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HUTCHMAN, C.A.
2000

PYTHAGOREANISM

PYTHAGORAS IS THE MOST WIDELY KNOWN of the Presocratic philosophers, yet modern scholars often view Pythagoras and Pythagoreanism with considerable suspicion. Instead, the history of Presocratic philosophy is told in terms of the great figure of Parmenides, his rejection of earlier accounts of reality, and the response to his challenge by figures such as Anaxagoras and Democritus. This ambivalent attitude toward Pythagoras is mirrored in the ancient evidence as well. Plato and Aristotle, writing one hundred to one hundred fifty years after his death, hardly mention Pythagoras himself. Aristotle is famous for reviewing the work of his predecessors in each field of philosophy before presenting his own views. Yet, although he discusses at some length the Pythagoreans of the 5th century, who were active fifty years after Pythagoras's death, he never in his extant works mentions Pythagoras himself as an important figure in any branch of philosophy. Aristotle did write a treatise on the Pythagoreans, surviving only in fragmentary form, in which he deals directly with Pythagoras himself, but only as a wonder-working religious teacher. Plato likewise shows influence from 5th- and 4th-century Pythagoreanism but mentions Pythagoras himself only once, as a famous teacher who left a way of life for his followers. However, if we jump six hundred years to the Neoplatonist Iamblichus writing in the 2nd century C.E., we find Pythagoras presented as a semidivine figure who brought all true philosophy from the gods to men and from whom Plato and Aristotle stole all their best ideas. So by 300 C.E. Pythagoras is viewed in some circles as the philosopher par excellence, while in the 4th century B.C.E. Plato and Aristotle could discuss the important philosophical issues of the day with no reference to Pythagoras.

Recent scholarship (most notably the work of Walter Burkert) has made enormous progress in unraveling the complex tradition about Pythagoras and Pythagoreanism, but inevitably much remains uncertain and controversial. Before giving a detailed assessment of Pythagoras and specific Pythagoreans, it is necessary to give a brief overview of the whole Pythagorean tradition, to identify the unique problems in arriving at an accurate portrayal of Pythagoras and his philosophy. Pythagoras spent his early years on the island of Samos off the coast of Asia Minor, probably around the age of forty (ca. 530 B.C.E.), he emigrated to Croton in southern Italy, and he spent the rest of his life there, dying around 490 in the town of Metapontum. One of the crucial things to remember about him is that he wrote nothing. It is true that by the

2nd and 3rd centuries C.E. books were attributed to him, and the seventy-one lines of the *Golden Verses* were confidently accepted as his on into the Renaissance, but these forgeries were part of the phenomenal growth of the Pythagoras legend. The first book written in the Pythagorean tradition was by Philolaus of Croton (ca. 470–390 B.C.E.). Fragments of this book survive, and it seems to have been the primary basis for Aristotle's reports on the Pythagoreans. The other important name in early Pythagoreanism was Archytas (fl. 400–350 B.C.E.), who was a contemporary of Plato and who is connected to serious mathematical work, particularly in relation to music theory.

Aristotle is careful to distinguish the Pythagoreans from Pythagoras himself and refers to them as the "so-called Pythagoreans," probably to show that this is the name commonly given them but to express doubts about the exact connection between their thought and Pythagoras himself. However, among the immediate successors of Plato in his school the Academy, Speusippus and Xenocrates, an important new attitude toward Pythagoras arises. Many of Plato's most mature ideas, and particularly his positing of the One and the indefinite dyad as the basic principles of reality, come to be regarded as simply developments of ideas that Pythagoras had originated. It is not completely clear why Plato's followers should want to see his philosophy as simply a development of Pythagoras's thought, but it seems to be part of a mind set that sees philosophy as a divine revelation given to a chosen man (Pythagoras). Such a view, of course, gives the authority of divine revelation to the philosophy of Plato insofar as it unfolds ideas of Pythagoras.

This attitude toward Pythagoras comes to dominate the entire later tradition, and Aristotle's distinctions both between Pythagoras and later Pythagoreans and between the Pythagoreans and Plato are ignored. One important result of this dominant attitude toward Pythagoras is that by the 2nd century C.E. a very large collection of writings had been forged in the name of Pythagoras and early Pythagoreans. These pseudographs are full of Platonic and Aristotelian ideas and terms and are clearly meant to provide the "early" texts from which Aristotle and Plato supposedly derived their whole philosophy. A few short treatises survive intact; the most famous is the purported original of Plato's dialogue *Timaeus*, supposedly written by Timaeus of Locrus. It is largely a précis of Plato's *Timaeus*, and like most of the pseudographs it adheres very closely to the doctrine of its model while possessing little of its literary merits. Plato's own dialogue *Timaeus*, with its powerful description of the structuring of the cosmos by a divine craftsman, was more frequently quoted but regarded as simply a report of Pythagorean doctrine. Many more fragments from spurious Pythagorean works survive than fragments that are likely to be from genuine works by early Pythagoreans.

It is not until the 3rd century C.E. that we have the appearance of the first detailed accounts of the life of Pythagoras and the Pythagorean way of life that have survived intact. These lives by Diogenes Laertius (fl. 200–250), Por-

phyry (232–ca. 305), and Iamblichus (ca. 250–325) are the sources that were used in the Renaissance and by many later scholars to provide a picture of Pythagoras and his accomplishments. They drew heavily on sources from the 4th century B.C.E. that have not survived. In particular they drew on Aristotle's two-volume work on Pythagoreanism and on works on Pythagoras by Aristotle's pupils Aristoxenus and Dicaearchus, as well as works by the Platonist Heraclides and the historian of southern Italy, Timaeus. Since these lives are rather uncritical compilations of earlier sources, they must be used with utmost caution. However, it is their portrait of Pythagoras as a philosopher of almost divine status that dominates the late antique world and carries over even into many modern interpretations.

