

Russian Studies in Philosophy, vol. 41, no. 2 (Fall 2002), pp. 50–69.
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1061–1967/2003 \$9.50 + 0.00.

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The Pythagorean Syndrome in Science and Philosophy

From numbers they make what has heaviness and lightness.
—Aristotle

The problem of the relationship between mathematics and objective reality, which arose in early antiquity, is still a subject of heated discussion. The discussions are mainly about the question that probably was posed most clearly by Immanuel Kant in his *Critique of Pure Reason*: “How do subjective conditions of thought have objective validity, that is, how do they become conditions of the possibility of all knowledge of objects?”¹ Is it because they are themselves elements of objective reality, or because they are contained in thought a priori, before and independently of experience, or, finally, because they are subjective images of corresponding facets and aspects of objective reality?

Underlying the first two responses to the question is the Pythagorean syndrome, which was first described in the famous thesis of Pythagorean philosophy: “All things are numbers.” What this meant was that numbers, existing only in human consciousness, were identified with existing things that are external and independent: “The ancient Pythagoreans

taught that things consist of numbers in the same sense in which, according to the teachings of their predecessors and contemporaries, things consist of water, air, fire, and so forth, that is, numbers are the very substance and the prime matter of all things. Nor did the Pythagoreans separate number from matter; they did not accept a special world of numbers, accessible to reason and independent of the sensuous world.”²

The present article discusses the subsequent fate of the Pythagorean syndrome; how it influenced the various domains of the world of culture, and above all science and philosophy; what effects this had; what is the reason for the endurance of this syndrome; what is its case history; is there any effective remedy for it, and if so, what? Of course, we cannot examine all the known manifestations of the Pythagorean syndrome in science and philosophy from Parmenides’s doctrine that “an idea and what it is about are one and the same” to the rejection of Einstein’s general theory of relativity by A.A. Logunov’s relativistic theory of gravitation in which abstract space, existing only as an element of the theory, is identified with real space and time existing external to and independent of the theory. I shall limit myself here to an examination of only those manifestations of the Pythagorean syndrome that in my view have been the most influential in our times.

Let me say, to begin with, that the Pythagorean syndrome from its inception in Pythagorean philosophy is not merely the identification of things with numbers, as it may seem at first glance. It includes also (although in implicit form) the thesis that things are identical with geometric figures. This is connected with the fact that for the Pythagoreans numbers themselves possess geometric form. As Brunschvicg wrote quite aptly: “Before saying that things are numbers the Pythagoreans began by understanding numbers as things. The expressions ‘square number’ or ‘triangular number’ were not metaphors. These numbers were literally square and triangular to the eye and the mind.”³ The transition from the identification of things with “figural numbers” to their identification with the corresponding geometric ideas occurred explicitly later in connection with the discovery of the incommensurability of magnitudes. This occurred first with the discovery of the incommensurability of the diagonal with the side of a square. As I.M. Iaglom writes, “The demonstration that the length of the diagonal of a square with a side of one unit cannot be expressed by any number was a shock to the Pythagoreans (to their credit, they promptly appreciated the significance of this discovery).”⁴

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Although I agree that the Pythagorean thesis that “all things are numbers” contradicted the thesis of the incommensurability of the diagonal with the sides of a square, I do not share Iaglom’s opinion about the Pythagoreans’ reaction to this discovery (rather, I see in it an element of black humor). The Pythagoreans, indeed, promptly appreciated the significance of this discovery: according to tradition, Hippasus, who told them of the incommensurability of the diagonal and the sides of a square, was immediately thrown overboard by the angry Pythagoreans. The transition from an old paradigm to a new one did not occur “quickly” in those days (nor does it today) . . .

From its inception the Pythagorean syndrome has appeared in one way or another as a form of logical-epistemological pathology and has had a corresponding influence on the further development of philosophy and science. It enters the history of philosophy and science as one of the epistemological roots of rationalism, which identifies what is logically proved with what really exists; it is the line of least resistance in dealing with the question of why real phenomena come under theories that describe directly only their idealized analogies; and it promotes the development of various philosophical systems and scientific theories in which physical objects and relationships between them are interpreted as what the main character of I.A. Goncharov’s novel *An Ordinary Story* [Obyknovennaia istoriia], Aleksandr Aduiev, describes as “the physical signs of nonphysical relationships.”

The first to attempt to unravel the riddle of the Pythagorean syndrome was, apparently, Aristotle. Concluding that the Pythagorean syndrome “is an impossible thing,”⁵ he countered it with the thesis that things actually contain not numbers, not mathematical concepts, but their prototypes; that mathematical concepts are abstracted from the real world and for that reason are applicable to it. (Later, Engels reproduces this thesis of Aristotle’s almost word for word in his *Anti-Dühring*). Admittedly, Aristotle understood the class of objects that are the prototypes in the real world of mathematical concepts in a narrow sense: it did not include all the properties of and interrelations among physical objects.

The Sophists also took exception to the Pythagorean syndrome, and refuted it with the thesis that mathematical concepts are not and cannot be in things, that the existence of numbers, nonextended points, lines without width, and so forth, in things contradicts experience and the evidence of the senses. For, as Protagoras put it, the measure of things is not the things themselves, “the measure of all things is man, of existing

things that they exist, and of nonexisting things that they do not exist.” The fact that neither Pythagoras himself nor his disciplines understood this (and much less) was one of the reasons for Heraclitus’s conclusion: “Learning of many things does not teach intelligence; otherwise it would have taught . . . Pythagoras.”⁶

The subsequent development of science and philosophy introduced certain refinements to Heraclitus’s statement. The knowledge of many things that mankind had acquired by the end of the twentieth century (that the Pythagoreans and their numerous disciples as well as their less numerous opponents had lacked), did teach us something (including how to diagnose and avoid the Pythagorean syndrome in science and philosophy, as will become clear below).

