality that brought him to the beginning of the vanguard, when he first introduced the concept of

⁸⁰ Xenakis had lost his mother in 1927 when he was five years old, so by the time of school he had lost all contact with music.

See Themeli, Pioneer composer Iannis Xenakis, from Aristotle University of Thessaloniki website (http://users.auth.gr/users/5/0/007005/public_html/text/xenakis.pdf) accessed 12/5/2018

⁸¹ See Legislative Decree 17/1974 - Government Gazette A-236 / 2-9-1974, from the website e-legislation.gr (<https://www.e-nomothesia.gr/kat-politike-prostasia-psea-pallaike-amyna/nd-17-1974.html>) accessed 17/5/2018

⁸² It was founded on 6 October 1919 by Alfred Cortot and Auguste Mangeot with the aim of providing excellent musical training and the cultivation of virtuosos, as well as the preservation and dissemination of the musical character of France. (see Historique, from: Alfred Cortot's Ecole Normale de Paris Musique (<http://www.ecolenormalecortot.com/ecole-et-etudes/historique>), accessed: 18-5-2018)

⁸³ See Xenakis Iannis, *Musiques formelles*, Paris: Stock, 1981 (reprint), p. 19

probability into musical composition, when he used the theory of the Golden Section and the Fibonacci⁸⁴ sequence, as we shall see in the next chapter.

⁸⁴ Mathematical sequence where the next number is the sum of the last two, 1,1,2,3,5,8,13,21 etc. It was discovered by Leonardo of Pisa also known as Fibonacci in the early 13th century and was the the solution to a mathematical reflection he had set himself, known as the "rabbit problem". See Carney Gies Frances, "Leonardo Pisano", from Britannica.com (<https://www.britannica.com/biography/Leonardo-Pisano>) accessed 31-05-2018

From 1953 onwards, Xenakis has turned to the way and style of his compositions. He focuses on a mechanism associated with eight different tonal heights with durations that follow the first four numbers of the Fibonacci sequence. He sends this composition to Pierre Schaeffer, to bring him at the studio of this specific music which he directs and eventually ends up to the Hermann Serhen Orchestra Director. The first big step has been made. Xenakis's manuscripts are within the synthetic circles of the era and despite minor failures (although Scheren was interested in *Metastaseis*, he eventually directed it, giving baguette to director Hans Rosbound), he is slowly taking his place in music life in Paris, but also in its history. So in the early 1960s, Xenakis began to shine as an excellent composer. Two facts are helping him, the scholarship in West Berlin and his hiring as a composition teacher, a summer course at an American university⁸⁵. From there the rise begins. His standard of living is greatly improved and as a result he abandons all different work and devotes himself entirely to his music.

He gave his art the name "stochastic music" because he believed that it started with mathematics, ending up with something random but possibly reversed. He used as we have mentioned, but we will see in more detail below, mathematics and probability theory, using at least 15 mathematical theories in his works.

The truth is that his music is difficult and not for everyone. He combined the sound with the lights and space making the listener feel united, integrating it within. His last work, he composed it in 1997, entitled "Omega", a purely symbolic title and was cut short due to serious health problems. As early as 1996 he was perhaps his biggest dream, the Center for Contemporary Music Research. In 2001 he was awarded an honorary doctorate in the Department of Music Studies of the Aristotle University and four days later (4-2-2001) he left his last breath in Paris.

Perhaps there, in Paris, Iannis Xenakis eventually took with him his stochastic music and hidden mathematical treasures to leave us with thought, research and honest listening.

3.2 *Theories*

Xenakis was not only the great musician and architect we know today. Xenakis was a thinker, a philosopher, and then everything else. Perhaps this is why his works each have their own 'energy'⁸⁶, their own vision and their own mathematical model. For Xenakis, composition from scratch is a struggle both in his own musical world and in his own life⁸⁷. That is why, from an early age, his main concern is how to differentiate himself, and how he will recreate each work each time⁸⁸, from a new perspective. And to justify the composer, it was not a matter of originality, but rather than a matter of ontology and reflective thinking.

⁸⁵ In 1963, he receives a proposal from A. Copeland to teach at the Berkshire Music Center in Tunkwood, Massachusetts.

⁸⁶ The word energy, used by the composer himself modestly, is found in the article "About Time" with Morton Feldman in 1988

⁸⁷ "I'm not sure I exist unless I do something different. The different is a proof of existence, knowledge, participation in the things of the world [...]"
See Varga, BA, *Conversations with Iannis Xenakis*, London: Faber & Faber, 1996, pp. 50

⁸⁸ This method was called Practice of Editing. The composer recaptures the material of a work that has been calculated based on one theory to integrate it into another work.
See Solomos Makis, *Iannis Xenakis: The Universe of a Peculiar Creator*, Athens: Alexandria, 2008

It was the unprecedented theorization of music that prompted him to write many musicological texts, the best known of which are in *Musiques formelles* (1963). But before all this and aiming at something different and revolutionary in the field of music, Xenakis proposes to mix the arts⁸⁹ and through logical pythagorism first introduces mathematics and physics into the world of music, thus creating something entirely new, such as we'll see later. The 'turn' in music had to include and be based on three basic 'parables' as he called them, space, numbers and gases⁹⁰.

At this point, as a first step he introduces the possibilities into his own music and then a variety of mathematical combinations and theories. This is also the reason why we could use his musical work for mathematical analysis with problem and solution. Just before we look in more detail at some of the mathematical puzzles he put into his works, I will give you a list of all the "Xenakis' theories" he used in all his works.