The present overview of the Pythagorean tradition is necessarily drastically simplified and selective. However, the crucial lesson to be learned is that from the time of Plato's successors in the Academy onward (ca. 350 B.C.E.), the Pythagorean tradition is dominated by a school of thought that assigns back to the divine Pythagoras all that is true in later philosophy. If we want to get behind that view to gain an appreciation of the actual significance of Pythagoras's thought in the late 6th century B.C.E., we must begin by looking only at the early evidence for Pythagoras. Initially, then, we must look only at the evidence of authors up to and including Plato and Aristotle, but not their pupils.

By modern standards there is not much evidence from this early period, a collection of twenty or so brief references in works of early philosophers, poets, and historians, apart from the fragments of Aristotle's book on the Pythagoreans, but this is in fact much more evidence than we have for many other early thinkers, and to some extent it is a reflection of Pythagoras's fame. What emerges most clearly from this evidence is that Pythagoras was famous for his knowledge of the soul and its fate after death, and more specifically for his belief in metempsychosis, the doctrine that our soul is reborn in another human or animal form after our death. Even these early reports about Pythagoras often have a polemical tone, either attacking some aspect of his doctrine or portraying him as possessing superhuman wisdom. Thus his contemporary Xenophanes (ca. 570–480) is clearly having fun at Pythagoras's expense when he reports the story that Pythagoras pitied a puppy that was being beaten by a man and said to the man, "Stop, don't keep beating it, for it is the soul of a friend of mine, I recognize his voice" (frg. 7). On the other hand, Ion of Chios (b. ca. 490 B.C.E.) is clearly serious in his praise of Pythagoras for his knowledge of the fate of the soul after death (frg. 4).

Heraclitus attacks Pythagoras, among others, for being a polymath without having attained any real understanding (frg. 40), calls him "the chief of swindlers" (frg. 81), and says that he practiced inquiry beyond all other men, making a wisdom of his own by picking things out of the writings of others, a wisdom that was in fact "a polymathy and evil trickery" (frg. 129). Some

scholars have thought that since Heraclitus links Pythagoras with Xenophanes and Hecataeus as polymaths (frg. 40), Heraclitus must have regarded Pythagoras as being engaged in the same sort of rational inquiry about the world that they pursued and that had arisen around the city of Miletus in Asia Minor, close to Pythagoras's home of Samos, in the 6th century. However, the fragment of Heraclitus in fact joins him more closely to Hesiod the poet, who was famous for his mythical account of the origin of the gods in *Theogony*. Nevertheless, the overall tone of Heraclitus's reports is clear. He regards Pythagoras as a charlatan of some sort. Empedocles (ca. 493–433), in a fragment that most agree refers to Pythagoras (frg. 129), adopts the exact opposite tone toward Pythagoras and begins by praising him in general terms as a man who knew "extraordinary things" and possessed "the greatest wealth of wisdom." He then goes on to praise him specifically for "wise deeds" and says that "whenever he reached out with all his intellect he easily beheld all the things that are in ten and even twenty generations of men." It is not necessary, but it is tempting, to see this last phrase as a reference to the idea of rebirth, and the mention of "wise deeds" calls to mind the image of a wonder-worker of some sort who here is venerated but who might well also elicit Heraclitus's description of him as a charlatan.

Other early evidence about Pythagoras shows a close connection to religious ritual. His connection with Orphism in Ion of Chios and Herodotus underlines his close connection to ritual (initiations) and his expertise on the soul and the afterlife. Plato's contemporary Isocrates says (*Busiris* 28) that Pythagoras journeyed to Egypt and makes the vague point that "he first brought other philosophy to the Greeks" before going on to say that he was particularly zealous about the matters of sacrifice and rites of the temple.

Other early evidence suggests that in connection with his interest in the soul and its afterlife and religious ritual Pythagoras showed a broader interest in how we should live our lives. Thus the atomist Democritus (b. 460) is reported as having been very influenced by Pythagorean ideas and as perhaps having studied with Philolaus. However, it seems clear that this influence was on his ethical views, since we are told that he wrote a book on Pythagoras and that this book was classified with Democritus's ethical works. Isocrates in the passage mentioned above said that Pythagoras's reputation was so great that young men wanted to be his pupils and that their parents were happier to have them associate with him than to attend to family affairs. Isocrates comments that even in his day, Pythagoreans were more marveled at for their silence than great orators for their speech. This statement suggests that the later tradition of a rule of silence among Pythagoras's pupils has some foundation. The 4th-century rhetorician Alcidas gives what is probably an apocryphal report, but which nonetheless represents early 4th-century attitudes, that Empedocles studied with both Pythagoras and Anaxagoras, and that while he got knowledge of nature from Anaxagoras, he got the dignity of

his bearing from Pythagoras. Once again it is striking how consistently Pythagoras is associated with the manner in which we live our lives rather than with theoretical knowledge.

