



Pythagoras' celestial spheres in the context of a simple model for quantization of planetary orbits [☆]

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Abstract

In the present article we attempt to search for a correlation between Pythagoras and Kepler's ideas on harmony of the celestial spheres through simple quantization procedure to describe planetary orbits in our solar system. It is reasoned that starting from a Bohr-like atomic model, planetary mean radii and periods of revolution can be obtained from a set of small integers and just one input parameter given by the mean planetary radius of Mercury. It is also shown that the mean planetary distances can be calculated with the help of a Schrödinger-type equation considering the flatness of the solar system. An attempt to obtain planetary radii using both gravitational and electrostatic approaches linked by Newton's dimensionless constant of gravity is presented.

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1. Introduction

The search for a way to represent the quintessence of physical sciences began in earnest with Pythagoras and his school, in their attempts to find mathematical relationships that might describe natural phenomena. The first of these attempts consisted of establishing a relationship between simple numerical proportions and musical intervals. Simple ratios between integers were used effectively to construct the entire musical scale produced by a string instrument.

The Pythagoreans took this notion further, extending ideas on musical harmony to the movement of the planets. The same ratios between the integers that rule musical notes might similarly rule the distances between the planets. In the geocentric system of ancient Greece, the Sun and the planets, each in their own orbit, might be described as a musical instrument generating a melodious set of divine sounds expressed by the harmony of the celestial spheres [1].

In 1618, more than 2000 years later, the brilliant German astronomer Johannes Kepler resurrected the Pythagorean idea of harmony in his book "Harmonice Mundi" (The Harmony of the World). By combining the orbital velocities of the different planets in a heliocentric system, he inferred that the ratio between these speeds could be compared to the numbers obtained in musical scales.

Slightly less than 300 years after that, the Danish physicist Niels Bohr proposed a model aimed at finding a relationship between atomic structure and the discrete nature of electromagnetic radiation emitted by atoms. This model, which was successfully applied to the hydrogen atom, consisted of a positive nucleus and of a negative electron orbiting

[☆] In honor of Professor Mohamed El Naschie on the occasion of his 60th birthday.
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around it in circular paths, like a solar system in miniature, where only certain orbits are permitted. The intrinsically discontinuous nature of the quantum world was imposed by the condition that the angular momentum of the electron (L) be described as integer multiples of Planck's constant divided by the factor 2π

$$L = n \frac{h}{2\pi}, \quad \text{where } n = 1, 2, 3 \dots$$

each orbit is then labelled with the integer n , starting with $n = 1$ for the fundamental state.

Therefore, the grandiose idea of the harmony of Pythagorians celestial spheres seems to be equally present in the beauty of the dance of electrons and the way this manifests itself in the magnificent existence of spectral lines characteristic for each element.

In accordance with the E infinity theory as developed by the Egyptian physicist Mohamed El Naschie, the elementary particles of nature arise with the same elegance as a harmonious set of musical notes [2–6]. All of the investigations carried out by this scientist seem to perceive physics through the beauty of the arts and poetry. Newton's dimensionless constant of gravity, fundamental to the theory [7,8], appears to play just as important a role in the description of planetary orbits [9].

The present article searches for a correlation between this set of historical facts and a model to describe planetary orbits in our solar system, starting from a simple quantization procedure analogous to that of Niels Bohr, taken in the context of a broader discussion than that presented in our previous works [9,10]. The central idea that led to the formulation of the model is based on the supposition that an observer standing on the frontiers of the universe and trying to obtain a description of our solar system, would establish a system of units for his measurements in such a way that the Sun and the set of bodies gravitating around it would have the dimensions of an atomic system [10].

We will show that the planetary orbits can be obtained equally well starting from the solutions to an equation that is similar to Schrödinger's wave equation.

2. A Niels Bohr-like atomic model

As presented in Ref. [9,10] from a Bohr-like atomic model the mean planetary radius r is linked to the integer n through the following equation:

$$r = n^2 g^{*2} / 4\pi^2 G M_s m_p^2, \quad (1)$$

where M_s is the Sun's mass and m_p is an average planetary mass taken from all known orbits; G is Newton's constant of gravity and g^* a Planck-like constant. The period of revolution T for a planet can then be determined from

$$T = n^3 [(g^{*3}) / (4\pi G M_s^2 m_p^2)]. \quad (2)$$

Kepler's second and third laws can then be easily derived from Eqs. (1) and (2). These equations show that the ratio between the mean radii of any two successive orbits is given by

$$r_i / r_j = n_i^2 / n_j^2 \quad (3)$$

and for the periods,

$$T_i / T_j = n_i^3 / n_j^3. \quad (4)$$

Eqs. (3) and (4) suggest the following principle:

The ratio between the mean planetary distances obeys the ratio between the square of the integers associated to the respective orbits, and for the periods of revolution, the ratio between the cubes of the same numbers.