- Stochastic Theorie
- Game Theory
- Symbolic Logic
- Group Theory
- Sieve Theory
- Dynamic stochastic Synthesis
- Dendroseis
- Brown Movement
- Honeycomb Automatic
- Computer System UPIC

Early works on mathematical labyrinths the *Herma* (1961), *Eoda* (1964) and *Law a* (1965) where he

⁸⁹ "Sur travail", was the title of his dissertation, in which he supported the mixing of the arts, which was published in a book in 1979
see Xenakis Iannis, «Arts / Sciences.Alliages», Paris: Casterman, 1979

⁹⁰ See Xenakis Iannis, "The Three Parables", MA (1958), pp. 17, 17-18, 18-19

used Boole's algebra theory⁹¹, set theory and sieve theory. Boole algebra is also called algebra of logical relationships and was originally used for the design and analysis of electronic circuits. This type of algebra uses two variables, true and false (0, 1), which are also called logical variables. On the other hand, the set theory⁹² is the theory that studies sets

⁹¹ Boole's algebra was first introduced and founded in 1854 by George Boole with his work *An Investigation of the Laws of Thought*. However, as a terminology, it was first used in 1913 by Sheffer. See Halmos, P., *Lectures on Boolean Algebras*, Princeton: Van Nostrand, 1963

⁹² The "set theory" was proposed by Georg Cantor in the 1870s. Although his theory was contradictory, in the 20th century new systems of axioms were proposed with the most well-known set theory of choice, or "Zermelo-Fraenkel". See Rotman, J., *An Introduction to Group Theory*, New York: Springer-Verlag, 1996

involving functions and relationships, such as a group, as opposed to other mathematical theories that study structure. For Georg Cantor, who laid the foundations of this theory, a set is defined as any collection of objects that are perceived through our experience or our intellect, are easily classified and distinguished.

On the other hand, set theory avoids referring to the nature of set elements and treats them purely with mathematical logic. Xenakis therefore followed these methods in these works. Taking into consideration all the vocals of the extent of a classical piano and their whole room, he divided it into basic groups-sets. After separating the sets, begins to add them together where other sets emerge. Thus each unit of time or set in the project contains at least one note of these sets. At the completion of these two works he called them as "symbolic music". In particular, his work *Law a'* was characterized by himself the most ambitious work, when he wrote an entire article devoted to it entitled "Towards a Philosophy of Music", while the same work was the subject of many theoretical analyzes of the time.

The next theory he used in his works was "The Brown Movement"⁹³. The Brown movement is simply the "dance" of the particles as they float in fluid materials. Originally this theory or more correctly this equation was formulated by Albert Einstein in 1905. He observed that large objects, when immersed in a humid environment, all the sides pressed and collided with the liquid molecules, and that the number of atoms impacting on each side of the body fluctuated. So based on this observation Einstein developed an equation that could calculate the size of the molecules but also predict the specific properties of the atoms of liquids and gases. Of course you will wonder how this equation helped to compose Xenakis' works and is very simple. Taking an example of the motion of the molecules, he designed a computer-assisted curve, drawing from there all the melodic lines of *N'Shima's* (1975) *Cendrees* (1974), taking into account all possible moves that could perform molecules and atoms of materials.

Of particular interest are the works by *Duel* (1959), *Strategy* (1962), and *Linaia -agon* (1972), as the composer inspired them and created them based on Game Theory⁹⁴.

This specific theory deals with the taking of rational decisions in a competitive environment. This theory is the most autonomous branch of mathematics, with a broad application to our daily lives.

⁹³ In 1827, a biologist named Robert Brown observed under the microscope irregular movements of pollen grains in the water. He called this dance of pollen the Brown Movement, but was unable to explain its causes. See Ford, Brian J., «Robert Brown, Brownian Movement, and Teethmarks on the Hatbrim», *The Microscope* (1991), Vol. 39 pp. 171

⁹⁴ The theory was first formulated in the 18th century by James Waldegrave in 1713 but becomes known through the book "Game Theory and Economic Behavior" in 1944 by John von Neumann and Oskar Morgenstern. It is considered one of the most important theories in the world of mathematics. John Forbes Nash identified the deficiencies in the theory and completed it in 1950, saying "Every game with a finite number of players and actions has at least one balance point, according to which all players think about what their opponent can choose, trying to understand the behavior of others and they choose their strategy accordingly. They choose the actions most advantageous to them, knowing the choices of their opponents". This theorem gave him the Nobel of economy in 1994. They have been awarded so far with the Nobel price on the theory of games, about ten other economists, as the theory has grown or improved over the years. See Osborne, Martin J. & Rubinstein Ariel, *A Course in Game Theory*, Cambridge: MIT Press, 12th edition, 1994, pp. 368

It studies various strategies that must be followed for individual or group good, with the victory as the one and only goal.

The best known problem in game theory is The Dilemma of Prisoner⁹⁵, coined by Merrill Flood and Melvin Dreshe, in California in 1950 during the Cold War.

Game theory was used by Xenakis to compose controlled improvisation projects such as the one above. Essentially, each audio event is freely created but subject to specific rules, as for example a football match. In the first of these, for example, the Duel, the orchestra is divided into two sets, each with a different conductor. Each maestro chooses from a palette of 6 different modules, always depending on the choice of the other maestro. In the Strategy process extends to a larger orchestra, but the rules are simplified for practical reasons.

But of all the theories and mathematical models that he incorporated into his compositions I find, both myself and his innumerable researchers, particularly attractive the theory of sieve. For many, even those who are intensively involved in mathematics, I'm sure that they hear it for the first time, let alone we who are just music researchers. And yet this theory is the most important in the composer's works because it is his first attempt at universal music. But first of all, let's explain what Sieves are.

The sieve theory was first formulated by the mathematician Eratosthenes of Cyrene (276 BC - 194 BC) and was first presented in writing by Nicomachus in his book "Introduction to Arithmetic" and was used to find all prime numbers. Assuming we have a shaft with specific dimensions with two available points. The available and feasible choices between the two points are called sieves, and the whole system can be used to build any scale. In music has as a result in an off-time construction that can be used in composition, for example the sieve of heights where it was called the scales where formed from numerical formulas based on the equivalents of m and three logical operations of conjugation, division and refusal. Like that Xenakis presents the theory in measure 18 of Law a. But the sieve theory does not end here. It is a huge part of Xenakis and the work itself, even if it was included only in one measure, and it requires special and individual analysis to make it understandable, which we do not need at this time. Sieve theory has been clearly mentioned to understand a few things about how it works.