Yet this syndrome continued to spread in one form or another in science and philosophy down to our own day. Perhaps the influence of the Pythagorean syndrome was manifested most clearly in Campanella’s and Galileo’s doctrine of “the two books,” Kant’s a priorism, Poincaré’s conventionalism, and Logunov’s relativistic theory of gravitation mentioned above.

The Pythagorean syndrome is responsible for the fact that the mathematization (geometrization) of science was explained in the course of the intellectual revolution of the sixteenth and seventeenth centuries in Europe as a consequence of the mathematization (geometrization) of nature. Ultimately, this is the sense of A. Koyré’s well-known comment that this revolution was based on the “mathematization (geometrization) of nature and, consequently, the mathematization (geometrization) of science.”⁷ Thus was born Campanella’s and Galileo’s doctrine of “the two books.” In his *Mathematical Discourses and Demonstrations Concerning Two New Sciences*, Galileo explained this doctrine as follows: it is based on the theory of double truth, according to which there are two classes of truth—the truths of theology and the truths of philosophy. The first are represented in the Bible, the Book of Divine Revelation, written in ordinary language, and the second in the Book of Nature, in the “greatest book that is always before our eyes (I am speaking of the universe), but cannot be understood without first learning to understand the language and to distinguish the signs in which it is written. It is, in fact, written in mathematical language.”⁸

The Pythagorean syndrome had a somewhat different influence in Kant’s philosophy, according to which mathematical concepts are contained in the human understanding a priori and appear in reality as a

consequence of the fact that in experience man introduces them into reality. According to Kant, the fact that mathematical concepts are confirmed by empirical facts does not mean that they are derived from them. In Kant's opinion this is confirmed by the universality of mathematical concepts, by the circumstance that they are clearly thinkable independently of any connection with concrete empirical content. This holds above all for Euclidian space: it is contained in the human understanding as an a priori form of sensibility and appears in reality as a consequence of the fact that man introduces it there. For this reason, according to Kant, all things as phenomena appear in Euclidian space: abstract space contained in the human understanding a priori is the space in which all things as phenomena are located.

The secret of the Kantian a priori (the fact that it once had been a posteriori) was cleared up finally only in the twentieth century, when it was demonstrated that mathematical concepts are not universal and that there are limits to their applicability. Neither Kant nor the science of his time were aware of this.

Some historians of science believe that "had Kant paid more attention to the developments in the mathematics of his time, perhaps, he would not have insisted that the ordering of spatial sensations in the image and likeness of Euclidian geometry is the only ordering reason can admit."⁹ One can hardly accept this opinion. First, the mathematics of Kant's time was not clear on this issue: the works of N.I. Lobachevskii and J. Bolyai on non-Euclidian geometry were published more than a quarter of a century after Kant's death, and K.F. Gauss, who had realized as early as 1792 that non-Euclidian geometry was possible, communicated this only in 1831 and, then, only in a letter to his friend H.C. Schumacher.

Second, in this comment M. Kline clearly confuses two different questions: (1) is "Euclidian geometry . . . the only one reason can admit?" and (2) is the "ordering of spatial sensations in the image and likeness of Euclidian geometry . . . the only ordering reason can admit?" As Kline assumes, had Kant paid greater attention to developments in the mathematics of his time, then possibly he would have answered the first of these two questions negatively. However, this would not have predetermined the answer to the second question, for its answer, in the final analysis, depends on what geometry is objectively realized in the spatio-temporal realm within the reach of our sense organs and, as a result, is necessarily manifested in our sensations.

* * *

The first thinker to arrive at the conclusion that geometry is a posteriori was, apparently, Gauss.¹⁰ In a letter to H.W. Olbers, dated 28 April 1817, he wrote: “geometry ranks not with arithmetic, which exists purely a priori, but rather with mechanics.”¹¹ He clarified this further thirty years later in a letter to F.W. Bessel: “It is my profound conviction that the theory of space occupies a completely different place in our knowledge than pure mathematics (which operates with numbers). In all of our knowledge there is nothing that would demonstrate in any convincing way the absolute necessity (and, consequently, the absolute truth value) that is so characteristic of pure mathematics. We can only humbly add that if number is the product of our reason then space is a reality lying outside our reason to which we cannot prescribe our own laws.”¹²

Lobachevskii raised the question whether there are limits to the domain of applicability of Euclidian geometry. He wrote: “We assume that certain forces in nature obey one geometry, while others obey their own special geometry.”¹³ This is why space is Euclidian only in the domain in which the corresponding forces are at work and can be non-Euclidian beyond its limits, where forces of a qualitatively different nature operate, “either beyond the visible world,” hypothesized Lobachevskii, “or in the compact sphere of molecular attractions.”¹⁴

The critics of this view of the foundations of the relation between geometry and physics point out that the metric of space cannot have an internal cause, for it is introduced into space from outside by massive objects. It is not difficult to see that such critical comments are based on the Pythagorean syndrome: the identification of real space with abstract space into which the theoretician imports a metric “from outside.” By means of massive objects, the investigator does not create, but only gains knowledge of the metric properties of real space.