The most famous evidence for Pythagoras as an influential teacher and founder of a way of life is Plato's only reference to him, which is found in *The Republic*. Pythagoras is not included in the list of those who have contributed to the good government of a city, as the lawgivers Lycurgus and Solon did, nor among the good generals, nor among the men wise in practical affairs and the producers of ingenious inventions, such as Thales. Instead Plato places Pythagoras's activity in the private sphere and presents him as the model of the sort of figure who was particularly loved as a teacher and whose followers even now are famous for their Pythagorean way of life (600a9–b5). Thus, the early evidence consistently portrays Pythagoras as a figure with an extremely loyal following who can tell you how to live your present life to best satisfy the gods and to ensure the best fate for your soul in the next life. At the same time, outsiders such as Heraclitus and Xenophanes could well view his pretensions to knowledge about ritual and the fate of the soul, as well as the glorification of him by his followers, as foolishness or even quackery.

Particularly problematic for nonbelievers would have been the "wise deeds" to which Empedocles refers and which may well be the miraculous actions that Aristotle assigns to Pythagoras. Aristotle reports that Pythagoras was supposed to have appeared in both Croton and Metapontum at the same hour on the same day; that when he crossed a river it gave him the greeting, "Hail, Pythagoras"; that he bit and killed a poisonous snake; and that he had a golden thigh. This last report probably picks him out as the favorite of the god and is tied to initiation rites in which a part of the body is dedicated to the divinity. Indeed, Aristotle reports that the people of Croton called him the Hyperborean Apollo. The other reports stress his special control over the animal and natural world. Often these miraculous stories are rejected as useless in understanding the historical figure of Pythagoras, but this attitude is mistaken because the stories show how he was conceived by his early followers and are thus crucial in understanding the type of figure he was.

The biggest lacuna in the picture of Pythagoras presented in the early evidence is the precise nature of the way of life he handed on to his followers. Once again some details can be filled in from Aristotle and other later 4th-century witnesses, but these sources contradict each other in important ways; accordingly, the following description of the Pythagorean life is very problematic. Particularly important are the many Pythagorean maxims preserved by Aristotle. Some of these are likely to go back to Pythagoras himself because they fit well with the picture of Pythagoras derived from the earliest evidence. These maxims are called *acusmata* (things heard) or *symbola* (pass-words, or things to be interpreted) in the later tradition. Iamblichus gives a

very large collection of them in his work *On the Pythagorean Life* (82–86), which is surely based primarily on Aristotle.

In Iamblichus's collection the maxims are broken up into three classes, corresponding to the three questions: (1) What is it? (2) What is best? (3) What must be done? Of the first sort are the maxims reported by Aristotle: "The ring of bronze when it is struck is the voice of a daimon trapped in it" or "The planets are the hounds of Persephone." Of the second sort Iamblichus lists, "What is wisest? Number"; "What is the most just? To sacrifice"; "What is the loveliest? Harmony." But the biggest number of maxims are of the third sort, which often have clear ties to rules of religious ritual. Aristotle is given as the authority for maxims against eating certain fish (red mullet and black tail) and certain parts of animals (heart and womb). Iamblichus gives a long list, including the injunctions that one must beget children, one must put the right shoe on first, one must not sacrifice a white cock, one must not use public baths. As early as 400 B.C.E. a tradition sprung up according to which these maxims were not to be taken literally but interpreted to find a deeper meaning. Thus the prohibition not to eat the heart is interpreted as meaning that one should not grieve. However, this is likely to be the attempt of later generations to explain away seemingly bizarre rituals rather than the original import of the maxims. Their connection with ritual and magical practices elsewhere in Greece shows that they were probably meant to be taken quite literally and also that many of them were hardly original to Pythagoras but were adopted by him.

One of the most famous of the maxims is the one that prohibits the eating of beans. All the early evidence supports the authenticity of this maxim except Aristotle's student Aristoxenus, who reports in his book on Pythagoras that "Pythagoras especially valued the bean among vegetables saying that it was laxative and softening. Wherefore he especially made use of it." This is probably an attempt on Aristoxenus's part to eliminate the stranger aspects from early Pythagorean doctrine, and it is likely that we should follow Aristotle's report that the eating of beans was prohibited. Aristotle gives several obscure explanations for the prohibition: that beans were like the genitals, or the gates of Hades, or the universe, or connected to oligarchy. It is also plausible that the problems beans cause for digestion, or indeed the fact that many people are allergic to a certain amino acid that is in this type of bean, was behind the prohibition.

Slightly more complicated is the issue of vegetarianism. There would seem to be a natural connection between the belief in metempsychosis and vegetarianism; the philosopher Empedocles writing in the generation after Pythagoras made the connection clear in graphically condemning the eating of all animal flesh as cannibalism. Moreover, Eudoxus of Cnidus, a famous mathematician and member of the Academy in Plato's day, reports that Py-

thagoras not only did not eat any meat but also even avoided the company of butchers and hunters. Aristoxenus, however, takes the iconoclastic view again and asserts that although Pythagoras avoided plow oxen and rams, he was particularly fond of young kids, suckling pigs, and cockerels. Moreover, in this case there is considerable support for Aristoxenus's view that Pythagoras was not a strict vegetarian. Some *acusmata* reported by Aristotle only ban eating specific parts of animals. Moreover, there is the maxim mentioned above that says that the most just thing is to sacrifice, and Iamblichus reports among the *acusmata* one to the effect that souls do not enter into sacrificial animals. Some scholars have argued that to reject animal sacrifice totally would be such a complete overturning of traditional ways as to be unthinkable, while others have tried to argue that sacrifice was acceptable but that the Pythagoreans limited themselves to a simple sacramental tasting of the victim. The fact that Pythagoreans were recognized as so distinctive in their way of life suggests that a prohibition on animal sacrifice is not so unthinkable and that, with Empedocles as the model, we should believe that Pythagoras was a strict vegetarian. This issue is an excellent example of the complexity of determining the actual nature of the Pythagorean way of life and shows the already contradictory nature of the evidence even in the 4th century B.C.E.