Thus, for successive orbits:

$$r_{i+1} = r_i [(n_{i+1}) / (n_i)]^2$$

and

$$T_{i+1} = T_i [(n_{i+1}) / (n_i)]^3.$$

However in order for the calculated orbits and those observed in the solar system to match, it is necessary to introduce a new set of numbers relating to the integer n . Such numbers refer to some orbits of the planets nearest the Sun with low values of angular momentum. These orbits were empirically obtained from a set of numbers m , associated with the value of the integer n corresponding to a given orbit, ranging from zero to n . In order to illustrate this relationship,

consider $n = 2$ with $m = 0$, $m = 1$ and $m = 2$; starting from the pair $(n = 2; m = 2)$, the orbits $(n = 2; m = 1)$ and $(n = 2; m = 0)$ can then be determined. For these orbits a relationship between the sum of the squares of n and m is what should be considered. If we consider, for all the orbits, that the mean distance is given by

$$r = [(n^2 + m^2)/2][(g^{*2})/(4\pi^2 GM_s m_p^2)] \quad (5)$$

and $m = n$, with the exception of the orbit of Venus ($n = 2; m = 0$) and that of Earth ($n = 2; m = 1$), which were obtained from the integer associated with the orbit of Mars ($n = 2$), just as the orbits of the inner limit ($n = 3; m = 0$) and the outer limit ($n = 3; m = 1$) of the Hungarias asteroids and the inner ring of the main asteroid belt ($n = 3; m = 2$) were obtained from that associated with the outer ring of this belt ($n = 3$), then the original expression determined for the calculation of distances remains unchanged. Therefore Eq. (5) can be written as

$$r = [(n^2 + m^2)/2]r_0, \quad (6)$$

where $r_0 = 0.387$ AU; which means that all planetary mean distance can be calculated from Mercury's radius and taking into account the pairs n and m associated to each orbit. Table 1 shows the set of planetary radii calculated to the third decimal point. Such theoretical values are in very satisfactory agreement with the observed mean planetary radius and period values [9,10]. The asteroids Vesta and Camilla, presented in Table 1, can be found in the inner and outer rings of the main asteroid belt that lies between the orbits of Mars and Jupiter, and which contains approximately 2000 bodies in orbit around the Sun. The asteroid Chiron was discovered in 1978 by the American astronomer Charles T. Kowal.

The mean calculated distance of 24.77 AU is associated with the recently discovered asteroids known as 1993HA2 and 1995DW2 [11].

Unfortunately the belt that corresponds to the Hungarias asteroids was not taken into account in our previous published studies [9,10]. These asteroids are located at a mean observed radius between 1.78 AU and 2.0 AU from the Sun. In Table 1, HIL and HOL denote the inner and outer limits of Hungarias asteroids, respectively.

Recently, other authors have discussed the possibility of describing planetary orbits based on an approach using quantum mechanics [12–22]. Ref. [23] gives a brief summary of these theoretical studies.

It is worth noting that at the time of the present author's first work on this subject [10], we did not yet know of the existence of bodies at a mean distance of 24.77 AU from the centre of the Sun. Thus, the empty state where $n = 8$ was later identified in the recent discovery, between the orbits of Uranus and Neptune [11], of asteroids 1993HA2 (named Nessus) and 1995DW2 (named Hylonomé), whose mean distances are 24.76 AU and 24.17 AU, respectively. Other considerations can be noted:

In accordance with the present model, the ratio between the orbital velocities is given by the inverse of the ratio between the integers that are used to label each orbit [9,10].

Jupiter's theoretical radius (6.2 AU) differs by approximately 19% from the observed value (5.2 AU). This can be attributed to the fact that Jupiter contributes 80% of the system's total planetary mass, so there is a very strong gravitational effect between this planet and the Sun.