At this point, we must say that Xenakis dated his works according to the Fibonacci number line, even the works of the late period, electro-acoustic music.

According to him, the kind of electronic music that inspired him from the beginning, as he hid great synthetic potential. After all, his contribution was also historic, as he came to fill the void of the two musical trends that emerged in the '50s, that of "specific" and "electronic" music. Due to his extensive involvement in 1979 he also created the UPIC system (Unité Polyagogique Informatique du CEMAM), which financed the French Ministry of Culture itself, creating a federation to oversee and promote it, known as Les Ateliers UPIC. The purpose of the composer was to prove that this machine could be an important tool for composers, since they could create more complex projects.

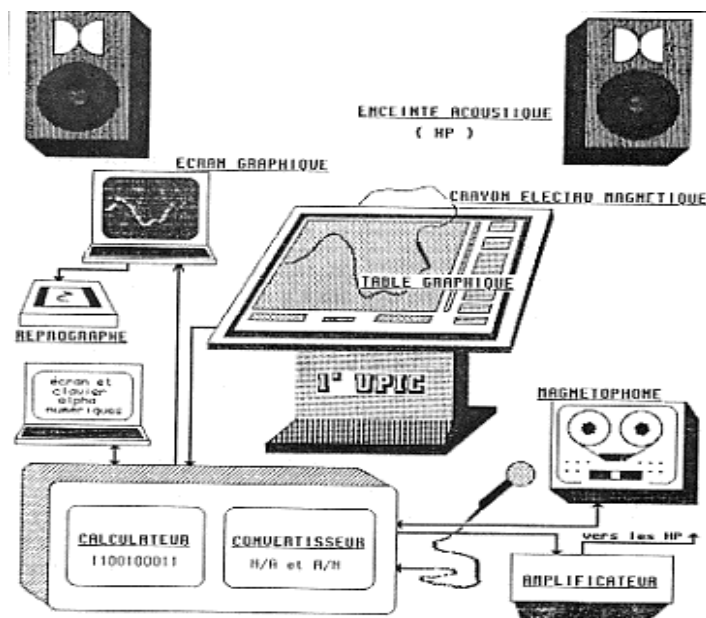
⁹⁵ See Boboula Angeliki, The Prisoner's Dilemma, the Most Famous Game Theory Problem from efsyn.gr website (<http://www.efsyn.gr/arthro/dilimma-toy-fylakismenoy-diasimotero-provlima-tis-theorias-paignion>) accessed 11-10-2018

He proved it in 1978 by writing the first opera through this system, entitled Mycenae-Alpha , where it premiered at the Polytope de Mycènes festival. But what is the UPIC system?

UPIC is a premium computing system consisting of two parts, an electronic drawing board and an electronic pencil. Both were connected to a computer with speakers. During the designing of any sketch, the computer would translate it at the same time into composition instructions. In this way, a sketch could also be a musical composition⁹⁶. The works compiled in this way belong to the second period of Xenakis's electronic composition.

Four are the works that belong to this period, Mycenae alpha (1978), Taurhiphanie (1987), Voyage absoludes Unarivers Andromède (1989) and the Pour la Paix tape (1981). This was only the beginning. He later used programming and complex calculations in his projects, as well as other electronic programs such as GENDYN⁹⁷, which completed in 1991 and examined the limits of algorithms. His insight into the mix of technology and music was obvious⁹⁸.

Image 1: The structure of the UPIC system



⁹⁶ See <https://www.youtube.com/watch?v=6KKIeg8tv1Q>

⁹⁷ Hoffmann, Peter, « The New GENDYN Program », Computer Music Journal (2000), Vol. 24, pp. 31-38

⁹⁸ "All my experience over the last few years has led me to believe that the future of music lies in the advancement of modern technology. This will affect both how we create and how we listen to music. But I also expect that technical progress will have a profound effect on individual listening. Thanks to computer systems, which can be integrated into the audio generator chain, it will be possible for the listener to individually, at home, adjust the sound quality as well as the volume of sound, benefiting from the ability to listen individually or specifically a particular organ or group of organs, while also looking at the image of the performers. There is no doubt. Thanks to technology, we can be sure that the music of the past, but also the music that will come, will be music that has never been heard again. See "Problematic scientists and compositions musicales", Unesco, Académie Européenne des Sciences, des Arts et des Lettres, 1980

Today, this system is still used, both in its original design and with more sophisticated machines by the Center de Creation Musicale Iannis Xenakis (CCMIX) .

At the same time, has been created a free graphics sequencer, Iannix, inspired by Xenakis' works of the digital age, aimed at digital art.



Image 4: The UPIC system today, such as presented in a museum!

Although his contribution to this genre of music is enormous, since the use of technology and science was significant, Xenakis has been associated with stochastic music, a genre he created himself and was followed by many. And as we said at the beginning, he was nothing more than a musical thinker and might not do anything about it unless he raise concerns and questions about what we call music. So we'll see later what really led the composer to create what he called stochastic music.

3.3 Iannis Xenakis and his Stochastic Music

What could anyone describe as stochastic music, and what could this music encompass within it? For some it refers to philosophy and mature thinking, for others creation through thought, to its creator is nothing more than architecture and mathematics, or it is all. Any gap that appears in the music of that time, Xenakis converts it into numbers and probabilities. And this is how he creates his own space in sound, with the ultimate goal of being somewhere.