One way of trying to reconcile the contradictions in the testimony about vegetarianism both in the ancient world and among modern scholars has been to suppose that there were different grades among the followers of Pythagoras, as the later tradition suggests, and to argue that vegetarianism was practiced to different degrees by members of different grades. This explanation raises directly the broader question of whether Pythagorean societies in a strict sense ever existed at all. Some have suggested that the notion of a community separated from regular society and governed by a strict rule along the lines of a monastery would be an anachronism in the 6th century B.C.E., and that such an idea was introduced into the tradition at a later point and perhaps even by Iamblichus. However, some of our early evidence, the reports of Plato and Isocrates, make it very clear that Pythagoras attracted a loyal following and showed them a way of life that made them easily distinguishable from ordinary citizens even as late as the 4th century. How strict the way of life was and how the community was related to the rest of society may remain open questions, but this evidence surely justifies our talking about Pythagorean communities as groups of people who lived a way of life that clearly set them apart from other people.

A number of reports from the 4th century B.C.E. indicate that Pythagoras and his followers had significant impact on the politics of the Greek city-states in southern Italy. It is difficult to derive any coherent Pythagorean political views out of these reports, which contradict one another on a number of points. Since Plato seems to clearly distinguish Pythagoras from lawgivers and other public figures, it seems most likely that there were not many spe-

cific Pythagorean political policies. Rather it was probably the case that many of the talented and prominent members of the communities became followers of the Pythagorean way of life while at the same time participating in the government of the city-state. Of course the Pythagorean societies may have looked like political organs to their opponents, and we know that there were two major attacks on them. One seems to have occurred in the lifetime of Pythagoras himself and to have led to the deaths of many of his followers in a fire set in the house where they were meeting in Croton, and to the flight of Pythagoras himself to Metapontum. Another attack on the societies seems to have occurred in the mid-5th century and to have led once again to many deaths and to the apparent dispersal of many of the communities.

A common feature of an exclusive society is some sort of secrecy associated with initiation and grades of membership. Isocrates' report lets us know that the Pythagoreans were famous for their silence, but it is unclear whether this refers simply to their self-restraint in not speaking too much, or to a sort of novitiate in which new members were not allowed to speak for the first five years, as mentioned already by the historian Timaeus in the 4th century. It is also possible that it refers to some sort of specifically secret doctrine. Both Aristotle and Aristoxenus give evidence that at least some doctrines were not for all ears. Aristotle reports that among their very secret doctrines was the division of rational animals into three groups, men, gods, and beings like Pythagoras. Aristoxenus says that the Pythagoreans used to say that not all of Pythagoras's doctrines were for all men to hear, which suggests both secret doctrine and perhaps grades of initiates. All the same, neither Aristotle nor Plato hints that the Pythagorean philosophy of the 5th and 4th centuries was in any way secret. The most likely scenario is that the Pythagoreans, like most exclusive societies, did have secrets such as Aristotle and Aristoxenus suggest. However, these were limited to a certain group of maxims, without putting any restriction on the publication of philosophical ideas such as we find in the book of Philolaus, or on mathematical proofs such as those that appear in Archytas. It is true that Hippasus, a Pythagorean active in the generation after Pythagoras, is supposed to have been killed for the impious act of publishing a construction of a dodecahedron. However, in terms of the history of mathematics it is unlikely that Hippasus gave a strict construction of the dodecahedron at this date; the story may instead relate to the dodecahedron as a cult object.

Hippasus is also connected to the problem of the existence of different sects of Pythagoreans. A report in Iamblichus that seems to go back to Aristotle suggests that in the 5th century there were two groups referred to as the *acusmatici* and the *mathematici*. The *acusmatici* were clearly connected with the *acusmata* and were said to be recognized as genuine Pythagoreans by the *mathematici*. However, the *acusmatici* argued that the *mathematici* were not true Pythagoreans but in fact stemmed from Hippasus rather than Pythago-

ras. The *mathematici* in turn argued that Pythagoras himself had presented his views to some people in simple commandments, while to others who had time for more study (young men), he had given the explanations. Hippasus, as they presented him, was publishing views that were originally Pythagoras's own, only for his own aggrandizement. Thus it is clear that by Aristotle's time, and probably earlier, there was already debate about what ideas in fact went back to Pythagoras himself, with the *mathematici* seeming to adopt the view found among the successors of Plato, that all ideas really go back to the master.

At this point we must confront the most controversial of all issues regarding Pythagoras: Was he a natural philosopher and mathematician as well as the founder of a way of life and expert on the gods, ritual, and the afterlife? After all, he is most famous among the educated public for his connection to the so-called Pythagorean theorem (see below). However, it should be clear from the evidence given above that, up to and including Plato and Aristotle, there is no direct evidence that shows him to be either a mathematician or a natural philosopher. The example of Empedocles shows that the roles of religious wonder-worker and natural philosopher can well be combined in 5th-century Greece, but were they so combined in the figure of Pythagoras? Pythagoras grew up on Samos, which was the site of great technological achievements involving the application of mathematics (most notably the tunnel of Eupalinus, which went under a mountain and was dug from both sides, missing only slightly in the middle). Samos was an easy journey from the city of Miletus, on the coast of Asia Minor, which was home to a series of early natural philosophers, such as Anaximander and Anaximenes. Accordingly some scholars argue that it is impossible to imagine that Pythagoras was not aware of such work or that he could have become famous in such a milieu if he were simply a wonder-worker. However, this argument fails spectacularly since, however probable it may be that Pythagoras knew of the natural philosophy that grew up at Miletus, this argument says nothing about whether or not he was interested in that sort of inquiry himself. Surely an expert on the fate of the soul after death might achieve as much or more fame as someone who speculates about whether water or air is the original state of the cosmos.