Table 1
Mean planetary radii calculated from Eq. (6)

Position	$r = [(n^2 + m^2)/2]r_0$	Mean distance in AU	Planet/asteroid
$n = 1; m = 1$	$r_1 = r_0 = 0.387$	0.387	Mercury
$n = 2; m = 0$	$r_2 = [(2^2 + 0^2)/2]r_0$	0.774	Venus
$n = 2; m = 1$	$r_3 = [(2^2 + 1^2)/2]r_0$	0.967	Earth
$n = 2; m = 2$	$r_4 = [(2^2 + 2^2)/2]r_0$	1.548	Mars
$n = 3; m = 0$	$r_5 = [(3^2 + 0^2)/2]r_0$	1.741	HIL
$n = 3; m = 1$	$r_6 = [(3^2 + 1^2)/2]r_0$	1.935	HOL
$n = 3; m = 2$	$r_7 = [(3^2 + 2^2)/2]r_0$	2.515	Vesta
$n = 3; m = 3$	$r_8 = [(3^2 + 3^2)/2]r_0$	3.483	Camilla
$n = 4; m = 4$	$r_9 = [(4^2 + 4^2)/2]r_0$	6.192	Jupiter
$n = 5; m = 5$	$r_{10} = [(5^2 + 5^2)/2]r_0$	9.675	Saturn
$n = 6; m = 6$	$r_{11} = [(6^2 + 6^2)/2]r_0$	13.932	Chiron
$n = 7; m = 7$	$r_{12} = [(7^2 + 7^2)/2]r_0$	18.963	Uranus
$n = 8; m = 8$	$r_{13} = [(8^2 + 8^2)/2]r_0$	24.768	Nessus, Hylonomé
$n = 9; m = 9$	$r_{14} = [(9^2 + 9^2)/2]r_0$	31.347	Neptune
$n = 10; m = 10$	$r_{15} = [(10^2 + 10^2)/2]r_0$	38.700	Pluto

This model presupposes a single empty state, found for the pair ($n = 1; m = 0$), which would correspond to the existence of bodies orbiting around the Sun every 28 days at a mean distance of 0.18 AU.

The theoretical results presented here for all the radii do not depend on the planetary mass and are in agreement both with the distances referring to asteroids and also with those of larger planets such as Neptune, Saturn or Uranus. Could such a statement be related to discretizing or quantizing the geometry inherent in the localization of planetary orbits?

In general, results like these draw our attention to those found for the mean values of electron distances associated with the atomic orbitals, the denomination given to those wave functions that are solutions to Schrödinger's equation for the hydrogen atom or other systems made up of a positive nucleus and a single electron. Therefore, these results suggest that the mean planetary distances can be associated with functions obtained from an equation that is similar to Schrödinger's bi-dimensional equation, in order to describe the solar system's geometry in the framework of a flat disc model.

3. A Schrödinger-type equation

A Schrödinger-type equation in a disc arises when we employ quantum mechanics to describe a planar system involving an attractive central field, with a body of mass m moving around a body of mass M under an interaction potential $V(r)$

$$-\frac{(\hbar^2)^2}{2\mu} \left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} \right) + V(r)\psi = E_s \psi. \quad (7)$$

The detailed treatment involving the resolution of this equation, which was considered as an equation of diffusion, and the interpretation of its solutions, was carried out in Ref. [23]. At this point it is worth reiterating how valuable and fundamental was the support of Dr. Liliane Maia from the Department of Mathematics at the University of Brasília in the numerical resolution of this equation, using the Maple programme, as was that of Dr. Saulo Carneiro from the Institute of Physics at the Federal University of Bahia in the improvements of the discussions on the physical aspects of the obtained results.

In the above equation, given that the potential V is a function only of the radial variable, we can resolve the equation by separating variables into the radial and angular parts

$$\psi(r, \theta) = f(r)\Theta(\theta). \quad (8)$$

Similarly to the resolution of Schrödinger's equation for the atomic case, Eq. (7) has ψ_{nl} solutions, from which

$$n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \dots,$$

$$l = 0, 1, 2, \dots, n - \frac{1}{2}.$$

As in the case of atomic wave functions the mean electronic radius is obtained by

$$\bar{r}_{nl} = \int_0^\infty \int_0^{2\pi} (\psi_{nl}^* r \psi_{nl}) r dr d\theta.$$

Table 2 shows the mean planetary distances calculated from the states associated with the pairs (n, l), indicating a high rate of coincidence between the majority of distances observed for extra-solar planets and the radii corresponding to the