Xenakis created his own stream in a Europe that has gone through two world wars, dominated by twelve-tone technique⁹⁹ and serialism¹⁰⁰. However, both streams fail to become the new common language that art circles have been calling for. Each creator is looking for ways to create a unique language, while science and technology of the time are so integrated into the musical composition and practice that each composer's work must be different in the way,

⁹⁹ The first to introduce the concept of twelve-tone technique was Arnold Schoenberg. This method is basic to the stream of atonality and uses a selected twelve-tone sequence called a series. The purpose of twelve-tone is to avoid hierarchical notes (eg tonal, dominant etc.), as in tonal music, but they all sound just the same. See Reti, Rudolph, *Tonality, Atonality, Pantonality: A Study of Some Trends in Twentieth Century Music*, Connecticut: Greenwood Press, 1978

techniques and means.

Xenakis, had all the means to fundamentally create what the era wanted, but first he had to prove that serialism and twelve-tone technique were saturated. That's why he heavily criticized their creators, resulting to removed him from the contemporary music community for a long time. This alienation seems to have completed him and led him to complete his music, introducing mathematics for the first time in his work.

Although the majority of his works seem to lean towards atonality and serialism, Xenakis never used them. What we hear in his works are sets of sound events and masses where they consist of innumerable sounds. How these sounds are distributed is random. Nevertheless, Xenakis does not leave everything to their own fate. The overall creation has guidelines and rules. Imagine a race being organized. You don't know the outcome and the moves of the players, but you know the system and the rules that already exist.

All in all, Xenakis's works are reminiscent of sophisticated linear polyphony. The new music, or rather the new language offered by Xenakis, was a superlative language, and the first work he uses is *Metastaseis* written for 61 instruments. It lasts eight minutes and was first presented in 1955 at the festival Donaueschingen¹⁰¹. For the first time, he used glissandi¹⁰² on strings, giving the impression that the sound is moving massively and does not remain stable in a tonality, which is why the listener himself feels that the melody is lost in this mobility of the sound masses. Below we will see the graphic score of the composer himself, from the first steps of the work, where the glissandi of the strings can be distinguished, as well as the beginning of the work that includes them.

¹⁰¹ It is a festival of contemporary music that takes place every year in the city of Donaueschingen, Germany and started in 1921. It is considered one of the most famous festivals of new music and has occasionally featured great names in contemporary music such as Alban Berg, Arnold Schoenberg, Igor Stravinsky, Pierre Boulez and many others, as well as Iannis Xenakis.

See Hausler Josef, *Spiegel der Neuen Music: Donaueschingen: Chronik - Tendenzen - Werkbesprechungen*, Kassel: Bärenreiter Verlag, 1996

¹⁰² The word comes from the Italian *glissando* and its root lies in the French *glisser* which means slip. It is a sequence of tones that slide, which means, they slide up or down, either diatonically or chromatically.

See Michels Ulrich, *Atlas of Music*, IEMA, Athens: Philip Nakas, 1994, Volume I, p. 75

Image 6: The part of glissandi

Later on, the technique of glissandi was used by other composers such as György Ligeti and Krzysztof Penderecki. I would like to mention that for the construction of the Phillips Pavilion in Brussels in 1958¹⁰³, he used the same mathematical theory as that of the *Metastaseis* project.

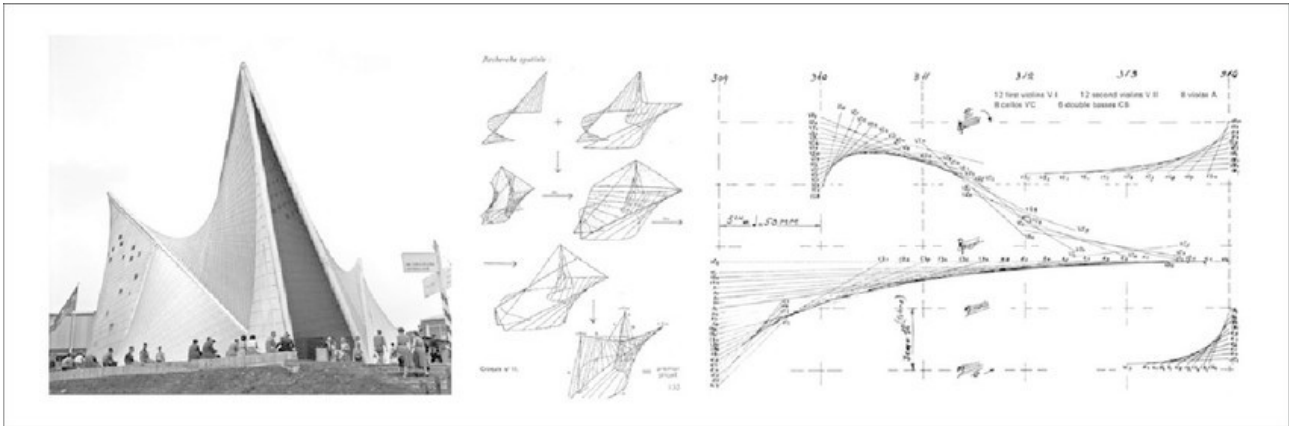


Image 7: Plans for the Phillips Pavilion and the "Metastaseis" project

In general, his work "*Metastaseis*" can be analyzed in its entirety, as it is the first mathematical musical creation, not only of the composer but of an entire era. It is also seen as a starting point for Xenakis' subsequent works and for the composers of his generation.

His next stochastic work, *Pithoprakta* in 1955-1956 for string orchestra, two trombones and percussion. Its name derives from the fact that it introduces the reasons for probabilities (probable actions).

Here, in this work, Xenakis again uses the same innovations as *Metastaseis*, glissandi and personalized melody, only to lift them up. And this is because he imagines six parallel worlds of sound colors opposing each other. So the vitality and movement of the project is clear. At the same time they are calculated with absolute precision, more than a thousand pizzicati¹⁰⁴ and glissandi which he again distributes by drawing on millimeter paper. His use of probabilities was originally derived from his innate Pythagorean mathematical nature, but also from the study of the behavior of molecules, as reported in Kinetic theory of gases.