The strongest reason for believing that Pythagoras had special interest in mathematics and the structure of the natural world is that Aristotle's evidence and Philolaus's book make clear that in the 5th century people called Pythagoreans gave an account of the natural world that showed it to be structured according to pleasing mathematical relationships. Furthermore, Archytas, the contemporary of Plato, who is also identified as a Pythagorean, was a distinguished mathematician. We must remember that Philolaus and Archytas may have been Pythagoreans only insofar as they followed a Pythagorean way of life, and that their views on the natural world and mathe-

matics may have been primarily influenced by non-Pythagorean writers such as Anaxagoras, or Hippocrates of Chios. However, it is legitimate to ask if the seeds of this interest in mathematics and the mathematical structure of the world were planted by Pythagoras himself.

The connection between Pythagoras and proposition 1.47 of Euclid's *Elements* (ca. 300 B.C.E.), popularly known as the Pythagorean theorem, is based on the commentary on Euclid written by Proclus in the 5th century C.E. It is reported that Pythagoras, on making a geometric discovery of some sort, sacrificed an ox. This tradition seems to rest on an epigram by an obscure Apollodorus: "When Pythagoras found the famous figure, for which he carried out the noble ox-sacrifice." This Apollodorus might date back to the 4th century, which would give the story considerably more authority, but this dating is far from certain. The story was actually as famous in the ancient tradition for the apparent contradiction with vegetarianism represented by the ox sacrifice as it was for its connection to a geometrical theorem. However, even if we accept the story as genuine 4th-century tradition, there are serious problems. First, it is clear from the history of Greek mathematics that it is unlikely that Pythagoras could have proven the theorem. A strict proof would have required some sort of structure of theorems and definitions, and the first hint of such a structure that we have is connected to Hippocrates of Chios in the later 5th century. However, it is known that the Pythagorean theorem had been part of Babylonian arithmetical technique for centuries, although the Babylonians had given no general proof of it. One might say that Pythagoras was the first Greek to discover the truth of the theorem or the first Greek to bring the knowledge from Babylon to the Greeks, but the truth of the theorem is likely to have come to Greece through several sources. Indeed the reports about Pythagoras do not stress that he was the one who discovered the theorem, and it is tempting to take the reports to emphasize the value he put on knowledge of the theorem. He was so impressed when he found out that the sides of the right triangle were related according to this precise mathematical rule that he sacrificed an ox, something that an ordinary Greek might do to celebrate a more material success, such as a safe voyage home.

Another set of late stories associated Pythagoras with the discovery that the basic musical concords of the octave, fourth, and fifth could all be expressed in terms of whole-number ratios of string lengths (1 : 2, 3 : 4, and 2 : 3, respectively). Supposedly he heard the musical concords in the sounds made by a blacksmith's hammers and discovered that hammers whose weights were in the ratios given above produced the musical concords when struck one after the other. He is also said to have hung weights equal to the hammers by strings so that the strings also produced the concords. However, none of these observations or experiments correspond to physical reality. The earliest full versions of the stories come from Nicomachus in the 2nd century C.E., but in the 4th century B.C.E. Xenocrates had already reported that "Py-

thagoras discovered that the musical intervals did not arise without number" (frg. 9). At the same time, there is a report that ascribes an experiment to Hippasus that does in fact work and thus may well be true. It is important to note that another report associates Lasus of Hermione with such an experiment. Lasus is in fact a contemporary of Pythagoras, although he is not a Pythagorean. In conclusion, it seems best to say that Pythagoras himself did not discover the whole-number ratios that underlie the musical concords, but that the knowledge may well have been available in his day and that Hippasus may have demonstrated it with a valid experiment in the next generation. As in the case of the Pythagorean theorem, it may well be that what was important was the value Pythagoras attached to this information when he learned of it from others.

What then can we conclude about Pythagoras's connection to mathematics? The key may be found in the maxims attested for the early Pythagoreans. One says that the wisest thing is number, and yet another reports that the oracle of Delphi is "the tetractys which is the harmony in which the Sirens sing." In the later tradition the "tetractys," the tetrad consisting of the numbers one through four whose sum equals the "perfect" number ten, was central to Pythagoreanism but clearly had become contaminated with ideas derived from Platonism. The Pythagoreans were supposed to have sworn oaths by Pythagoras as "the one who handed on to our generation the tetractys, which has the source and root of everflowing nature." It is doubtful that the oath in this form can go back to the time of Pythagoras, but the tetractys itself as mentioned in the *acusmata* might. The connection of the tetractys to the Sirens, who were famed for their singing, suggests a connection to music, and it becomes just possible that the tetractys was in part venerated because its four numbers were all involved in the whole-number ratios that govern the musical concords. If we connect the Sirens in turn with the famous doctrine of the harmony of the spheres, which says that the heavenly bodies produce musical concords as they move (as Plato does in *The Republic*), then Pythagoras starts to emerge as someone impressed with the power of number—particularly as embodied in the first four numbers, which seem to be the basis of the musical concords, which in turn seem to structure the cosmos. The doctrine of the harmony of the spheres is discussed by Aristotle in *De caelo* but once again is ascribed only to Pythagoreans and not to Pythagoras himself; it remains quite possible that it too belongs to 5th-century Pythagoreanism and not to Pythagoras. It was never worked out in any mathematical detail by the early Pythagoreans but was, rather, a very general conception.