Table 2
Mean radii as the solutions of a Schrödinger-like equation

Position	Pairs (n, l)	Calculated mean radius (AU)
Fundamental radius	(1/2, 0)	0.055
Mercury	(3/2, 0) (3/2, 1)	0.387; 0.332
Venus	(5/2, 2)	0.83
Earth	(5/2, 0); (5/2, 1)	1.05; 0.995
Mars	(7/2, 3)	1.54
Hungarias	(7/2, 0); (7/2, 1); (7/2, 2)	2.04; 1.99; 1.82
Asteroid belt	(9/2, 0); (9/2, 1); (9/2, 2); (9/2, 3); (9/2, 4)	3.37; 3.32; 3.15; 2.88; 2.49

fundamental state ($r = 0.055$ AU) and those associated with Mercury until the asteroid belt [24]. The fundamental radius is not found in the framework of the formulation of Bohr's model, possibly due to the three-dimensional nature of our solar system.

For higher n values, a large number of states is found [23,25]. However, for extra-solar systems, there were no orbits found with a radius greater than that of Jupiter's. In the case of our solar system, as is pointed out by Nottale et al. [15] to explain the circularity of the orbits, a large number of states occupying the same region in space, most of them presenting significant eccentricities, leads to strong chaos and to the crossing of orbits, which on a large time scale would lead to the condensation of states on the observed, approximately circular, orbits. Thus, for $n > 9/2$, we only consider the states that present rotational symmetry, that is, with $l = 0$.

From this, if we assign to (n, l, r) the resulting numbers n, l as obtained above and the corresponding mean planetary radius r_{nl} (in AU), then the states $(11/2, 0, 5.03)$ and $(15/2, 0, 9.34)$ can be associated with the orbits of Jupiter and Saturn, respectively. The states $(17/2, 0, 12.0)$ and $(19/2, 0, 15.0)$ correspond to the value of the mean radius of Chiron's orbit, at 13.7 AU from the Sun. The state $(21/2, 0, 18.3)$ can be associated with the orbit of Uranus, while the state $(25/2, 0, 25.9)$ presents a mean distance close to the value of the orbital radius for the recently discovered asteroids Nessus and Hylonome. The next state, $(27/2, 0, 30.2)$, clearly corresponds to the orbit of Neptune and, last of all, the state $(31/2, 0, 39.9)$ can be easily linked to that of Pluto. One of the three empty states, mentioned in Ref. [23], relating to the trio $(23/2, 0, 21.9)$ can be associated to the orbit of the asteroid Pholus (1992 AD), which executes its orbit with a mean radius of 20.35 AU, between the orbits of Saturn and Uranus, every 91 years.

Similarly to what happens with the Niels Bohr model, only the radius of Mercury needs the introduction of an input parameter, in order to obtain remaining radii, suggesting that planetary or asteroid localization is not dependent on planetary mass throughout the solar system.

4. Planetary orbits from Newton's dimensionless constant of gravity

As Mohamed El Naschie has reported in his investigations [7,8], Newton's dimensionless constant of gravity) $\bar{\alpha}_G$ can be defined using descriptive set theory, in a very beautiful way, as illustrated below

$$\bar{\alpha}_G = \frac{\hbar C}{G m_p^2} \cong 1.7(10)^{38} = 2^{\bar{\alpha}_{ew}-1} = 2^{128-1} = 2^{127},$$

where C is identified by "physical" space-time and $\bar{\alpha}_{ew}$ is the weak electron coupling constant.

In accordance with El Naschie's theory $\varepsilon^{(\infty)}$, Newton's dimensionless constant $\bar{\alpha}_G$ can also be derived from a complex formula which involves various exponents of scale and fundamental coupling constants [26], as follows:

$$\bar{\alpha}_G = (2)^{(\bar{\alpha}_0-10)} \left(1 - \frac{1}{\alpha_{gs}} \right)^{\bar{\alpha}_g},$$

where $\bar{\alpha}_0 = 137.082039325$, which corresponds to the inverse of the electromagnetic dimensionless fine structure constant, $\alpha_{gs} = 26.18033989$, which corresponds to the supersymmetric coupling of quantum gravity, and finally $\bar{\alpha}_g = 42.36067977$ is the coupling constant of non-supersymmetric unification of all fundamental forces.