The work is not only innovative in its theories and techniques but also in its overall effect. If we concentrate during its listening, we will observe how the whole work mimics a sound, a noise, which eventually reaches a clear sound. So he writes two works in one.

However, the work was disapproved of its premiere in Munich in 1957, resulting in maestro Serhen directing the work to omit the latter part of the project, which contains long pauses.

¹⁰³ The Phillips Pavilion was designed by architect L e Corbusier, in collaboration with his friend and collaborator Iannis Xenakis, in 1958 for the World Exhibition in Brussels. Visitors entered a dark place on an eight-minute route, where they first heard an introductory composition by Xenakis and then various sounds. The interior is said to have tended to resemble a cow's stomach because of the multidimensional space. See Clarke, Joseph L., "Iannis Xenakis and the Philips Pavilion", from Academia Edu website (https://www.academia.edu/5090254/Iannis_Xenakis_and_the_Philips_Pavilion) accessed 9-11-2018

¹⁰⁴ The term pizzicato means pinched and is of Italian origin. It is a string instruction, where the performer snaps the string. See Michels Ulrich, *Atlas of Music*, IEMA, Athens: Philip Nakas, 1994, Volume I, p. 77

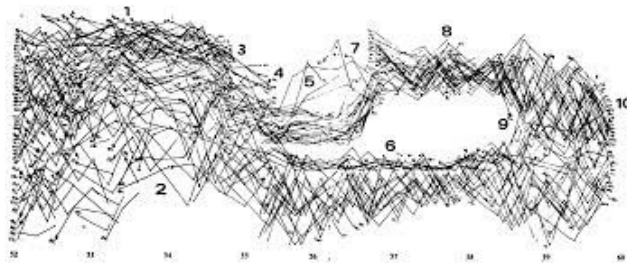


Image 8: The pizzicati-glissandi as designed by Xenakis.

But it would be frivolous on my part since we analyze Xenakis' stochastic music, not to mention the work on the purely stochastic laws of Achorriopsis.

Its name derives from the words ἄχος + πίψις in Greek, which means audio jacks. It was composed in 1956-1957 and was first presented in Buenos Aires by his great friend and maestro Hermann Scherchen¹⁰⁵. Its organization and composition are multileveled by the stochastic laws it has applied, and it is also the reason it has been introduced so few times.

Primarily in this work he uses Poisson's discrete distribution or otherwise the law of small numbers or distribution of rare events as we will find in many bibliographies. It was named after the French mathematician Smeon Dennis Poisson and belongs to the theories of statistics and probability. It is used to model binomial situations where we are interested in identifying the number of rare occurrences in large populations, such as if we had a staff of 500, how many of them could have been born on New Year's Day.

But how does Xenakis use it?

The project lasts 7 minutes. Before proceeding to the musical composition, Xenakis created a table of 28 columns containing the 28 time units in which the whole work is divided, that is to say we have 15 seconds in each column, each corresponding to 6.5 meters, if our rate is in 52 halves. The whole project contains 182 meters, so if we divide $28/6.5$ we get the result of 4.30 seconds. However, if we analyze the work we can see that the composer did not fully apply the division effect, probably for the sake of better musical flow. In the 28 boxes that the project divides, it distributes the possible musical events initially by: $P_k = \frac{\lambda^k}{k!} e^{-\lambda}$ isity, at 0, 1, 2, 3, 4 and 5. With λ (corresponding to cell) being 0.6 seconds, it calc bility of a single double or quadruple event based on Poisson's equation:

¹⁰⁵ Born in Berlin in 1891, Hermann Scherchen was a violinist and conductor. He was mainly involved in contemporary music, as he was at the heart of its evolution, although most people know him because of his involvement and recordings with Bach's work, particularly of the work Art of fougua. At the same time, he helped many young artists in the beginning of their careers, including Iannis Xenakis. See "Hermann Scherchen", from Encuclopaedia Britannica website (<https://www.britannica.com/biography/Hermann-Scherchen>) accessed 12-11-2018

The 7 vertical lines represent the different sound colors. Each line contains a set of instruments or sound colors for example

1. Flute (Piccolo in Mi b, Clarinet, Bass Clarinet)
2. Oboe (Oboe, Fagotto, Contrafagotto)
3. Glissando strings (Violin, Cello, Contrabass)
4. Percussion (Xylophone, WoodBlock, Grancasa)
5. Pizzicato (Violin, Cello, Contrabass)
6. Bronze (2 Trumpets, Trombone)
7. String arco (Violin, Cello, Contrabass)

With proper mathematical operations we will see that at least half the cells do not contain any sound events. As for how he calculated the density of each cell, he considered that if an ensemble plays an average of 10 sounds per second, then a single event can equal three parameters, $\delta = 2.2$ / second, or $\delta = 5$ sounds / meter 26 millimeters or $d = 4432.5$ / sounds / unit of time.