It should be clear that the evidence does not support the conclusion that Pythagoras was any sort of serious mathematician or that he was a natural philosopher or scientist. Instead he is to be seen as a charismatic teacher who attracted a large group of followers for whom he prescribed a way of life that included instruction about the future life of the soul, the gods, ritual, and a

bewildering variety of rules that applied to many aspects of daily life, including what foods to eat and how to dress. Some of this instruction encouraged certain moral virtues (such as silence) and probably included the memory training and emphasis on friendship attested in the later tradition. It may be that it also instilled a certain attitude toward the cosmos and promoted reverence for the concept of number and particularly for certain significant numbers. The followers of Pythagoras stood out in the communities of which they were a part, attracting both praise and ridicule as well as having some influence on the politics of southern Italy.

The real significance of Pythagoras, as Walter Burkert points out, is that he was the first person to establish a set of rules that apply not just to the special moments of life that are marked by religious ritual but that give direction as to how each day of our life ought to be lived, as well as how our life in this world relates to the life in the next. It is in this sense that he is a great moral teacher and a true precursor of Socrates and Christ, however bizarre to modern sensibility some of the teachings of the *acusmata* may seem to be. It is with the so-called Pythagoreans of the 5th century B.C.E., and in particular with Philolaus of Croton, that we see the important contributions of Pythagoreanism to natural philosophy.

We know virtually nothing about Philolaus's life (ca. 470–390) except that he came from Croton (or perhaps Tarentum) in southern Italy and visited Thebes in mainland Greece sometime before 399. Nonetheless he is the central figure in early Pythagoreanism, since his book was the first to be written by a Pythagorean. The book was still available in the late 4th century B.C.E. to Aristotle's pupil Meno, who reports Philolaus's medical views in his history of medicine. The whole book has not been handed down to us, although more than twenty fragments and a number of secondhand reports (*testimonia*) have survived. A consensus has emerged that, although more than half of the fragments that survive in his name are from spurious works, a central core of fragments from his genuine book survive (1–7, 13, 17). These fragments show that, although Philolaus's book was the primary source for Aristotle's account of Pythagoreanism, Aristotle recast Pythagoreanism in important ways to fit his own purposes, and that Aristotle's evidence thus must be used with caution. In addition it becomes clear that Philolaus's book was a significant influence on Plato's *Philebus*.

The book appears to have followed a typical Presocratic pattern in that it presents an account of the natural world, beginning with a cosmogony, and goes on to present astronomical, psychological, and medical theories. It is striking that in the fragments there is no discussion of morality or how to live one's life, although Philolaus is mentioned in Plato's *Phaedo* as arguing that suicide is not permissible. Philolaus explains the cosmos in terms of two basic types of things, limiters and unlimiteds. Unlimiteds include the elements, such as water and air, that had been used by earlier Presocratics to ex-

plain the natural world, but Philolaus does not focus on any one specific element or group of elements of this sort. Instead he argues simply that a supply of such unlimiteds (continua that are in themselves without any limit, including water, air, earth, and also things like void and time) must be presupposed in order to explain the world that we see. But Philolaus's most noteworthy innovation is his claim that earlier thinkers had been mistaken to think that the world could be adequately explained in terms of such unlimiteds alone. He argues that limiters, things that set limits in a continuum—e.g., shapes and other structural principles—must also be regarded as basic principles. Objects that we observe in the world, such as a tree, are manifestly combinations of continua that are unlimited in their own nature (the wood) and structural principles (the shape and structure of the tree). Thus individual objects in the world and the cosmos as a whole are combinations of limiters and unlimiteds.

However, limiters and unlimiteds alone are still not sufficient to explain the world. Limiters and unlimiteds are not combined in a chance fashion but are instead held together by a *harmonia*, a fitting together in accordance with pleasing mathematical relationships. The first sentence of Philolaus's book sets out his basic view of the world: "Nature in the cosmos was fitted together both out of limiters and unlimiteds, both the cosmos as a whole and everything in it" (frg. 1). Philolaus gives as the model of this fitting together a musical scale. The unlimited continuum of sound is limited by certain notes picked out in that continuum. However, these notes are placed not at random intervals but in accord with the whole-number mathematical ratios that govern the basic intervals of the diatonic scale. Thus, to really understand the world or anything in it, we must not only observe the limiters and unlimiteds that make it up but also the number in accordance with which those limiters are fitted together. Writes Philolaus, "And indeed all things that are known have number. For it is not possible that anything whatsoever be understood or known without this" (frg. 4). Of course he is putting forward a grand scheme here and was unable to specify many of the numbers that governed the structure of individual things. It is to his credit that there is little evidence that he tried to postulate an arbitrary set of numbers that governed all things.

Aristotle recasts this emphasis on number as giving us knowledge of things into the claim that the Pythagoreans thought that things were made up of numbers in some way, but Philolaus says nothing of the sort. In trying to distinguish the Pythagoreans from Plato, who thought that numbers had existence separate from things, Aristotle supposed that the Pythagoreans must have identified numbers with things. However, this anachronistically supposes that the Pythagoreans had asked the questions, what sort of existence do numbers have, and how are they related to things? These questions probably first directly arose in the 4th century in Plato's Academy. Philolaus was content to argue that numbers give us knowledge of things in the world,

without focusing on the ontological status of numbers. What is crucial is the insight that, in principle, both the cosmos as a whole and also everything in it are knowable with the determinacy of perspicuous numerical relationships.