As shown in Ref. [9], Newton's dimensionless constant can equally well be written as

$$\bar{\alpha}_G = (\bar{\alpha}_{eg})(\bar{\alpha}_0),$$

where $\bar{\alpha}_{eg}$ represents the ratio between the intensities of the electrostatic and gravitational forces of a proton/anti-proton system in interaction,

$$F_e/F_g = (Ke^2)/(Gm_p^2) = 1.2361602(10)^{36}$$

and $\bar{\alpha}_0$ is the inverse of the fine structure constant,

$$\bar{\alpha}_0 = \frac{1}{\alpha}, \quad \text{where } \alpha = \frac{Ke^2}{\hbar c} = \frac{1}{137.036},$$

these calculations result in

$$\bar{\alpha}_G = (1.6939844)(10)^{38},$$

which is in accordance with the experimental value of this dimensionless constant [26],

$$\bar{\alpha}_G = (1.693)(10)^{38}.$$

In the case of the present investigation, such a constant takes the form of

$$\bar{\alpha}_G = \left(\frac{Ke_p^2}{Gm_p^2} \right) \left(\frac{\hbar c}{Ke_p^2} \right) = \frac{\hbar c}{Gm_p^2},$$

where $\hbar = h/2\pi$; c = speed of light in vacuum; m_p = proton mass; G = Newton's gravitational constant.

The same value for $\bar{\alpha}_{eg}$ can also be obtained if we take the value $M_P = 34.38EM$ to the mean planetary mass associated with each asteroid/planet and a mean asteroid/planetary charge of $\bar{e}_p = 1.976(10)^{34} C$, in the context of “electrostatic interactions” between the Sun and the planets, as in atomic systems [9].

In Ref. [9] it was likewise shown that a constant similar to that of Planck, considering “electrostatic interactions” between the Sun and the planets, can be determined from the relation

$$h^* = (\bar{\alpha}_{eg})^{1/2} g^*.$$

The constant h^* can then be determined and, as reported earlier, $g^* = 0.8462(10)^2(AU)^2(EM)(EP)^{-1}$ for the planetary system, which leads us to the value $h^* = 94.36(10)^{18}(AU)^2(EM)(EP)^{-1}$ in a unit system named AUC (astronomic unit of calculation). Certainly, the correct values of g^* and h^* must be improved since they will depend upon on further discovered bodies in orbit around the Sun that not yet been discovered, mainly those beyond Pluto.

In this way, the mean planetary distances can be theoretically calculated starting from an atomic Bohr model, considering the “electrostatic” hypothesis from a similar equation in order to determine the radii of the electron orbits given by

$$r_n = \frac{n^2 h^{*2}}{4\pi^2 K^* Z M_P \bar{e}_p^2}.$$

If such an equation is applied for the integer $n = 1$, the result gives us the radius of Mercury, $r_0 = 0.39 AU$, which corresponds to the mean distance of this planet from the Sun. In this way, the same procedure that was employed to obtain mean planetary distances through a gravitational approach, as shown in Table 1, can also be used to obtain all the planetary radii based on the “electrostatic” hypothesis applied to the solar system.

5. Discussions and conclusions

Taking into account our assumption of a planar nature of the solar system, the resolution of a Schrödinger-like equation has shown that only one input parameter—given by the mean Sun–Mercury distance—is needed in order to determine the other mean radii. One interesting result refers to the prediction of a fundamental radius given by $r = 0.05 AU$, which was also presupposed in studies done by Nottale et al. [14,16] and Agnese and Festa [17]. It is worth mentioning that many planets in extra-solar systems have been discovered in this position [24].

This fundamental radius does not appear as one of the radii obtained from applying Bohr-like rules of quantization to the solar system, that is by taking into account the three-dimensional nature of the solar system [9]. Nevertheless, this model was useful in establishing all the planetary orbits and also in predicting the existence of other celestial bodies with mean radii situated between the orbits of Uranus and Neptune. Likewise, only the radius of Mercury is necessary as an input parameter to obtain the other orbits. This radius was theoretically calculated from a single constant analogous to that of Planck and for $n = 1$; the other planetary distances can be determined from a set of small integers.

The so-called 10th planet, recently discovered and announced in July 2005, designated 2003 UB313, executes its orbit at maximum distance 97 AU and minimum distance 38 AU from the Sun, every 560 years. The observed mean distance is then 67.5 AU and would be associated to the integer $n = 13$ that predicts orbits approximately 66 AU from the Sun. Therefore two other orbits are predicted from the model, associated to integer $n = 11$ at a mean radius of 47.2 AU and $n = 12$ at a mean radius of 56.2 AU.