The reference to Riemann, who in his Göttingen lecture “On the Hypotheses on Which Geometry Is Based” was to have stated that the metric is introduced into real space from outside, is based on a misunderstanding. Actually, Riemann did not speak about this, but about explaining the metric of space by the “forces of cohesion” or “the inner cause of the occurrence of metric relations in space.”¹⁵ I would explain the continuing references to Riemann by recalling merely that, according to mathematical tradition, no one present at Riemann’s lecture un-

derstood it and only old Gauss left the lecture immersed in thought. It is difficult for me to judge whether supporters of the thesis that the metric is “introduced” into space by massive objects belong among those “immersed in thought,” but the fact that many have still not understood Riemann’s lecture seems incontestable. Riemann’s lecture ended with the recommendation to pay particular attention to facts that go beyond the domain of applicability of classical physics, whose metric properties are Euclidian. “Our only hope,” said Riemann, “of finding the answers to these questions is to take the present empirically verified conception, whose foundations were laid by Newton, and progressively to refine it guided by the facts it cannot explain. . . . We stand here at the threshold of a domain belonging to another science—physics, and the present time gives us no cause for crossing it.”¹⁶

The further development of physics proceeded precisely along this path, leading to the discovery of Einstein’s special and general theories of relativity, which proved beyond a doubt that the metric properties of space–time, in which gravitational interactions play a determining role, differ from Euclidian properties because of the definite subordination of the metric properties of space to the corresponding properties of the physical interactions that prevail in a given domain, the dependence of the former on the latter, and their conditioning by the “forces of cohesion,” which Riemann predicted in his Göttingen lecture. In the final analysis this is precisely what H. Weyl had in mind when he later wrote that “a full understanding of Riemann’s closing remarks concerning the inner essence of the metric of space became possible only after Einstein formulated his general theory of relativity.”¹⁷

Something similar (*mutatis mutandis*) happened sooner or later with a number of other mathematical concepts, which also, eventually, lost their universal “innocence” and encountered limits to their domain of applicability. Let me illustrate this by just one more example, by the statement, familiar to every schoolboy, that the order of multipliers does not affect the product. This is the commutative law of multiplication, according to which $uv - vu = 0$, which for a long time seemed self-evident.

The fact that noncommutative magnitudes in which $uv - vu$ is not equal to zero can exist in mathematics was realized first by W.R. Hamilton. “The light dawned on him (as his admirers like to recount) one October day in 1843 as he was crossing a bridge in Dublin and he discovered quaternions.”¹⁸ All hitherto known mathematical magnitudes

from real to complex numbers) were commutative. The fact that complex numbers consisted not of one but of two components (real and imaginary) was another matter. In studying the multiplication of complex numbers it became obvious to Hamilton that there must be mathematical magnitudes consisting of four components (quaternions).

The discovery that the commutative multiplication of complex numbers leads to the noncommutative multiplication of quaternions was made in connection with the geometric interpretation of the multiplication of complex numbers. As N. Bourbaki wrote: "Once the multiplication of complex numbers was interpreted with the help of planar rotation it was found that to extend this concept to space it is necessary to consider noncommutative multiplication. . . . This was one of the ideas that guided Hamilton in his discovery of quaternions, which were the first examples of a noncommutative entity."¹⁹

The question of limits to the domain of applicability of commutative mathematical magnitudes arose many years later, when physics entered into the realm of quanta, a realm characterized by relativity and a specific link among certain fundamental properties of physical objects and their interrelations, which within the domain of applicability of classical physics appeared as absolute and independent of one another. It was found that in theory these properties of quantum entities and the interrelations among them could be described mathematically by using noncommutative mathematical magnitudes. There had never been, nor is there now, anything like this in the domain of applicability of classical physics: all the known properties of classical macroscopic entities and their relations are described mathematically using commutative magnitudes.

It is natural, therefore, that, historically, the study of the properties and interrelations of physical entities in the domain of applicability of quantum mechanics began with the extrapolation of the commutative paradigm of classical physics to it. The fact that the paradigm proved incapable of adequately representing in theory a number of fundamental properties of physical entities in this domain was taken by physicists as one proof of the untenability of quantum mechanics as a theory of quantum entities. P.A.M. Dirac later recalled: "Heisenberg became extremely upset when he found that uv was different from vu After discovering noncommutativity, Heisenberg decided that this was the unavoidable end of the theory and that it had to be renounced."²⁰ It took some time to realize that in fact the discovery of this noncommutativity

merely showed that in quantum mechanics science had stumbled on the limits of the domain of applicability of commutative mathematical magnitudes, a domain in which the commutative paradigm of classical physics had undivided rule.²¹

Thus, the development of physics in the twentieth century fully confirmed the brilliant insights of the mathematicians of the nineteenth century. But they had never been understood by their contemporaries. I have already mentioned how Riemann's Göttingen lecture was received by his listeners. The discovery of non-Euclidian geometries and noncommutative mathematical magnitudes and, especially, the attempts to discover what corresponds to them in objective reality were viewed by the contemporaries of Gauss, Lobachevskii, and Hamilton as, at best, an eccentricity of genius.

When Gauss measured the angles between mountain peaks in Western Germany to discover whether the sum of the angles of large triangles was equal to 180° , and when Lobachevskii calculated the sum of the angles of triangles whose apices were the stars, both of them were told patiently that practice could be the criterion of the truth of our knowledge in any realm whatsoever except in mathematics and that even schoolchildren knew that the sum of the angles of a triangle (large or small) is equal to 180° . Yet Gauss and Lobachevskii, not their contemporaries, were right: the subsequent development of physics proved that the sum of the angles of a triangle in spatiotemporal regions in which gravitational interactions prevail is indeed not equal to 180° .

When Hamilton said that noncommutative mathematical magnitudes were the key to understanding the unified physical universe and that "this discovery seems to me as important for the mid-nineteenth century as the discovery (calculation) of derivatives was for the end of the seventeenth century"²² this perplexed his contemporaries. But whereas we can understand the latter, it is quite difficult to explain rationally the analogous reaction of our contemporaries to such an outstanding discovery. L.S. Polak had this to say about the above-quoted words of Hamilton: "Never has a great mathematician been so hopelessly wrong."²³ And this was said in a book published in 1993, more than two-thirds of a century after the brilliant success of the noncommutative paradigm in quantum physics, a victory fully comparable to the brilliant success of Newton's calculus in modern physics. Polak's comment, slightly rephrased, could be readdressed to him: never has a historian of science been, in my view, so hopelessly wrong . . .