The cosmos began with a fire (an unlimited) in the center of a sphere (a limiter). This central fire then breathed in another group of unlimiteds (time, void, breath), which were fitted together with limiters to produce Philolaus's famous astronomical system. With this theory, for the first time the Earth is made a planet, and Copernicus refers to Philolaus as one of his precursors. However, rather than orbiting around the Sun, the Earth orbits around the central fire along with the Sun, the Moon, the five planets, the fixed stars, and a counter-Earth that was evidently introduced to bring the number of orbiting bodies up to the perfect number ten. The system thus combines a reverence for principles of order that fly in the face of observation with a desire to accommodate the basic observable facts. Night and day are explained by the Earth's moving around the central fire in one twenty-four-hour period, while the Sun takes a year, thus turning our side of the Earth away from the Sun for a portion of the twenty-four-hour period. The Earth rotates so that our side of the Earth is always turned away from the central fire and counter-Earth, explaining the fact that we never see them. This is also the first astronomical system that identifies a set of five planets and arranges them in proper order according to observations of their periods.

After presenting his view of the generation of the cosmos as a whole, Philolaus turned to a discussion of life. Just as he was interested particularly in the structure of the cosmos, so he emphasizes the structure of the animate world, in ways that prepared for the later Platonic and Aristotelian systems. He regarded all life as ordered in the hierarchy of plant, animal, and human beings, where each higher level shares the abilities of the lower but has a unique ability of its own. All life shares the ability to reproduce itself; plants have the ability to send out roots (which is paralleled by the human umbilical cord), animals can move and have sensation, but humans alone have reason. Philolaus saw a clear analogy between the birth of the cosmos and the birth of individual human beings. Humans were seen as initially consisting of just the hot, which then at birth naturally breathes in the cold, just as the central fire breathed in to generate the cosmos. The health of the human body seems to have depended on the proper limitation of the hot by cooling breath. Philolaus is said to have explained disease in terms of the three humors, bile, blood, and phlegm, which are also prominent in the contemporary theories of medicine found in the Hippocratic writings.

Philolaus's view of the cosmos is recognizably Presocratic, just as his medical views employ the same concepts as the early Hippocratic writings. However, the thematic emphasis on the structural features of the cosmos (limiters) and on the role of number in making the cosmos intelligible are important original steps that had influence on both the great philosophers of the next

century, Plato and Aristotle. Indeed, Philolaus is the first philosopher to present a coherent view of the natural world that calls for its explanation in terms of mathematical relationships. However, the growth of the Pythagorean legend that began in Plato's Academy made it imperative that the Pythagorean cosmos appear to be not merely an important precursor of Plato and Aristotle but the true revelation of reality. Hence, Philolaus's system, after being relatively faithfully represented by Aristotle, was pushed aside and neglected, and Plato's mature presentation of the cosmos in *Timaeus* came to be regarded as in reality the work of Pythagoras; accordingly, the term *Pythagorean* in the Renaissance usually in fact refers to the Platonism of *Timaeus*.

Archytas (fl. 400–350), who belonged to the generation after Philolaus and who was a contemporary of Plato, is the Pythagorean who comes closest to matching the modern stereotype of a Pythagorean as the master mathematician. Whereas there is no evidence that Philolaus made any contribution to the advancement of Greek mathematics, Archytas is primarily known for important mathematical proofs and as the founder of the discipline of mechanics. There are in fact only three fragments (1–3) that can with some confidence be regarded as coming from genuine works, amid a much larger group of fragments that derive from spurious works. We know of his mathematical achievements through secondhand reports.

Archytas came from Tarentum in southern Italy, and the details of his life are largely unknown to us. He has tended to be better known than Philolaus because he is closely associated with Plato in several Platonic letters, whose authenticity, however, is controversial. He sent a ship to rescue Plato from the clutches of the tyrant Dionysius II of Syracuse in 361. However, even if we accept the Platonic letters as providing accurate information, the exact relationship between Plato and Archytas and the nature of their influence on one another is unclear. He is never mentioned in the Platonic dialogues.

As in the case of Philolaus, and in contrast to the tradition about Pythagoras himself, we have little information about Archytas's views on moral issues or the proper way to live one's life. Two anecdotes suggest that he emphasized the importance of acting solely on rational judgments and never in anger or under the influence of the enticements of pleasure. When Archytas came back from war to find his lands neglected, he reportedly told his slaves that they were lucky that he was angry, because otherwise they would have been punished. In another case, when someone had argued that pleasure was the ultimate good, Archytas responded that reason was the greatest gift of the gods to human beings, but that in the throes of pleasure we are unable to use it. In fragment 3 he praises "correct calculation" as the source of political harmony.

It is not unlikely that Archytas did have something to say about the basic principles of the cosmos, but we have very little evidence about his views on

this topic. There is nothing in the tradition about Archytas that mentions limiters and unlimiteds. However, Aristotle does say that Archytas gave definitions that appealed to both form and matter. Thus, calm weather is defined as "lack of movement in a mass of air," where the air is the matter and the lack of movement is the form. Such a definition could well have arisen out of the Philolaic distinction between unlimiteds and limiters. Again it is unclear whether Archytas accepted Philolaus's view of the structure of the cosmos or not. He is reported as arguing that what is outside the cosmos is unlimited in extent, by asking, "If I were at the extremity, for example at the heaven of the fixed stars, could I stretch out my hand or my staff into what is outside or not?" The study of such sciences as astronomy, geometry, arithmetic, and music is for him crucial to the understanding of reality. His reference to these sciences as "sisters" may have influenced Plato (*Republic* 530d). But the fragment goes on to present a theory of acoustics rather than discussing the cosmos as a whole.