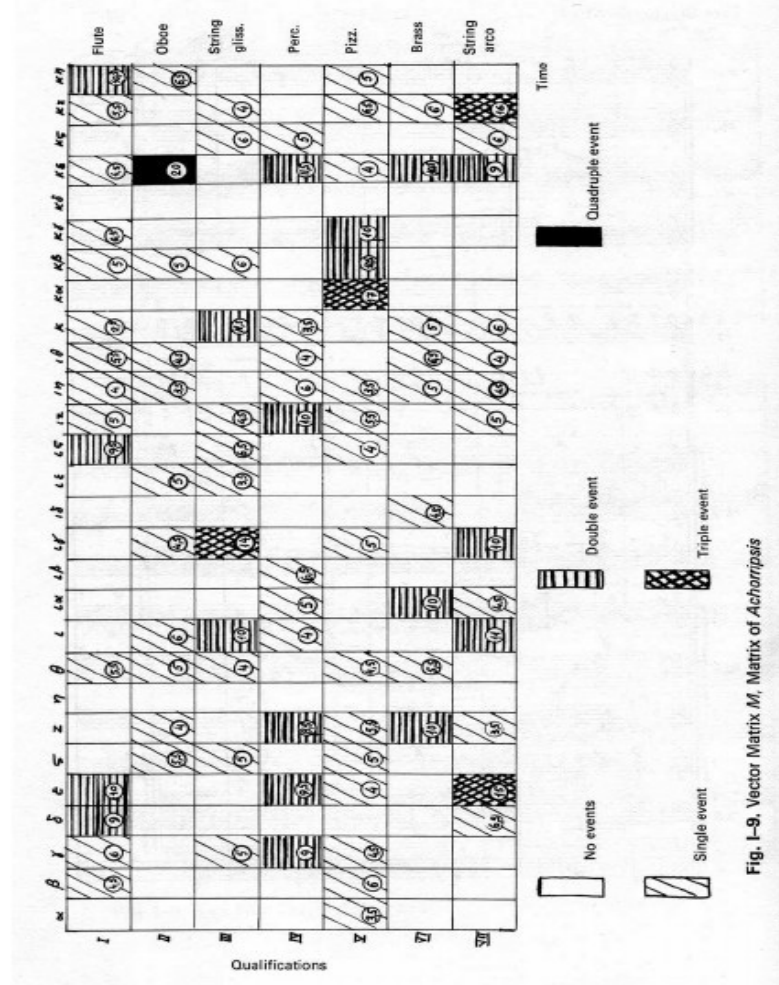


Fig. I-9, Vector Matrix M, Matrix of Achorripsis

Image 9: Table of the project "Achorripsis", as designed by Iannis Xenakis.

For a musician and for any music researcher not involved in mathematical science it is almost inconceivable to correlate and understand these calculations according to the musical notes. After all, Xenakis himself might never have done all of this if his original skill, that of an architect, had not helped him.

From this period onwards all his works include mathematical and physical theories that are sometimes proved within his works and sometimes not. From now on, all of his audio material is shaped by one-way statistics, with a specific purpose. His stochastic music, then, beyond the dual meaning of the term, which also refers to philosophy and mathematics, had a real purpose in each of his works. For this reason, it has been widely criticized by many analysts for its limitations on the composer. And for this reason, after the end of the *Achorripsis* project, Xenakis publishes a text explaining whether the work is ultimately coincidental after all this mathematical analysis, as well as how the analyst and listener could see it musically¹⁰⁶.

It seems that the composer did not like to leave anything to chance, or anything to create doubt. He stands out in 20th century music history, not for his innovation, but for his talent for taking a step forward and not pushing the boundaries of music composition and science simply because it is art. For Xenakis, it is obvious that art must be understood and imprinted in the full sense of the term, and art for him seems to be a human mind practiced without width and length, without tonalities and armor, without pentagram or millimeter papers, but with passion, talent and love for what he wants to do.



Image 10: Iannis Xenakis, at his studio in Paris, 1987

¹⁰⁶ Xenakis, I., *Formalized Music: Thought and Mathematics in Music*, Hillsdale, N: Pendragon Press, 1992, p. 37

CONCLUSION

Reaching the end of the project, I understand that all that I have said, all these great theories and discoveries, have not only become important because someone has formulated them and promoted them to humanity, but because they already exist, because they existed from the beginning of creation. And it's really impressive, the result of the technological tools of our time in our field. What is most striking, however, is that although each season lagged behind theories, instruments and techniques from the next, one confirmed the other. Pythagoras talked about the "musical" stars, and four thousand years later, technology confirmed him. The one who possessed only logical and mathematical intelligence. Euclid's mathematics, Boil's, Newton's physics, and many other theories that seemed impossible to relate to music were found united and well integrated, in some of the most famous music works of the 20th century.

Especially the works of the last century, aurally seem to be scattered outside form and tonal center and yet they are structured in perfect mathematics.

In this work, in addition to the knowledge I gained, as well as my infinite respect and admiration for the field of natural sciences, but especially mathematics, made me infer something concrete.

Many times, I would say, most of us new music scholars, but also the earlier ones, are mistaken in dealing with our own art. We focus much more on its rules, its forms and its complexity, so we lose the essence. And the essence my friends, hides in places that have no harmonious solutions mathematical, but in the "puzzles" we certainly hear, but we can't measure them or spread them over five lines.

In her mystery, her beauty is hidden!

REFERENCES

- The editors of Encyclopaedia Britannica, Pythagoras, from the Encyclopaedia Britannica website (<https://www.britannica.com/biography/Pythagoras>) accessed: 23-4-2018
- Arnold Hermann, Think Like a God: Pythagoras and Parmenidis, Mt Penelope Triada, Athens: Enalios, 2008, p.65
- Barbera, A. «The eleventh consonant and the expansion of musical tetraktys», Journal of Music Theory (1984), Vol. 28 pp.191-224
- Barker, A. The science of Harmonics in classical Greece, Cambridge: Cambridge University Press, 2007, pp. 3-30
- Bernard, Pierre;The UPIC as a Performance Instrument, Contemporary Music Review (2009), Vol.6 pp. 47-57
- Bussy, Pascal, John Coltrane (Paris: SEE, 1999) Caroline Usher , Pythagoras, from Academia (<https://www.academia.edu/12353896/Pythagoras>), accessed: 23-4-2018
- Caswell, Estelle, « The most feared song in Jazz,explained », [audiovisual / Video], from the Vox website (<https://www.youtube.com/watch?v=62tIvfP9A2w&feature=youtu.be>) *access date: 14/1/2019*
- Childs, E. , Achorriopsis: A Sonification of Probability Distrubtions, Kyoto: Proceedings of the 2002 International Conference on Auditory Display, 2002
- Childs, E., "The Matrix of Achorriopsis" (image), from Research Gate (https://www.researchgate.net/figure/The-Matrix-of-Achorriopsis-1_fig1_228783866) accessed : 14-11-2018
- Creese, D., The Monochord in Ancient Greek Harmonic Science, Cambridge: Cambridge University Press, 2010, pp. 131-177
- Crocker, R., «Pethagorean mathematics and music», Journal of Aesthetics and Art Criticism (1964), Vol.22, pp. 189-198 & 325-335
- Clarke, K., "A Decade of Cellular Urban Modeling with SLEUTH: Unresolved Issues and Problems ", Lincoln Institute of Land Policy, Cambridge 2008