* * *

It would seem that in the course of its history science, and above all physics, should have developed a certain immunity to the Pythagorean syndrome. But this has not happened. The identification of theoretical structures, mathematical forms, abstract symmetries, and scientific laws that are elements of a theory with corresponding structures, forms, symmetries, and laws of the objective world existing outside and independently of any theory continues to this day.²⁴ Thanks to the Pythagorean syndrome physics has “populated the objective world with scalars, vectors, tensors, spinors, abstract spaces, singularities, strange attractors, and so forth.

The Pythagorean syndrome played a decisive role in the rise of both the old reaction of Poincaré’s approach to the problem of the interrelationship between geometry and physics against Einstein’s program of the geometrization of physics as well as of the present reaction, which grew out of the former, of Logunov’s relativistic theory of gravitation to Einstein’s general theory of relativity.²⁵ Poincaré’s approach to the problem of the relationship between geometry and physics is based on the identification of abstract space with real space and time. For him the metric properties of space are independent of and totally unrelated to the properties of physical objects and the relations among them; hence, it is totally unimportant what geometry is used in physics; what is important is simply that without it, it is impossible to express physical laws. This understanding of the role of geometry in physics, of course, leads to the denial of its cognitive function and is one of the sources of Poincaré’s conventionalism.²⁶

Einstein’s program of the geometrization of physics is based on the notion of abstract space as a theoretical model of real space and time. For Einstein, the “geometric properties of space are not independent: they are conditioned by matter.”²⁷ For him the choice of geometry in building a physical theory is subordinate to the higher aim of physics, namely, to know the physical world. The transition from Euclidian geometry to Riemannian geometry, which accompanies the transition from classical physics to Einstein’s general theory of relativity, was conditioned not only and not so much by the awareness of the fundamental role of geometry in the formulation of physical laws as by the realization of the intimate connection between the geometry used in physics and the problem of physical reality. From Einstein’s vantage point,

in physics, geometry not only determines the structure of physical theory, but is also determined by the structure of physical reality. Only if geometry fulfils both functions at the same time is it possible to avoid conventionalism.²⁸

As Poincaré wrote: "By virtue of natural selection, our mind has adapted to the conditions of the external world. . . . It has assimilated the geometry most suited or, in other words, most convenient for the species. . . . Geometry is not true, it is only useful."²⁹ Man's mind has effectively adapted to the conditions of the external world, including the metric properties of real space and time in the corresponding domain of the external world and hence, has appropriated the geometry that proved to be adequate to reality and was more convenient only because of this.³⁰ Geometry as an element of theory is another matter. It may reflect the metric properties of real space and time, but then again it may not, in which case it would be a geometry of some abstract space, which would serve to reconstruct in theory the properties of physical interactions. In the first case, it is a question of its truth or falsity, in the second of its convenience. The absolutization of the second solution and the reduction of the problem of the relationship between geometry and reality to it is, in the final analysis, a consequence of the Pythagorean syndrome, of the mistaken identification of abstract space with real space and time; it cannot lead physics beyond the bounds of conventionalism.

Only the first solution makes it possible to save physics from the Pythagorean syndrome and conventionalism. The fact is that the process of recreating the properties of physical interactions according to the corresponding metric properties of space and time is not an experimental but a purely theoretical procedure. As a purely theoretical procedure it does not differ in principle from the process of recreating in theory the same properties of physical interactions with the aid of the metric properties not of real space and time, but of the corresponding suitably organized abstract spaces. Hence, we have, on the one hand, the illusion that a theoretician can arbitrarily choose a geometry as the background for studying physical interactions and, on the other, the rational core of Poincaré's conception of the relationship between geometry and physics: as components of theories with which the theoretician recreates the properties of physical interactions the geometries really can differ and in this sense theory contains an element of conventionality.

However, this is not conventionalism. First, because the metric properties of space and the corresponding properties of physical interactions

are not independent of one another, not only in objective reality but also in theory: we cannot arbitrarily choose a geometry in a theory. We always select it in such a way as to use it to recreate in theory the corresponding properties of real interactions. Second, because the question of which of the geometries used to recreate the properties of physical interactions in a theory adequately represents in it the metric properties of real space and time cannot be solved within the theory: the question goes beyond theory to the domain of experiment.

I believe this point is central to the relationships between Einstein's general theory of relativity and Logunov's relativist theory of gravitation.³¹ The latter is based on: (a) the so-called principle of geometrization according to which the properties of gravitational interactions can be reconstructed in theory on the basis of either the properties of Riemann's curved space-time or Minkowski's planar space-time; (b) the proposition that only Minkowski's planar space-time adequately represents real space and time in theory whereas Riemann's curved space-time functions in it only as an "effective" space. Logunov proposes that "the geometry of space-time is pseudo-Euclidian (Minkowski space) for all physical fields."³² He reproaches Einstein and D. Hilbert, "these two great scientists rejected the astonishingly simple Minkowski space and entered the labyrinth of Riemannian geometry, which has enticed subsequent generations of physics."³³

It is, of course, true that the pseudo-Euclidian geometry of Minkowski's planar space is simpler than Riemann's non-Euclidian geometry of space. However, the main question is what are the properties of real space and time; are real space and time planar, Euclidian, or nonplanar, non-Euclidian in objective reality, external to and independent of theory? Einstein and Hilbert rejected the "astonishingly simple Minkowski space . . . and entered the labyrinth of Riemannian geometry" precisely because they were interested not only and not so much in the metric properties of abstract space, which one could use to describe in theory real space and time, as in the latter's metric properties.³⁴ The Pythagorean syndrome played a cruel joke on Logunov. It is this that underpins the initial thesis of the relativistic theory of gravitation, according to which the laws of conservation of energy, impetus, and momentum of a quantity of motion define unambiguously the properties of symmetry of space and time and thus their metric properties in the objective world (this directly entails the assertion that only Minkowski's planar space-time adequately represents in theory real space and time).