Table 1 invokes Pythagoras' harmony of the celestial spheres. A clear connection between the integers and the mean planetary radii and periods can be seen. The number 10, the last of the “principal” number n , associated with the last planetary orbit, that of Pluto, was sacred to the Pythagoreans. It is striking how the expressions found for the radii and periods in connection with the integers, assuming circular orbits, confirms Kepler's second and third laws. Further investigations, however, should seek to find the foundations that establish the nature of semi-integers, as determined in an equation analogous to that of Schrödinger for a planar disc, and establish the integers for a model analogous to that of Bohr. That means it would be interesting to examine the physical foundations that rule the three-dimensional or bi-dimensional nature of a planetary orbit.

The present work raises another curious question when we consider a model like Bohr's. This is the fact that the orbit that corresponds to the inner ring of the main asteroid belt is determined when we take into account the pair $(n = 3; m = 2)$ based on the orbit that corresponds to the outer ring of this belt $(n = 3)$, as well as the orbits associated with the outer $(n = 3; m = 1)$ and inner $(n = 3; m = 0)$ limits of the Hungarias asteroids. Analogously, the orbit of Earth described by the pair $(n = 2; m = 1)$ and that of Venus $(n = 2; m = 0)$ are derived from the orbit of Mars $(n = 2)$.

Considering the mean planetary distances obtained from the functions that are solutions to an equation like Schrödinger's, it is interesting to observe that the set of numbers l associated with the number $n = 7/2$ includes the inner and outer limits of Hungarias asteroids and that the set of numbers associated with the number $n = 9/2$ includes precisely the main asteroid belt at its upper and lower limits. For the planet Earth the pairs $(n = 5/2; l = 0)$ and $(n = 5/2; l = 1)$, and for Venus the pair $(n = 5/2; l = 2)$ are found. Such evidence, taken in the context of the present model and its approaches, might lead one to pose this question: could the planets Venus and Earth (or even Venus, Earth and Mars) have originated from an asteroid belt in the primordial formation of our solar system? It is also worth questioning if the four states corresponding to the inner and outer ring of the main asteroid belt and inner and outer limits of Hungarias asteroids $(n = 3; m = 0, 1, 2, 3, 4)$ would not have been associated with a single planet if these belts were not localized so close to Jupiter's orbit.

One aspect of some relevance, which is indicated as much by the planar approach as by the three-dimensional one, is that the results point to there being no dependence between the mean radii of the orbits and the mass of the bodies, whether they be heavy or light planets or indeed asteroids.

These theoretical investigations have continually tried to show, by using a model similar to Bohr's atomic model, that Newton's dimensionless gravitational constant $\tilde{\alpha}_G$, given by El Naschie employing descriptive set theory [7,8], can be deduced from a combination of the ratio of the electrostatic and gravitational force between a proton and an anti-proton and the fine structure constant [9]. Such a result, associated with a quantum rescaling procedure, is shown to be important as it infers a description of the planetary orbits based on an "electrostatic" approach.

In El Naschie's descriptive set theory it can be seen that a very precise physical description of the elementary particles of nature is closely linked to the dimensionless fine structure constant. This theory also allows us to show that the ratio of the squares of Planck's mass to the proton mass leads to Newton's dimensionless constant of gravity [8]. In the current work, Newton's dimensionless constant connects the ratio of the squares of the constants g^* and h^* by means of the fine structure constant. The constants g^* or h^* allow us to determine the mean radius of Mercury, the only input parameter needed to obtain the whole set of mean radii of planetary orbits in the solar system. In other words, if it is possible to imagine an observer standing on the frontiers of the universe, it is indifferent to this observer if he uses the gravitational or the "electrostatic" Niels Bohr-like atomic approaches to deal with calculations on planetary orbits of the solar system. It would be interesting to question the origin of the physical foundations that connect these facts. That is, the foundations that govern the results need to be examined further, given that the current investigations were carried out in the scope of simple atomic models. Further studies should be carried out so as to provide stricter physical foundations that in turn aim to establish a more solid base for the many aspects found during the use of the models presented here.

The present author is a graduate in physics, took his master's degree in elementary particle physics and his doctorate in quantum chemistry. The ideas presented in this study have come out of many years teaching both old and modern atomic theory to his students and of a lifelong passion for astrophysics. This article, therefore, ends with an appeal to the modern theories that deal with questions involving chaos, fractals and complexity theory to treat the present subject in a way similar to the investigations already performed by Nottale et al. [13–16] and Agnese and Festa [17–19]. It is worth mentioning intriguing questions recently arisen by El Naschie on different interpretations of our classical concept of space–time geometry and topology that would be in line with the development of quantum mechanics and general relativity [27,28]. Certainly, these recent approaches may bring greater understanding and better interpretations of the results presented throughout this work.

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