- Diaz-Jerez, G. , Algorithmic Music: Using Mathematical Models in Music Composition, NY: Manhattan School of Music, 2000

- Gino, Loria, History of Mathematics, translated by Michael L. Kovaio, (Athens: Greek Mathematical Society, 1990), pp. 183-190

- Godwin, J., The Harmony of Spheres, Inner Traditions, 1992.

- Goodheart, Matthew, «The" Giant Steps "fragment», Perspectives of New Music (2001), Vol. 39 pp. 63-95

- Heath, Thomas , A History of Greek Mathematics, (New York: Dover Publications, 1981)

- Henderson, I. «Ancient Greek Music», Wellesz, E. Ancient and Oriental Music, London: Oxford University Press, 1957, pp. 336-403

- Huffman, Carl A., Archytas of Tarentum: A Pythagorean, Philosopher and Mathematician King, Cambridge: Cambridge University Press, 2005

- Ivon Thomas (translator), Greek Mathematics vol. I, from Thales to Euclid , London: Harvard University Press, 1967

- Jamie, J., The music of the spheres, London: Abacus, 1995, pp. 20-40

- Karpati, A. «Greek Music Theory in the IV Century BC», International Journal of Musicology (1994), Vol.3 pp. 57-88

- Kraut, Richard, The cambridge companion to Plato, Cambridge: Cambridge University Press, 1996

- Laha, RG and Rohatgi, VK. *Probability Theory*. New York: John Wiley & Sons, 1979

- Leonard Robert, Von Neumann, Morgenstern, and the Creation of Game Theory: From Chess to Social Science, 1900-1960, Cambridge: Cambridge Univeristy Press, 2012

- Levin, FR, The harmonics of Nicomachus and the Pethagorean tradition, American Park, PA, American Philological Association, 1975

- Lippman, EA, Musical Thought n Ancient Greece, New York: Columbia University Press, 1964, pp. 87-110

- Marie-Hélène Serra, «Stochastic Composition and Stochastic Timbre: GENDY3 by Iannis Xenakis», Perspectives of New Music (1993), Vol.31 pp.236-257

- Matossian Nouritza, Xenakis, London: Kahn & Averill, 1986

- Matossian Nouritza, Iannis Xenakis, (Paris: Fayard, 1981)

- Mattei JF, Pythagoras and the Pythagoreans, Kapsabeli K., Athens: M. Kardamitsa,

- Neubecker, AJ, Music in Ancient Greece, translated by Simota - Fidetz M., Athens: Odysseas EP ..E, 1986
- Nolan, C., «Music theory and mathematics,» Christensen, T., Western music theory, Cambridge: Cambridge University Press 2002
- Olsen Scott, The Golden Ratio: The Greatest Secret of Nature, translated by Diamianidou Despina, Athens: Alexandria, 2012
- Reinach, T., Greek Music, translated by Karastathi, A., Athens: Institute of the Book - A. Kardamitsa, 1999
- Riffenburgh, R. H, Statistics in medicine, San Diego: Academic Press, 3rd edition, 2012
- Santa, Matthew, "Nonatonic Progressions in John Coltrane's Music", Annual Review of Jazz Studies (2003), Vol.13, pp.13-25
- Sir Thomas L. Heath, A Manual of Greek Mathematics, New York: Dover, 1931
- Squibbs Ron, "Images of Sound in Xenakis's Mycenae-Alpha", from Academia Edu website (https://www.academia.edu/3529884/Images_of_Sound_in_Xenakiss_Mycenae-Alpha) accessed 31-10-2018
- Stamou, Lelouda, «Plato and Aristotle on Music and Music Education: Lessons from Ancient Greece», International Journal of Music Education (2002), Vol.39, p. 3-16
- Von Neuman, John & Morgenstern Oskar, Theory of Games and Economic Behavior, New Jersey: Princeton University Press, 1944
- Walfram Stephen, A New Kind of Science, Walfram Media.Inc, 2002
- Warren, DA, Music and Musicians in Ancient Greece, Ithaka, Ithaka [NY]: Cornell University Press, 1994, pp.248

HELLENIC LANGUAGE BIBLIOGRAPHY

- Anapolitanos D., Introduction to the Philosophy of Mathematics, 4th Edition, Athens: Nefeli, 2009
- Antonopoulos, A., From Tonic to Modern Music Theory, Athens: Print, 1999
- Aristoxenus of Taranta, All : Harmonious Elements, Rhythmic Elements, Excerpts, translated by Cactus Philological Group, (Athens; Cactus, 2005)
- Bormann, Karl, Plato, translated by Kalogerakos G. Ioannis, Athens: A. Kardamitsa, 2006
- Gaitani, Antonia, "The Relation of Mathematics and Music through Ancient Greek Texts"

[PDF], Web site of the National Kapodistrian University of Athens (http://www.math.uoa.gr/me/dipl/dipl_gaitani.antonio.pdf) accessed 13-12-2018