But in J. Neter's theorem on differential invariants, to which Logunov refers, the issue is totally different: it deals with the properties of symmetry of the laws of motion that are reflected in the invariance of equations of motion or field equations (the Lagrange–Euler equations) relative to the corresponding transformations.³⁵ But the thesis on which the relativistic theory of gravitation essentially rests does not follow from Neter's theorem: analysis shows that neither Neter's theorem nor the theory of which it is an element is sufficient for an unambiguous solution to the question of the metric properties of space and time in the objective world.³⁶ Einstein's general theory of relativity and the relativistic theory of gravitation give what appear to be at first glance different answers to this question. However, on closer scrutiny we find that this difference is only apparent. Essentially, both Minkowski space and Riemann's "effective" space, on the basis of whose properties the properties of gravitational interactions are reconstructed in the relativistic theory of gravitation, function in it only as abstract spaces, as elements of the theory that do not exist external to and independently of it. Only experiments to determine the metric properties of real space and time can answer the question of what corresponds to them in objective reality. As has been demonstrated elsewhere,³⁷ experiments to determine the metric properties of space and time show that it is not Minkowski's planar space–time but what Logunov calls the "effective" Riemannian space that is the abstract space that properly represents real space and time in both the relativistic theory of gravitation and the general theory of relativity: "The attempt to interpret the metric relationships of the planar world as observable and the concrete observational predictions based on this interpretation lead only to contradiction with experiment."³⁸

Logunov's rejection of Einstein's general theory of relativity, his replacing it with the relativistic theory of gravitation, his use of criteria internal to the theory to answer the question of which of the abstract spaces of the theory adequately represents in it real space and time and the identification the abstract space with real space and time all are manifestations of the Pythagorean syndrome. As A.D. Sakharov said quite rightly in a postscript to Ia. B. Zel'vich's article "Can the Universe Be Created Out of Nothing?" [*Vozmozhno li obrazovanie Vselennoi "iz nichego"?*],³⁹ it is not Riemann's curved space–time, but Minkowski's planar space–time that functions as an "auxiliary planar space in the relativistic theory of gravitation. However, it is wrong to interpret magnitudes defined in terms of this space as observable. The statement by

the authors of the relativistic theory of gravitation that the conclusions of the general theory of relativity are unambiguous is wrong. And their refusal to examine topological structures of space–time other than those in Minkowski’s world is unfounded.”⁴⁰

* * *

The secret of the Pythagorean syndrome is to be found in the distinctive features of the subject of mathematics. Scientific reflection, which plays an auxiliary role in other sciences, plays a defining role in mathematics. By absolutizing this feature of mathematics we overlook the fact that through reflection, through self-study, mathematics studies what is not itself; its subject matter is identified with the subject matter of its reflection; what is found in it is ontologized and identified with what is found outside and independently of it.

For some this is physical reality, for others it is the so-called mathematical reality, which exists alongside of physical reality. “Inasmuch as the separation of form from content, quantity from quality, is impossible in empirical physical reality,” writes A. Nysanbaev, “yet inasmuch as this separation is a very real fact, the natural conclusion is that this fact occurs in a special reality not identical with the reality revealed in nature. This reality is the so-called mathematical reality.”⁴¹ It makes no difference whether we are speaking of physical or mathematical reality, for in either case the result is a conception of mathematics that is expressed in Bertrand Russell’s famous aphorism: “Mathematics is a doctrine in which we do not know what we are talking about and whether what we say is true.”⁴²

The fact that this conception of mathematics is in patent contradiction with the results of the development of science (in the course of which the apparently unmotivated original assumptions lying at the basis of mathematics and the corollaries flowing from them on closer examination proved to be theoretical images of the corresponding facets and aspects of objective reality), has been gradually understood not only by some natural scientists and philosophers, but also by many mathematicians such as P.S. Aleksandrov, who described “mathematics as singing about the general forms proper to physical being,”⁴³ and A.N. Kolmogorov, who pointed out that “mathematics studies the general (‘pure’) forms of concrete being.”⁴⁴

V.I. Arnol’d is undoubtedly right when he writes about the “harm

done by the view of mathematics as . . . the analysis of the implications of arbitrary systems of axioms.”⁴⁵ Commenting ironically about the proponents of such a conception of mathematics, he describes it as a “deductive-axiomatic scholastic style” that “consists in the fact that the account of mathematical theory begins with an unmotivated definition. The psychological difficulties into which this thrusts the reader are almost insurmountable for a normal human being: “Why should I give anyone two apples?” here is the main difficulty that turned Buratino away from the study of arithmetic.”⁴⁶

However, Buratino is not the only one who has psychological difficulties with this. In his article “The Ruinous Influence of Mathematics on Science” [Pagubnoe vliianie matematiki na nauku], D. Shvarts called attention to mathematics’ characteristic “willingness to carefully develop any idea, no matter how absurd it may be, and to dress up brilliant achievements as well as scientific absurdities alike in an impressive uniform of formulas and theorems.” “Unfortunately,” says Shvarts, “an absurdity in uniform is much more persuasive than naked absurdity.”⁴⁷ The numerous relapses of the Pythagorean syndrome in science and philosophy—some of which I have discussed in this article—are connected in large measure with this fact. On closer examination, one can detect in each of them the specific “absurdity in uniform.”⁴⁸

By the way, in other areas of culture, in contrast to science and philosophy, the Pythagorean syndrome is used consciously in this very capacity to characterize various absurdities in relations between people. For example, it is often associated in literature with what is known as female logic. A typical expression of this is found in Turgenev’s novel *Rudin*, where one of the characters, Pigasov, spells out how male logic differs from female logic: “A man might, for example, say that two plus two is not four but five or three and a half, but a woman will say that two times two is a wax candle.”