It is for his work in the specific sciences that Archytas is particularly famous. Ptolemy (2nd century C.E.) in his *Harmonics* reports that Archytas was the most dedicated of all the Pythagoreans to the study of music and then presents Archytas's mathematical accounts of the diatonic, enharmonic, and chromatic scales. Archytas's proof that numbers in a superparticular ratio ($n + 1/1$) have no mean proportional is important for ancient musical theory in that it shows that neither the whole tone (9 : 8) nor the central musical intervals of the octave (2 : 1), fifth (3 : 2), and fourth (4 : 3) can be divided in half. Fragment 2 shows Archytas working with the arithmetic, geometric, and harmonic means that are also important in music theory.

He was regarded as the founder of mechanics and was said to have given method to the discipline by applying mathematical principles to it. This interest is manifested in the charming reports of his making toys for children, including a flying wooden dove. It is also reported that he was the first to use mechanical motion in geometrical constructions. Thus, in his proposed solution to the famous problem of doubling the cube, which was supposedly first posed by inhabitants of the island of Delos, who were commanded by the god to double the size of an altar, he used a striking construction in three dimensions that determined a point as the intersection of three surfaces of revolution.

After Archytas there are no prominent Pythagoreans. Aristotle's pupil Aristoxenus says that the Pythagoreans of the first half of the 4th century were the last Pythagoreans. However, this is not in fact the end of Pythagoreanism in the ancient world, although it is probably the end of the Pythagorean societies. As has been mentioned above, there was a strong revival of what were regarded as Pythagorean ideas in the 2nd and 3rd centuries C.E., although there were some important figures even earlier. The revival was

connected with Neoplatonism to such an extent that it is sometimes hard to know whether to call an individual thinker a Neopythagorean or a Neoplatonist.

Some Neopythagoreans focused primarily on Pythagoras's special connection to the gods, his wonder-working, and his way of life. Most prominent is Apollonius of Tyana (1st century C.E.), whose writings are largely lost but whose way of life is described in Philostratus's *Life of Apollonius*. Apollonius was a wandering ascetic wonder-worker who claimed a divine wisdom derived from Pythagoras. Some of his miraculous achievements were clearly modeled on stories about Pythagoras, such as his ability to be in two places at one time and the ability to speak the language of animals. Most Neopythagoreanism, however, focused on Pythagoras as the archetypal philosopher to whom the divine revelation of the true philosophical doctrine had been given. Two figures will serve as central examples of this tradition, Nicomachus and Iamblichus.

Nicomachus (ca. 50–150 C.E.), who wrote a very influential *Introduction to Arithmetic*, is a good example of the tendency to treat Platonism as if it were in fact Pythagoreanism. Nicomachus begins his book with Pythagoras and presents him as the first person to correctly define philosophy as knowledge of the immaterial and unchangeable reality that lies behind the flux of bodily existence. He is thus ascribing to Pythagoras Plato's central distinction between the world of unchanging forms and the constantly changing phenomena. Nicomachus then illustrates Pythagoras's supposed philosophy by quoting from Plato's *Timaeus*, thus exemplifying a pattern that will be repeated many times in the later tradition. While some authors do quote the Pythagorean pseudepigrapha to illustrate Pythagorean doctrine, it is more common to treat Plato's *Timaeus* as the central text for Pythagorean doctrine. Nicomachus also portrays Pythagoras as the master mathematician and scientist who instituted the quadrivium of mathematical studies that must be mastered to understand reality. These are the studies of arithmetic, music, geometry, and astronomy. Nicomachus himself wrote works on at least three of the sciences in the quadrivium.

The Neoplatonist Iamblichus (ca. 250–325 C.E.) wrote a ten-volume work entitled *On Pythagorean Doctrine*. The first volume, *On the Pythagorean Life*, was, as its title suggests, more than a biography of Pythagoras. Iamblichus's goal was to show Pythagoras as the archetype of the sage who had close contacts with divinity, and to present the way of life that he handed on to his followers. Many scholars have suggested that it would not be inappropriate to call Iamblichus's work a gospel, and it may be that Pythagoras was consciously set forth as a competitor to Christ. Iamblichus went on to write a volume on each of the four sciences in the quadrivium and then a volume each on the role of mathematics in physics, in ethics, and in theology. These last three works are lost and known only secondhand. But a later treatise of

uncertain authorship entitled the *Theologumena arithmeticae* seems to have drawn extensively on Iamblichus's and Nicomachus's work on the theology of number and comprises a discussion of the properties of each of the first ten numbers, including their connection with various divinities. Iamblichus had a number of important predecessors in his work on Pythagoreanism, including Nicomachus and a 2nd-century Neopythagorean named Numenius. Iamblichus's teachers Porphyry, who was the pupil of the great Neoplatonist Plotinus, and Anatolius, who wrote a work titled *On the Decad*, also fostered his Pythagoreanism. But Iamblichus himself is a crucial figure insofar as he appears to be responsible for the Pythagoreanizing and mathematizing of Neoplatonism that is still prominent in the important Neoplatonists Proclus and Syrianus in the 5th century C.E.

Neopythagoreanism has some connections to early Pythagoreanism: the inspiring figure of the semidivine sage is based to some extent on the historical Pythagoras; the picture of the cosmos ordered according to number does go back, in spirit at least, to Philolaus; and the notion of a set of sciences necessary to understand reality is found in Archytas. However, it was only in Platonism that these general ideas were given a sophisticated metaphysical foundation and a compelling literary embodiment. Neopythagoreanism is not so much a revival as a transformation of Pythagoreanism.

CARL HUFFMAN

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