- Georgiou, M.P, Searching for Universal Sound, Nicosia: In Press 2005
- Goudouvas Chr. , Sotiris, Geometric Routes, Athens: Private, 2015
- Zika Christina, "What Theories Albert Einstein Formulated", from the website To Bhma (2008) (<https://www.tovima.gr/2008/11/24/archive/poies-theories-diatypwse-o-albertos-ainstain>) accessed 7-10-2018
- Theodorakopoulos Ioannis, Introduction to Plato, University Traditions, Athens, 1964
- Kopsachilis, Stelios I., Music Philosophy: Aristoxenus, music student of Aristotle and Music Theory, (Thessaloniki: Mayandros, 1996)
- Mauroeidis, Marios. , The Musical Ways in the Eastern Mediterranean, (Athens: Fagotto Books, 1999)
- Mousoulidis Fotis, « Iannis Xenakis - The Search for Global Music and the Pythagorean template ', from the Academia Edu website (https://www.academia.edu/36126510/%CE%99%CE%91%CE%9D%CE%9D%CE%99%CE%A3_%CE%9E%CE%95%CE%9D%CE%91%CE%9A%CE%99%CE%A3_%CE%97_%CE%91%CE%9D%CE%91%CE%96%CE%97%CE%A4%CE%97%CE%A3%CE%97_%CE%93%CE%99%CE%91_%CE%9C%CE%99%CE%91_%CE%A0%CE%91%CE%93%CE%9A%CE%9F%CE%A3%CE%9C%CE%99%CE%91_%CE%9C%CE%9F%CE%A5%CE%A3%CE%99%CE%9A%CE%97_%CE%9A%CE%91%CE%99_%CE%95%CE%99%CE%9F_%CE%A0%CE%A1%CE%9F%CE%A4%CE%A5%CE%A0%CE%9F?auto=download) , accessed : 19-10-2018
- Ballas Kostis, Pythagoras and the Pythagoreans, Athens: Forecast, 2001
- Bonis Constantine, Saint Augustine Bishop Opponos: Life and Texts, (Athens: Gregory, 1964)
- Djouni GE, Spyridis Charalambos Ch., "Two-day Conference: Pythagorean Theory of Music: Mathematical and Philosophical Approaches", by the National and Kapodistrian University of Athens website (<http://users.uoa.gr/~hspyridis/ntziouni.pdf>) accessed 25-08-2018
- Papaeconomou - Khpourgou, K. Music in Ancient Greece, Athens: Georgiadis, 2007, pp. 43-60
- "Pythagoras the Samos", Pellegrinis Thodoris, Dictionary of Philosophy, Athens: Greek Letters, 2004, p. 1144
- Porphyrios, Pythagoras Life, by Makris Constantinos, Athens: Katarti, 2001, pp. 72-149

- Poulos, Panagiotis, Music in the Islamic World: Sources, Theories, Practices, (Athens: Association of Greek Academic Libraries, 2015)
- Sakellariou, G. Th., Pythagoras: The Teacher of the Ages, Athens: Efstathiou Drivopylou 1963 pp.342
- Solomos Makis, Iannis Xenakis: The Universe of a Peculiar Creator, Athens: Alexandria, 2008
- Spandagos K. Evangelos, The Golden Cross in Ancient Greece, Athens: Aethra, 2004
- Sotiriou G.N, (under publication) "Pythagorean Theorem. Testimonies and Reconstructions ", Mathematical Review, Issue 88

IMAGES

- Image 1: 'John Coltrane' Giant Steps 'analysis', from Raphael Jezukiewicz website (<https://raphaeljezukiewicz.wordpress.com/2016/04/16/john-coltrane-giant-steps-analysis/>)
- Image 2: « De Coltrane Cirkel & Geometrie », από τον ιστότοπο: Sax wereld (<https://saxwereld.nl/coltrane-toon-cirkel-geometrie/>)
- Image 3: Lauten, Elodie, « Concepts », from website: Elodielauten.net (<http://www.elodielauten.net/concept.html>)
- Image 4: « 'UPIC system' (Unité Polyagogique Informatique du CEMAMu) Patrick Saint-Jean & Iannis Xenakis, France, 1977. », από τον ιστότοπο: 120 years of electronic music (<http://120years.net/upic-system-iannis-xenakis-france-1977/>)
- Image 5: « 'UPIC system' (Unité Polyagogique Informatique du CEMAMu) Patrick Saint-Jean & Iannis Xenakis, France, 1977. », από τον ιστότοπο: 120 years of electronic music (<http://120years.net/upic-system-iannis-xenakis-france-1977/>)
- Image 6: From website *Uoregon.edu* (<https://blogs.uoregon.edu/222s16/2016/04/04/see-and-hear-this-diagram-iannis-xenakis-metastasis/>)
- Image 7: Kristy W., « Project 3—Sound/Ambience: Iannis Xenakis », from website: *Medium.com* (<https://medium.com/@risingskies2/project-3-sound-ambience-iannis-xenakis-65d62dc4b640>)
- Image 8: Parthenios, Panagiotis, « Reciprocal transformations between music and architecture as a real-time supporting mechanism in urban design » [figure 1], από τον ιστότοπο: Research gate (https://www.researchgate.net/figure/Philips-Pavilion-Metastaseis-B-Iannis-Xenakis-1953-1958_fig1_308759695)
- Image 9: Beilharz, Kirsty, « Designing Sounds and Spaces: Interdisciplinary Rules & Proportions in Generative Stochastic Music and Architecture », [figure 11], from website (*Semantic Scholar* (<https://www.semanticscholar.org/paper/Iannis-Xenakis's-Achorripsis%3A-The-Matrix-Game-Arsenault/244f2e35a63410d44def20feb8f931e0b24d9bfa>)

- **Image 10:** Arsenault, Linda M., « Iannis Xenakis's Achorripsis: The Matrix Game », [figure 1] , από τον ιστότοπο: *Semantic Scholar* (<https://www.semanticscholar.org/paper/Iannis-Xenakis's-Achorripsis%3A-The-Matrix-Game-Arsenault/244f2e35a63410d44def20feb8f931e0b24d9bfa>)
- **Image 11:** « Xenakis in his studio », Circa, 1989, από τον ιστότοπο: *Iannis-xenakis.org* (<http://www.iannis-xenakis.org/xen/look/photos.html>)