But manifestations of the Pythagorean syndrome in personal interrelations are not limited to innocent associations of this sort. The results of numerous attempts to base various models of human social organization from the religious-philosophical brotherhood created by Pythagoras in Croton to analogous social structures in certain utopias and anti-utopias on the Pythagorean syndrome are inevitably “absurdities in uniform” (of a patently totalitarian-profascist cult). Typical in this regard is the Unified State in E.I. Zamiatin’s anti-utopian novel *We* [My] with its “square harmony,” “mathematically flawless happiness,” “mathemati-

cally perfect life,” and its identification of any deviations from these norms (such as art, love, and freedom) with imaginary numbers (the narrator in the novel *We*, number D-503, recalls his first encounter with $v-1$: “I cried, beat my fist against a table, and shouted, ‘I don’t want $v-1$! Free me from $v-1$!’ This irrational root grew into me as something strange, alien, and horrible, it devoured me.”⁴⁹).

In conclusion, I would like to say what must be done to protect science and philosophy from the influence of the Pythagorean syndrome. Of course, we can reconcile ourselves with its existence and, at our next encounter with the syndrome in science and philosophy, recall Chatskii’s words from his dialogue with Famusov in A.S. Griboedov’s comedy *Grief from Wit* [Gore ot uma]: “The houses are new, but the prejudices are old” (or anything else along this line) or, paraphrasing Hegel’s famous aphorism, comment that the history of science, philosophy, and other areas of culture shows that people learn nothing from it.

We could place some hope on the greater rigor of mathematics (the so-called mathematical precision) or even on greater demands of the mathematicians (recalling, for example, the code of laws of the Roman and Byzantine emperor Justinian titled *On Malefactors, Mathematicians, and Their Like*), if it were not for one fact that undercuts the effectiveness of either measure. What I have in mind is René Thom’s relation of indeterminacies, “the more rigor, the less meaning,” which, in my opinion, applies far beyond mathematics and mathematical logic.

It seems to me that there is only one effective measure against this illness. It was Aristotle who first drew attention to it. He understood that the conception of mathematics that is based on the identification of its object with the object of its reflection can be contrasted with the conception of mathematics as a system of concepts abstracted from the real world in which only their concrete prototypes, not the concepts themselves, exist. It becomes evident here that, as Hilbert wrote, “mathematics, like any other science, cannot be based on logic alone. On the contrary, in order to apply logical inference and to put logical operations into effect something must already be given, namely, definite extra-logical concrete objects that tangibly exist in immediate experience prior to any thought whatsoever.”⁵⁰

I think that this is the only way to protect science and philosophy from the influence of the Pythagorean syndrome—by realizing that, as Poincaré rightly noted, although “mathematics has to reflect on itself . . . the main forces of our army have to be directed toward . . . the study of

nature,”⁵¹ by realizing that the subject of mathematics is not reducible to the object of its reflection, that by means of reflection and self-analysis mathematics comes to know that which is not itself (but that of which it is the theoretical image)—the objective world that exists prior to, outside of, and independently of mathematics.

Of course, deliverance from the Pythagorean syndrome, as from any other chronic ailment, is not without losses. The main loss, in my view, is the notion of theoretical simplicity and perfection that accompanies the Pythagorean (let us recall how Logunov compared “the astonishingly simple Minkowski space” with the “labyrinth of Riemannian geometry”). But if the overriding purpose of a theory that uses a mathematical apparatus is knowledge of the objective work, its description and explanation, then the losses will turn into unquestionable gains that are linked with corresponding changes in notions about theoretical simplicity and perfection. “Our unwavering aim,” wrote Einstein, “is an increasingly better understanding of reality. . . . The simpler and more fundamental our assumptions become, the more complex are the mathematical tools of our reasoning and the longer, more subtle and complex, the path from theory to observation. Although it sounds paradoxical, we can say that modern physics is simpler than the old physics and, hence, it seems more difficult and complicated.”⁵²

My assessment of the losses and gains of delivering science and philosophy from the Pythagorean syndrome and my comparison of various theories that are free of the Pythagorean syndrome with theories that in one way or another are under its influence lead me to a conclusion that in many respects echoes Shakespeare’s famous comment in the 130th sonnet:⁵³

I love to hear her speak, yet will I know
That music hath a far more pleasing sound.
I grant I never saw a goddess go,
My mistress, when she walks, treads on the ground.
And yet, by Heaven, I think my love as rare
As any she belied with false compare.

Notes

1. I. Kant, *Soch. v. 6-ti tomakh* (Moscow, 1964), vol. 3, p. 185.
2. A.O. Makovel’kii, *Dosokratiki* (Kazan’, 1919), pt. 3, p. xix.
3. L. Brunschvicg, *Les étapes de la philosophie mathématique* (Paris, 1947), p. 34.

4. I.M. Iaglom, *Matematika i real'nyi mir* (Moscow, 1978), p. 19.
5. Aristotel' [Aristotle], *Metafizika* (Moscow/Leningrad, 1934), p. 232.
6. Diogen Laertskii [Diogenes Laertius], *O zhizni, ucheniakh i izrecheniakh znamenitkh filozofov* (Moscow, 1979), p. 359.
7. A. Koire [Koyré] *Ocherki istorii filosofskoi mysli* (Moscow, 1965), p. 130.
8. G. Galilei [Galileo], *Besedy i matematicheskie dokazatel'stva, kasaiushchiesia dvukh novykh otraslei nauki, otnosiashchikhsia k mekhanike i mestnomu dvizheniiu* (Moscow/Leningrad, 1934), p. 25.
9. M. Klain [Kline], *Matematika. Utrata opredelennosti* (Moscow, 1984), p. 93.
10. On this point see R.A. Aronov, "O metode geometrizatsii v fizike. Vozmozhnosti i granitsy," in *Metody nauchnogo poznaniia i fizika* (Moscow, 1985), p. 345.
11. K.F. Gauss, "Pis'mo k Ol'bersu (28 apreliia 1817 g.)," in *Ob osnovaniakh geometrii* (Moscow, 1956), p. 103.
12. Idem, "Pis'mo k Besseliu ot 9 apreliia 1830 g.," quoted in Klain, *Matematika. Utrata opredelennosti*, p. 104.
13. N.I. Lobachevskii, "Novye nachala geometrii s polnoi teorii parallel'nykh," in *Ob osnovaniakh geometrii*, p. 64.
14. Ibid., p. 65. See in this connection R.A. Aronov, "On the Foundations of the Hypothesis of the Discrete Character of Space and Time," in *Time in Science and Philosophy* (Prague, 1971), p. 265.
15. B. Rimann [Riemann], "O gipotezakh, lezhashchikh v osnovanii geometrii," in *Ob osnovaniakh geometrii*, p. 323.
16. Ibid., p. 324.
17. "Komentarii G. Veilia k memuaru Rimana," *ibid.*, p. 340; see in this context R.A. Aronov, "Evoliutsiia predstavlenii o prostranstve i vremeni," *Fizika v shkole*, 1961, no. 3, p. 11; idem, "Mogut li prostranstvo i vremia razdelit' sud'bu teploroda i flogistona?" in *Fizicheskaiia teoriia i real'nost'* (Voronezh, 1976), p. 101; idem, "Iavliaiutsia li prostranstvo i vremia abstraktsiiami?" in *Dialekticheskii materializm i filosofskie voprosy estestvoznaniia* (Moscow, 1981), p. 3.
18. D.Ia. Stroik, *Kratkii ocherk istorii matematiki* (Moscow, 1968), pp. 238–39. For more details on this point see L.S. Polak, *Uil'iam Gamil'ton* (Moscow, 1993).
19. N. Burbaki, *Ocherki po istorii matematiki* (Moscow, 1963), pp. 79–80.
20. P.A.M. Dirak [Dirac], *Vospominaniia o neobychnoi epokhe* (Moscow, 1990), p. 19; for more details on this point see J. Mehra and H. Reichenberg, *The Historical Development of Quantum Theory* (New York, 1982) vol. 4, pp. 129–30.
21. See on this point R.A. Aronov, "Kvantovyi paradoks Zenona," *Priroda*, 1992, no. 12, p. 76. I skirt the question how all these changes can affect other quantities. The change from the commutative to the noncommutative paradigm, for example, can lead to a more precise determination of what constitutes the basis of the idea of probability. In the domain of classical physics this is the independence of the manifestation of the properties of one element of reality from the manifestation of the properties of another element, while in the quantum domain the manifestations of the properties of the corresponding elements of reality (described by means of the noncommutative component of the mathematical apparatus of the theory) do not exist independently of each other (see R.A. Aronov, "Nekommutativnaia paradigma i real'nost'," in *Logika, metodologiya, filosofiia nauki* (Moscow/Obinsk, 1995), vol. 8, p. 14).

22. Quoted in Polak, *Uil'iam Gamil'ton*, p. 53.
23. Ibid.
24. See R.A. Aronov, "O nekotorykh rezul'tatakh postizheniia vremeni," *Voprosy filosofii*, 1994, no. 5, p. 195; R.A. Aronov, S.E. Kamenetskii and N.V. Sharonova, "Mnenie o mnenii," *Fizika v shkole*, 1995, no. 4, p. 35.
25. See A.A. Logunov and M.A. Mestvirishvili, *Osnovy relativistskoi teorii gravitatsii* (Moscow, 1985); A.A. Logunov, "Reikhenbakh, Einshtein i sovremennye predstavleniia o prostranstve i vremeni," in G. Reikhenbakh (H. Reichenbach), *Filosofia prostranstva i vremeni* (Moscow, 1985), p. 314; idem, "Osnovnye printsipy relativistskoi teorii gravitatsii," *Teoreticheskaia i matematicheskaia fizika*, 1989, vol. 80, no. 2, p. 165; idem, "Teoriia klassicheskogo gravitatsionnogo polia," *Uspekhi fizicheskikh nauk*, 1995, vol. 165, no. 2, p. 187.
26. See R.A. Aronov, "Dva podkhoda k otsenke filosofskikh vzgliadov A. Puan-kare," in *Dialekticheskii materializm i filosofskie voprosy estestvoznaniia* (Moscow, 1985), p. 3.
27. A. Einshtein [Einstein], *Sobranie nauchnykh trudov* (Moscow, 1967), vol. 4, p. 280. In this connection see R.A. Aronov, B.M. Bolotovskii, and N.V. Mitskevich, "Elementy materializma i dialektiki v formirovanii filosofskikh vzgliadov A. Einshteina," *Voprosy filosofii*, 1979, no. 11, p. 56.
28. See R.A. Aronov, "Dve tochki zreniia na prirodu fizicheskoi real'nosti," *Filosofskie nauki*, 1991, no. 6, p. 178; R.A. Aronov and V.M. Shemiakinskii, "O dvukh podkhodakh k probleme vzaimootnosheniia geometrii i fiziki," in *Dialekticheskii materializm i filosofskie voprosy estestvoznaniia* (Moscow, 1991), p. 28; R.A. Aronov, "Einshtein i fizicheskaia real'nost'," *Filosofskie nauki*, 1995, nos. 2-4, p. 63.
29. A. Puankare [Poincaré], *O nauke* (Moscow, 1983), p. 62.
30. See R.A. Aronov and V.V. Terent'ev, "Sushchestvuiut' li nefizicheskie formy prostranstva i vremeni?" *Voprosy filosofii*, 1988, no. 1, p. 78.
31. See R.A. Aronov, "Reikhenbakh, Einshtein i sovremennye predstavleniia o prostranstve i vremeni," in *Dialekticheskii materializm i filosofskie voprosy estestvoznaniia* (Moscow, 1987), p. 3; R.A. Aronov and V.N. Kniazev, "K probleme vzaimootnosheniia geometrii i fiziki," in *Dialekticheskii materializm i filosofskie voprosy estestvoznaniia* (Moscow, 1991), p. 28.
32. Logunov and Mestvirishvili, *Osnovy relativistskoi teorii gravitatsii*, p. 8.
33. Ibid., p. 6.
34. See R.A. Aronov, "O filosofskoi otsenke nauchnogo naslediiia Einshteina," *Uspekhi fizicheskikh nauk*, 1980, vol. 132, no. 3, p. 589; idem, "Prostranstvo i vremia i prostranstvo—vremia," in *Problemy i metodologii nauchnogo poznaniia* (Moscow, 1974), p. 267.
35. See E. Neter, "Invariantnye variatsionnye zadachi," in *Variatsionnye printsipy mekhaniki* (Moscow, 1959), p. 611.
36. See R.A. Aronov, "K voprosu o sviazi prostranstva i vremeni s dvizheniem materii," in *Nekotorye voprosy filosofii* (Kishinev, 1959), no. 1, p. 48; R.A. Aronov and V.A. Ugarov, "Prostranstvo, vremia i zakony sokhraneniia," *Priroda*, 1978, no. 10, p. 99; idem, "Teorema Neter i sviaz' zakonov sokhraneniia so svoistvami simmetrii prostranstva i vremeni," in *Filosofskie voprosy sovremennogo estestvoznaniia* (Moscow, 1978), p. 3.
37. See Ia.B. Zel'dovich and L.P. Grishchuk, "Tiagotenie, obshchaia teoriia

otnositel'nosti i al'ternativnye teorii," *Uspekhi fizicheskikh nauk*, 1986, vol. 149, no. 4, p. 695; L.P. Grishchuk, "Obshchaia teoriia otnositel'nosti—znakomaia i neznakomaia," *Uspekhi fizicheskikh nauk*, 1990, vol. 186, no. 8, p. 147.

38. Zel'dovich and Grishchuk, "Tiagotenie, obshchaia teoriia otnositel'nosti i al'ternativnye teorii," pp. 695-696.

39. A.D. Sakharov, "Posleslovie," *Priroda*, 1988, no. 4, p. 26.

40. Ibid. In this connection see R.A. Aronov, "OTO i fizika mikromira," in *Klassicheskaiia i kvantovaia teoriia gravitatsii* (Minsk, 1976), p. 55.

41. A. Nysanbaev, "Rol' printsipa protivorechiia v razvitiia matematiki," in *Metodologicheskie problemy razvitiia i primeneniia matematiki* (Moscow, 1985), p. 14.

42. On this see R.A. Aronov, "Pifagoreiskii sindrom v sovremennoi fizike," in *Tezisy dokladov i vystuplenii na X Vsesoiuznoi konferentsii po logike, metodologii i filosofii nauki (seksii 6-7)* (Minsk, 1990), p. 3; idem, "M. Klain. Matematika. Utrata opredelennosti," *Voprosy filosofii*, 1986, no. 5, p. 170; idem, "Filosofskie osnovaniia matematiki i sindrom Khlodviga," *Priroda*, 1992, no. 3, p. 87.

43. P.S. Aleksandrov, "O novykh techeniakh matematicheskoi mysli, vznikshykh v sviazi s teoriei mnozhestv," in *Sbornik statei po filosofii matematiki* (Moscow, 1936), p. 14.

44. A.N. Kolmogorov, "Sovremennaiia matematika," in *Sbornik statei po filosofii matematiki* (Moscow, 1936), p. 10.

45. V.I. Arnold'd, "Matematika s chelovecheskim litsom," *Priroda*, 1988, no. 3, p. 117; see in this connection idem, "IaB i matematika," in *Znakomyi neznakomyi Zel'dovich* (Moscow, 1993), p. 212; A.D. Myshkis, "'Ochelovechivanie' matematiki," in *ibid.*, p. 219.

46. Ibid., p. 118.

47. Quoted in V.A. Fabrikant, "Zametki starogo pedagoga," *Fizika v shkole*, 1988, no. 2, p. 12.

48. On the point that Logunov's relativistic theory of gravitation is no exception to this rule, see Grishchuk, "Obshchaia teoriia otnositel'nosti—znakomaia i neznakomaia"; D.E. Burlankov, "Ob'iasniaet li RTG gravitatsionnye efekty?" *Iadernaia fizika*, 1989, vol. 50, no. 1, p. 278; R.A. Aronov and V.M. Shemiakinskii, "K voprosu o paradoksal'nosti programmy geometrizatsii fiziki," in *Filosofiia, chelovek, nauka* (Moscow, 1992), p. 101.

49. E. Zamiatin, *Izbrannoe* (Moscow, 1989), p. 332.

50. D. Hilbert [Hilbert], *Osnovaniia geometrii* (Moscow/Leningrad, 1948), pp. 365–66; see in this connection R.A. Aronov, "K probleme vezdesushchnosti soznaniia," *Voprosy filosofii*, 1995, no. 3, p. 182.

51. Puankare, *O nauke*, p. 302.

52. Einstein, *Sobranie nauchnykh trudov*, vol. 4, pp. 492–93.

53. S. Marshak, *Sochineniia v 4 tomakh* (Moscow, 1959), vol. 3, p. 138; see in this connection R.A. Aronov and V.M. Shemiakinskii, "Adaptatsiia fiziki v sisteme kul'tury," in *Fizika v sisteme kul'tury* (Moscow, 1996), p. 37.

